Synthetic aperture interferometry: measurement of steep aspherical surfaces using an anamorphic

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Synthetic Aperture Interferometry:
Measurement of Steep Aspherical Surfaces
using an Anamorphic Probe

by
Amiya Biswas

Doctoral Thesis
Submission in partial fulfilment for the award of Doctor of Philosophy of
Loughborough University

October 2007

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ABSTRACT

Synthetic aperture interferometry (SAI) is a novel way of measuring aspherical optics (aspherics) where a scanning probe measures the optical path difference (OPD) between light reflected from the front (test) and rear (reference) surfaces of the aspheric optic. The surface form of the aspherical surface is subsequently computed assuming that the form of the reference surface is known. This method is straightforward to implement, does not require null compensators and is inherently insensitive to vibration. Consequently it has the potential to measure aspherics as they are being polished. When SAI was originally proposed (Tomlinson, Coupland & Petzing 2003), bare fibres (NA ~ 0.12) were used to construct the probe, however this configuration was unable to measure steep aspherics and had poor light gathering efficiency.

In this thesis, a new probe has been designed to measure the surface form of steep aspherics by increasing the NA of the probe using supplementary optics. In addition, the light gathering efficiency of the probe has been increased by adopting an anamorphical design. A single source and receive point of the probe was devised and it is shown that this configuration reduces the computational complexity. Alternative measurement configurations were investigated and their relative performance compared. A robust and fast phase evaluation process using a-priori information has been developed to extract the phase from measured interference pattern. Several steep surfaces have been measured to assess the feasibility of the SAI technique. Finally a detailed error analysis has been carried out to identify the major sources of error in measurement of OPD.

Keywords: Aspherical optics (aspherics), synthetic aperture interferometry, interferometry, phase evaluation, optical path difference, surface form, metrology, error analysis.

ACKNOWLEDGEMENTS

This thesis would not have been possible without the support of many people. First and foremost I would like to express gratitude to my supervisor Professor Jeremy Coupland who was abundantly helpful and provided invaluable guidance, encouragement and motivation throughout the course of this research. I would like to thank the Wolfson School of Mechanical and Manufacturing Engineering for providing the scholarship which enabled me to undertake this study.

I acknowledge the support given by my office, Space Applications Centre (Indian Space Research Organization) by granting me study leave to complete the PhD degree. I would like to express my sincere thanks to all the academic and technical staff of the Wolfson School who provided support during the course of this research. My thanks are also extended to my office colleagues for their suggestions and feedbacks on various aspects of my thesis.

A special thanks to my parents for their lifelong commitment and enthusiasm towards my education. I owe special thanks to my sister, Ashima for her support and encouragement. I am indebted to my wife, Amrita, for her endless love and encouragement throughout my PhD work. I am particularly thankful to her for patiently spending time alone which otherwise we would have shared together. Finally this thesis is dedicated to Amrita and my son, Amrith, who keeps reminding me that I have to play with him more often and which I could not do enough of during the preparation of this thesis.
### ABBREVIATIONS

<table>
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<tr>
<th>Abbreviation</th>
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<tr>
<td>ADC</td>
<td>Analogue-to-digital converter</td>
</tr>
<tr>
<td>CGH</td>
<td>Computer generated holograms</td>
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<tr>
<td>CNC</td>
<td>Computer numerically controlled</td>
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<tr>
<td>FC</td>
<td>Fibre cable</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>He-Ne</td>
<td>Helium-Neon</td>
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<tr>
<td>HP</td>
<td>Hewlett Packard</td>
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<tr>
<td>MRF</td>
<td>Magnetorheological finishing</td>
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<tr>
<td>NA</td>
<td>Numerical aperture</td>
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<tr>
<td>OPD</td>
<td>Optical path difference</td>
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<tr>
<td>OPL</td>
<td>Optical path length</td>
</tr>
<tr>
<td>PLL</td>
<td>Phase locked loop</td>
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<td>PSI</td>
<td>Phase shifting interferometry</td>
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<tr>
<td>SAI</td>
<td>Synthetic aperture interferometry</td>
</tr>
<tr>
<td>SNI</td>
<td>Sub-Nyquist frequency interferometry</td>
</tr>
<tr>
<td>TWH</td>
<td>Two wavelength holography</td>
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<tr>
<td>VCO</td>
<td>Voltage controlled oscillator</td>
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NOMENCLATURE

λ  Wavelength of light
φ  Phase
φₙ  Phase for the negative values of the probe position
φₚ  Phase for the positive values of the probe position
φᵢ  Ideal phase
φᵢᵢ  Ideal phase (positive values reversed)
φₘ  Wrapped phase
Δφ  Difference between measured and ideal phase
Δd  Probe distance measurement error, longitudinal misalignment, lateral misalignment
Δh  Linear movement in a direction perpendicular to the translation
Δl  Distance between two measurements
ΔOPD  Difference between ideal OPD and measured OPD
Δx  HP interferometer data
Δxₚ  Distance between successive probe positions
Δy  Surface deviation
(xᵢ, yᵢ)  Co-ordinates of the ray intersection point at the front surface for the ray that is reflected back from the rear surface (in the backward configuration)
(xᵢᵢ, yᵢᵢ)  Co-ordinates of the ray intersection point at the front surface for the ray that is reflected back from the front/rear surface (in the forward/backward configuration)
(xᵢᵢᵢ, yᵢᵢᵢ)  Co-ordinates of the ray intersection point at the front surface for the ray that is reflected back from the rear surface (in the forward/backward configuration)
(xᵢᵢᵢᵢ, yᵢᵢᵢᵢ)  Co-ordinates of the ray intersection point at the front surface for the ray that is reflected back from the rear
surface (in the forward configuration)

$(x_p, y_p)$ Co-ordinates of the probe position

$(x_{p1}, y_{p1})$ Co-ordinates of the probe source point

$(x_{p2}, y_{p2})$ Co-ordinates of the probe receive point

$(x_r, y_r)$ Co-ordinates of the ray intersection point at the rear surface for the ray that is reflected back from the rear surface

$(y_{Li})_j, (y_{Hi})_j$ Surface heights (on a regular grid) on either side of $(y_{Li})_i$

$(y_{L2})_j, (y_{H2})_j$ Surface heights (on a regular grid) on either side of $(y_{L2})_i$

$A(u)$ Fourier Transform of $a(x)$

$a(x)$ Additive disturbances

$A_4, A_6, A_8, A_{10}$ Aspheric coefficients of higher order

$b$ Blind vector

$b(x)$ Multiplicative noise

$c$ Centre thickness, curvature of the base of an asphere

$C(u)$ Fourier Transform of $c(x)$

$c(x)$ Analytic signal

$C^*(u)$ Complex conjugate of $C(u)$

$c^*(x)$ Complex conjugate of $c(x)$

$d$ Linear movement, probe distance, distance between two corner cube reflectors, distance between two probe passes

$F_{p'}, F_{p0}, F_{p^*}$ Finite impulse response filters

$I(u)$ Fourier Transform of $I(x)$

$I(x)$ Intensity of interference signal

$I_r$ Intensity of light reflected from front/rear surface

$k$ Conic constant

$m$ Number of samples, multiplication of two complex-valued functions

$M, M_1, M_2$ Matrices

$n$ Refractive index, number of surface height points

$N$ Number of cycles, noise in the system

$n_d$ Refractive index at d helium line (587.5618 nm)
O

OPD measurement points

OPD_{meas} Measured optical path difference

OPL_f Optical path length for the ray reflected from the front surface

OPL_r Optical path length for the ray reflected from the rear surface

p Integer consisting of values ranging from \(-P\) to \(+P\)

q Local data point

r Surface height position

S Ray intersection points

sag Sagitta

u Spatial frequency

x Spatial position

x_1, x_2 Probe paths

y_0, y_1, y_2 Surface heights defined at regular grid

z Sag of an asphere

z_q, z_q^0, z_q^+ Fourier signals

z_q^{\text{max}} Maximum value of Fourier signals
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Chapter 1

Introduction

1.1 Background

In general, optical systems such as cine zoom lenses use optics which have spherical curvatures. This is due to the fact that spherical surfaces are easier to fabricate and test. The techniques used to produce spherical surfaces date back to the ancient Romans, and since the times of Galileo, spherical lenses have been produced in volume. In the optical fabrication process a spherical surface is first roughly generated by rotating the optic while it is pressed against a cutting tool which rotates in the opposite direction (Horne 1972). This surface is then smoothed by the grinding process in which the surface is brought into an abrasive contact with a similar spherical surface of opposite sign. In this way, for the fabrication of a convex surface, a concave surface of similar radius is used as a grinding tool. The grinding process removes subsurface damage and brings the surface figure close to that required. In the final step, the ground surface is polished to an optical quality surface using a liquid slurry. With care, this simple procedure results in a smooth spherical surface. In essence this is because the only form that can maintain contact across the whole tool area is a spherical form. This method is relatively insensitive to machine accuracy, and since the introduction of sophisticated computer control, smoothing is often not required at all; the optic can be directly polished after the surface generation. Also, a large number of small spherical optics can be fabricated together under a single lap process making this method relatively fast and cheap for mass production of spherical surfaces.
A spherical element is generally tested in a standard interferometer configuration such as the Fizeau or Twyman-Green interferometer (Chen, Murata 1988, Kocher 1972), and a measurement relative to a reference surface (either a flat or spherical surface) is made. In this way the quality of the spherical surface can be assessed to an accuracy of a fraction of a wavelength using interferometers which are quite easy to set up.

The objective of optical surfaces in an imaging system is to modify the object wavefront so as to obtain an image of that object. In general, spherical optics can produce aberrations which distort the image, and must be offset by other optics, at the expense of added weight and size. A high quality optical system often consists of several spherical surfaces. This adds weight and precision mechanics that are required to align these surfaces increases the cost considerably. Aspherical optics (aspherics) are known to be an effective means to increase the number of degrees of freedom in an optical system and are a valuable tool to improve the quality of an optical system (Stevens 1992). Also, one asphere can replace several spherical surfaces without compromising quality. Aspherics can be used to correct aberrations (spherical aberration, coma, astigmatism, distortion and chromatic aberration) that exist in most optical systems. The aberrations in a complicated optical system are dealt with as an integral whole and not separately (as is frequently the case with spherical surfaces). The benefit of aspheric surfaces from this context is that more aberrations can be corrected to obtain significant improvements in the optical system such as smaller f-number, larger field, lower weight and smaller size. Aspherics can be used effectively in both reflective and refractive optical systems, but to date, they have been used only in limited areas due to prohibitive costs and difficulties in mass production.

The spherical optical fabrication process is unsuitable for aspheric surfaces. Aspheric optics generally require some form of diamond milling or ultra precision grinding, however, the process is very sensitive to the manufacturing environment and usually requires significant smoothing prior to final polishing (Fahnle, Brug & Frankena 1998, Heynacher 1979, Heynacher 1979).
Many strategies have been tried to overcome this deficiency including CNC generated aspheres (Erdei, Szarvas & Lorincz 2004, Nicholas, Boon 1981, Ruckman, Fess & Van Gee 1999), diamond-turning techniques (Xie Jin et al. 2004), microcomputer controlled polishing (Doughty, Smith 1987), sub-aperture polishing (Hao-Bo Cheng et al. 2005), and ion-beam figuring (Schindler et al. 2001). Diamond-turning techniques use point-contact contouring to fabricate aspherics, but its use has been limited due to poor surface finish and high frequency diamond-turning marks in visible optics. Recent advances in CNC techniques use small tool or high speed grinding to produce an aspheric surface. These methods also does not yield an optically smooth surface and an additional polishing step is required. In general, polishing is done using a computer-controlled small-area technique to produce optical smoothness. These methods rely on feedback and are very difficult to control. Recently the technique of magnetorheological finishing (MRF) was proposed and shows considerable promise (Golini et al. 1996, Shorey, Kordonski & Tricard 2004). In this method, optical surfaces are polished in a computer-controlled magnetorheological finishing slurry. Unlike conventional rigid lap polishing, the MR fluid acts as a compliant polishing lap, whose shape and stiffness can be magnetically manipulated and controlled in real time. They can remove the high frequency diamond-turning marks and shows better control than other polishing processes. However, the MRF technique is not perfectly deterministic and can be termed as "open loop" controlled because repeated testing is required to obtain the final surface form. One way to reduce the cost of aspheric fabrication is to make moulded lenses using aspheric mould tools (Ouyang Miao-an 2006). In fact, this method is extensively used for small size aspheric lenses inside many products such as digital cameras, mobile phones and DVD players. However, this method is only appropriate to specific (having low coefficient of thermal expansion) materials and cannot be used for most of the optical glass types. The high temperature involved in moulding causes shrinkage
when the lens cools and this results in surface form errors. A direct machining method is required for fabricating large diameter (> 25 mm) aspheric lenses.

In optical fabrication it is often stated that one can manufacture a part only to the accuracy that one can measure it and the cost of quality aspheric production is largely due to the difficulty involved in testing (Tiziani et al. 2001). Interferometry used to test spherical optics is not suitable for testing aspherics as the resulting interferogram contains high frequency fringes that cannot be resolved easily if the asphere has a significant deviation from a spherical form. An alternative way of testing aspherics is to use a contact stylus method, where the vertical movement of the stylus is detected by a differential transducer which converts mechanical displacement into an electrical signal (Scott 2002). This method is not suitable for soft materials as they tend to induce surface damage due to surface contact. The test optic needs to be removed from the fabrication unit to be tested and is not suitable for in-process measurement. Many methods have been suggested in the literature to overcome the problem of testing aspherics and are described in detail in the next chapter.

1.2 Objective and outline of the thesis

The main objective of this research is to develop a method of testing steep aspherics which is simple, low cost and can be used for in-process measurement. The concept of Synthetic Aperture Interferometry (SAI) was chosen to develop a system for measuring steep aspherics since it is straightforward to implement and has the potential to be used in-process. SAI is basically a scanning interferometer where the interferogram is recreated by superposition of signals obtained by a probe which synthesizes the aperture (Tomlinson, Coupland & Petzing 2003). The fundamental advantage of the SAI is that the scanning path of the probe can be changed to suit the test optic so as to reduce the spatial bandwidth of the interference pattern without the
need for null lenses. In this thesis investigations of SAI are presented in eight chapters.

- **Chapter 2** gives description of various testing techniques currently employed to measure aspherics and introduces SAI.
- **Chapter 3** presents the detailed theory of SAI to compute optical path difference (OPD) and subsequently the surface form. It also describes the associated phase evaluation methods to convert intensity signals to phase and OPD values.
- **Chapter 4** describes improvements to SAI and in particular the design of a new anamorphic probe. It also describes the application of the interferometer on a CNC lathe and the initial results obtained.
- **Chapter 5** explains the development of an off-line experimental set-up to assess measurement accuracy. It also presents a new phase evaluation process suitable to be used in our case of SAI.
- **Chapter 6** gives the details of alternative experimental configurations and analyses the errors obtained in these cases. A large diameter test optic and an aspheric surface with four higher order coefficients are measured to illustrate the new configurations.
- **Chapter 7** presents a detailed error analysis along with measurement of two steep surfaces of similar profiles but of opposite signs to assess the errors.
- **Chapter 8** summarizes the work done and provides suggestions for future work.
2.1 Introduction

This chapter reviews methods of testing aspherical surfaces (aspherics). Aspheric testing can be performed both interferometrically and noninterferometrically (Malacara 1992a). In the interferometric tests, two beams (usually one is a reference beam of known quality and other the beam reflected from the test surface) are made to combine to produce an intensity pattern which in turn gives rise to interference fringes (Peters, Boyd 1920). The surface quality of the test surface can be interpreted by analyzing the fringes. Standard interferometric tests such as the Fizeau interferometer can be easily used to test spherical surfaces using a spherical reference surface (Chen, Murata 1988). Figure 2.1 shows a schematic of Fizeau interferometer for testing a concave surface using a convex reference surface. They provide high resolution and can easily give sub-wavelength accuracy. In essence standard interferometers are not suitable to test aspheric surfaces because the resulting interference pattern contains too many fringes to analyze. For this reason different techniques have been developed to overcome this problem.

Interferometric tests to measure aspherics can be subdivided into null and non null tests. In null tests, supplementary optics are used to provide a reference wavefront which is similar to the test aspheric wavefront with opposite sign to reduce the number of fringes in the interference pattern. The supplementary optics can be either conventional or holographic elements including computer generated holograms (CGH). Non null tests such as shear interferometry attempt to modify the test wavefront to obtain an interference
pattern to assess the surface quality of test surface. A major advantage of interferometric tests is that there is no surface to surface contact and hence no possibility of surface damage.

![Fizeau interferometer diagram](image)

**Figure 2.1: Fizeau interferometer for testing a spherical surface**

Non-interferometric tests can also be divided into contact and non-contact methods. Non-contact methods are usually optical methods which utilize some kind of structured light, focused spot or intensity modulation to measure surface form without obtaining interference. In general, these types of measurement are mainly used for qualitative purposes and are limited to measuring near-flat polished surfaces. It is fair to say that most aspheric profilometry is done using contacting methods. In these methods, a stylus or ball tip is made to traverse the test surface along a line. The mechanical movement thus obtained is converted into an electrical signal to obtain the
test surface profile. This type of test is not suitable for soft materials as they are prone to induce surface damage.

The following sections present the different ways of measuring aspherics in some detail. First, we discuss some of the non-interferometric tests which is followed by a discussion of interferometric tests. A section on stylus profilometry is then presented since it is most regularly used in industry to test aspherics. Finally, we introduce the concept of synthetic aperture interferometry (SAI) which we later modify in this thesis to measure steep aspherics.

2.2 Non interferometric tests

One of the first uses of aspheric surfaces was in telescopic mirrors. Until early 1900s knife-edge tests were extensively used to test these mirrors. Knife-edge tests are a type of Schlieren tests which are quite useful to test optical surfaces. The basic principle of Schlieren techniques is to calculate the lateral displacement of a ray by blocking or modifying it. It was first introduced by Foucault in the year 1858 (Ojeda-Castaneda 1992) and may be considered as a method for detecting transverse aberrations in an optical surface. The method for testing a lens using this technique is shown in Figure 2.2.
In this method, one part of a plane traversed by rays or diffracted light is blocked so that a shadow appears over the aberrated region. The quality of the test lens can be checked by observing the variations in the shadow pattern when the knife-edge is placed at different positions. Although this process was lengthy and less accurate because the figure errors were obtained from measurements at a large number of zones, it was the main method of testing aspherics in the early stages of aspheric development. The only alternative was to test the mirror in autocollimation using an optical flat. The size of the optical flat had to be larger than the mirror being tested and with a higher surface figure accuracy than the desired accuracy of the test mirror.

The concept of introducing a compensating lens for null tests to be used in the knife-edge test was first introduced by Couder in 1927 (Offner, Malacara 1992a). The basic principle of a null test is to compensate the aberration emanating from an aspherical surface to produce a stigmatic image using a specially designed auxiliary system. The auxiliary system is called a null corrector or compensator which aids in formation of a stigmatic image. Couder placed a two-element compensator between a paraboloidal mirror.

**Figure 2.2: Knife-edge test**
and the image of a point source placed at its centre of curvature to remove the 
aberrations and get a stigmatic image. In 1936, Burch used a spherical mirror 
beyond the centre of curvature to compensate the paraboloid aberrations, 
while placing the source near its centre of curvature (Burch 1936). Two years 
later Burch developed a solution for a refracting compensator for 
compensating third-order aberrations of a paraboloidal mirror (Burch 1938). 
Another refractive null corrector, consisting of a spherical refracting element 
and an aspheric corrector plate, was developed by Ross in 1943. This 
compensator forms a coma-free & stigmatic retroreflected image near the 
mirror centre of curvature (Ross 1943).

In the tests mentioned in previous paragraph, an aspheric element was 
needed to compensate for spherical aberrations of an aspheric mirror to 
achieve high accuracy in measurements. The aspheric component in the 
corrector had to be manufactured to a greater accuracy than the test aspheric 
surface. Abe Offner (Offner 1963) overcame this problem by designing a 
simple optical system consisting of two small lenses, imaging lens and field 
lens as shown in Figure 2.3. The field lens forms an image of the imaging lens 
at the aspheric mirror where the spherical aberration of the lenses matched 
the aberration of the ray normal to the aspheric mirror so that a stigmatic 
image is obtained without using an aspheric null compensator.
The null tests mentioned previously were mainly used for testing concave aspheric mirrors. For a convex mirror, Hindle proposed stigmatic set-up for different types of conics (Offner, Malacara 1992b). A schematic of Hindle arrangement for a convex hyperboloid is shown in Figure 2.4. The stigmatic image is formed by retroreflection from a sphere whose centre coincides with virtual focus of the test hyperboloid.
The invention of the Offner null corrector led to frequent testing of aspheric surfaces using these types of null correctors. Paul L. Ruben (Ruben 1976a, Ruben 1976b) surveyed and described different types of null correctors. He put forward pros and cons of various types of one-element and two-element null correctors for testing concave and convex aspheric surfaces. This work provides a guidance in both the design of an aspheric surface to assess its nullability and the design of null correctors suited to different aspheric surfaces.

A test based on the knife-edge technique without using supplementary null optics to measure symmetric aspheres has been developed where the knife edge is replaced by a circular stop as shown in Figure 2.5 (Handojo, Frankena 1998). The test surface is illuminated by a spherical wave and a small circular stop is placed around the centre of curvature of the best-fitting sphere so that only rays characteristic of a deviation from the spherical surface remain unobstructed. Light and dark regions appear in the image of the test surface whose boundaries are correlated to the surface profile, the stop size and the stop position along the symmetry axis. This method requires inspection at several annular zones which then need to be joined together to obtain the surface profile of the test surface. The accuracy of the test is severely limited by the number of zones inspected, diffraction at the stop and measurement accuracy of the stop movement. In addition, the test is insensitive to small slope variations making this test useful only for quick qualitative analysis.
The use of Fresnel correctors for testing aspherics was first described by Meinel & Meinel (Meinel, Meinel 2001). The Fresnel corrector consists of a zone plate which has concentric rings of increasingly narrow spacing outward from its centre. They compared the Fresnel correctors with both classical Offner and Hindle null correctors. One advantage of Fresnel correctors over that of conventional null correctors is that the Fresnel null corrector can independently generate third, fifth and higher order aspheric terms to match the aberrations of the aspheric surface whereas in conventional null lenses all orders of aberrations are closely coupled. In practice, getting high accuracies from Fresnel correctors depends on the fabrication and testing of these elements and to date they have not been used widely.

