A genetic algorithm based
topology optimisation
approach for exploiting rapid
manufacturing’s design
freedom

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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

By

Darren Michael Watts
B.Eng.(Hons) M.Sc.(Eng)

Doctoral thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

Wolfson School of Mechanical & Manufacturing Engineering

September 2008

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Current product structures are designed to meet predefined specifications and are therefore rarely optimal. Instead, they are frequently over-engineered to ensure fitness for purpose, which results in excess weight and uneven stress distributions throughout their structures. Designs are then often compromised further in terms of optimality by the inherent process limitations of conventional manufacturing. Rapid Manufacturing (RM), due to its vastly increased design freedom, can overcome these restrictions and become the enabling technology for fabricating uncompromised, optimal products.

This thesis describes the design, creation, testing and evaluation of a new design optimisation system capable of exploiting the high design freedom afforded by RM technologies. Inspired by the design rules that Nature follows, the system combines the stochastic search behaviour of a Genetic Algorithm (GA) with finite element analysis in order to evolve optimal topological structures via a survival of the fittest process. The novelty of this approach is that 3D unit cell structures varying in volume fraction are used to simulate different densities of a single material, which are then efficiently distributed throughout the problem domain by the GA to yield an improved stress distribution. Furthermore, the system can consider different unit cells mixed together within the same problem, thereby substantially expanding the topology optimisation research field. Following a series of experiments of increasing complexity, the performance, stability and computational demands of the system are discussed.

Experimental results indicate the system works, however, the presence of unit cells was found to cause localised stress concentrations to occur, which tend to inadvertently steer the overall optimisation process. Suggestions to address this issue were made. In addition, the mixing of different unit cells together was found to improve trade-offs between system objectives but did not always improve stress distribution.

Keywords: Rapid Manufacturing, Genetic Algorithm, Topology Optimisation, Functionally Graded Structure, Unit Cell, Uniform Stress Distribution.
ACKNOWLEDGEMENTS

This section should read more like a long register of names, as there are numerous people that have contributed to this work in some form or another. However, whilst it is not possible to individually name all that have helped, particular thanks must go to the following:

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Certain individuals beyond Loughborough also warrant individual thanks, these include Paul Smith from Delphi Diesel Systems for help with the frontplate optimisation; Georges Fadel of Clemson University for helpful discussions regarding the implementation of GAs; Chris Williams, Scott Johnson and David Rosen from the Georgia Institute of Technology for sharing their research ideas; Markus Gross from Cambridge University for vast amounts of help and time trying to get the supercomputer to work, and Jim Pattison for providing me with a bed when it inevitably didn’t.

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Craig for continually showing an interest in my work, and for always making time to deal with the many queries and requests that I had. I would also like to extend thanks to the people at Logmein.com, for creating really useful freeware which enabled me to run many of my experiments remotely from the comfort of my home. If it wasn’t for this fantastic little bit of software I would probably still be waiting for my experiments to finish even now!

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Darren Watts
7th August 2008
GLOSSARY