The null compensators mentioned above compensate the spherical aberration with different degrees of perfection. The chromatic aberration is not corrected in any of the compensators, so use of monochromatic light is essential in using them. The amount of aberration depends on the axial position of the compensating lens. The figure of the aspheric surface cannot be inferred accurately if the position of the compensator is not measured very accurately. Different null compensators have to be fabricated for different types of aspherics. In addition, they themselves have to tested to a higher degree of accuracy than the test surface. This makes them very costly to use and is mainly used in large industrial labs.
Screen tests represent another form of qualitative test where the basic principle is that a wavefront can be sampled in various predetermined locations which can be subsequently recreated assuming that the sampling points are related to each other (Ghozeil 1992). The wavefront is sampled by a number of rays normal to it, whose height at a desired location is recorded. The wavefront error can be measured by calculating the deviation of the actual ray height with respect to the ideal ray height. This technique was first used by Hartmann in 1900 who sampled a wavefront with a perforated screen. The pattern of a Hartmann screen is radial as shown in Figure 2.6. The holes on the screen are spaced evenly along a number of diameters of the circular aperture which makes analysis easier in polar coordinate system.

![Hartmann pattern](image)

**Figure 2.6: Hartmann pattern**

Malacara (Malacara 1972) devised a novel method to test aspherical mirrors using the Hartmann test. In his method, small glass wedges are placed over each hole of the Hartmann screen to bend each beam such that all the beams pass through a common point. In this way, a null test is obtained. Pfund (Pfund, Lindlein & Schwider 2001) modifies the Hartmann test by replacing the holes with a microlens array which measures the local propagation vector of the wavefront. In principle, this is a Shack-Hartmann sensor that measures
the derivative of a test wavefront which can be reconstructed using numerical integration. The limitations of the Shack-Hartmann sensor for testing aspherics were described by Rocktaschel (Rocktaschel, Tiziani 2002) and he showed that the dynamic range of the sensor depends on the curvature of the incident wavefront. An analytical expression was developed to calculate optimum value for microlens parameters to measure an approximately known test surface. It was observed that this method is suitable for wavefronts with low curvature but it fails if the measured wavefront has high curvature. Moreover, in screen tests, it is not possible to detect small scale surface changes between successive screens, and errors due to numerical integration affect the accuracy of this method.

In this section, we have described some of the non-interferometric tests used to measure aspherics. Conventional null tests are still being used to test large telescopic mirrors. However, null tests are used sparingly in practice because different mirrors require different null correctors and it is quite costly to fabricate and test them. The following section describes some of the interferometric methods used to test aspherics.

### 2.3 Interferometric tests

In the previous section, we have described null tests as one of the non-interferometric tests. Null tests are also used in interferometric configurations where the basic principle is to compensate the aberration emanating from an aspherical surface to produce interference pattern with less number of fringes so that it can be analyzed. Holograms can be used here as an alternative to conventional null optics (Creath, Wyant 1992a). Wavefronts produced by master optical systems can be stored in a hologram and this hologram is then used to perform null tests of similar optical systems. Holographic tests can be carried out both with standard interferometers and with setups having a large tilt angle between object and reference wavefronts. Figure 2. 7 shows an interferometer that can be used for making a hologram of a concave aspheric
mirror. Holograms can be recorded on photographic plates, thermoplastic materials or photorefractive crystals. A hologram is normally made in a plane conjugate to the test surface. Once the hologram is made, it can be replaced in the same location and reconstructed by illuminating it with a plane wave. The plane reference wave then interferes with the hologram to produce the wavefront due to the test surface.

In holographic tests, the storage of a holographic wavefront is very critical. It is required that the reconstructed wavefront to test aspheric surface is identical to the recorded hologram. Errors may creep in to the test system due to differences in reconstructing and recording geometry, recording material deformation and aberrations due to recording material substrate. If it is not possible to make a real hologram due to unavailability of master optical systems, it can be replaced by a CGH to provide the reference wavefront.

A CGH is a synthetic representation of an actual hologram obtained by interference between an ideal wavefront from the test system and a tilted plane wavefront. Generation of CGH's has been explained by many authors (Lee 1970, Lohmann, Paris 1967). In essence the test setup to obtain the interference fringes between an ideal test wavefront and a tilted plane wavefront in the hologram plane is raytraced in a computer where the hologram plane is at the conjugate of the exit pupil of the test system.
Application of CGH’s in testing aspherics has been investigated by McGovern & Wyant (MacGovern, Wyant 1971). The test setup is similar to that for a real hologram to interferometrically test aspherics. It is carried out by the interference of a test wavefront with the reference wavefront stored in the hologram. The interference is usually done in the Fourier plane of the hologram between the zero-order test wavefront and first-order reference wavefront. A CGH is used to modify either the reference or the object wavefront so that wavefronts exiting the interferometer are closely matched.

One of the major problems in testing aspherics using a CGH is the generation of extra diffraction orders, which cause “disturbing areas” in the interferogram. Norbert Lindlein (Lindlein 2001) provided an approximative analytical expression for the spatial frequencies of the disturbing light in the interferogram coming from the different diffraction orders of the CGH. This expression is useful for calculating the size and shape of the “disturbing areas” in the interferogram.

In general, null tests using CGH’s are used to test single monolithic surfaces. In recent times, the size of mirrors used in telescopes have been steadily increasing to collect more light. As handling a single large mirror is extremely difficult, numerous segmented pieces are combined together to give the effect of a single mirror, but there are many tradeoffs for using this type of optic in terms of fabrication and testing. Feenix Y. Pan and Jim Burge (Pan et al. 2004a, Pan et al. 2004b, Pan, Burge 2004) developed a method of measuring large quantities of segmented pieces using an interferometric test plate and CGH. In the test they proposed, the segmented pieces are tested interferometrically using a best-fit spherical test plate to control the radius of curvature where the aspherical departure is compensated with a CGH that is imaged onto the test plates.

For aspherical testing, a CGH has similar sensitivities to error as a real hologram. In addition, errors which may affect the CGH wavefront are incorrect hologram size and position, plotter & photoreduction lens distortion and quantization errors. A detailed analysis of the errors (Wyant, Bennett
1972) shows that once again all the errors are proportional to the slope of the aspheric wavefront and calibration of the errors present in a CGH is not straightforward.

Null test methods described previously use a static optic as a null corrector. The continuous improvement in adaptive optics had an influence on testing of aspherics as well. A new approach of testing aspherics with a dynamic null lens was proposed by Pruss (Pruss, Tiziani 2004). The dynamic null corrector is a membrane mirror consisting of a thin aluminium coated silicon nitride membrane stretched over an array of electrodes, the shape of which can be controlled. This method allows the null lens to adapt quickly to different aspheric forms providing flexibility in optical shop testing. The stability of the membrane mirror in this test, however, is limited by the stability of the applied voltage.

One way to reduce the number of fringes without using a null compensator is to use a longer wavelength light source such as a CO\textsubscript{2} laser in a standard interferometer (Kwon, Wyant & Hayslett 1980). This reduces the aspheric departure from the best-fit reference sphere (in units of probing wavelength). There is a corresponding decrease in sensitivity, but in many cases, it is quite adequate. The major disadvantage in using a longer wavelength is that it causes practical difficulties since it cannot be seen and cameras to record the interferogram directly are not readily available.

The problems associated with longer wavelength interferometry can be solved by using two wavelength holography (TWH) (Wyant 1971). TWH provides a means of obtaining an interferogram similar to that obtained using a longer wavelength by using only visible light. In the first step of TWH, a hologram is photographically recorded using the test surface in an interferometer using a wavelength $\lambda_1$. The second step consists of replacing the developed hologram in the interferometer and illuminating it with fringe pattern obtained by testing the test surface using a different wavelength $\lambda_2$. In this way, Moire interference is observed between the stored holographic fringes (using $\lambda_1$) and the live fringes (using $\lambda_2$). These fringes are identical to
those that would be obtained using a long effective wavelength given by \((\lambda_1\lambda_2)/|\lambda_1-\lambda_2|\). A wide range of equivalent wavelengths can be obtained using this method.

A major drawback of TWH is creation of an intermediate hologram, which means that optical setup should be exactly same while recording the fringes for both the wavelengths. Any change in the setup including air turbulence will lead to errors. Creath (Creath, Wyant 1985) showed that the TWH can be used with phase shifting to remove the necessity of intermediate recording. In this method, data is taken at the first wavelength while shifting the phase appropriately and calculating the phase modulo \(2\pi\) for that wavelength. Similarly, phase modulo \(2\pi\) is calculated for the second wavelength. These two-phase measurements are then combined to produce a phase corresponding to a long wavelength. The problems associated with recording holograms limit its use in testing aspherics.

The problem of producing a separate reference wavefront is totally eliminated in shearing interferometry which can be of two types called lateral and radial shearing interferometers. Lateral shearing interferometry consists of shifting the test wavefront laterally by a small amount to obtain the interference fringes between the original and displaced wavefronts (Mantravadi 1992). This is obtained by displacing a planar wavefront in its own plane as shown in Figure 2.8. The lateral shear fringes appear in the common area.
Lateral shearing interferometry using gratings has been reported by many authors (Patorski, Ulinowicz 1987, Rimmer, Wyant 1975, Thomas, Wyant 1976, Wyant 1973). In these systems, two shifted images of the test wavefront interfere with each other to obtain interference fringes. The amount of shear determines the dynamic range of the measurement. In this technique, the resulting fringe patterns represent the first derivatives of the phase distribution of the light beam transmitted or reflected from the test surface. Quantitative evaluation of aspherics using double grating shear interferometry was presented by Betend-Bon (Betend-Bon, Wosinski & Breidne 1992). Two high frequency sinusoidal diffraction gratings were used in collimated light to prevent different diffraction orders from overlapping. The first diffracted order is used to record the shearing interferograms of the test surface. A phase stepping technique was added to increase the test accuracy. Further, a second shear perpendicular to the first was introduced to measure both components of the phase derivative. The amount of shear could be adjusted by changing the distance between two gratings. The disadvantage of this test is that it gives lower measurement accuracy compared to conventional tests but presents higher dynamic range and is therefore suitable for testing deep aspherics.

A lateral shearing interferometer using square prisms for optical testing of aspheric lenses was proposed by Kim (Kim, Cho & Kim 1998). The
interferometer consists of four square prisms whose sliding motions provide the lateral shearing and phase shifting necessary to evaluate the wavefront of the beam collimated by the lens being tested as shown in Figure 2.9. The prisms were attached face to face using index-matching oil so that undesirable disturbances from external mechanical vibration and atmospheric turbulence could be minimized. A special least-squares phase-measuring algorithm was adopted to compensate for the phase-shifting errors caused by the variation in thickness of the index-matching oil holding the prisms.

Figure 2.9: Lateral shearing interferometer using four prisms

In conventional lateral-shearing interferometry, it is necessary to record two interferograms with orthogonal directions of shear. The interferogram does not cover the entire pupil and it is laborious and time consuming to evaluate the actual shape of the surface from measurements on photographs of the fringes. These problems can be overcome by using radial shear interferometry in conjunction with a microcomputer-controlled digital electronic system for measurements of the phase difference at a matrix of points covering the interferogram. A radial shear interferometer produces two interfering wavefronts with identical deformations, but one of the wavefronts is contracted or expanded with respect to each other (Malacara 1992b) as shown in Figure 2.10.
This system also permits the measured values of the phase difference to be processed directly to obtain the shape of the test surface (Hariharan, Oreb & Zhou 1984). The sensitivity of radial shear varies with the amount of shear and is limited by the quality of wavefront fitting, sample spacing and reading errors. Both lateral and radial shearing interferometers introduce measurement errors during the reconstruction of the surface profile as this is done by integrating the slopes obtained by the interferometers.

A Ronchi test can also be used to measure the quality of a test mirror as the fringes produced by this test depend on the aberrations of the mirror. In the Ronchi test, fringes are produced when a ruling is placed near the centre of curvature of a mirror, and the image of the grating is superimposed on the grating itself as shown in Figure 2.11 such that a kind of Moire pattern is produced. It was discovered by Italian physicist Vasco Ronchi (Cornejo-Rodriguez 1992) and he named the pattern as combination fringes. The Ronchi test can be explained by two equivalent models: one is geometrical, interpreting the fringes as projections of the ruling bands, and the other is physical, interpreting the fringes as interference between several wavefronts produced by the ruling acting as a diffraction grating. Both models give similar results when the frequency of the ruling is not very high (Cornejo, Malacara 1970).
A method to calculate the geometrical Ronchi pattern of any spherical or aspherical mirror produced with a point source at any point along the optical axis was proposed by Daniel Malacara (Malacara 1965). He calculated deviations of a given mirror by measuring the position of the fringes in the Ronchigram produced by the defective mirror. In a Ronchi test, light passing through each of the Ronchi ruling interferes with that passing through the other slits. This produces an effect of many laterally sheared pupils. This effect reduces the test accuracy because of numerous interfering images and reduced precision in location of the Ronchi pattern over the pupil. An improved Ronchi Test using holographic sinusoidal gratings instead of conventional ruled gratings was proposed by Lee (Lee, Kim 1999). In this test, measurement precision was improved as the sinusoidal gratings produces only 0th and 1st order diffraction unlike the conventional rulings which exhibit higher order diffractive noise. Although useful, this test is limited by alignment difficulties as a small misalignment causes large disturbances in beam diffraction.

Subaperture testing with annular zones is another method of testing aspherics (Ying-Moh Liu, Lawrence & Koliopoulos 1988). Here the density of
interferometric fringes is effectively reduced by dividing the surface into annular sub aperture zones as shown in Figure 2.12 which are measured independently. The zones are then stitched together to obtain a continuous profile.

Figure 2.12: Subaperture configuration

A similar method has been developed to test aspherics using multiple annular interferometry (Melozzi, Pezzati & Mazzoni 1993). In this method, successive overlapping phase maps are recorded from a set of annular interferograms of an aspheric surface using a conventional phase-shifting interferometer and a precision translator stage. The aspherics surface is moved from the focus of an interferometer reference sphere so that its radius of curvature matches the aspheric surface on rings of increasing or decreasing diameter. These overlapping annular interferograms for different positions of asphere are then stitched together and analysed to recreate the surface error. Sub-aperture stitching interferometers are prone, however, to significant measurement errors during the stitching process.

In addition to the interferometric tests mentioned in this section, several other techniques have been developed to test aspherics. Moire
techniques (Creath, Wyant 1992b) have been used to complement holographic interferometry in testing aspherics. Recent innovations include the use of compensatory liquid (Yun, Liu & Li 1998) or liquid crystal (Yun et al. 1999) to reduce the fringe density. In addition to errors caused by misalignment, uncertainty in the measurement of fringe order and phase unwrapping, these methods are also sensitive to refractive index changes and practical difficulties such as surface contamination and cleaning.

The development of phase shifting interferometry (PSI) has given impetus to automated fringe analysis. PSI is not a specific hardware configuration but a data collection and analysis method that can be applied to several standard interferometers (Greivenkamp, Bruning 1992). The disadvantage of single frame (static) interferometry is to deduce the phase from the spatial change in intensity. In PSI the phase difference between reference and test wavefronts is varied and recorded in a known manner and direction. The wavefront phase is encoded as the variation in the intensity pattern of the recorded interferograms, and a simple point-by-point calculation recovers the phase. Reference of this technique to test aspheric lenses was made by Koliopoulos (Koliopoulos, Steijn 1982). A linear detector array is used to obtain surface profile of an aspheric wavefront by measuring the optical phase at each element. It was assumed that the changes in azimuth of the wavefront is slowly varying and the wavefront can be quantified by taking a small number of profiles at different orientations.

A technique for continuous-phase determination of interferograms was applied to test aspheres in a null setup by Servin (Servin, Malacara & Rodriguez-Vera 1994). In this method, the phase detection and unwrapping is done simultaneously using a phase-locked-loop (PLL) interferometer which uses a CGH compensator. The compensator is calculated and used within the computer to phase demodulate a sample interferogram obtained from the asphere being tested. The demodulated phase contains information about the wave-front departures from the ideal computer-stored aspheric
interferogram. It was assumed that the recorded frequency in the computer hologram and the sampled hologram are below Nyquist frequency.

Sub-Nyquist frequency is a data collection and analysis method that increases the measurement capability of PSI using an a-priori information (Greivenkamp 1987, Greivenkamp, Lowman & Palum 1996). The Nyquist frequency is defined as the inverse of twice the pixel spacing. PSI works effectively assuming that the sampling of fringes can be done for a frequency less than the Nyquist frequency which means that the wavefront slope should change by less than \( \pi (\lambda/2) \) per pixel. This assumption limits the range of measurement of PSI when applied to aspherics. In Sub-Nyquist frequency interferometry (SNI), it is assumed that the test wavefront slope is smooth and has continuous derivatives. This information leads to analyse high density fringes which exceeds Nyquist frequency by a large amount. The advantage of SNI becomes apparent during phase unwrapping of the modulo 2\( \pi \) data. SNI assumes that the derivatives of reconstructed wavefront do not exhibit any large change from pixel to pixel, which limits the wavefront slope change to \( \pi \) per pixel and large changes of the wavefront height are allowed. To satisfy this condition, an appropriate number of 2\( \pi \)'s are added so that there is a unique solution for each pixel. This method gives a much larger measurement range than PSI using same number of measurement points. It should be noted that the calibration of such systems are critically important to obtain a good measurement accuracy.

It should be noted that most of the methods described previously in this chapter measure the aspheric wavefront reflected by a mirror or refracted by a lens rather than the direct measurement of surface form. All the methods have their limitations and involve a trade-off which either increases the cost, makes calibration difficult or reduces the precision and accuracy. It is fair to say that to date no generally applicable interferometric method has been established for measurement of aspherics. For these reasons, the testing of aspherics is usually accomplished using a mechanical probe instead of
interferometry which measures the surface form directly and is discussed in
the next section.

2.4 Surface profilometry

Surface profilometry is a method for testing surfaces and usually provides
quantitative data in the form of a surface height map over the measurement
area (Creath, Morales 1992). Profiler types can be split into contact and
noncontact devices. Huang (Huang, Xu 1999) proposed a non contact optical
probe which measures the slope of an aspheric surface by measuring the
displacement of a reflected laser beam. The surface profile is obtained using
numerical integration of the measured slopes. As with Hartmann sensors the
use of these type of probes is limited by the error caused due to integration.

A contact profiler scans a probe across the surface and determines
height by looking at the height variations of the probe as it is scanned. The
most common type of surface profilometer is the stylus. Stylus profilometry is
a common contact approach for testing aspheric surfaces during the stage of
generation and grinding in the manufacture of aspheric surfaces (Lee et al.
2005). In this process, a stylus or ball tip is traversed at constant speed along a
workpiece as shown in Figure 2.13.

![Stylus probe](image)

**Figure 2.13: Stylus probe**

Vertical movement of the stylus is detected by sensing the height variations of
the stylus. Generally, it is simple, accurate and relatively low cost, but
measurements are slow compared with the interferometer. The size of the
stylus needs to be quite small to provide accurate measurement but
decreasing the size of the stylus increases the local force on the test surface which can deform and damage the surface. More advanced styluses use tip radii of less than a tenth of a micrometer, with minimal tip loadings, but need to be used in vibration isolations systems and take a long time to scan a surface. Improvements in the stylus have reduced the risk of surface damage and they are used widely in industry to test aspherics in the absence of suitable interferometric methods (Bennett, Mills, Scott 2002).

For this reason, our main aim was to find an alternate interferometric method to test aspherics which can be used for measuring most aspherics of interest. Synthetic aperture interferometry (SAI) is a straightforward, low cost interferometric method which has the potential to measure aspherics in the production environment. The following section describes the basic method of SAI to test aspherics. This method is later modified and developed in this thesis to enable it to measure steep surfaces.

2.5 Synthetic aperture interferometry

SAI is a novel way to measure aspherics which was reported by Tomlinson in 2003 (Tomlinson, Coupland & Petzing 2003). An SAI produces an interference pattern that is similar to a Fizeau interferometer. The difference in the two interferometers is that the Fizeau interferometer uses a whole field interference pattern which means that the interference fringes are recorded simultaneously over the entire surface of interest. Whereas, SAI is a single point scanning method where the fringe pattern is reproduced by combining the intensity of interference obtained by a scanning detector. The sampling rate of the detector can be adjusted to scan the aperture at a rate appropriate to the spatial bandwidth of the interference pattern produced by the test surface. One of the major advantages of the SAI is that it is straightforward to implement and has a potential for on-line measurement of aspherics. In general, aspheric testing is done off-line using a standard interferometer or a contact stylus probe. This leads to added time and possibility of damage to
the test surface. Due to these reasons, SAI has a great potential to be of immense commercial benefit to the optical manufacturing industry and user community.

In Tomlinson’s work, a probe was used to measure the optical path difference (OPD) between wavefronts reflected from the front and back surface of the aspheric to be tested. A pair of source and receive fibre was used to construct the probe which scanned the full aperture of the test optic along a defined path. The interferogram was recreated by synthesizing the test aperture from the data collected along the path of the probe. If the shape of either the front or rear surface is known (as a reference), it was shown that the form of the unknown surface can be deduced.

The simplest form of SAI to test an aspheric optic is shown in Figure 2.14. The interferometer consists of a single send-receive fibre that collects the light from a laser source and scans the test optic along a nominally circular arc. The interference between light reflected from the test surface and fibre termination is recorded using a detector. The OPD is twice the surface deviation similar to a conventional Fizeau interferometer.

![Figure 2.14: Simple synthetic aperture interferometer](image)

The fundamental advantage of SAI is that the scanning path of the fibre can be changed to suit the test optic so as to reduce the spatial bandwidth of the interference pattern as the path of the fibre termination acts as a reference surface. This means that theoretically, the fibre can be made to follow a path similar to the ideal aspheric surface form to minimize the spatial bandwidth.
of the interference pattern. In this simplified case, the measurement is inherently sensitive to vibration as the measurement is made relative to fibre path. A more practical SAI configuration (as used by Tomlinson) is shown in Figure 2.15. The configuration consists of separate source and receive fibres to avoid ghost reflections from the fibre end. The fibres were held rigidly together to form a probe that traverses along a defined path. Light reflected from the front and rear surface is collected at the receive fibre. The inherent sensitivity to vibration is reduced as a differential measurement is made between the test and reference surface.

![Figure 2.15: Practical synthetic aperture interferometer](image)

Here, it is assumed that the probe path and surface form of the rear surface is known. This assumption is used to find the ray intersection points on the front and rear surface of the test optic for different probe positions. The optical path lengths corresponding to the rays reflected at front surface and the rear surface can then be calculated to find the ideal OPD.

The measured intensity signal is then used to compute the measured OPD. The difference between the measured OPD and the ideal OPD represents an error in surface form which is calculated using numerical
methods. SAI using bare fibres suffers from two major problems. In order to collect the light, the normal of the front surface must be included within the cone defined by fibre NA. A typical fibre has a numerical aperture of 0.12, which means that the surface normal must be within a cone with half angle of 7°. This puts a severe limitation on slope of the aspheric surface to be tested. Therefore, this method is not suitable for measuring steep aspherics. Secondly, the light efficiency of the system is very poor as shown in Figure 2.16 because only a very small fraction of the light in the illumination cone comes back to the receive fibre. Clearly, the capability of the probe to measure steeper slopes can be increased by increasing its NA, but this reduces the light gathering efficiency further as the larger the NA, the larger the cone of illuminating light and consequently the efficiency diminishes.

![Illumination cone of the source fibre](image)

**Figure 2.16: Illumination cone of the source fibre**

### 2.6 Summary

Several methods of testing aspherics were discussed in this chapter. Null interferometric tests are some of the few methods which provide accuracy and have a high resolution. These tests can be done either using conventional optics or CGH. Null tests are used in large industrial labs where the accuracy
is of utmost importance. Because of the high cost of null tests, methods using contacting stylus probes are widely used in industry. Being a contact measurement method, however, it has a tendency to damage the test surface. Sub-aperture interferometry is another method which has been proposed in literature. The accuracy of this interferometer depends on the accuracy of stitching various sub-aperture zones. The precision mechanics and corresponding software required to control the interferometer increases the cost of such interferometers significantly.

It is fair to say that, to date, no generally applicable interferometric method has been established for in-process measurement of aspherics. SAI is a novel interferometric method to test aspherics where a probe scans the test surface to obtain its form. The major advantage of SAI is that it is straightforward to implement and has a potential to measure aspherics during the polishing process. This method is inherently tolerant to vibration as a differential measurement between the test and reference surfaces is made.

Preliminary work has only considered the application of the technique to optics of relatively small curvature and had a relatively low light gathering efficiency. The main objective of this thesis is to remove these problems in SAI. It has been observed that the major limitation of the original method to measure steep surfaces is the low NA of the probe. This problem has been alleviated by the use of supplementary optics. An anamorphic design for the probe has been devised to increase the light gathering efficiency and this is described in chapter 4. Before discussing practical implementation, however, the theory that is the basis of SAI is discussed in the following chapter.
Chapter 3

Synthetic Aperture Interferometry: Theory

3.1 Introduction

In the previous chapter, methods to test aspherics were reviewed and synthetic aperture interferometry (SAI) was introduced. It was demonstrated that although SAI is straightforward to implement, preliminary systems did not have sufficient numerical aperture (NA) to measure steep surfaces and increasing the NA significantly decreases the light gathering efficiency. The problem of measuring steep surfaces can be addressed through the use of supplementary optics to increase the NA of the fibre probe, the details of which are given in the next chapter. Before covering this topic, however, we introduce the detailed theory of SAI in this chapter. This theory is described for the case of a single source and receive point for the probe and it is shown that this configuration reduces the computational complexity of the dual fibre configuration of Tomlinson (Tomlinson, Coupland & Petzing 2003).

In essence the probe signal is collected as a discrete number of samples as the probe scanned the test optic, the movement of which is monitored using Michelson interferometer as shown in Figure 3.1. The intensity signal obtained by the probe is converted to optical path difference (OPD) values by evaluating and unwrapping the phase embedded in the signal. A phase evaluation process using the Fast Fourier Transform (FFT) is used to convert the signal from Michelson interferometer to distance measurement and is described in section 3.3.1. Because of inherent noise, this method is not suitable for evaluating the phase of signal collected by the probe and
therefore a phase evaluation method based on a phase-locked-loop (PLL) is used to extract the phase information and is explained in section 3.3.2.

![Block diagram of the measurement set-up](image)

Figure 3.1: Block diagram of the measurement set-up

3.2 Theory

The theory of SAI using an anamorphic probe is similar in many respects to the original bare-fibre theory presented by Tomlinson (Tomlinson, Coupland & Petzing 2003). It is simplified however because the source and receive points of the probe are coincident. The probe is oriented relative to the test optic as shown in Figure 3.2 and traversed across it in order to make a surface measurement. The probe provides a measure of the OPD between the rays reflected from the test (front) surface and reference (rear) surface. The surface profile is recovered in two steps. In the first step, the OPD is calculated...
assuming that the ideal form of test surface, lens thickness and the refractive index of the material of the test optic is known. We refer to this step as the forward problem. In the second step, the difference between measured and calculated OPD is used to calculate the surface profile of the test surface. This is referred to as the inverse problem.

![Ray Diagram for SAI](image)

**Figure 3.2: Ray Diagram for SAI**

### 3.2.1 The forward problem

Let us define the probe position by the co-ordinates \((x_p, y_p)\) and the ray intersection point at the front surface by \((x_{f1}, y_{f1})\) where light is reflected back to reach the probe. Accordingly, the ray intersection points at the front and rear surfaces for the ray that is reflected from the rear surface are \((x_{f2}, y_{f2})\) and \((x_r, y_r)\) respectively. Using these definitions the optical path lengths for the ray reflected from the front surface \((OPL_f)\) and that reflected from the rear surface \((OPL_r)\) can be written as
\[ OPL_f = 2\left\{[(x_p - x_{f1})^2 + (y_{f1})^2]^{1/2}\right\} \]

\[ OPL_r = 2\left\{[(x_{f2} - x_p)^2 + (y_{f2})^2]^{1/2} + [n(y_r - y_{f2})]\right\} \]

where \( n \) is the refractive index of the test optic. We assume that the probe moves along a straight line and let the functional form of the probe path is defined by \( y_p = 0 \).

Equations (3.1) and (3.2) define \( OPL_f \) and \( OPL_r \) in terms of the ordinates of the ray intersections at the front surface and, according to Fermat's principle, these ordinates are those which minimise the optical path lengths. Therefore, we can write,

\[ \frac{\partial OPL_f}{\partial x_{f1}} = \frac{\partial OPL_r}{\partial x_{f2}} = 0 \]

\[ (3.3) \]

For a given surface form, the ray intersections are most easily calculated using a line search procedure to find the turning points. It is worth pointing out that in general a surface may exhibit more than one turning point. We note, however, that most useful optical surfaces are monotonic and consequently have only one turning point. Consequently only one ray is retro-reflected to the fibre from each of the front and rear surfaces.

For illustrative purposes, we give an example of an OPD calculation for the plano-convex lens that we measure in section 4.5. The lens has a nominal focal length of 160 mm and is made of BK7 \( (n_d = 1.517) \). The optical specifications of the test lens are given in Table 3.1.

**Table 3.1: Optical specifications of the test lens**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>100 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>82.688 mm</td>
</tr>
<tr>
<td>Centre Thickness</td>
<td>4 mm</td>
</tr>
<tr>
<td>Edge Thickness</td>
<td>1.5 mm</td>
</tr>
</tbody>
</table>
In this example, the distance between probe and front surface of the lens is 8.5 mm. The x-coordinates of the ray intersections, \((x_{1f}, y_{1f})\), \((x_{1r}, y_{1r})\) and \((x_r, y_r)\) are shown as a function of probe position, in Figure 3.3.

Figure 3.3: Ordinates of the ray intersections as a function of probe position

It can be seen that the ray intersections due to the reflection from the front surface increasingly lags behind the probe position whereas that from the rear surface leads the probe position. It is noted that the ray intersections \(x_{12}\) and \(x_r\) are the same. This is due to the planar surface profile of the rear surface and the ray is retro reflected from the rear surface. These intersections will be different if the rear surface deviates from a planar profile or if the lens is tilted such that the rear surface is not parallel to the probe path.

Figure 3.4 shows the optical path lengths of the rays reflected from the front and rear surface. Here, it can be seen that the optical path length of the ray reflected from the front surface increases with probe position whereas the
optical path length of the ray reflected from the rear surface decreases with probe position.

![Diagram showing optical path lengths](image)

**Figure 3.4:** Optical path lengths of the rays reflected from the front and rear surface

The forward OPD can be calculated once the ray intersections are found and optical path lengths calculated, and is the difference between the two optical path lengths. We can write

\[
OPD = 2\left\{\sqrt{(x_p - x_{f1})^2 + y_{f1}^2} - \sqrt{(x_{f2} - x_p)^2 + y_{f2}^2}\right\} - n(y_r - y_{f2})
\]

(3.4)

Figure 3.5 shows the change in OPD with respect to the lens centre for the lens specified earlier.
The total change in OPD for a lens aperture of 30 mm is approximately 6300 fringes at a wavelength of 632.8 nm. We can calculate the intensity distribution resulting from the interference signals with this OPD. If we assume equal intensity, $I_r$, of the light reflected from the front and rear surfaces, then the interference signal, $I(x_p)$, as a function of probe position, can be written,

$$I(x_p) = 2I_r \left( 1 + \cos \left( \frac{2\pi OPD(x_p)}{\lambda} \right) \right)$$

(3.5)

The resulting fringes, for the first 1 mm from the centre of the specified lens, are shown in Figure 3.6.
The rate of change in OPD near the lens centre is quite small compared to near the lens edge where it increases sharply. The OPD change for a scan length of 1 mm across the lens centre is approximately $17.5\mu$ (27 fringes), whereas for a similar distance at the edge of the lens, OPD change is approximately $510\mu$ (810 fringes). This means that the density of fringes increases by almost 30 times for such a lens across the surface and this must be taken into account in the digitization of the signal. This will be, in general, discussed further in section 3.3.2.

3.2.2 The inverse problem

The objective of the inverse problem is to obtain the actual OPD from interference signals of the type described in equation (3.5), and subsequently to calculate the surface form. The measured optical path difference, $\text{OPD}_{\text{meas}}$ can be deduced from this measurement by inverting equation (3.5) such that,
\[ OPD_{\text{meas}}(x_p) = \frac{\lambda}{2\pi} \left[ \cos^{-1}\left( \frac{I(x_p)}{2I_x} - 1 \right) \right] \]  

(3.6)

where the inverse cosine should be interpreted as a phase unwrapping operator. It should be noted that in practice, with the addition of noise and variations in intensity, estimation of phase and phase unwrapping is quite a complicated procedure and will be discussed further in section 3.3. In the following, however, we will assume that the signal is of sufficient quality and the phase can be unwrapped without error. The difference between ideal OPD (obtained from the forward problem) and measured OPD (obtained from the experiment) can be written as

\[ \Delta OPD = OPD_{\text{meas}} - OPD \]  

(3.7)

Since the ideal form of the test optic is known we can linearize the forward problem and consider small changes in optical path difference, \( \Delta OPD \), resulting from small changes in the surface form. Given equations (3.3), we can write

\[ \Delta OPD = \frac{\partial OPD}{\partial y_{f1}} \Delta y_{f1} + \frac{\partial OPD}{\partial y_{f2}} \Delta y_{f2} \]  

(3.8)

Differentiating equation (3.4), we have

\[ \Delta OPD = 2(\{x_{f1} - x_p\}^2 + (y_{f1})^2)^{1/2} y_{f1} \Delta y_{f1} - \{\{x_{f2} - x_p\}^2 + (y_{f2})^2\}^{1/2} y_{f2} \Delta y_{f2} \]  

(3.9)

It is clear from the above equation that for a particular probe position, the change in OPD is a function of surface deviation at two different points \( (x_{f1} \text{ and } x_{f2}) \) of the test surface. In the original bare fibre theory, the change in OPD was a function of surface deviation at three different points and therefore the computation was more complex. Equation (3.9) can be rewritten as

\[ \Delta OPD = M_{y_{f1}} \Delta y_{f1} - M_{y_{f2}} \Delta y_{f2} \]  

(3.10)
where

\[ M_1 = 2\left[ (x_{f1} - x_p)^2 + (y_{f1})^2 \right]^{1/2} y_{f1} \approx 2 \]  

\[ M_2 = 2\left[ (x_{f2} - x_p)^2 + (y_{f2})^2 \right]^{1/2} - n] y_{f2} \approx -1 \]  

(3.11)  

(3.12)

In practice, measurements are made at each probe position to give a discrete set of data \( \Delta \text{OPD}_i \). It is noted that \( \Delta \text{OPD}_i \) is linked to changes in height at two defined points \( (\Delta y_{i1}) \) and \( (\Delta y_{i2}) \) as shown in Figure 3.7. In this figure, \( P_i \) and \( P_{i+1} \) are successive data measurement points and it can be seen that the ray intersections corresponding to the two probe positions are generally independent. Consequently, each measurement then depends on two unknowns and it is impossible to solve equation (3.10). In the case of a test surface which is nearly flat (as presented in the original paper by Tomlinson (Tomlinson, Coupland & Petzing 2003)), \( (\Delta y_{i1}) \) and \( (\Delta y_{i2}) \) are coincident and therefore equation (3.10) can be easily solved, but in general, such a straightforward solution is not possible. It is to be noted that near the lens centre, \( (\Delta y_{i1}) \) and \( (\Delta y_{i2}) \) are again coincident and the surface form error will be approximately one third of the OPD error.

![Figure 3.7: Independent ray intersection points for different probe positions](image)
For the general case, we assume that the height profile is band-limited. This means that there is some correlation between the ray intersection points for different probe positions and this is true if the surface height is slowly varying compared to the sampling interval. Let us assume that we have a set of measurements that are made at regularly spaced intervals along a linear probe path. Let us attempt to find the surface height points on a regular grid with an equal spacing that is the distance between probe positions ($\Delta x_p$) as shown in Figure 3.8.

In this figure, $y_0$, $y_1$, $y_2$ are the surface heights defined once again on a regular grid. Let us consider the probe position at $P_1$ where the ray intersection for the ray reflected at the front surface is represented by $(y_{l1})_i$ and the corresponding intersection point for the rear surface is $(y_{r2})_i$. Although it can be seen that $(y_{l1})_i$ is not coincident with the grid, it can be written in terms of surface heights on either side of $(y_{l1})_i$, $(y_{l1})_i$ and $(y_{r2})_i$. Similarly, for $(y_{r2})_i$, the surface heights on the grid on either side can be termed as $(y_{l2})_i$ and $(y_{r2})_i$ such that
\[
\Delta OPD_i = M_1[a(\Delta y_{f1})_i + b(\Delta y_{r1})_i] + M_2[c(\Delta y_{f2})_i + d(\Delta y_{r2})_i]
\]  
(3.13)

where

\[
a = 1 - b = 1 - \frac{(y_{r1})_i - (y_{f1})_i}{(y_{r1})_i - (y_{f1})_i}
\]
(3.14)

and

\[
c = 1 - d = 1 - \frac{(y_{r2})_i - (y_{f2})_i}{(y_{r2})_i - (y_{f2})_i}
\]
(3.15)

Let us consider that the interference of the intensity reflected by the test and reference surface was collected for \(m\) number of samples and the corresponding surface heights are represented by \(n\) number of points. In this case, equation (3.13) can be written as a matrix equation.

\[
\begin{pmatrix}
\Delta OPD_1 \\
\Delta OPD_2 \\
\Delta OPD_3 \\
\vdots \\
\Delta OPD_m
\end{pmatrix} =
\begin{pmatrix}
M_{11} & M_{12} & M_{13} & \cdots & \cdots & M_{1n} \\
M_{21} & M_{22} & \cdots & \cdots & \cdots & \cdots \\
M_{31} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
M_{m1} & \cdots & \cdots & \cdots & \cdots & M_{mn}
\end{pmatrix}
\begin{pmatrix}
\Delta y_1 \\
\Delta y_2 \\
\Delta y_3 \\
\vdots \\
\Delta y_n
\end{pmatrix}
\]  
(3.16)

An image of this sparse matrix \(M\) for the case of plano-convex lens specified earlier (on page 36) is shown in Figure 3.9.
Given the matrix $M$ from knowledge of the ideal form, the surface deviation then can be computed by inverting equation (3.16) such that

$$
\begin{pmatrix}
\Delta y_1 \\
\Delta y_2 \\
\Delta y_3 \\
\vdots \\
\Delta y_n
\end{pmatrix} = \text{inv} \begin{pmatrix}
M_{11} & M_{12} & M_{13} & \ldots & \ldots & \ldots & M_{1n} \\
M_{21} & M_{22} & \ldots & \ldots & \ldots & \ldots & \ldots \\
M_{31} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
M_{m1} & \ldots & \ldots & \ldots & \ldots & \ldots & M_{mn}
\end{pmatrix} \begin{pmatrix}
\Delta \text{OPD}_1 \\
\Delta \text{OPD}_2 \\
\Delta \text{OPD}_3 \\
\vdots \\
\Delta \text{OPD}_n
\end{pmatrix}
$$

(3.17)

The following plot shows the actual difference in measured and ideal OPD as a function of probe position for a particular scan for an actual measurement of the plano-convex lens described in Table 3.1. Here, the probe scans from the centre to 15 mm circumference of the test lens.
Figure 3.10: Difference between ideal and measured OPD

Due to non-square nature of the matrix $M$ in equation (3.17), a Moore-Penrose pseudo-inverse method (Albert 1972) is used. As we are dealing with significantly large number of samples, the size of matrix $M$ becomes very large and in some cases, the computer memory is incapable of handling such large amount of data. In this case, the total number of data points can be reduced in actual calculation so that it can be handled by the computer memory by filtering and under sampling. If equation (3.17) is used to obtain the surface form error as a function of radial position corresponding to the OPD error shown in Figure 3.10, the following output results.
Figure 3.11: Surface form error using under-determined matrix

Noting the scales in Figure 3.11, it can be seen that the output bears little resemblance to the surface form error and noise, that increases exponentially as the radial position of the probe increases is apparent. This noise is mainly due to the nature of the inversion process using the pseudo-inverse. It is noted that for a convex (or concave) lens the number of unknowns exceeds the number of measurements \( n > m \) and the system is under-determined. In this case, the pseudo-inverse provides the solution that exactly explains the data and has minimum magnitude (Albert 1972). In the presence of measurement noise, the magnitude of the solution increases dramatically in order to provide an exact solution.

By defining the surface height on a coarser grid, the system of equations can be made to be over-determined such that \( m > n \). In this case the pseudo-inverse is the solution that best fits the data in a least squares sense. If we reduce the resolution of the surface heights by a factor of 2 such that the distance between successive surface height position is twice the distance between probe positions \( (2\Delta x_p) \) and use equation (3.17) to calculate the
surface form using this OPD error, the plot of surface form error as a function of radial position is shown in Figure 3.12.

![Graph of surface form error](image)

Figure 3.12: Surface form error using over-determined matrix

It can be seen that in the region 0-14 mm, the measurements shows a small deviation from the nominal surface form. However, the deviation increases approximately to 12µ at the edge of lens. This is considerably more than the change in OPD (shown in Figure 3.10) and will be discussed later in chapter 6. In the following sections, however, we return to a discussion of phase evaluation techniques to find the phase of the interference signals obtained by the probe and the Michelson interferometer.

### 3.3 Phase evaluation

The intensity signal obtained by the Michelson interferometer is converted to a distance measurement to find the distance between successive samples by measuring its phase. Similarly, the intensity signal reflected from the test lens
and collected by the probe needs to be converted to OPD to compare it with ideal OPD and again this is done by deducing its phase. For the case of the Michelson signal, noise is negligible, amplitude variations are small and the frequency is nominally constant. For the case of the probe signal, noise affects the phase and amplitude and the frequency varies by a factor of 30 times or more over a lens diameter of 30 mm. In the Michelson case, phase unwrapping was accomplished using the MATLAB routine unwrap(). For the case of the probe signal an algorithm based on PLL was written to extract phase. These are discussed in the following sections.

3.3.1 Distance measurement from Michelson interferometer

The intensity of the interference pattern obtained by the detector in Michelson interferometer has the form

\[ I(x) = a(x) + b(x) \cos[\phi(x)] \]

(3.18)

where \( a(x) \) represents additive disturbances such as background intensity and electronic noise and \( b(x) \) represents multiplicative, amplitude noise. The intensity pattern for 1000 samples obtained by the interferometer is shown in Figure 3.13.
Equation (3.18) can be written as

\[ I(x) = a(x) + c(x) + c^*(x) \]  

(3.19)

where

\[ c(x) = b(x)e^{i\phi(x)} / 2 \]  

(3.20)

and \( c^*(x_p) \) is the complex conjugate of \( c(x_p) \).

Fourier Transformation of this equation gives us the signal in the frequency domain which is

\[ I(u) = A(u) + C(u) + C^*(u) \]  

(3.21)

where \( u \) is the spatial frequency and \( A, C \) and \( C^* \) are complex Fourier amplitudes. The corresponding spectrum in the Fourier domain is shown in Figure 3.14. The presence of a range of frequencies and amplitude noise in the power spectrum means that the tool holding the probe was not traversing
at a constant speed and the tool stock was vibrating as it traversed across the test lens.

![Power spectrum in the Fourier domain](image)

**Figure 3.14: Power spectrum in the Fourier domain**

In equation (3.21), $I(u)$ is a Hermitian function in the frequency domain as the intensity $I(x)$ in equation is a real-valued function. The property of a Hermitian function states that its complex conjugate is equal to its original function with an opposite sign. Therefore, we can write,

$$I(u) = I^*(-u)$$

(3.22)

This means that the spectrum in the Fourier domain is symmetric to the origin and the real part of $I(u)$ is even and the imaginary part is odd. The term representing the zero peak and the low frequency component originating from the background intensity is the term $A(u)$ in equation (3.21). In Figure 3.14, the peaks on either side of the zero peak are due to the ambient light radiating at 50 Hz. It is clear from equations (3.21) and (3.22) that the phase information encoded in $C(u)$ and $C^*(u)$ are the same. A band pass filter in the frequency domain is used to eliminate the term $A(u)$ and $C^*(u)$. The resulting
amplitude spectrum does not remain real-valued after this filtering in the reciprocal space. The inverse Fourier Transform of the isolated term results in a complex-valued function \( c(x) \) with a real and imaginary part. If the signal is relatively free of noise, the phase modulo \( 2\pi \) can be obtained as the arctangent of the ratio of imaginary and real term.

\[
\phi(x) = \arctan \frac{\text{Im} c(x)}{\text{Re} c(x)}
\]

(3.23)

The phase which is generated is defined between \( \pm \pi \) and referred to as a wrapped phase map. A continuous phase map is required to convert it into linear distance measurement. Let us consider that the wrapped phase map is depicted by \( \phi_w(x) \) and the continuous phase map by \( \phi(x) \). The relationship between the two phases is shown in the Figure 3.15. It is clear from the figure that the only difference between the wrapped and unwrapped phase is the \( 2\pi \) jumps. A phase unwrapping program is required to unwrap the wrapped phase to a continuous phase map. We have used the MATLAB routine unwrap() for this function.

Figure 3.15: Relationship of wrapped and unwrapped phase
The continuous phase thus obtained is then converted to OPD values using
the following equation

\[ OPD = \frac{\lambda}{2\pi} \times \phi \]

(3.24)

It should be noted that the conversion of OPD to linear distance measurement
will incorporate a scale factor of 2 as the difference between two paths
represents twice the linear movement, \( d \), of the probe such that

\[ d = \frac{OPD}{2} \]

(3.25)

### 3.3.2 OPD calculation from the probe signal

The intensity signal collected by the probe represents the interference of the
light beam reflected from the test and reference surface. A typical signal at the
lens centre is shown in Figure 3.16.

![Figure 3.16: Probe signal across the lens centre](image)

Figure 3.16: Probe signal across the lens centre
The phase information encoded in this signal cannot be extracted using the method described in the previous section as it contains significant amount of noise and the fringe frequency varies considerably over the entire test surface. For this reason, an algorithm based on PLL was written to extract phase from the test signal. In essence, PLLs are circuits in which the phase of a local voltage controlled oscillator (VCO) is maintained close to the phase of an external signal (Brennan 1996). This is achieved by following the phase changes of the input signal and changing the applied voltage in VCO so that there is no phase error between VCO's signal and the input signal. A typical block diagram of a PLL is shown in Figure 3.17.

![PLL block diagram](image)

**Figure 3.17: PLL block diagram**

In our algorithm, finite impulse response filters containing 4 complete cycles were used to measure the local phase of the fringes. The number of cycles can be increased to decrease the influence of noise, but taking a larger number of cycles both increases the computation time and smoothes the recovered phase. In practice, 4 cycles appeared to be the best compromise. The frequency of the filter was then varied according to the rate of change of phase in a manner analogous to the VCO in a PLL. This type of adaptive
filtering was used to extract the phase change over the entire data length and subsequently to calculate the change in OPD.

In this method, three sets of filters are created, as shown in the following equations

\[ F_p^- = \exp \left( \frac{2\pi i (0.9N)p}{2P+1} \right) \]
\[ F_p^0 = \exp \left( \frac{2\pi i Np}{2P+1} \right) \]
\[ F_p^+ = \exp \left( \frac{2\pi i (1.1N)p}{2P+1} \right) \]

(3.26)

where \( p \) is an integer consisting of values ranging from \(-P\) to \(+P\) and \( 2P+1 \) is the number of discrete points that contain \( N \) (or \( 0.9N \) for \( F_p^- \) and \( 1.1N \) for \( F_p^+ \)) cycles in a particular part of the data.

The next step involves creation of Fourier signals for each data point \( q \) for the set of three filters,

\[ z_q^- = \sum_{p=-P}^{P} F_p^- I_{p+q} \]
\[ z_q^0 = \sum_{p=-P}^{P} F_p^0 I_{p+q} \]
\[ z_q^+ = \sum_{p=-P}^{P} F_p^+ I_{p+q} \]

(3.27)

The maximum \( z_q^{\text{max}} \) value returned by the 3 filters provides the local phase of the fringe at the data point \( q \), such that

\[ \phi(q) = \arctan \left( \frac{\text{Im}(z_q^{\text{max}})}{\text{Re}(z_q^{\text{max}})} \right) \]

(3.28)

Here, the \( \arctan \) is a four-quadrant inverse tangent and its value lies between \(-\pi\) and \(+\pi\). The following plot shows a comparison between the Fourier signal obtained using the filter and the normalized actual signal.
Figure 3.18: Comparison between the Fourier and actual signal

The length of the filters are changed accordingly for the next data point. This type of adaptive process is followed for each data point to calculate the phase map over the entire surface. The phase map thus obtained is wrapped and is subsequently unwrapped as described in the previous section. The unwrapped phase difference is then converted to OPD using equation (3.24) to obtain the actual OPD. It is to be noted that the adaptive filtering was started from the edge of the lens and continued to its centre. For creating filters with appropriate length, the dominant frequency present in the signal was calculated by analysing the power spectrum of the corresponding portion of the signal.

3.4 Summary

A modified theory of SAI was developed and presented in this chapter. The major difference between the new theory and the one presented by Tomlinson (Tomlinson, Coupland & Petzing 2003) is that only a single point of source
and receive were considered in the new theory, whereas the source and receive points in the original theory were different. This assumption reduces the computation complexity considerably. The computation of surface form consists of two parts - the forward and inverse problems. In the forward problem, the ideal OPD between the test and reference surface is calculated. The difference between the ideal OPD and the measured OPD is then used in the inverse problem to obtain the surface form of the test surface. The results obtained using normal inversion method show some edge artefacts, the reason for these will be discussed later in chapter 6.

Two different phase evaluation methods were used to extract the phase in the intensity signal obtained from the Michelson interferometer (used for monitoring probe movement) and that from the probe itself. As the probe is assumed to move at approximately constant speed, a method based on the MATLAB unwrap() function is quite sufficient to evaluate the phase from the intensity signal obtained by the Michelson interferometer. But as the fringe frequency varies considerably for the intensity signal obtained by the probe, and the influence of noise is more significant, this method was not suitable to extract phase from the probe signal. A new algorithm, based on PLL method was developed and used to evaluate the phase from this signal. The theory of both the phase extraction methods has been described.
Chapter 4

An Anamorphic Probe for Steep Surfaces

4.1 Introduction

The theory of synthetic aperture interferometry (SAI) and the associated phase evaluation methods were described in the previous chapter. In this chapter, we present a new probe design which uses supplementary optics to increase its numerical aperture (NA) so as to enable it to measure steep surfaces. The light gathering efficiency decreases with increasing NA but this loss is substantially reduced by using an anamorphical design. In the new design it is also noted that the source and receive points of the probe were made to be coincident by using additional optics. As stated previously this reduces the complexity of the inverse problem.

An opto-mechanical design of the probe has been described which takes into account various controls needed for alignment of optics inside the probe. The capability of the new probe was demonstrated by testing a spherical optic on a CNC lathe. The lathe was used to hold the test optic and the probe and a Michelson interferometer was used to monitor the probe movement. In this chapter, an experimental set-up on a CNC lathe is described and the preliminary results obtained using this set-up are discussed.

4.2 Optical design of the anamorphic probe

The main requirement of the anamorphic probe was to increase the NA. Although increased NA is desirable, it is accompanied by a proportional loss
in light gathering efficiency as explained in section 2.5. However, the gradient of a rotationally symmetric (or nominally rotationally symmetric) optics varies principally in the radial direction and a large NA is only required in this direction. Consequently, the light gathering efficiency can be increased significantly through the use of an anamorphic design.

A schematic of our anamorphic fibre probe is shown in Figure 4.1 for planes that are radial and tangential to the test optic respectively. The test optic is shown on the right of these diagrams.

In the radial plane a collimated beam of 6 mm diameter illuminates the aspheric lens to utilize its full NA. This was achieved by collimating the emission from a source fibre plated at the front focal point of a doublet lens having a clear aperture of 9 mm and focal length of 25 mm. In this way, a demagnified (x0.18) image of the fibre source is produced in the rear focal plane of the aspheric objective. In the plane tangential to the test optic, the illuminating beam is collimated by the doublet and brought to focus in the front focal plane of the objective, using a cylindrical lens of 160 mm focal length. The objective collimates the beam in this plane such that a sheet of
light approximately 175 µm thick radiates from the probe. The objective lens is a single aspheric element with a focal length of 4.6 mm and was used to increase the NA of the probe to NA = 0.53. Consequently, the surface gradient restrictions are now relaxed to be within the range ±32° enabling the measurement of most aspheric surfaces of interest whereas it was only ±7° in the earlier work of Tomlinson (Tomlinson, Coupland & Petzing 2003).

The advantage of the anamorphic design over a rotationally symmetric design of the same NA is its light efficiency. For example, a rotationally symmetric probe with an NA of 0.53 will project a circular beam having an area 123 mm² at a distance of 10 mm. An anamorphic probe with similar specifications will project an elliptical beam having an area about 1.7 mm². The photon density in the beam produced by an anamorphic probe is about 70 times more than that produced by a symmetric probe. Considering a 4µ core diameter of a single mode fibre, for a given measurement set-up, the propagation losses of the anamorphic system are reduced by approximately 18dB relative to a symmetric system of the same NA.

The second innovation in our new anamorphic probe design is coincidence of the source and receive points. To achieve this we have introduced a beam-splitter behind the objective lens (in the return path). The doublet and cylindrical elements are duplicated in the optical train to the mono-mode receive fibre. In this way, the beam-splitter and the objective are the only elements common to both paths. To avoid unwanted reflections from the objective, it is tilted by 3° about an axis normal to the tangential plane. The reflected path of this configuration is shown in Figure 4.2. Because the illuminated aperture is only 175 µm in the tangential plane, diffraction limited performance of the objective is maintained and this was verified by raytracing (using the OSLO package) (Lambda Research Corporation 2001).
Figure 4.2: Back reflections from objective lens tilted at an angle of 3°

The use of a conventional plate beam splitter to divert the reflected beam to the receive fibre will result in ghost reflections, thereby adding additional interference terms and distorting the desired signal. This can be avoided by the use of a pellicle or cube beam splitter. We have used a custom made pellicle beam splitter to split the light to avoid ghost reflections. Pellicle is a high tensile strength membrane made of Nitrocellulose. It was stretched and bonded over a custom made mount. The comparison between a pellicle beam splitter and a glass beam splitter is shown in Figure 4.3. The chromatic and spherical aberrations are negligible in a pellicle and there are no ghost reflections.

Figure 4.3: Comparison between pellicle and plate beam splitter
The following table depicts the comparison between our new anamorphic probe and the earlier bare fibre probe configuration of Tomlinson (Tomlinson, Coupland & Petzing 2003).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Anamorphic Probe</th>
<th>Bare fibre probe</th>
</tr>
</thead>
<tbody>
<tr>
<td>NA</td>
<td>0.53</td>
<td>0.12</td>
</tr>
<tr>
<td>Surface Gradient Restriction</td>
<td>±32°</td>
<td>±7°</td>
</tr>
<tr>
<td>Light Gathering Efficiency</td>
<td>+18dB</td>
<td>-</td>
</tr>
<tr>
<td>Source &amp; Receive Points</td>
<td>Coincident (Simplified Analysis)</td>
<td>Separate</td>
</tr>
</tbody>
</table>

It is clear from the above table that the anamorphic probe should be capable of measuring most optics of interest.

### 4.3 Opto-mechanical design of the probe

A probe based on the anamorphic lens design was constructed and used to measure a curved surface to assess the feasibility of this technique. This section describes the opto-mechanical design of the probe. The mechanical design of the probe determines the ease of alignment of its components. Care has been taken to allow sufficient tolerances for each component to achieve this. The following 3D (Solid Edge) (UGS 2005) model depicts a schematic of the probe to hold different optical components.
The source fibre collects the light from a He-Ne laser and delivers it to the collimating lens. The fibre end is inserted inside the fibre holder using a FC to FC mating adaptor. The distance between fibre end and the collimating lens is maintained using the adaptor so that a collimated output is obtained. The collimating and the cylindrical lens is mounted in a tube using retaining rings.

A pellicle beam splitter was attached directly on to one end of a tube, cut at an angle of 45°. The refractive index of the pellicle was 1.5 and its uniformity was within 4 fringes (2 waves) per inch. The coating used on the pellicle was NPX9 50/50 whose characteristics are shown in Figure 4.5.

The aspheric lens was glued to the end of another tube at an angle of 3 degrees with respect to the optical axis. The length of the tube was chosen so that the rear focus of the cylindrical lens matches with the front focus of the
aspheric lens so as to obtain a thin parallel sheet of light in the tangential plane. Minor adjustments were done by moving the tubes into and out of the block.

The reflected beam from the test lens was directed towards the fold mirror by the beam splitter. The fold mirror was a reflecting prism coated with aluminium so that the light signal is reflected towards the receive fibre through the cylindrical and achromatic doublet. The prism was attached to one end of the tube, cut specifically at right angles so that the prism can be glued to it. Rotational adjustments were achieved by rotating the tube holding the prism. The attachment of the receive fibre to the probe is similar to that of the source fibre using an ADAFC1 adaptor.

A CNC lathe is used to mount the test optic and the probe. The probe is mounted using a custom made tool inside the lathe. The probe mount was designed to allow sufficient movement in three degrees of freedom as shown in Figure 4.6 so that the light emanating from the probe can be properly aligned with the lens. The three degrees of freedom were

1. Vertical movement of the probe (to maintain its height with respect to the centre of the test lens).
2. Rotation about the probe axis (to ensure that the NA is greatest in the radial direction).
3. Rotation perpendicular to the axis (to ensure that the probe axis is held normal to the lens).

The control of first and third degree of freedom was done using an elongated hole on the probe holder. The second degree of freedom was controlled by the screw holding the probe through the cutout in the probe holder.
4.4 Experimental set-up

To show the robustness of the technique, a numerically controlled lathe was used to mount the test optic and the probe. The lens was fixed in a cylindrical mount which was held rigidly by the jaws of the lathe chuck. The probe was mounted on the tool stock of the lathe and aligned with the lens in such a way that the light reflected from the test and reference surfaces was collected along its full traverse of the tangential plane. Figure 4.7 shows a photograph of the experimental set-up, the block diagram of which has been depicted in Figure 3.1.
Figure 4.7: Photo of the experimental set-up
In practice, it was found that the precision of the numerically controlled lathe was insufficient to accurately control the position of the probe as shown in Figure 4.8. The probe movement was monitored using a Michelson interferometer for a probe traverse of 15 mm and this figure shows the difference between the data shown by CNC lathe and predicted by the Michelson Interferometer. It can be seen that the difference can be as large as 80µ and consequently a Michelson interferometer is essential to measure the linear movement of the probe as it traverses across the lens surface. The calculation of probe position using the Michelson interferometer was as explained in section 3.3.1.

![Graph](image_url)

**Figure 4.8: Difference between CNC and interferometer data**

In our experiment, the moving mirror of the Michelson interferometer is a corner cube reflector mounted on the probe. Another static corner cube reflector is used as the reference fixed mirror. A He-Ne laser is used to illuminate the reflectors and the interference signal is separated from the
original signal using a glass plate kept at an angle of 45°. A photodiode is used to record the resultant intensity of the interference signal.

The intensity signal collected by the probe was measured using a Hamamatsu H6780 photomultiplier. The probe was translated at a feed rate of 6 mm/min in the radial direction. Signals from the photodiode (the interference signal from Michelson interferometer) and the photomultiplier (the interference signal from test lens) were simultaneously digitised by a National Instruments analogue-to-digital converter. The sampling rate of data collection was 10 KHz so that the distance between successive samples at a feed rate of 6 mm/min is approximately 0.01 µm. A low pass filter was used to limit the signal bandwidth to 5 KHz. The interference signal for a probe traverse of 1 mm starting from the lens centre is shown in Figure 4.9. The evaluation of phase and subsequently the calculation of OPD from this type of interference signal was described in section 3.3.2.

![Figure 4.9: Interference pattern for a probe traverse of 1 mm from lens centre](image)
4.5 Results

The measurement of the curved surface of a plano-convex lens was carried out to assess the theory of the anamorphic probe. The test lens used was a plano-convex lens of 160 mm focal length. The optical specifications of the test lens is given in Table 3.1 and an optical schematic is shown in Figure 4.10.

![Optical schematic of the test lens](image)

<table>
<thead>
<tr>
<th>Left Surface</th>
<th>Material Specification</th>
<th>Right Surface</th>
</tr>
</thead>
<tbody>
<tr>
<td>R 82.688 CX</td>
<td>Schott BK7</td>
<td>R ∞</td>
</tr>
<tr>
<td>ø e 40.0</td>
<td>n d 1.5168±0.001</td>
<td>ø e 39.3407</td>
</tr>
<tr>
<td>Comar Planovex Lens</td>
<td></td>
<td></td>
</tr>
<tr>
<td>160 PQ 40</td>
<td>Ind. acc. ISO 10110</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4.10: Optical schematic of the test lens

The interference pattern formed by the surface reflections from the test lens will be symmetric as the test lens is axially symmetric. Practically, this means
that the sampling of interference pattern can be done sparsely around the circumference to obtain the interference signal. In our experiment, the probe measures the contour of the test optic along one radius from the centre of the lens to its edge at any particular instant. The probe was made to traverse along 12 different radial paths, each separated by an angle of 30° to cover the entire surface of the lens assuming that the surface form difference between successive radial paths is negligible. Figure 4.11 shows the pattern of the scanning process.

Figure 4.11: Scanning pattern

The difference between the ideal and measured OPD for a particular scan is shown in Figure 4.12. The two plots shown in the figure correspond to two different measurements taken for the same portion of lens. It can be seen that the repeatability of the experimental set-up is quite good with a maximum error of approximately 1µ in the measurement of OPD error considering that the tool stock holding the probe was oscillating by an amount of approximately 80µ over its entire traverse length.
Figure 4.12: Measured OPD error for same track

The OPD error was computed for the 12 tracks represented in Figure 4.11. It was seen that all the results show similar trend and is illustrated in Figure 4.13.

Figure 4.13: Measured OPD error for the 12 radial tracks
The surface form error corresponding to the OPD errors in the above figure is shown in Figure 4.14.

![Figure 4.14: Surface form for the 12 radial tracks](image)

It can be seen that all the results show high values towards the edge and the reason for this is discussed in section 6.2. If we consider the surface data for a major portion of the test lens (0-14 mm), the surface form shows a small (±3µ) deviation from surface profile. Although, this value is close to the surface form tolerance stated by the manufacturer, there is a characteristic periodicity which is worth further consideration. It is unlikely that a symmetric periodic deviation is a true artefact of the lens surface because of the nature of the spherical polishing process. However, because the inverse problem is non-linear in nature it is not clear how changes in focal length etc. will affect the measured results. For this reason the optimisation was reproduced with different combinations of lens thickness, surface radii, refractive index and probe stand-off distance. It was found that changes in these parameters resulted in small monotonic increase or decrease in the measured surface
form, but did not explain the observed periodicity. Other than a real variation in surface form, it seems that a small periodic variation of this type can only be introduced by a periodic variation in either refractive index or the probe stand-off distance. A sensitivity analysis reveals that refractive index variations would result in greater error in the centre of the lens (4 mm thickness) than at the edge (1.5 mm). It is straightforward to show that the measurement is independent of the probe stand-off at the centre of the lens and increases toward the edge. The most likely reason for the periodic deviation in the experimental results therefore is a change in probe stand off distance due to the traverse mechanism. The periodicity of the error is similar to that shown in Figure 4.8 and this suggests that the tool stock may be rotating as it translates. For this reason a more controlled set of experiments was initiated (see section 5.2).

4.6 Summary

The original SAI configuration proposed by Tomlinson (Tomlinson, Coupland & Petzing 2003) was straightforward to implement but was only capable of measuring very shallow surfaces (surface gradient restrictions were within ±7°) and had a poor light efficiency. A new probe was designed with supplementary optics to increase the NA of the probe so as to enable it to measure steep surfaces. The surface gradient restriction was increased to be within ±32° which is suitable for measuring most aspherics of interest. An anamorphic lens was introduced in the probe to increase the light efficiency of the probe significantly and the source and receive points of the probe were made coincident to reduce the computation complexity. To show the robustness of the probe, experiments were carried out on a CNC lathe to measure a curved surface of a plano-convex lens. The details of experimental set-up was explained.

The difference between the ideal and measured OPD was computed from which the surface form error was calculated for a test surface. It was
seen that the surface form error has some noise towards the edge. A detailed
analysis of this problem will be given in chapter 6. The value of the surface
form error for the most part of the test lens shows that the surface form was
within a value comparable to the tolerance provided by the manufacturer,
however, we believe that this is not due to the actual surface error, but due to
error in the experimental configuration in traversing the probe. The
experimental configuration on CNC lathe suffers from two major problems -
error in the alignment of the probe with respect to the test optic and error in
the probe movement, both of which could not be controlled. To remove the
problems associated with the CNC lathe, an off-line experimental set-up was
developed and is explained in the next chapter. Though the phase evaluation
process based on phase-locked-loop is suitable for extracting the phase from
the probe signal, it is quite time consuming as essentially, adaptive filtering is
required at each measurement point. For this reason, a new phase evaluation
process, using a-priori knowledge of the surface form, has been developed
and is also described in the next chapter.
Chapter 5

Development of an off-line Synthetic Aperture Interferometer

5.1 Introduction

In the previous chapter, the technique of synthetic aperture interferometry (SAI) was demonstrated using an anamorphic probe. The experiment was carried out on a CNC lathe where the test lens was held in the lathe chuck and the probe was fitted to the tool stock. Although this set-up demonstrated the feasibility of SAI as an in-process measurement technique, proper alignment between the probe path and the test lens was difficult to assess. The accuracy of the traversing tool was also unknown along the two perpendicular directions of the traverse path. For these reasons, an off-line SAI was developed consisting of a rotary stage and a manual translation stage. The rotary stage consists of a mount to hold test optics and has mechanical tilt & rotational controls. The manual translation stage is used to traverse the probe across the test lens. A pair of micrometer translation stages were attached to it for controlling the height & distance of the probe with respect to the test optic. A detailed description of the experimental set-up is given in the following section.

One of the most critical steps in SAI is the phase extraction process from the intensity pattern obtained by the probe. A phase extraction process based on phase-locked-loop (PLL) was used in our first experiment conducted on lathe and is explained in chapter 3. It was found, however, that though the process based on PLL works reasonably well with the obtained intensity pattern, the process was quite time consuming and was prone to some errors
in case of noise. These problems prompted the development of a new phase extraction process. Since in our case the ideal surface form of the test surface is known to us in advance, this a-priori information allows a more robust phase extraction process to be performed and is explained in this chapter.

5.2 Experimental set-up

The experimental set-up consists of two sub-assemblies, one holding the test optic and the other holding the probe. The base of the first sub-assembly was a rotary stage to control and mount the test optic. The test lens was fixed on a custom made mount which can hold lenses up to 100 mm in diameter. A sketch of the lens mount is shown in Figure 5.1. The lens is held rigidly using four slides that accommodate lenses of various sizes. This lens mount was attached to a rotary stage to complete the first sub-assembly.

![Figure 5.1: Lens mount](Image)

The first sub-assembly was designed to align the test optic with respect to the probe. To achieve this, the rotary stage has following degrees of freedom.

- A rotation axis to allow scanning of different portions of test lens.
• Slide adjustment on the lens mount to centre the lens with respect to the axis of rotation.

• Tilt adjustment on the lens mount to ensure the lens axis is parallel to the axis of rotation.

• Tilt of the axis of rotation through a horizontal axis to achieve vertical alignment between the probe and the test lens so that the reflected light from the test lens is collected by the probe throughout its traverse across the test lens.

The rotary stage has the capability of rotating the lens about its axis through 360 degrees. This was used to measure the test lens along 8 successive tracks, each separated by its previous track by 22.5 degrees to cover the full surface of the test lens as shown in Figure 5.2.

![Figure 5.2: Scanning tracks](image)

The second sub-assembly consists of a manual translation stage that holds the probe, and was used to align the probe with respect to the test optic. This sub-assembly was designed to have the following controls.

• Rotational adjustment to ensure that the major axis of the anamorphic beam passes through the axis of rotation.

• Vertical adjustment of the probe to ensure that the probe translates through the axis of rotation.

• A graduated focus adjustment to measure probe distance with respect to the test optic.
The first control was achieved with a probe mount which was designed to hold the probe, a schematic of which is shown in Figure 5.3. The probe mount has a cutout to hold the probe tube and is held rigidly using a screw and can be rotated along its optical axis to adjust the beam coming out of the probe to make the beam horizontal with respect to the probe path.

![Figure 5.3: Probe mount](image)

This mount was then attached to an assembly consisting of two micrometer translation stages which were used to achieve the second and third controls to align the probe with respect to the test optic. The vertical micrometer translation stage was used to move the probe up and down so that the probe passes through the lens centre. The distance of the probe focus from the test surface was measured using the second micrometer translation stage. This stage was translated towards the lens until a point where the focal point of the probe lies on the test surface. At this point the source and receive fibre are confocal such that there is a substantial increase in the intensity of reflected light received by the probe as almost all of the light energy is retro-reflected and collected by the receive fibre. Thereafter, the probe was moved back by a certain amount, \(d\), which is the probe distance as shown in Figure 5.4.
The summary of all the controls in the off-line SAI is depicted in Figure 5.5.

In this figure, the numbers correspond to the following adjustments:

1. Slide adjustment to centre the lens
2. Tilt adjustment to match the lens axis with rotary stage axis
3. Rotation of the lens about its optical axis
4. Tilt adjustment of the rotational axis through a horizontal axis
5. Rotational adjustment of the probe
6. Vertical adjustment of the probe
7. Focus adjustment of the probe

The block diagram of the experimental set-up is shown in Figure 5.6 which is similar to the set-up in Figure 3.1 except for the following changes. The fixed and movable corner cube reflectors in the Michelson interferometer have been placed on the same path to reduce the error due to environmental instability. A mirror was used to separate the return signal instead of a glass plate to direct the signal towards the photodiode. Another difference between the two set-ups is that we have used a band pass filter instead of a low pass to filter out the unwanted lower frequency drift that was apparent in the signals.

Figure 5.6: Block diagram of the experimental set-up
When aligned, the probe is translated across the test lens along a line that passes through the axis of the lens. The Michelson interferometer was used to measure the linear movement of the probe. To achieve this, a corner cube reflector was attached to the top of the probe mount. Light from a He-Ne laser is made to fall upon a beam splitter which splits the beam into two, one of which (the object path) is delivered to the corner cube reflector on the probe. The other beam (the reference path) is reflected back from a fixed corner cube reflector and the interference between the object and reference beam was recorded using a PIN photodiode.

The interference signal reflected from the test optic was collected by the receive fibre of the probe and transferred to a photomultiplier for intensity measurement. A band pass filter was used to limit the signal frequency in the band of interest between 1 Hz and 5 KHz. The two interference signals from the photodiode (signal from Michelson interferometer) and the photomultiplier (signal from the test lens) were simultaneously digitized at 10 KHz by a dual channel National Instruments analogue to digital converter similar to the previous experiment described in section 4.4. The software used to record this data was the signal processing toolbox of MATLAB (The Mathworks Inc. 2006).

A photo of the actual set-up consisting of the two sub-assemblies and the Michelson interferometer is shown in Figure 5.7.
5.3 Calibration

The linear translation stage was calibrated using a standard Hewlett Packard (HP) 5519A interferometer to assess its linear movement along the three perpendicular directions as shown in Figure 5.8. The probe scans the test optic along the translation axis. Ideally, there should not be any movement along the two perpendicular directions when the probe is being traversed along this axis. The HP 5519A is a heterodyne interferometer, which consists of a He-Ne laser with automatically tuned, Zeeman-split, two frequency output. The standard resolution of the interferometer is 10 nm in a controlled environment. The calibration of the probe traverse along translation axis is explained in section 5.3.1 and the measurement of pitch and yaw is explained in section 5.3.2.
5.3.1 Calibration along translation axis

The block diagram of the experimental setup to calibrate the linear movement of the manual translation stage along the translation axis is shown in Figure 5.9.

The linear translation stage was moved over a distance of 100 mm and data was recorded simultaneously by the HP interferometer and the Michelson...
interferometer. Two corner cube reflectors were used to reflect the light back to the respective interferometers where different portions of the reflectors were used by the two interferometers to separate the two measurements as shown in Figure 5.9. One of the reflectors was kept stationary (upper) to reflect the reference beam and the other (lower) was placed on the probe mount to represent the linear stage movement. The distance measurement method and phase unwrapping using the Michelson interferometer was explained in section 3.3.1. The difference between the two measurements for a stage movement of 100 mm is shown in Figure 5.10. It can be seen that the random error between the two successive measurements is within 0.1 microns, but a steady upward trend is observed with the maximum difference between two measurements being about 1.2µ for a 100 mm traverse. A wavelength value of 632.8 nm was taken to calculate the distance from the Michelson interferometer. An error in the second decimal place of the wavelength (which has not been considered for calculation) will result in such an error. It can also be observed that for the first 10 mm traverse, the random error between the two measurements is almost zero. The experiment was repeated many times and all the results show similar trend. This may be due to a poor quality region of the thread after approximately 10 mm. The calibration of the linear translation stage shows that the stage is quite robust and the Michelson interferometer can be used successfully to measure its linear movement to within the tolerances required.
5.3.2 Measurement of pitch and yaw

In addition to the linear movement calibration, the linear stage movement was characterized in the other two perpendicular directions to ensure that the stage is moving along a line. The HP heterodyne interferometer was used again to measure pitch and yaw of the translation stage. The schematic of the calibration for yaw measurement is shown in Figure 5.11. This schematic is similar for calibration for pitch measurement where the mirrors and the beam splitter/fold mirror combination is kept side by side (instead of on top of each other). In this configuration, HP interferometer shows a value representing the tilt measurement of the corner cube reflectors. To convert the tilt measurement to a distance measurement, the measurements were taken at 10 mm steps of interval to cover a translation of 100 mm. If the HP interferometer gives a value of $\Delta x$, then using the Moody method (Reinshaw
1993), the linear movement in a direction perpendicular to the translation can be calculated,

\[ \Delta h = \frac{\Delta x}{d} \Delta l \]  

(4.1)

where \( d \) is the distance between the two corner cube reflectors and \( \Delta l \) is the distance between two measurements, which is 10 mm in this case.

**Figure 5.11: Calibration set-up for yaw measurement**

In this way, the linear movement along the two perpendicular directions was calculated for a total translation of 100 mm along the translation axis. The deviation of the probe from a straight line in the vertical (y) direction is shown in Figure 5.12 and the corresponding deviation in the horizontal (z) direction is shown in Figure 5.13.
It is clear from the two plots that the maximum deviation of the translation stage along the perpendicular directions is about 0.025µ for a stage translation.
of 100 mm. These deviations may arise due to vibration and environmental instability during the measurements, but we have shown that the translation stage is of sufficient accuracy for our measurements. In the following section, we discuss a new phase evaluation method to extract phase from the intensity signal obtained by probe.

5.4 Phase evaluation based on a-priori model

A method based on a phase-locked-loop (PLL) was used previously to extract phase from the probe signal. It should be noted that the phase extraction process based on a PLL is suitable for an arbitrary intensity pattern and is quite slow as adaptive filtering is required at each data point. This method is prone to failure if it is unable to maintain the phase lock at any instant. We have developed a new, more robust phase extraction method based on the assumption that the ideal surface form is known in advance. This priori information is used to create an a-priori signal which represents the signal that is calculated using the forward model from the ideal surface form data.

As an example, the application to an f/5 plano-convex lens (that is characterized in chapter 6) is considered. As described previously, the intensity of the interference pattern as a function of probe position can be written as

\[ I(x_p) = 2I_r \left[ 1 + \cos[\phi(x_p)] \right] - N = a(x_p) + b(x_p) \cdot \cos[\phi(x_p)] \]

(5.1)

where \(I_r\) is the reflected intensity, \(\phi(x_p)\) is the phase at \(x_p\), \(N\) is the noise in the system, \(a(x_p)\) represents additive disturbances such as background intensity and electronic noise and \(b(x_p)\) represents multiplicative disturbance such as visibility.

The intensity of the interference pattern reflected by the test optic was recorded by the probe at a sampling frequency of 10 KHz while the probe was manually scanned across the test lens. A small portion of the intensity pattern near the lens centre is shown in Figure 5.14 illustrating that the frequency of
the fringes increases as we move away from the centre of the lens (assuming that the probe is traversed at a constant speed).

![Intensity of the interference pattern](image)

**Figure 5.14: Intensity of the interference pattern**

The conversion of the intensity signal in equation (5.1) to an analytic signal has been explained in section 3.3.1 and is given by

\[
c(x_p) = b(x_p) e^{i(\theta(x_p)) / 2}
\]

(5.2)

If the signal is relatively free of noise, the phase modulo \(2\pi\) can be obtained as the arctangent of the ratio of imaginary and real term. But in SAI, the signal is not always noise free, and more often than not, it is not possible to obtain the phase information in this manner. The assumption that the ideal surface form of the test surface is known in advance is used to extract the phase information from this complex valued spectrum. It is to be noted that in the new process, a change in phase from the ideal value is computed rather than the absolute phase change obtained in other methods. The ideal OPD and subsequently ideal phase information as a function of probe position can
easily be computed from equations (3.1) to (3.4). Let us refer to this ideal phase as $\phi_i$ and a plot of this phase as a function of probe position is shown in Figure 5.15.

\[ \phi_i(x_p) = \phi_-(x_p) + \phi_+(x_p) \]  
(5.3)

where $\phi_-(x_p)$ refers to the phase for the negative values of the probe position (-45 to 0 mm) and $\phi_+(x_p)$ for the positive values (0 to 45 mm). It is noted that in practice, the phase of the analytic signal that is obtained from the intensity data (as discussed in section 3.3.2) is monotonically increasing. A comparable signal is computed by reversing the ideal phase for positive values of probe position such that

\[ \phi_i'(x_p) = \phi_-(x_p) - \phi_+(x_p) \]  
(5.4)
This is shown in Figure 5.16.

An analytic signal with this phase is therefore,

$$c_i(x_p) = e^{-i\phi_i(x_p)}$$

(5.5)

The real part of the above function for the central portion of the lens is shown in Figure 5.17.
Multiplying the two complex-valued functions representing the measured and ideal phase respectively gives us

\[ m(x_p) = c(x_p)c_1(x_p) = b(x_p)e^{i(\theta(x_p)-\theta_1(x_p))}/2 \]

where \( \Delta \phi \) is the difference between measured and ideal phase. The above function was Fourier transformed and frequencies above 20 cycles/mm were filtered out to eliminate noise. Inverse Fourier Transform of the resultant signal yields a complex valued amplitude \( m(x) \). The difference in phase (modulo \( 2\pi \)) between the measured and ideal signal can then be computed using the following equation.
This signal is relatively free from noise as shown in Figure 5.18 and can be unwrapped using MATLAB to generate a continuous phase change. The change in OPD between the light reflected from the front and rear surface of the test optic can then be computed using equation (3.24).

\[
\Delta \phi(x_p) = \arctan \left( \frac{\text{Im} m(x)}{\text{Re} m(x)} \right)
\]

(5.7)

5.5 Summary

In this chapter, development of an off-line experimental set-up was explained. This new set-up was required to alleviate the difficulties arising from the previous experiment performed on a CNC lathe. Mechanical tilt controls present in the rotary stage and the lens mount were used to achieve a proper alignment between the test optic and probe. Micrometer stages were used to
control the position of the probe with respect to the test optic. The alignment procedure was quite fast and reliable in the new setup.

The linear movement of the manual translation stage was monitored using a Michelson interferometer which was calibrated by a HP heterodyne interferometer. The movement of the linear translation stage along the two perpendicular directions that are normal to the translation was also calibrated using the HP interferometer. It was found out that the maximum error was $1.2\mu$ for a linear movement of 100 mm along translation axis. The maximum deviation from a straight line was less than $0.025\mu$ for a probe traverse of 100 mm. This shows that the linear translation stage was of good quality and is quite robust. It is shown in chapter 7 that this characteristic is sufficient for sub-micron form measurement.

In this chapter, we have also introduced a new phase evaluation method suitable for lens measurement. This process is based on a-priori model based on the assumption that the ideal surface is known in advance. This model can be successively used to extract the phase information in the intensity pattern observed by the probe while scanning the test optic. This method is better than the method based on PLL due to speed and robustness in handling noisy signals.
Chapter 6

Alternative Measurement Configurations

6.1 Introduction

The theory of synthetic aperture interferometry (SAI) was explained in chapter 3 in terms of the forward and inverse problems. In the forward problem, the optical path difference (OPD) between the light reflected from the test and reference surface is computed for an ideal test surface. The inverse problem deals with extracting the surface form from the difference in OPD between the ideal and test surface. During the inverse process, it was seen that the calculated surface form showed some unexpectedly high values near the edge of the convex test surface. This value is more than the OPD error and is somewhat unexpected since the form error is approximately one third of the OPD error at the centre of the lens (see section 3.2.2). In this chapter, we explore the inverse problem further and the reason for edge artefacts is discussed in section 6.2. Subsequently, we develop a new method to find the surface form based on a double pass method in which the probe traverses along two different paths. This is described in section 6.3.

The configuration of the experimental set-up mentioned previously consists of the probe facing the test (curved) surface and the reference (plane) surface positioned at the rear. We term this configuration, the forward configuration. An alternate set-up where the position of the test optic is reversed such that the reference surface faces the probe is also possible. We will show that this configuration totally alleviates the problem faced in forward configuration and an unambiguous result can be obtained. We refer
to this configuration as backward configuration which is described in section 6.4.

The modified theory of the forward configuration and the theory of the backward configuration have been demonstrated by measuring the surface form of two different surfaces. First, we have measured the curved surface of a 100 mm diameter F/5 plano-convex lens in both the forward and backward configurations, results of which are given in section 6.5.1. Finally, a steep (F/1.5) aspherical surface with 4 higher order coefficients was measured to show the capability of the probe in measuring a generalized aspherical form and the corresponding results are given in section 6.5.2.

6.2 Error investigation in the forward configuration

The inverse problem of computing surface form using an over-determined set of equations was described in section 3.2.2. Although the results using this method showed expected values for a major portion of the test surface, edge artefacts were observed near the edge. This phenomenon is investigated further in this section. Although in practice we collect data at discrete points, let us consider a sequence of probe measurements that hypothetically, are taken in such a way that the ray intersection at the front surface by the ray reflected at the rear surface coincides with the ray intersection from the ray reflected at the front surface of the next probe position. This type of selective scheme is shown in Figure 6.1 where the ray diagram shows the last 4 probe positions represented by $O_i$. 
In this figure, $S_i$ represents the ray intersection points at the test surface in which $O_m$ represents the last measurement point. The value of OPD at $O_m$ depends on the height of test surface at $S_m$ and $S_{m+1}$. This means that any change in measured OPD from the ideal OPD at $O_m$ depends on the change in surface form at $S_m$ and $S_{m+1}$. It is clear from equations (3.11) and (3.12) that the change in OPD at $O_m$ is approximately twice as sensitive to change in surface form at $S_m$ than at $S_{m+1}$. Similarly, change in OPD at $O_{m-1}$ is twice as sensitive to change in surface form at $S_{m-1}$ than at $S_m$ and so on. Thus we can formulate a matrix equation which shows this relation and is given below

$$
\begin{pmatrix}
\Delta OPD_{m-3} \\
\Delta OPD_{m-2} \\
\Delta OPD_{m-1} \\
\Delta OPD_m
\end{pmatrix}
= 
\begin{pmatrix}
2 & 1 & 0 & 0 \\
0 & 2 & 1 & 0 \\
0 & 0 & 2 & 1 \\
0 & 0 & 0 & 2
\end{pmatrix}
\begin{pmatrix}
\Delta y_{m-3} \\
\Delta y_{m-2} \\
\Delta y_{m-1} \\
\Delta y_m
\end{pmatrix}
$$

(6.1)

In this equation, $\Delta OPD_i$ is the difference between the measured and ideal OPD at $O_i$ and $\Delta y_i$ is the surface deviation of the measured surface from the ideal one at $S_i$. We note that in this case 4 values of OPD are linked with 5 surface height variables. It is clear that the system is undetermined and there
is no unique solution. The system has an additional degree of freedom and we note that the change in OPD is not affected by addition of a vector, $b$, to the surface height, where $b$ has the form

$$b = \begin{pmatrix}
1 \\
-2 \\
4 \\
-8 \\
16 \\
\end{pmatrix}$$

(6.2)

Because the system cannot measure this form, we term the vector (or any multiple of it), the blind vector. The effect of the blind vector is clear from equation (6.3) which shows that the multiplication of the system matrix with the blind vector results in a null output.

$$\begin{pmatrix}
2 & 1 & 0 & 0 & 0 \\
0 & 2 & 1 & 0 & 0 \\
0 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 2 & 1 \\
\end{pmatrix} \begin{pmatrix}
1 \\
-2 \\
4 \\
-8 \\
16 \\
\end{pmatrix} = \begin{pmatrix}
0 \\
0 \\
0 \\
0 \\
\end{pmatrix}$$

(6.3)

In practice this means that our measurement system is insensitive to surface form errors of the type illustrated in Figure 6.2 which results in instability in the computation of surface form.

![Figure 6.2: Effect of blind vector on surface form](image)

Figure 6.2: Effect of blind vector on surface form
We note, however, that the blind vector increases in spatial frequency, that is, the distance of the surface intersections between the surface intersection points decreases towards the centre of the lens. The blind vector therefore breaks the assumption of band-limited surface profile. It can be deduced that reducing the grid resolution of surface heights in equation (3.17) suppresses the effect of blind vector for a major portion of the test surface but noise in OPD can lead to edge artefacts that increase exponentially in the same manner as the blind vector.

Nevertheless when we measure convex front surfaces we must be aware of the blind vector effect. In the case of concave surfaces, although blind vector is present, the exponential increase is towards the centre of test surface unlike the case of convex surface where it increases towards the edge. In practice the blind vector effect does not affect the surface form calculation for concave surfaces as the boundary condition defines the surface form value at centre to be equal to zero and this eliminates solutions containing the blind vector.

It is noted that for convex surfaces the blind vector affects the surface form values near the edge. Considering equation (6.1), it can be shown that the blind vector effect can be nullified if we take an additional measurement at a different probe distance. To make the system determinate, we take another OPD measurement without changing the number of surface height variables as shown in Figure 6.3.
Figure 6.3: Ray diagram for a determinate solution

In this case, $O_{m+1}$ is a OPD measurement point where the probe is moved away from the test surface such that the OPD change at this point depends on change in surface form at $S_{m-1}$ and $S_{m+1}$. Accordingly, equation (6.1) can be rewritten as

\[
\begin{align*}
\Delta OPD_{m-3} &= 2 \Delta y_{m-3} \\
\Delta OPD_{m-2} &= 0 \Delta y_{m-2} \\
\Delta OPD_{m-1} &= 0 \Delta y_{m-1} \\
\Delta OPD_{m} &= 0 \Delta y_{m} \\
\Delta OPD_{m+1} &= 0 \Delta y_{m+1}
\end{align*}
\]

(6.4)

It can be seen that in this case, the equation is in the form of a square matrix with equal number of unknown (surface heights) and known (OPD measurements) variables. The solution to this equation is unique and in principle does not have a blind vector. In this case the distance between the two probe stand-off positions is crucial to remove the blind vector effect. However, calculation of distance required by the probe to be moved to achieve this configuration is quite complex and in practice it was not possible
for us to make the probe follow a path other than a straight line. The following section discusses the modified theory of forward configuration based on two sets of measurements for same segment of the test surface. Although the modified theory does not follow the exact principle of the method illustrated in Figure 6.3, it suppresses the blind vector effect for a major portion of the test surface and is discussed in the next section.

6.3 Two-pass forward configuration

The modified theory of the forward configuration is illustrated in Figure 6.4. In this configuration measurements were taken for two probe stand-off distances, both of which followed a straight path.

![Figure 6.4: Two-pass method](image)

First the probe traverses the path defined by \( x_1 \) to measure the test optic. Then it is moved backwards by a distance \( d \) and traversed across the lens defined
by path $x_2$. The point of intersections at front surface are $S_1$ and $S_2$ for the probe path defined by $x_1$ whereas the corresponding intersection points are $S_3$ and $S_4$ for the second probe path. The two matrix equations for the two sets of measurement are:

$$
\begin{pmatrix}
\Delta OPD_1 \\
\Delta OPD_2 \\
\Delta OPD_3 \\
\vdots \\
\Delta OPD_m
\end{pmatrix}
= \begin{pmatrix}
M_{11} & M_{12} & M_{13} & \ldots & \ldots & M_{1n} \\
M_{21} & M_{22} & \ldots & \ldots & \ldots & \ldots \\
M_{31} & \ldots & \ldots & \ldots & \ldots & \ldots \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots \\
M_{m1} & \ldots & \ldots & \ldots & \ldots & M_{mn}
\end{pmatrix}
\begin{pmatrix}
\Delta y_1 \\
\Delta y_2 \\
\Delta y_3 \\
\vdots \\
\Delta y_n
\end{pmatrix}
$$

(6.5)

and

$$
\begin{pmatrix}
\Delta OPD_1 \\
\Delta OPD_2 \\
\Delta OPD_3 \\
\vdots \\
\Delta OPD_m
\end{pmatrix}
= \begin{pmatrix}
M_{11} & M_{12} & M_{13} & \ldots & \ldots & M_{1n} \\
M_{21} & M_{22} & \ldots & \ldots & \ldots & \ldots \\
M_{31} & \ldots & \ldots & \ldots & \ldots & \ldots \\
\vdots & \ldots & \ldots & \ldots & \ldots & \ldots \\
M_{m1} & \ldots & \ldots & \ldots & \ldots & M_{mn}
\end{pmatrix}
\begin{pmatrix}
\Delta y_1 \\
\Delta y_2 \\
\Delta y_3 \\
\vdots \\
\Delta y_n
\end{pmatrix}
$$

(6.6)

In these equations, $x_1$ represents the first set of measurement and $x_2$ represents the second set of measurement with the probe moved backwards by a certain distance. The two equations can be combined to generate a single equation by considering that for same set of reflection points at the front surface for the two sets, the intersection point at the front surface by the ray reflected from the rear surface will be different as shown in Figure 6.5. This property will change the elements of matrix in equation (6.5) and (6.6), thereby reducing the blind vector effect.
Using this property, we can modify equations (6.5) and (6.6) by taking the common $\Delta y$ coordinates in the two sets of measurement such that

$\begin{pmatrix}
\Delta y_1 \\
\Delta y_2 \\
\Delta y_3 \\
\vdots \\
\Delta y_n
\end{pmatrix} = \text{inv} \begin{pmatrix}
M_{11} & M_{12} & M_{13} & \cdots & \cdots & \cdots & M_{1n} \\
M_{21} & M_{22} & \cdots & \cdots & \cdots & \cdots \\
M_{31} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
M_{n1} & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{pmatrix}_{x_1} \begin{pmatrix}
\Delta \text{OPD}_1 \\
\Delta \text{OPD}_2 \\
\Delta \text{OPD}_3 \\
\vdots \\
\vdots \\
\Delta \text{OPD}_n
\end{pmatrix}_{x_1}$

(6.7)

It should be noted that this equation once again gives us an over-determined set of equations as number of unknown variables (surface heights) is less than
the known variables (OPD measurements). This is similar to the over-
determined solution provided by the method explained in section 3.2.2 where
coarser resolution for surface heights was considered. However, it was found
that this method is more robust and gives more consistent results. It is noted
however that the suppression of edge artefacts depends on the separation $d,$
between the two passes. This is discussed with reference to the results
presented in section 6.5.

6.4 Backward configuration

As mentioned in the introduction, a different experimental set-up is also
possible, where the reference surface (plane) faces the scanning probe instead
of the test surface (curved). We refer to this configuration as the backward
configuration. This configuration does not suffer with the problem of blind
vector associated with the forward configuration. In this case a simple
arithmetic ratio is used to obtain the surface form. In essence this is because
only rays reflected from the rear surface depend on the unknown surface
parameters. The theory of the backward configuration is illustrated in Figure
6.6.
Figure 6.6: Backward configuration

Similar to the forward configuration, a pair of rays from the probe will retrace their path after getting reflected from the front (reference) surface and rear (test) surface. Let the probe position be defined by the coordinates \((x_p, y_p)\). The coordinates of the ray intersection points at the front surface are \((x_f, y_f)\) and the rear surface are \((x_r, y_r)\) for the ray retracing its path after reflection from the rear surface. As the front surface is planar, the ray reflected from the front surface will be perpendicular to the reference surface and the probe path. If the distance between the probe and the front surface is given by \(d\), the optical path length of the ray reflected from the front surface will be given by,

\[
OPL_f = 2d
\]

(6.8)

The optical path length of the ray reflected from the rear surface can be written as,

\[
OPL_r = 2\left(\left[(x_f - x_p)^2 + (y_f - y_p)^2\right]^{1/2} + n\left[(x_r - x_f)^2 + (y_r - y_f)^2\right]^{1/2}\right)
\]

(6.9)

Here,
\[ y_p = 0, \ y_f = d, \text{ and } y_r = d + c - sag(x_r) \]

(6.10)

where \( c \) is the centre thickness of the test lens, \( n \) is the refractive index of the material of the test optic and \( sag \) is the sagitta of the test surface. For illustration purposes, we give an example of plano-convex lens (which is measured later in section 6.5.1). The optical specification of the lens is shown in Table 6.1.

Table 6.1: Optical specification of the large diameter spherical lens

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>500 mm</td>
</tr>
<tr>
<td>Lens Diameter</td>
<td>100 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>258.4 mm</td>
</tr>
<tr>
<td>Centre Thickness</td>
<td>6.4 mm</td>
</tr>
<tr>
<td>Edge Thickness</td>
<td>1.5 mm</td>
</tr>
<tr>
<td>Glass</td>
<td>BK7</td>
</tr>
</tbody>
</table>

The optical path lengths corresponding to the ray reflected at the front and rear surfaces are shown in Figure 6.7.
Figure 6.7: Optical path lengths in backward configuration

It can be seen that in the backward configuration, the optical path length for the front surface is a constant due to the planar surface and the optical path length for the rear surface decreases with probe position. In a similar manner to equation (3.4) the OPD can be written as

\[
OPD = 2\left(d - \left([x_f - x_p]^2 + d^2\right)^{1/2} + (n\left(x_r - x_f\right)^2 + (y_r - d^2)^{1/2})\right)
\]

(6.11)

The change in OPD with respect to the surface vertex is shown in Figure 6.8.
Figure 6.8: OPD change in backward configuration

The figure shows that the total change in OPD in this configuration is about 12 mm for a probe traverse of 45 mm. Once again equation (6.11) can be linearized as we have assumed that the ideal form of the test optic is known and small changes in the surface form results in small changes in the optical path difference, ΔOPD. Differentiating equation (6.11), we can write

$$\Delta OPD = \frac{\partial OPD}{\partial y_r} \Delta y_r$$

(6.12)

or,

$$\Delta OPD = 2n[(x_r - x_f)^2 + (y_r - d)^2]^{1/2} (y_r - d) \Delta y_r$$

(6.13)

There is a major difference between the ΔOPD equations of the forward and backward configuration as evident from equations (3.9) and (6.13). In the backward configuration, the change in OPD at a given point is related to the surface height at a single point (unlike the forward configuration where the change in OPD was due to a change in surface heights at two different
This fact significantly simplifies the inverse problem of calculating the surface form error from the change in OPD and, from equation (6.13) we find that the surface form error can be written directly

$$
\Delta y' = \frac{\Delta \text{OPD}}{2n[(x_1 - x_f)^2 + (y_1 - d)^2]^{-1/2}(y_1 - d)}
$$

(6.14)

It is clear from equation (6.14) that the surface form error in this configuration is simply a ratio of change in OPD and a constant value representing a particular probe position, which we term the backward quotient. It is noted that the surface form calculation for concave surfaces in the backward configuration will be similar to the calculation of convex surfaces.

The next section gives the results of surface form for two different types of test surfaces in both forward and backward configurations. One is a large diameter spherical surface and the other one is a steep aspherical surface.

### 6.5 Results

Measurements of two different types of lenses were carried out to assess the modified theory of the forward configuration and the backward configuration. First, a plano-convex lens having a diameter representing the maximum size capability of the mount (100 mm) was measured in both the configurations. To show the capability of the probe in measuring a generalized aspheric profile, a steep aspherical surface was then measured in the forward and backward configurations.

#### 6.5.1 Large diameter spherical lens

The first test surface to be measured in the new experimental set-up was the convex surface of a plano-convex lens where the planar surface acts as a reference. The test optic is specified in section 6.5.1.1. The surface forms
measured in the forward and backward configurations are given in section 6.5.1.2 and 6.5.1.3 respectively.

6.5.1.1 Test optic specifications

The test optic is a 500 mm focal length planovex lens. A photo of the test optic is shown in Figure 6.9 and its specification in Table 6.1. An optical schematic of the test lens is depicted in Figure 6.10.

![Figure 6.9: Large diameter spherical lens](image)

6.5.1.2 Forward configuration

In this configuration, the intensity reflected by the test optic was recorded for a probe traverse of 30 mm as 5 mm was left on each side of the lens to avoid a collision between the probe and the lens mount. The phase-extraction process (based on an aperture model explained in section 5.4) was used to measure the phase from this signal along 8 different tracks (shown in Figures 5.5-7). The plot of change in OPC for a particular track is shown in Figure 6.11 for a probe distance of 0.6 mm (measured from the vertex).
6.5.1.2 Forward configuration

In this configuration, the intensity reflected by the test optic was recorded for a probe traverse of 90 mm as 5 mm was left on each side of the lens to avoid a collision between the probe and the lens mount. The phase-extraction process based on an a-priori model (explained in section 5.4) was used to evaluate the phase from this signal along 8 different tracks (shown in Figure 5.2). The plot of change in OPD for a particular track is shown in Figure 6.11 for a probe distance of 0.6 mm (measured from the vertex).
To validate the OPD error obtained from the probe, it was necessary to check the computation process. This was done by changing the radius of curvature of the test surface by 0.2 mm from the manufacturer’s specification such that the maximum OPD error for the ideal case is about 9\(\mu\) over a lens diameter of 90 mm. The measured signal was then used in the computation process to obtain the measured OPD error using the changed value of radius of curvature. The plot of the ideal OPD error and the measured OPD error for this case is shown in Figure 6.12. It can be seen that the two plots are almost overlapping which shows that the computation process is valid and can be successfully used to compute the OPD errors.
To reduce the blind vector effect, we have taken another set of measurements along the same track for a different probe distance. The probe was moved backwards by 0.4 mm so that the probe distance for the second set of measurement was 1 mm. A plot of OPD error with respect to the ideal surface form for this probe distance is shown in Figure 6.13.
These two sets of OPD change are then used in equation (6.7) to find the error in surface profile of the curved surface. The plot of the surface form error as a function of radial position is shown in Figure 6.14.
The plot shows that the maximum surface error is within ±0.2µ over the surface height range of ±45 mm. However, some edge artefacts are apparent at either end of the test surface (between 45 and 45.5 mm). It is noted that in the computation of surface form in this case and in all other results shown in this thesis, it has been assumed that the reference surface has no form errors and other parameters such as refractive index and the thickness of the test lens have the exact values provided by the manufacturer. The values obtained for the surface form error are also uncorrected for the instrument alignment errors and these errors will be explained in detail in chapter 7. It should also be noted that to calculate the surface form we have to reduce the number of sampling points due to the memory limitation of the computer in handling large matrices. The sampling resolution for the surface form calculation was 0.1 mm. The surface form was thus calculated for 8 different tracks and extrapolated to give a grey scale image and is shown in Figure 6.15.
Figure 6.15: Grey scale image of the surface form error of a spherical surface in the forward configuration (uncorrected for alignment errors)

This plot shows that the surface error is within \( \lambda/2 \) (\( \lambda \approx 0.633 \mu \)) for a majority of the surface segment. It is noted that for a symmetric optic the surface form error shall be symmetric with respect to the lens centre. In this case however, it is apparent that this is not so due to the various alignment errors encountered during the experiment. A detailed error analysis is described in chapter 7. The next section gives the results of the same surface in the backward configuration.

### 6.5.1.3 Backward configuration

For reasons discussed earlier the calculation in this configuration is more straightforward as the surface form is a ratio of the OPD error to a slowly varying term (backward quotient) that is a function of the ideal form (the denominator in equation (6.14)). A plot of this term as a function of radial position for the plano-convex lens is shown in Figure 6.16.
Figure 6.16: Backward quotient

It can be seen that the surface form is approximately one third of the change in OPD. The OPD error is calculated in the same way as the forward configuration using a-priori information (as described in section 5.4). Similar to the forward configuration, intensity was recorded by the probe along 8 different tracks. The plot of OPD change as a function of probe position for a probe traverse of 90 mm for a particular track is shown in Figure 6.17.
Figure 6.17: Change in OPD for a spherical surface in the backward configuration

The resulting surface form calculated using equation (6.14) is shown in Figure 6.18.
The interpolated surface form error of the curved surface for a surface diameter of 91 mm is shown in Figure 6.19.
The surface form is within ±0.15µ in the backward configuration. It can be seen that the backward configuration shows slightly less error than the results obtained in forward configuration. This is probably due to better alignment between the probe and test lens in the backward configuration and will be discussed in next chapter. In this section we have measured a large diameter lens having a spherical profile. The next section describes the measurement of a steep aspherical surface.

**6.5.2 Steep aspheric lens**

This section shows the results of measurement of the surface form of a steep aspheric surface. The aspheric surface is hyperbolic in surface profile with four higher order aspheric coefficients. The measurement was carried out in both the forward and backward configurations. The specification of the test optic is given in section 6.5.2.1 followed by the surface form results in the
forward and backward configurations in sections 6.5.2.2 and 6.5.2.3 respectively.

6.5.2.1 Test optic specifications

The aspheric surface to be measured is the curved surface of a plano-convex lens where the planar surface acts as a reference. The plano-convex lens we have measured is the precision asphere from Edmund Optics (Part Number 47732), a photo of which is shown in Figure 6.20.

![Figure 6.20: Edmund Optics Precision Asphere (47732)](image)

An aspheric surface is represented by the sag of the surface at a particular point $r$, where $r$ is the height of that point from the optical axis. The equation to describe the aspheric surface consists of two parts - conic section departure from a sphere and aspheric deviation incorporating higher order polynomial terms. The equation of the sag of an asphere can be written as

$$z = \frac{cr^2}{1 + \sqrt{1 - (k + 1)c^2r^2}} + A_4r^4 + A_6r^6 + A_8r^8 + A_{10}r^{10} + \ldots$$

(6.15)

where

c is the curvature of the base sphere at the vertex and is equal to the inverse of radius,
k is the conic constant,
A4, A6, A8 and A10 are the aspheric coefficients of higher order.

The aspheric test lens has the following specifications.

**Table 6.2: Optical specifications for the steep aspheric lens**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>37.5 mm</td>
</tr>
<tr>
<td>Lens Diameter</td>
<td>25 mm</td>
</tr>
<tr>
<td>Clear Aperture</td>
<td>22.5 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>22.09 mm</td>
</tr>
<tr>
<td>Centre Thickness</td>
<td>6 mm</td>
</tr>
<tr>
<td>Glass</td>
<td>Ohara - L-BAL35</td>
</tr>
<tr>
<td>Conic Constant</td>
<td>-2.271309</td>
</tr>
<tr>
<td>4th Order Term</td>
<td>1.954456E-5</td>
</tr>
<tr>
<td>6th Order Term</td>
<td>-1.756349E-8</td>
</tr>
<tr>
<td>8th Order Term</td>
<td>2.597437E-11</td>
</tr>
<tr>
<td>10th Order Term</td>
<td>-2.414068E-14</td>
</tr>
</tbody>
</table>

An optical schematic of the aspheric lens is shown in Figure 6.21.
6.5.2.2 Forward configuration

Aspheric surface measurement is similar to the spherical surface measurement except for the fact that the sagitta calculation is more complex (as given in equation (6.15)) and the same model has been used to test it. The probe was traversed for a distance of 15 mm leaving aside 5 mm on each side of the lens to avoid collision between the probe and the lens mount similar to
the previous experiments. A probe traverse of 15 mm results in surface form calculation of about 16 mm as the ray reflected from the rear surface strikes the test surface at about 16 mm for the probe position at 15 mm. Two sets of measurements were taken for all 8 tracks corresponding to the probe distance of 0.3 and 0.55 mm. The change in OPD as a function of probe position for a particular set of measurement is shown in Figure 6.22.

![Figure 6.22: Change in OPD for an aspherical surface in the forward configuration](image)

The above plot shows discontinuity at the centre. This may be due to a misalignment between the source and receive point inside the probe. A detailed error analysis for this misalignment is given in chapter 7. The corresponding error in surface form as a function of radial position is shown in Figure 6.23.
Figure 6.23: Surface form error of an aspherical surface in the forward configuration (uncorrected for alignment errors)

The surface form over the entire surface is within ±1µ except for one point on either side of the surface, although some noise is observed near the edge which again depends on the separation between the two probe stand-off distances and noise in the OPD measurements. The surface form obtained from the eight tracks is then interpolated to construct a grey scale image and is shown in Figure 6.24.
The plot shows that the surface form calculation is not same for all the tracks, but is within $\pm 3\mu$ for all the tracks. This is probably due to the presence of tilt between the probe path and the test optic. A detailed analysis of the error due to the relative tilt between the probe path and the test optic is given in chapter 7.

6.5.2.3 Backward configuration

This section describes the measurement of the aspheric surface in the backward configuration. In this configuration, it was not possible to measure the surface form over the desired lens diameter of 15 mm. This is due to the fact that the probe was unable to collect the light reflected from the test optic near the edge. The reason for this limitation is explained below.

A pair of rays from the probe will retrace its path after getting reflected from the front (reference) surface and rear (test) surface. The probe position is defined by the coordinates $(x_p, y_p)$ and the coordinates of the ray intersection...
points at the front surface by \((x_f, y_f)\) and the rear surface by \((x_r, y_r)\) for the ray retracing its path after reflection from the rear surface. These ray coordinates as a function of probe position is shown in Figure 6.25.

![Diagram showing ray coordinates in the backward configuration](image)

**Figure 6.25: Ray coordinates in the backward configuration**

The angle at which the probe receives the reflected light is critical in this configuration. The x coordinate at the front surface for a probe position of 7.5 mm is 7.65 mm. In this example, we have considered the probe distance to be 0.17 mm. In such a case, the receiving angle is about 42° as shown in Figure 6.26.
The numerical aperture of our probe is 0.53 which means that the probe can accept a ray having a maximum angle of ±32°. Therefore, in this case, the probe will not be able to detect the interference between the two reflected beams at the edge. The receiving angle is less than the probe acceptance angle only up to a probe distance of 5.8 mm. This means that the surface form of the test surface in the backward configuration can be measured only up to a surface height of 7.5 mm (value of \(x_f\) at a probe position of 5.8 mm). The experiments in this configuration were carried out for a probe traverse of 10 mm. The OPD error in the backward configuration for a particular track is shown in Figure 6.27.
Figure 6.27: Change in OPD for an aspherical surface in the backward configuration

This plot is similar to the result obtained in the forward configuration and shows discontinuity at the lens centre. The corresponding surface form error as a function of radial position up to a radius of 6.5 mm (value of $x_r$ at $x_p = 5$ mm) is shown in Figure 6.28 and is approximately a third of the OPD error as explained in section 6.5.1.3.
Figure 6.28: Surface form error of an aspherical surface in the backward configuration (uncorrected for alignment errors)

The interpolated grey scale image of the surface form of the test surface obtained from the different tracks over the entire lens is shown in Figure 6.29.
Figure 6.29: Grey scale image of the surface form error of an aspherical surface in the backward configuration (uncorrected for alignment errors)

The figure shows that the surface form error is within ±1μ for most of the surface part taking into account all the associated errors, a detailed analysis of which will be described in the next chapter.

6.6 Summary

In this chapter, the reason for edge artefacts in the surface form obtained for a convex surface measured in the forward configuration was discussed. It was noted that this is due to a vector, termed as the blind vector, which the system is unable to see, and which increases exponentially towards the edge. A conceptual method to remove the blind vector was suggested but this is not practically feasible. An alternative method based on two sets of measurements was described to reduce the blind vector effect. In this method, the same track of test optic is scanned twice, but with two different probe distances which are parallel to each other. In this case, suppression of edge artefacts depends on the separation between the two probe distances. The results for a convex surface are similar to that given in section 3.2 using
coarser resolution for surface heights. However the two-pass method is more robust and gives more consistent solutions. The solution for the surface form of a concave surface in the forward configuration is not affected by the blind vector problem. This is because the blind vector for concave surfaces increases toward the lens centre unlike the convex surfaces where it increases towards the edge. Consequently boundary condition that the surface form error is zero at the centre eliminates solutions containing the blind vector for concave surfaces.

The computation required to calculate the surface form in the forward configuration is quite complex and time consuming due to the involvement of large matrices. The surface form calculation for both convex and concave surfaces in the backward configuration is quite straightforward. In this configuration, the plane surface faces the probe instead of the curved one. It was found out that the surface form calculation is just a ratio of the OPD change and a slowly varying term (backward quotient). The major disadvantage in the backward configuration is that the probe will be unable to collect light at the edge for extremely steep convex surfaces and for concave surfaces, surface form for a small segment near the edge cannot be obtained. However, this configuration is quite useful for measuring moderately steep convex surfaces and a major portion of concave surfaces.

Two test surfaces were measured in both the double pass forward and backward configurations. The first surface was a relatively slow (f/5) surface with a relatively large diameter (100 mm) and it was shown that the surface form error measured is within ±0.3μ in both the configurations. The second surface to be measured was a steep (f/1.5) aspherical surface. In this case, the forward configuration is capable of measuring the entire surface (f/1.5) (though in actual practice 5 mm near the lens edge is left out of the measurement due to lens mount constraints). The entire surface could not be measured in the backward configuration due to the steep incoming angle received by the probe. This configuration is only able to measure a surface height of ±7.5 mm which approximately represents an f/no. of 2.5. The errors
associated with alignment in the experiments have not been taken into account in the surface form measurements presented in this chapter. The measurement errors due to alignment are analysed in detail in the next chapter.
Chapter 7

Error Analysis

7.1 Introduction

This chapter describes an investigation into the errors in the set-up that may be encountered during the measurement of optical path difference (OPD) in synthetic aperture interferometry (SAI). The major sources of error in the interferometer system can be classified into the following categories.

- Error due to uncertainty in the probe distance measurement with respect to the front surface of the test optic.
- Error due to tilt between the probe path and test optic (non-perpendicularity between the optical axis of the test optic and the probe path).
- Error due to non-coincidence of the source and receive point of the probe.
- Error in determining the lens centre.

The theory of error calculation is explained in the following sections along with a quantitative error analysis with reference to two different surfaces. The first one is the curved surface of a plano-convex lens and the second one is the concave surface of a plano-concave lens, both having a focal length of 125 mm and a diameter of 50 mm. The analysis has been done for both forward and backward configurations and are presented as errors in measured OPD. Once again it is noted that errors in surface form measurement will be approximately one third of the OPD measurement error in both the configurations.
7.2 Uncertainty in probe distance measurement

The measurement of the distance between the probe and the front surface of the test optic accurately is crucial for the calculation of OPD in our implementation of SAI. In the experimental set-up presented in this thesis, the front surface refers to the test surface in the forward configuration and to the reference surface in the backward configuration, assuming that we are testing the curved surface of the test optic and the reference surface is planar in profile. As discussed in section 5.2 and Figure 5.4, the measurement of probe distance (from the vertex of test lens) is carried out by moving the probe such that the probe focus lies on the front surface of the test optic. This position is indicated by a sudden increase in the reflected intensity that occurs when the source and receive points of the fibres are confocal. In practice it has been found that the repeatability of this method is less than the resolution of the micrometer used to measure it (10µ).

The accuracy of probe distance measurement with respect to the confocal point is better in the backward configuration than in the forward configuration in the case of a plano-convex/plano-concave lens. This is because it is not necessary to determine the lens centre in the backward configuration as shown in Figure 7.1. The only alignment needed is, parallelism between the probe path and the front surface. If the probe path and the front surface are parallel, the confocal position remains constant irrespective of the probe position with respect to the test lens in the vertical and horizontal direction.
Figure 7.1: Probe distance measurement in the backward configuration

This is not true for the forward configuration as the front surface in this case, is curved. In addition to the need for parallelism between the probe path and test lens, the probe needs to be placed at the lens centre in both the vertical and horizontal directions. Any deviation from this position will add an error in the measurement of probe distance. The error will always be positive in the case of a convex surface and will be equal to the sag of the curved surface where the probe is placed as shown in Figure 7.2. If the misalignment between the probe and the lens is \( y \), then the sagitta at \( y \) will be added to the error in measurement of probe distance. This error increases with increased steepness of the curved surface. Thus unlike the backward configuration, the error in probe position measurement in the forward configuration is due to two factors – the sagitta of the test surface at the measurement point and the inaccuracies of the measuring micrometer stage.
Figure 7.2: Probe distance measurement in the forward configuration

An uncertainty in the measurement of the probe distance will give rise to an error in the measurement of OPD for both the forward and backward configurations. In the forward configuration, the optical path length (OPL) for the ray reflected from the front and rear surface is given by equations (7.1) and (7.2)

$$OPL_f = 2\left[ (x_{f1} - x_p)^2 + (y_{f1} - y_p)^2 \right]^{1/2}$$  \hspace{1cm} (7.1)

$$OPL_r = 2\left[ (x_{f2} - x_p)^2 + (y_{f2} - y_p)^2 \right]^{1/2} + n(y_r - y_{f2})$$  \hspace{1cm} (7.2)

where, $y_r = c + d$, $c$ being the centre thickness of the test optic and $d$ is the probe distance. Any error in the measurement of probe distance will affect the calculation of $y_r$. In addition to the effect in $y_r$, all the ray coordinates at the point of reflection at the front and rear surfaces will have respective errors as shown in Figure 7.3.
Figure 7.3: Ray diagram for probe distance measurement error in the forward configuration

In this figure, $\Delta d$ is the error in measurement of probe distance. If the error is positive (probe moving outwards from the front surface), the point of reflection at the front surface from the ray reflected at the front surface will move towards the centre of the test optic and the point of reflection for the ray reflected from the rear surface will move towards the edge of the lens. These will affect the calculation of OPLs, thereby producing an error in the calculation of OPD.

In the backward configuration, the OPLs are given by

\begin{equation}
OPL_f = 2d
\end{equation}

\begin{equation}
OPL_r = 2\left[(x_f - x_p)^2 + (y_f - y_p)^2\right]^{1/2} + n\left[(x_r - x_f)^2 + (y_r - y_f)^2\right]^{1/2}
\end{equation}

If the error in measurement of probe distance is $\Delta d$, then the change in $OPL_f$ is $2\Delta d$. The value of $OPL_r$ will also incur an error as the ray coordinate points at
the front and rear surface will change as shown in Figure 7.4. The ray intersection points will shift towards the edge of the lens as the probe is moved outwards.

Figure 7.4: Ray diagram for probe distance measurement error in the backward configuration

A quantitative analysis was done for a probe distance measurement error of 10µ for a plano-convex and a plano-concave lens of 125 mm focal length having a diameter of 50 mm and is illustrated in Figure 7.5 and Figure 7.6.
Figure 7.5: Plot of error in measurement of OPD for probe distance measurement error of 10µ with a plano-convex lens.

Figure 7.6: Plot of error in measurement of OPD for probe distance measurement error of 10µ with a plano-concave lens.
It can be seen that the forward configuration is less sensitive to error in probe distance measurement than the backward configuration for both the lenses. An error of 10µ in probe distance measurement in the forward configuration will cause a maximum error of about 0.72µ in OPD measurement (at 25 mm) whereas this error is 4.5µ (of opposite sign) for the backward configuration in case of this plano-convex lens. Remembering that the surface form error is approximately one third of the change in OPD this amounts to approximately a surface form error of 0.24µ for forward configuration and 1.5µ for backward configuration in measurement over a lens diameter of 50 mm.

The accuracy in the measurement of the probe distance in the forward configuration depends on the alignment of the probe with respect to the lens centre. For the plano convex and plano concave lenses a centration error of 1 mm will cause the probe distance measurement error to increase or decrease by about 8µ in addition to the limiting resolution of micrometer stage. Clearly a 1 mm centration error would not be expected in practice so the probe distance error in the forward configuration is dominated by errors introduced by the micrometer used to measure it.

### 7.3 Tilt between the probe path and the test optic

The calculation of OPD as a function of probe position for an ideal surface is made with an assumption that the optical axis of the test optic is perpendicular to the probe path. This means that the planar surface of the test optic is parallel to the probe path. Any deviation from this parallelism will give rise to a tilt between the probe path and test optic as shown in Figure 7.
In the backward configuration, this tilt can be observed by measuring the confocal distance of the probe from the reference surface at various points across the test optic as shown in Figure 7.8. In the case of proper alignment (the probe path and reference surface being parallel to each other for a particular lens position), all the measurements should give same value for the confocal position. If they give different values, the translation stage needs to be tilted so that the probe distance remains constant at any measurement point for a particular track.
Figure 7.8: Measurement of tilt using confocal positions in the backward configuration

In the experiments, the measurement of OPD was done for 8 different tracks as shown in Figure 5.2 to cover the full lens. The alignment procedure explained above is suitable to achieve parallelism between the probe path and reference surface only for a particular track. In an ideal case, it should remain aligned if the test optic is rotated about its optical axis to cover other tracks. If the optical axis of the test optic and the axis of the rotary stage do not coincide, however, it will give rise to different tilts between the probe path and test optic for different tracks.

To achieve the same tilt values for all the tracks, the optical axis of the test optic should coincide with the rotational axis of the rotary stage onto which the test optic is attached. This alignment can be achieved in the backward configuration by a simple alignment procedure similar to an autocollimator configuration. A schematic of this alignment procedure is shown in Figure 7.9.
A beam of light from a He-Ne laser is made to fall on the planar surface of the test optic and the reflected light is observed on a screen. The rotary stage holding the test optic is then rotated about its axis. If the optical axis of the test lens and the rotational axis of the stage coincide, then the reflected spot of light in the observation plane will remain stationary. If the two axes do not coincide, then the reflected spot will move by a certain amount. The tilt screws present in the lens holder are then adjusted to make the reflected spot stationary as the rotary stage is rotated along its axis by 360°. The stationary spot indicates that the optical axis of the test lens and the rotary stage coincides. It should be noted that this holds true only for the backward configuration.

The same alignment procedure will work for the forward configuration only if the test lens is centred with respect to the rotational axis. If the lens is not centred, the alignment procedure remains same for the backward configuration because the front surface is planar in profile where the angle of
reflection remains same at all points in this surface. In the case of forward configuration, the rotation of reflected spot is due to two factors, decentration and non-coincidence of two axes. As the centring of the test lens is done using manual slides, it is difficult to properly centre the lens with respect to the rotational axis in both the configurations. Due to this, the test lens is only approximately centred by manually adjusting the slides using the linear scale provided on the slides. In the forward configuration, proper tilt alignment can be achieved only when both the light reflected from the front and rear surface remains stationary when the test optic is rotated. Therefore, even if the reflected spot (from either the front or rear surface) remains stationary for a 360° rotation of the test lens, the tilt will change for different tracks for the forward configuration. A plot of error in measurement of OPD is shown in the following plots for a tilt of 1 arc min for both the plano-convex and plano-concave lens.

Figure 7.10: Plot of error in the measurement of OPD for a tilt error of 1 arc min with a plano-convex lens
A tilt error of 1 arc min generates an OPD measurement error of ±0.52µ in the forward configuration whereas the same tilt error produces an OPD error of ±3.4µ in the backward configuration. It is clear that the backward configuration is more than six times more sensitive to a tilt error than the forward configuration in the case of a plano-convex lens. The tilt error can be minimized to a large extent in the backward configuration unlike the forward configuration where its very difficult to achieve tilt alignment. The tilt error sensitivity of the plano-concave lens when compared to the plano-convex lens shows a similar trend as the probe distance measurement error. The sensitivity of the forward configuration to a tilt error is similar for convex and concave lenses whereas in case of backward configuration, the plano concave lens shows less sensitivity to a tilt between probe path and test optic.
7.4 Non-coincidence of source and receive points

The theory of SAI assumes that the source and receive points of the anamorphic probe are coincident. The simplest way to achieve this is to use a single fibre for both source and receive functions. Though more straightforward in theory, it has practical difficulties such as ghost reflections from the fibre end interfering with the reflected intensity of interest. In the anamorphic probe design, separate source and receive fibres were used to overcome the problem of ghost reflections. The source and receive fibres, however, needed to be aligned to make them coincident. Any improper alignment will give rise to an error in OPD measurement.

The block diagram for aligning the source and receive points is shown in Figure 7.12. A laser beam from a He-Ne laser is split into two beams using a beam splitter and fold mirrors. These beams are then carried inside two separate fibres which are then attached to the source and receive fibre of the probe using fibre to fibre connectors. This ensures that the beam entering the two fibres are from the same source and have exactly the same properties. These two beams are then made to traverse the same optical path inside the probe. The output is a highly elliptical beam due to the anamorphic nature of the probe. The two light beams are then adjusted using grub screws so that the interference pattern of the two beams contains no fringes.
No fringes in the interference pattern means that the two fibres are aligned perfectly and there is no linear misalignment between them. Any linear misalignment (longitudinal or lateral) will cause an error in measurement of OPD and is described below for both longitudinal and lateral misalignment.

### 7.4.1 Longitudinal misalignment

A schematic diagram of longitudinal misalignment between the source and receive point is shown in Figure 7.13. In this figure \((x_{p1}, y_{p1})\) represents the source point and \((x_{p2}, y_{p2})\) represents the receive point. The ray intersection point at the front surface for the ray reflected at the front surface is represented by the coordinates \((x_{fl}, y_{fl})\). The ray reflected at the rear surface
has three reflection points namely \((x_{f2}, y_{f2})\) and \((x_{f3}, y_{f3})\) at the front surface and \((x_r, y_r)\) at the rear surface. The ray travels from \((x_{p1}, y_{p1})\) to \((x_{f2}, y_{f2})\) where it is refracted and gets reflected at \((x_r, y_r)\). The reflected ray is again refracted in its reverse path at \((x_{f3}, y_{f3})\) to reach the receive point at \((x_{p2}, y_{p2})\).

The OPLs for the ray reflected at the front and rear surface can be written as

\[
OPL_f = \sqrt{(x_{p1} - x_{f1})^2 + (y_{p1} - y_{f1})^2} + \sqrt{(x_{p2} - x_{f1})^2 + (y_{p2} - y_{f1})^2}
\]

\[
OPL_r = \sqrt{(x_{f2} - x_{p1})^2 + (y_{f2} - y_{p1})^2} + \sqrt{(x_{r} - x_{f2})^2 + (y_{r} - y_{f2})^2}
\] + \sqrt{(x_{r} - x_{f3})^2 + (y_{r} - y_{f3})^2} + \sqrt{(x_{f3} - x_{p2})^2 + (y_{f3} - y_{p2})^2}
\]

In these equations,

\[
x_{p2} = x_{p1}, y_{p2} = y_{p1} + \Delta d \quad \text{and} \quad y_r = c + d
\]

where \(\Delta d\) = longitudinal misalignment between the source and receive points,

\(c\) = centre thickness of the test optic,

\(d\) = distance between the probe and front surface and
The OPD measurement in this case at a particular instant becomes a function of three variables represented by the ordinates $y_{f1}$, $y_{f2}$ and $y_{f3}$ instead of two as in the case of coincident source and receive points. The corresponding schematic for the backward configuration is shown in Figure 7.14.

In this case the equations for the OPL for the ray reflected from the front and rear surfaces are

$$OPL_f = (y_{f1} - y_{p1}) + (y_{f1} - y_{p2}) = 2d - \Delta d \quad (7.7)$$

$$OPL_r = [(x_{f1} - x_{p1})^2 + (y_{f1} - y_{p1})^2]^{1/2} + n[(x_r - x_{f1})^2 + (y_r - y_{f1})^2]^{1/2}$$

$$+ n[(x_r - x_{f2})^2 + (y_r - y_{f2})^2]^{1/2} + [(x_{f2} - x_{p2})^2 + (y_{f2} - y_{p2})^2]^{1/2} \quad (7.8)$$

It is clear from the equations that the front OPL decreases by an amount of $\Delta d$. The rear OPL is a function of three ordinates similar to the forward configuration. The error in measurement of OPD for a longitudinal
misalignment of 10µ between the source and receive points for the forward and backward configuration is shown in Figure 7.15 and Figure 7.16.

Figure 7.15: Plot of error in measurement of OPD for longitudinal misalignment of 10µ with a plano-convex lens
The maximum error in OPD measurement is ±0.36µ for the forward configuration and ±2.4µ for the plano-convex lens. It can be seen that the forward configuration is less sensitive (more than six times) to misalignment than the backward similar to the tilt error. The plano-concave lens shows a similar trend with similar sensitivity in forward configuration and the backward configuration being less sensitive than that of the plano-convex lens. It is noted that a 10µ longitudinal misalignment error is quite severe and would result in approximately 16 fringes across the field of view with our particular probe.

7.4.2 Lateral misalignment

The lateral misalignment schematic is similar to the longitudinal misalignment except the fact that the source and receive points are misaligned sideways. The schematic of the lateral misalignment is shown in Figure 7.17.
The OPLs for the rays reflected at the front and rear surfaces are shown in equations (7.9) and (7.10).

\[ OPL_f = \sqrt{[(x_{p1} - x_{f1})^2 + (y_{f1} - y_{p1})^2] + [(x_{p2} - x_{f2})^2 + (y_{f2} - y_{p2})^2]} \]

\[ OPL_r = \sqrt{[(x_{f2} - x_{p1})^2 + (y_{f2} - y_{p1})^2] + n[(x_r - x_{f2})^2 + (y_r - y_{f2})^2]} + n[(x_{f3} - x_r)^2 + (y_{f3} - y_r)^2] + [(x_{f3} - x_{p2})^2 + (y_{f3} - y_{p2})^2]} \] (7.10)

In these equations, \( x_{p2} = x_{p1} + \Delta d \), \( y_{p2} = y_{p1} \) and \( y_r = c + d \).

The corresponding schematic for the backward configuration is shown in Figure 7.18 and the equations for the OPLs are given in equations (7.11) and (7.12).
Figure 7.18: Ray diagram for lateral misalignment in the backward configuration

\[
OPL_f = \left[ (x_{f1} - x_{p1})^2 + (y_{f1} - y_{p1})^2 \right]^{1/2} + \left[ (x_{p2} - x_{f1})^2 + (y_{f1} - y_{p2})^2 \right]^{1/2} \\
= 2\left( (d + \Delta d / 2) \right)^{1/2} \tag{7.11}
\]

\[
OPL_r = \left[ (x_{f2} - x_{p1})^2 + (y_{f2} - y_{p1})^2 \right]^{1/2} + n\left[ (x_r - x_{f2})^2 + (y_r - y_{f2})^2 \right]^{1/2} \\
+ n\left[ (x_{f3} - x_r)^2 + (y_{f3} - y_{r})^2 \right]^{1/2} + \left[ (x_{f3} - x_{p2})^2 + (y_{f3} - y_{p2})^2 \right]^{1/2} \tag{7.12}
\]

In these equations,
\[x_{p2} = x_{p1} + \Delta d, \ y_{p2} = y_{p1}\] and \(y_i = c + d - \text{sag}(x_r)\)

The OPD error for a lateral misalignment of 10µ between the source and receive points is depicted in Figure 7.19 and Figure 7.20. It is noted that in this case this error will result in approximately 26 fringes across the field of view for our probe.
Figure 7.19: Plot of error in measurement of OPD for lateral misalignment of 10µ with a plano-convex lens

Figure 7.20: Plot of error in measurement of OPD for lateral misalignment of 10µ with a plano-concave lens
The maximum error for the forward configuration is 6.05µ and for the backward configuration, it is 6.44µ for the plano-convex lens. Unlike other errors, it can be seen that both the configurations have approximately similar sensitivity to the lateral misalignment error. Also, the measurement of OPD is more sensitive to lateral misalignment than the longitudinal misalignment. The same applies to the plano-concave lens with both configurations having similar sensitivity to lateral misalignment. It can be seen that this type of error (discontinuity at centre) has been observed in the surface form plot of aspheric surface (see section 6.5) for both the forward and backward configurations. It is worth mentioning here that the alignment between the source and receive points was not checked prior to measurement of the aspheric surface. The He-Ne laser used to deliver the light was often removed from the probe to be used in other experiments which may have caused a misalignment inside the probe. Therefore it can be said that the lateral misalignment between the source and receive points of the probe was the most probable cause of nature of the surface form while measuring the aspheric surface specified in section 6.5.2.1.

### 7.5 Uncertainty in determining the lens centre

In our SAI, the intensity of the reflected light is collected while the probe scans the test optic from one edge to another after passing through the lens centre. The probe movement is monitored using a Michelson interferometer as explained in section 3.3.1. A Michelson interferometer measures the probe position at each instant with respect to its previous position. The absolute position of the probe with respect to the test optic is needed by the set-up to use the probe position values obtained by the Michelson interferometer to calculate the OPD.

The absolute position of the lens centre is determined in practice using the plot of intensity fringes obtained by the probe. As the test optic is symmetric, the number of fringes at each side of the lens centre should be the
same for same amount of probe movement at each side. This information was used to determine the lens centre with respect to the probe movement. It should be noted that this condition is true only if the probe and the test optic are aligned and the lens is axi-symmetric. Any tilt between the probe path and the test optic will give rise to asymmetric fringes with respect to the lens centre. This will cause an error in determining lens centre which then gives rise to error in the measurement of OPD.

A quantitative analysis was carried out to find out this effect for a decentration of 1µ in both the lenses. The plot of OPD error as a function of probe position is shown in Figure 7.21 and Figure 7.22.

![Plot of error in measurement of OPD for centration error of 1µ with a plano-convex lens](image)

**Figure 7.21:** Plot of error in measurement of OPD for centration error of 1µ with a plano-convex lens
Both the lenses show similar sensitivity to the centration error in forward and backward configurations, but of opposite signs. These errors are approximately proportional to the amount of decentration over a diameter of 50 mm and shows that the OPD error will be approximately 1µ for a decentration of 1µ over a range of 50 mm diameter.

7.6 Results

The previous sections presented the theoretical error analyses for two different types of test optic (planovex and planocave), both having the same focal length and f/#. The error in OPD was measured for the curved surfaces of both the lenses in forward and backward configurations. The results are then analysed to identify the major sources of error in the experimental set-up.
7.6.1 Plano-convex lens

The test surface is the convex surface of the plano-convex lens where the plane surface acts as a reference surface. The specification of the test optic is given in section 7.6.1.1 followed by the results in the forward and backward configurations in sections 7.6.1.2 and 7.6.1.3 respectively.

7.6.1.1 Test optic specifications

The test optic is a 125 mm focal length plano-convex lens. A photo of the test optic is shown in Figure 7.23 followed by its specification in Table 6.1. An optical schematic of the test lens is shown in Figure 7.24.

![Figure 7.23: Edmund Optics f/2.5 plano-convex lens](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>125 mm</td>
</tr>
<tr>
<td>Lens Diameter</td>
<td>50 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>64.6 mm</td>
</tr>
<tr>
<td>Centre Thickness</td>
<td>10 mm</td>
</tr>
<tr>
<td>Edge Thickness</td>
<td>4.97 mm</td>
</tr>
<tr>
<td>Glass</td>
<td>BK7</td>
</tr>
</tbody>
</table>

Table 7.1: Optical specifications of the f/2.5 plano-convex lens
7.6.1.2 Forward configuration

The convex surface of the test optic was measured for 8 tracks for a probe traverse of 40 mm leaving aside 5 mm on each side of the lens as in previous experiments. The changes in OPD as a function of probe position are shown for 4 different tracks below to show the difference between different tracks and assess the associated errors.
It can be seen from above figures that there is a pattern of change in OPD as we go from track 1 to 7. Track 5 shows the minimum OPD error. If we compare these results with the error analysis, it shows that the optical axis of the test lens and the axis of rotation do not coincide as we rotate the lens to measure different tracks. This results in different tilts for different tracks. A quantitative analysis shows that the maximum tilt is observed for track 1 where the angle between the probe path and test optic is approximately 0.22 degrees. The probe path and the test optic become approximately parallel to each other at track 5. The following plot shows the angle between the probe path
path and the test optic as the rotary stage holding the test lens is rotated by an angle of 360 degrees starting from track 5. The angle between the probe path and the test optic has been calculated by an iterative method where the tilt value is adjusted to minimise the OPD error.

![Figure 7.26: Tilt between probe path and test optic for various tracks](image)

If we include this estimation of tilt in the calculation of OPD change to compensate the effect of tilt between the probe path and test optic, the corresponding changes in OPD for the four tracks (as shown in Figure 7.25) are illustrated in Figure 7.27.
The above plots show that maximum OPD change is sub micron, if the tilt is compensated. This OPD change is less than the random errors associated with the set-up. This illustrates that the major source of error in the experimental set-up is the tilt between the probe path and the test optic for the forward configuration.

**7.6.1.3 Backward configuration**

The change in OPD of the measured surface from the ideal surface was also measured in the backward configuration. It was observed that all eight tracks...
show similar plots in this case unlike the forward configuration where different tracks had substantially different plots. A plot of the measured OPD change as a function of probe position in this configuration for a particular track is shown in Figure 7.28.

![Figure 7.28: Change in OPD for the plano-convex lens in the backward configuration](image)

The error analysis shows that the backward configuration is more sensitive to errors in the experimental set-up than the forward configuration. Since all the tracks present a similar trend it may be concluded that it is easier to obtain a consistent result in the backward configuration and this is a consequence of the fact that alignment is easier. The above plot shows a small tilt between the probe path and the test optic which remains the same for all the tracks. This indicates that the linear traverse was not perpendicular to the axis of rotation.

Further analysis shows that the other major source of error in this configuration is the error in measuring the probe distance. This shows that the measurement in forward configuration is mainly limited by tilt whereas in the
backward configuration, it is important to achieve the alignment in all respects due to its low tolerance to misalignment.

7.6.2 Plano-concave Lens

This section gives the measurement of the concave surface of a plano-concave lens where the plane surface acts as a reference surface similar to the planovex lens. The specification of the test optic is given in section 7.6.2.1 followed by the results in the forward and backward configurations in sections 7.6.2.2 and 7.6.2.3 respectively.

7.6.2.1 Test optic specifications

The test optic is a 125 mm focal length plano-concave lens. A photo of the test optic is shown in Figure 7. 29 followed by its specification in Table 7.2. An optical schematic of the test lens is shown in Figure 7. 30.

![Figure 7.29: Edmund Optics f/2.5 plano-concave Lens](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>Focal Length</td>
<td>-125 mm</td>
</tr>
<tr>
<td>Lens Diameter</td>
<td>50 mm</td>
</tr>
<tr>
<td>Radius of curvature</td>
<td>-64.6 mm</td>
</tr>
<tr>
<td>Centre Thickness</td>
<td>5 mm</td>
</tr>
</tbody>
</table>
**Figure 7.30: Optical schematic of the f/2.5 plano-concave lens**

### 7.6.2.2 Forward configuration

The concave surface of the test optic was measured for 8 tracks for a probe traverse of 40 mm leaving aside 5 mm on each side of the lens as in previous experiments. The change in OPD as a function of probe position is shown for 4 different tracks below to show the difference between different tracks and assess the associated errors.
The results for the plano-concave lens in the forward configuration show a similar trend for different tracks similar to the plano-convex lens. As the mechanical axis of the rotary stage does not coincide with the optical axis of the test lens, the plot shows a different tilt for different tracks. Tilt compensation as described in section 7.6.1.2 can be used to compensate for this error and a plot similar to plot for track 5 will be observed for all the tracks as shown in Figure 7.32, the errors of which are within the random errors associated with the experimental set-up.
7.6.2.3 Backward configuration

The plots of the measured OPD change in the backward configuration show a similar trend to the plano-convex lens. All the tracks show a similar profile and indicate a small tilt between the probe path and test optic. Once again this is because the linear traverse was not perpendicular to the axis of rotation. The OPD error for a particular track is shown in Figure 7.33.
It can be seen that the OPD error is less than a micron over the surface height of 40 mm which means that the surface form error is less than 0.3μ for the plano-concave lens.

7.7 Summary

A detailed error analysis was carried out to assess the effect of various errors on the measurement of OPD in SAI. Five major sources of error were analysed and their effect on the measurement were computed. The first source of error that was examined was the error in measurement of probe distance. It was found that the sensitivity of the forward configuration to this error was approximately six times less when compared to the backward configuration. Tilt error between the probe path and the test optic was the second error to be explored. The error plot in this case shows a different shape when compared to the previous error plot, but the tilt sensitivity of the forward configuration
remains less than the backward configuration (similar to the probe distance measurement error).

The first two errors dealt with the alignment and measurement errors in the experimental set-up. The next two errors dealt with the misalignment within the probe itself. The measurement of OPD change incurs an error in the case of non-coincident source and receive points of the probe. A longitudinal misalignment causes an error shape similar to the probe distance measurement error. A lateral misalignment between the source and receive points produces a triangular shaped error with discontinuity at the centre. In this case, the forward and backward configuration shows similar sensitivity to the misalignment error.

The final error to be analysed was the uncertainty in determining the lens centre. The lens centre was determined after collecting the data and counting the fringes on either side of the centre of lens. Any error in finding lens centre produces an error in measurement of OPD which follows a straight line passing through the centre.

The errors were analysed for two lenses, piano-convex and plano-concave lens having the same power. It was seen that the planovex lens was more sensitive to errors in the backward configuration than the planocave lens, whereas in the forward configuration, the planocave lens was more sensitive to errors compared to the planovex lens. Generally, the backward configuration showed more sensitivity to errors than the forward configuration.

It was also noticed that the major source of error in the forward configuration was the tilt between the probe path and the test optic. If the tilt is compensated in the OPD calculation, the measurement errors are within the random errors present in the experimental setup. In the backward configuration, however, the presence of other errors also affects the OPD calculation and the measurement of probe distance and probe path need to be defined to a greater accuracy. It can be said that though the forward configuration is more tolerant to most of the errors than the backward
configuration, the alignment procedure is simpler in the backward configuration.
8.1 Conclusions

Aspherical optical elements play an important role in reducing the weight and size of high quality optical systems, but are not widely used due to the high cost in fabrication and replication. A significant proportion of the cost of aspherics is due to the difficulty in testing them. Many methods have been described in the literature to solve the problem of testing aspherics. Null tests are one of the interferometric methods which are particularly useful in testing aspherics, but they are used sparingly due to the high cost of fabricating and testing the null compensators themselves. It is fair to say that, to date, there is no generally applicable interferometric method to test aspherics. Stylus profilometry is currently the most common approach to test aspherics and is widely used in the industry during the fabrication stage. As a contacting method they are not suitable for soft materials as they tend to induce surface damage.

Synthetic aperture interferometry (SAI) is a non-contact interferometric method and has the potential to measure aspherics as they are being polished. In this technique, light reflected from the test and a reference surface is collected by a scanning probe. This signal is then used to find the optical path difference (OPD) between the two light beams which is subsequently converted to surface form of the test surface. The probe used in the original SAI investigations of Tomlinson (Tomlinson, Coupland & Petzing 2003) was made up of a pair of bare fibres, one acting as a source and the other as a receive fibre. Due to its low numerical aperture (NA) (~0.12), the probe was
not capable of measuring steep surfaces. The light gathering efficiency of the probe was also poor because only a very small fraction of light reaches the receive fibre.

The aim of the work described in this thesis was to modify the original SAI technique to enable it to measure steep aspherics while improving the light gathering efficiency. The NA of the probe was increased using external optics. An aspheric lens was used as an objective lens increasing the NA to 0.53. This means that the surface gradient restrictions of the test surface are relaxed to be within ±32°, which is suitable for most aspherics of interest (whereas for bare fibres it was only ±7°). The light gathering efficiency of the probe was increased by adopting an anamorphic design. A cylindrical lens was used in the probe to achieve this and the light efficiency of the probe was increased by about 18dB compared to a symmetrical probe. In addition, the image of the source and receive fibres were made coincident by the use of a beam splitter and additional optical components. This reduces the computation complexity compared to the original configuration since the OPD at a given probe position is a function of only two (rather than three) surface height parameters.

An experiment was carried out on a CNC lathe to validate the new probe design for SAI, where a steep spherical surface was measured. The tool stock of the CNC lathe was used to support the probe and the test optic was placed in a custom made mount which has held by the jaws of the lathe chuck. The interference pattern obtained by the probe while it scanned the test optic was converted to phase values by a phase extraction process based on a phase-locked-loop (PLL). The OPD obtained from this phase was used to compute the surface form. The original set of equations to compute surface form was under-determined as the number of known variables were less than the number of unknown variables. This system was made over-determined by taking coarser resolution for unknown variables to compute the surface form. Although it was noted that edge artefacts were present in the solution, the surface form for a major portion of the test surface was within ±3µ. Although,
this is comparable with the tolerance provided by the manufacturer, we believe that this value was more representative of alignment errors due to straightness errors in the probe path.

Alignment errors encountered on the CNC lathe were investigated further by developing a new off-line experimental setup. In this set-up, the probe was mounted on a linear translation stage to traverse it across the test optic. Two additional micrometer translation stages were used to control the vertical and horizontal movement of the probe. The test optic was mounted on a rotary stage having two degrees of freedom. The first degree of freedom was rotation about the optical axis of the test lens and this was used to rotate the test optic so as to collect data from different radial paths across the lens. The second degree of freedom was the tilt of the axis of rotation through a horizontal axis and this was utilized to achieve parallelism between the probe path and reference surface. Straightness of the probe path and the quality of displacement measurement was confirmed using an HP heterodyne calibration interferometer.

A new robust phase extraction process was developed for our use of SAI. The basic assumption that the ideal test surface is known in advance was used to demodulate the signal. An a-priori signal was created using this information and was compared with the obtained experimental signal. The difference between the two phase maps was used to obtain the OPD difference between the ideal and test surface. This phase extraction process was more robust and faster than the phase extraction process based on PLL.

The problem of edge artefacts in the computed surface form were analysed and it was found that the forward configuration of SAI for a convex surface is in fact blind to a characteristic surface form. Using a vector derivation we call this form the blind vector. For a convex surface the blind vector is a characteristic oscillating form that exponentially increases toward the edge. In practice, it is found that the inversion process introduces a substantial amount of this form to the solution to account for noise in the data. It was noticed that the system can be made to recognize the presence of
blind vector by taking additional OPD measurements. In practice, sensitivity to the blind vector is substantially reduced by using a two measurement set up where the same portion of the test surface is measured twice, with different probe stand off distances. In this case, an over-determined set of equations is obtained and a unique solution is possible. This method is similar to the method used earlier (using coarser resolution of surface heights), but is more robust and gives more consistent solutions. It was noticed that the blind vector does not affect the surface form computation of concave surfaces. This is because, for concave surfaces, the blind vector increases in magnitude and frequency towards the lens centre and the boundary condition that the surface form is zero at the lens centre eliminates solutions containing the blind vector.

An alternative configuration, known as the backward configuration, was also used to measure the test surface. In this configuration, the test lens was reversed so that the plane (reference) surface faces the probe instead of the curved (test) surface. It was shown that there are no blind vectors in this configuration and the calculation of surface form from the difference in OPD between the ideal and experimental signal is much more straightforward. In addition, the alignment procedure is much simpler in this configuration. Although this configuration was quite adept at measuring moderately steep surfaces, it was not suitable for measuring steep convex surfaces. It was also observed that for concave surfaces, a small portion near the edge cannot be measured in this configuration.

Two different surfaces were measured to assess the new set-up in the forward and backward configurations. The first test optic to be measured was a 100 mm diameter planovex lens. It was seen that the surface form error was within ±0.3µ over the entire surface in the forward configuration. This value was limited only by the random errors associated with the alignment of the experimental set-up. The backward configuration shows slightly better resolution (within ±0.15µ) as the alignment between the probe path and the test optic was easier in this configuration. The second test surface that was
measured had a steep aspheric profile with four higher order coefficients. The measurement of an aspheric surface shows the capability of the anamorphic probe in measuring a generalized aspheric surface. In this case, the surface form error was submicron, but with a discontinuity at the centre. An error analysis shows that this was probably due to misalignment between the source and receive points of the probe.

A detailed error analysis was subsequently carried out for the forward and backward configurations. Two surfaces of similar profile, but of opposite signs were used to illustrate the various errors. The error analysis was conducted to assess the effect on OPD of five different parameters. These parameters are, the error in the measurement of probe stand off distance, tilt error between the probe path and test optic, longitudinal and lateral misalignment between the source and receive points of the probe and error in determining the lens centre. It was observed that in most cases, the forward configuration was less sensitive to alignment errors than the backward configuration. The major source of error in the forward configuration was the tilt between the test lens and probe path due to difficulty in obtaining a good alignment between the two in this configuration. The alignment procedure was more straightforward in the backward configuration and consequently the observed OPD errors were less in this configuration than those in the forward configuration.

Finally, it can be concluded that the SAI technique using an anamorphic probe can be successfully used to measure steep aspherics. For moderately steep convex surfaces, the backward configuration would be the preferred method to test them. The simpler alignment procedure and ease in computation of surface form makes this configuration ideal for measuring test surfaces even though it is more sensitive to the errors than the forward configuration. It is noted that backward configuration cannot be always used on a machine because the test surface which is being polished is kept at the front and it may not be possible to access the rear surface. For extremely steep convex surfaces, the forward configuration is the preferable method.
Although, there is an increase in computation complexity and it is more susceptible to edge artefacts, it is less sensitive to misalignment than the backward configuration. For concave surfaces, as a small portion is left out in surface form computation in the backward configuration, the forward configuration is the preferable method as in this configuration the blind vector is suppressed by the boundary condition and a unique solution can be obtained in a single pass. It is also noted that in case the reference surface is curved and not planar, the theory of SAI still holds, albeit, with different set of equations and different receiving angles for the probe. In general, as observed in this thesis, both the configurations can be successfully used to measure most aspherics of interest. The next section describes some of the future work to improve the technique further so that it can be widely used to measure aspherics during the production stage.

8.2 Future Work

There are quite a few areas where there is a scope of further work to improve the technique of SAI so as to enable this technique to integrate into the production environment. These are discussed in this section.

8.2.1 Measurement using spiral sampling

Measurement of five different test surfaces of various speeds were discussed in this thesis to assess the theory of the anamorphic probe using the SAI technique. In all cases, the measurements were taken along different radial tracks representing different portions of the test optic. Subsequently, the data requires interpolation to get the surface form of the entire test surface.

Although this sort of sampling technique shows continuous data along a particular radial track, no information is available for portions between the different tracks. Measurement of surface form continuously throughout the surface, albeit, with a loss of resolution along a particular track, will reveal more information. This can be achieved by rotating the test optic along its
optical axis while the probe scans the test surface. The probe will follow a
spiral path with respect to the test surface to collect the interference signal as
shown in Figure 8.1. In this figure, the rotation of the test optic and the probe
traverse was done in such a way that the distance between two successive
points along any radial track is 1 mm for a surface diameter of 100 mm. It is to
be noted that this technique was in fact used by Tomlinson in the original
work of SAI, however, we could not apply this technique on our set-up
because of the vibration created by the CNC lathe. The probe traverse in our
case was monitored using a Michelson interferometer and the vibration
produced by the lathe affected this interferometer. In Tomlinson’s work, the
accuracy of CNC lathe was sufficient to measure the test surface used by him
as it was nearly flat and the requirement on the resolution of probe traverse
was not so stringent.

![Figure 8.1: Spiral sampling](image)

Measurement along a spiral path allows a trade-off between radial resolution
and transverse resolution. Clearly, for a nearly axis symmetric optic the
transverse resolution requirement is not as great as the radial requirement.
Moreover, a spiral path would allow asymmetrical profiles, for example gross decentration, to be measured more easily. A preliminary analysis shows that a centration error can be eliminated by examining the difference between the ideal and measured OPD. This difference will follow a sinusoidal path of increasing magnitude as we spiral towards the edge of the optic. The magnitude of the curve represents the amount of decentration and can be easily calculated. Once the magnitude of the decentration is known, this value can be incorporated in the forward process to calculate the forward OPD suitable for this type of surface. This OPD can then be compared with the measured OPD in the inverse process to obtain the surface form of the test surface.

**8.2.2 Wavelength modulation**

The technique of SAI can be improved in several areas by modulating the wavelength of the light delivered by the probe as discussed in the following sections.

**8.2.2.1 Phase polarity measurement**

In this thesis, measurements were carried out for a monotonically increasing or decreasing curved (convex or concave) surfaces. Phase extraction was done using an a-priori signal as we had prior knowledge of the test surface. In general, the sign of phase can be unambiguously detected using phase shifting or heterodyne detection methods. These techniques cannot be implemented directly in our case as the return paths from the front and rear surfaces are fixed for a particular probe position. A tunable laser diode can be used to modulate the wavelength of the probe beam to find the polarity of the phase change if a path length imbalance is present in the interferometer. The technique was originally proposed by Laming (Laming et al. 1986) by modulating the current to the laser.
8.2.2.2 Separation of spurious reflections

The concept of modulating beams can also be used to separate the reflected beam of interest from the unwanted reflections from other surfaces. It can be shown that for a mean wavelength of $\lambda$, an optical path length of $\lambda^2/\Delta\lambda$ can be resolved if the light is modulated by $\Delta\lambda$ (Ishii, Chen & Murata 1987). In principle, this means that if we use a diode (which can be tuned upto 100 kHz) over a wide range of wavelengths (resolution ~ 3 nm at $\lambda = 1510$ nm), then reflections separated by optical path length differences of less than a mm can be distinguished.

8.2.2.3 Generating a reference beam within the probe

In the forward configuration, we have considered the rear planar surface as the reference surface. This surface should be of high quality so that the surface form of the test surface can be deduced correctly. It may not always be possible to have a high quality reference rear surface. Wavelength modulation can be used to alleviate this problem. An internal reflection within the probe can be engineered so that it acts as a reference. Then wavelength modulation can be used to identify the interference between this reference surface and the test surface. It is noted that in this case the theory of backward configuration holds as the reference surface will be in front of the test surface. It is noted, however, that the resulting interferometer will be more prone to the affects of vibration and probe path.

8.2.2.4 In-process backward configuration for convex surfaces

It was observed that the computation of the surface form was quite straightforward in the backward configuration without displaying blind vectors. It is preferable to use the backward configuration to measure the test surface for moderately steep convex surface. However, it may not always be possible to use this configuration if the test surface is intended to be measured during the fabrication process as the rear reference surface may not be accessible in this case. This problem can be alleviated by using a reference
surface in front of the test surface and using wavelength modulation so that only the interference between the reference and test surface is collected by the probe.

8.2.2.5 Measurement of Fresnel surfaces

SAI has the potential to measure stepped aspherical Fresnel surfaces. Fresnel surfaces consists of a set of concentric annular sections (Fresnel zones) to reduce the lens thickness and weight compared to a conventional lens of equivalent power as shown in Figure 8.2. Currently, there is no interferometric method to test these types of surfaces and contact stylus profilometers have problems with steep gradients due to the profile constraints. As the surfaces in Fresnel optics do not increase or decrease monotonically, the probe in SAI may collect signals from one or more steps. A-priori knowledge can be used with wavelength modulation to identify the multiple reflections potentially allowing interferometric quality measurements of Fresnel surfaces.

Figure 8.2: Cross-section of a Fresnel and conventional lens of same power
8.2.3 Comparison with standard measurement methods

Through a straightforward measurement set-up, the surface form error computed for most of the test surfaces by SAI technique, shows a value which is less than a wavelength. Although these values are adequate considering the limitations of the present set-up, a comparison with other standard aspheric measurement set-ups needs to be done to assess the traceability of SAI. Two of the most widely used aspheric measurement instruments with which the results can be compared are the “Form Talysurf” from Taylor-Hobson Ltd and “Zygo Verifire” from Zygo Corporation Inc.

The “Form Talysurf” is a contact stylus probe to measure aspherics through mechanical means. It is widely used in industry to measure aspheric surface form with $\lambda/10$ accuracy. “Zygo Verifire” uses a Fizeau interferometer along with phase shifting techniques to stitch various annular apertures to get the surface form of an asphere. It also has a form uncertainty of $\lambda/10$ for most aspherics and can measure test optic upto 6 inches in diameter.

8.2.4 Alternate measurement configuration

The forward configuration is preferable method for in-process measurements because of the way the aspheric surface is polished on a CNC machine. This is due to the fact that the test surface which is being polished is kept in front and it is not always possible to access the rear surface. Edge artefacts were observed in the surface form when measuring the test surface in this configuration and it was observed that this was apparently due to the blind vector. We have been able to reduce edge artefacts due to the blind vector by using a two pass measurement set-up, but some edge artefacts still remain in the measurements presented in this thesis. Clearly further studies are required to derive the optimum measurement strategy.

In essence, the measurement uncertainty is greater at the edge of the lens. It was noticed that the number of unknown variables (surface form) are
more than the number of known variables (OPD errors) in the original configuration thereby making the system under determined. If the number of known and unknown variables can be made equal, then the system of equations is determinate but measurements uncertainty is not necessary equal across the lens aperture. However, things are improved if additional measurements are made near the edge. The ideal measurement strategy is likely to depend on the lens profile and further work is required to model the effect.

8.2.5 Measurement of reflective surfaces

All the measurements described in this thesis were for refractive test surfaces. A different experimental set-up is required for the measurement of a reflective surface. This set-up will need the introduction of a reference surface in front of the reflective test surface. The intensity of the beam reflected by the test surface also needs to be reduced so as to obtain an interference pattern with an appreciable contrast. This can be achieved by the use of a neutral density filter as shown in Figure 8.3.

![Neutral density filter setup](image)

**Figure 8.3: Measurement of a reflective surface**
8.2.6 Minitiaturization of the anamorphic probe

We have used an anamorphic probe in SAI to measure steep aspherics and increase light gathering efficiency. The source and receive points of the probe were two fibres which were made coincident by supplementary optics. The probe was made to be anamorphic by using a single cylindrical lens of 160 mm focal length which made the probe relatively big in size. A smaller size probe will be easier to integrate and control in a production environment. The probe can be redesigned using multiple cylindrical lenses to reduce the overall length of the probe. The design should also take care of alignment issues to maintain coincidence of the source and receive point of the probe. An optical design software package such as OSLO (Lambda Research Corporation 2001) could be used to achieve this.

8.2.7 Automated experimental set-up

Clearly, a more robust, automated and sophisticated experimental set-up is required to obtain accurate and precise results in a commercial environment. We have used a manual translation stage for traversing the probe and a manual rotary stage to achieve alignment between the test optic and probe path, the control of which was quite coarse. An automatic high resolution calibrated translation stage is required to reduce the problems of uncertainty in probe movement and enhance the measurement accuracy. A rotary stage with fine tilt controls is required to complement the translation stage to obtain a good alignment between the probe path and test optic to reduce the problem of alignment errors in the measurement of test surfaces. In addition, the rotational axis of the rotary stage needs to be matched with the optical axis of the test surface if the data is collected from a rotating lens. The control software needed to obtain the surface form needs to be integrated with the set-up. A front-end user interface will also be required to enable users to easily perform the experiments and understand the results.
8.2.8 In-process measurement

It is clear that SAI has a great potential to measure aspherics during the fabrication process as a "closed loop" method. In practice, some changes will be required in the CNC polishing machine to realize this. Equipments will be required to integrate the probe into the system to traverse it across the test optic and monitor its movement precisely. As residual particles such as cerium oxide slurry is left over the surface during the polishing process, the test optic needs to be cleaned for each measurement to avoid spurious reflections from reaching the probe. Successful implementation of SAI technique into the polishing process will reduce the cost of testing and fabrication of aspherics substantially while keeping the measurement process simple.

8.2.9 Measurement of steep surfaces using two probe paths

The NA of our anamorphic probe was increased to 0.53 using supplementary optics. This relaxed the surface gradients of the reflected rays to be within ±32°, which is sufficient for measuring several aspherics of interest. In case of a plano-convex lens made of BK7, this means that a lens of f/# upto 1.8 can be measured in the forward configuration. The probe will be unable to measure the test surface for faster lenses. In the backward configuration, the capability of probe to measure steep surfaces reduces further.

In essence, this problem can be overcome by increasing the numerical aperture of the probe further, but is not easy to implement in practice. An alternate strategy can be devised to overcome this problem by changing the angle of the probe traverse for the lens portion near the edge which cannot be measured as shown in Figure 8.4. Here, for the first half of test optic, the probe traverses across it over the path x₁. For the 2nd half of the test optic the probe will be unable to collect the light if it continues along the path x₁ due to the steep incoming angle of reflected rays. To avoid this problem, the probe is tilted and then made to traverse the path x₂ to reduce the incoming angle of
the reflected rays. It is noted that the forward model to compute OPD for the two paths will be different and the computation of forward OPD will be more complex for the 2nd probe path.

![Diagram of two probe path method]

**Figure 8.4: Two probe path method**

### 8.2.10 Measurement of steep surfaces using a curved reference surface

It was observed that for extremely steep surfaces, the incoming angle of the reflected rays to the probe was quite large. In the backward configuration (as is evident from section 6.5.2), the probe was unable to collect light near the lens edge. The reference surface used in this case was the planar surface of the plano-convex lens. The use of an appropriate curved reference surface instead of a planar one will reduce the incoming reflected angle such that the probe can collect the interference across the lens. The calculation of a suitable curved reference surface is the key to solve this problem. The only disadvantage of this technique is that different curved reference surfaces may be required to cover a range of different test surfaces.
8.2.11 Solid body compensation

A detailed error analysis was carried out (see chapter 7) to find the effect of different alignment errors on OPD measurement. It was observed that measurement of OPD has different sensitivity to different types of errors. Figure 8.5 shows the OPD measurement errors for probe distance error (10µ), tilt error (1 arc min) and decentration error (1µ). It can be seen that the error curves show specific characteristics. For example the decentration error can be fitted to a polynomial of 1st order whereas the probe distance measurement and tilt error shows characteristics of 2nd and 3rd order polynomials respectively. This information could in principle be bundled together in an algorithm to create a solid body model. This means that in practice, one will be able to find different alignment errors from the observed OPD error using a solid body compensation displacement estimation. Successful implementation of this model will ease the requirement of precision alignment. Use of solid body compensation should be done carefully to avoid compensating the actual OPD error itself.

Figure 8.5: Alignment errors showing specific characteristics
REFERENCES


Bennett, R.W. *Metrology*, 738/658; 33559 edn, G01B 500.


The Mathworks Inc. 2006, MATLAB.


UGS 2005, Solid Edge.