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<th>Abbreviation</th>
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<tr>
<td>2D</td>
<td>Two Dimensional</td>
</tr>
<tr>
<td>2½D</td>
<td>Two and a half Dimensional (simple extrusion)</td>
</tr>
<tr>
<td>3D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>3DP</td>
<td>Three Dimensional Printing</td>
</tr>
<tr>
<td>ACO</td>
<td>Ant Colony Optimisation</td>
</tr>
<tr>
<td>ADM</td>
<td>Advanced Digital Manufacturing</td>
</tr>
<tr>
<td>AF</td>
<td>Automated Fabrication</td>
</tr>
<tr>
<td>AML</td>
<td>Adaptive Modelling Language</td>
</tr>
<tr>
<td>AMOpt</td>
<td>Adaptive Modelling Optimisation</td>
</tr>
<tr>
<td>BESO</td>
<td>Bi-directional Evolutionary Structural Optimisation</td>
</tr>
<tr>
<td>CAD</td>
<td>Computer Aided Design</td>
</tr>
<tr>
<td>CAO</td>
<td>Computer Aided Shape Optimisation</td>
</tr>
<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
</tr>
<tr>
<td>CM</td>
<td>Constructive Manufacturing</td>
</tr>
<tr>
<td>CPU</td>
<td>Central Processing Unit</td>
</tr>
<tr>
<td>DCI</td>
<td>Distributed Computing Interface</td>
</tr>
<tr>
<td>DFA</td>
<td>Design For Assembly</td>
</tr>
<tr>
<td>DFM</td>
<td>Design For Manufacture</td>
</tr>
<tr>
<td>DM</td>
<td>Direct Manufacturing</td>
</tr>
<tr>
<td>DMLS</td>
<td>Direct Metal Laser Sintering</td>
</tr>
<tr>
<td>EFD</td>
<td>Evolutionary Form Design</td>
</tr>
<tr>
<td>ESO</td>
<td>Evolutionary Structural Optimisation</td>
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<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
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<tr>
<td>FGM</td>
<td>Functionally Graded Material</td>
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<tr>
<td>FOC</td>
<td>Freedom Of Creation</td>
</tr>
<tr>
<td>FOS</td>
<td>Factor Of Safety</td>
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<tr>
<td>GA</td>
<td>Genetic Algorithm</td>
</tr>
<tr>
<td>GB</td>
<td>Giga Byte</td>
</tr>
<tr>
<td>GUI</td>
<td>Graphical User Interface</td>
</tr>
<tr>
<td>Acronym</td>
<td>Definition</td>
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<td>-----------</td>
<td>-------------------------------------------------</td>
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<tr>
<td>HEX</td>
<td>Hexahedron (Brick shaped 3D finite element)</td>
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<tr>
<td>KBE</td>
<td>Knowledge Based Engineering</td>
</tr>
<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>LENS</td>
<td>Laser Engineered Net Shaping</td>
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<tr>
<td>LMT</td>
<td>Layered Manufacturing Technologies</td>
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<td>MB</td>
<td>Mega Byte</td>
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<tr>
<td>MD</td>
<td>Metamorphic Development</td>
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<tr>
<td>MOGA</td>
<td>Multi-Objective Genetic Algorithm</td>
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<tr>
<td>NPGA</td>
<td>Niched Pareto Genetic Algorithm</td>
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<tr>
<td>NSGA</td>
<td>Non-dominated Sorting Genetic Algorithm</td>
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<tr>
<td>PC</td>
<td>Personal Computer</td>
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<td>PFU</td>
<td>Page File Usage</td>
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<td>PSO</td>
<td>Particle Swarm Optimisation</td>
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<td>RAM</td>
<td>Random Access Memory</td>
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<tr>
<td>RM</td>
<td>Rapid Manufacture</td>
</tr>
<tr>
<td>RP</td>
<td>Rapid Prototyping</td>
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<tr>
<td>RT</td>
<td>Rapid Tooling</td>
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<tr>
<td>SA</td>
<td>Simulated Annealing</td>
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<tr>
<td>SC</td>
<td>Stress Concentration</td>
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<tr>
<td>SFF</td>
<td>Solid Freeform Fabrication</td>
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<tr>
<td>SIMP</td>
<td>Solid Isotropic Material with Penalisation</td>
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<tr>
<td>SKO</td>
<td>Soft Kill Option</td>
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<tr>
<td>SLA</td>
<td>StereoLithography Apparatus</td>
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<tr>
<td>SLM</td>
<td>Selective Laser Melting</td>
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<tr>
<td>SLS</td>
<td>Selective Laser Sintering</td>
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<tr>
<td>STL</td>
<td>STereoLithography file format</td>
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<tr>
<td>TET</td>
<td>Tetrahedron (Pyramid shaped 3D finite element)</td>
</tr>
<tr>
<td>TSCRM</td>
<td>TechnoSoft Computing Resources Manager</td>
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<tr>
<td>TSMOGA</td>
<td>TechnoSoft Multi-Objective Genetic Algorithm</td>
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<tr>
<td>TSP</td>
<td>Travelling Salesman Problem</td>
</tr>
<tr>
<td>TSXMM</td>
<td>TechnoSoft Execution Manager</td>
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<tr>
<td>VEGA</td>
<td>Vector Evaluating Genetic Algorithm</td>
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### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>E</td>
<td>Young's Modulus of Elasticity</td>
<td>[N/mm²] or [MPa]</td>
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<tr>
<td>N_p</td>
<td>Total number of permutations</td>
<td>[-]</td>
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<tr>
<td>X</td>
<td>Total number of cell types available</td>
<td>[-]</td>
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<tr>
<td>Y</td>
<td>Number of non-void cell types available</td>
<td>[-]</td>
</tr>
<tr>
<td>a</td>
<td>Number of non-loaded discrete variables</td>
<td>[-]</td>
</tr>
<tr>
<td>b</td>
<td>Number of loaded discrete variables</td>
<td>[-]</td>
</tr>
<tr>
<td>δ</td>
<td>Displacement</td>
<td>[mm]</td>
</tr>
<tr>
<td>σ</td>
<td>Stress</td>
<td>[N/mm²] or [MPa]</td>
</tr>
<tr>
<td>σ_vmi</td>
<td>Von Mises Stress</td>
<td>[N/mm²] or [MPa]</td>
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CHAPTER ONE

INTRODUCTION

1.1 Research Need

Current product components are not optimal in the true sense of the word, as the term *optimal* suggests that the best possible result has been achieved. Instead, components are generally designed to meet a set of product specifications, which are identified in the conceptual design stage. Hence the improvement process of optimisation is stopped as soon as the design meets the acceptable product specifications, which is generally before the best possible result is reached. Current design optimisation is generally limited to high performance areas such as military aircraft, spacecraft, turbine blades, and Formula 1 racing. This optimisation involves the use of both Finite Element Analysis (FEA) and Computational Fluid Dynamics (CFD). Both of these methods often require vast amounts of computing power for complex scenarios. Also, these tools are still not used in isolation, where, for example, CFD may be used to identify a general design shape, as seen in Figure 1.1, but wind tunnels are still used to verify solutions and tweak the final design.

*Figure 1.1 – CFD analysis of a Formula 1 car design, [1].*
Additionally, most industrial optimisation work is usually applied via a manually iterative approach (often termed *over engineering*) which, although may improve a design’s performance, does not yield a truly optimal solution as parts are often unnecessarily overweight [2]. During operation, such a component will require a greater amount of energy to move, energy that is effectively wasted. Over engineering is achieved by adding extra material at known locations of high stress. An example of this is illustrated in Figure 1.2 using a simple bracket, where the high stress at the corner fillet (shown by the red region of Figure 1.2a) is reduced in Figure 1.2b by increasing the radius of the fillet. The thinking behind this concept is to reduce the regions of high stress concentration by spreading them over larger areas. This method requires little skill and the components it yields will have an uneven stress distribution in addition to being overweight. It should be noted that a more dramatic improvement in distributing the stress more uniformly throughout the bracket can be achieved by a simple alteration of the design’s *topology*, shown here in Figure 1.2c by the introduction of a rib feature, despite the fillet radius remaining unchanged.

![Figure 1.2 - Bracket design stress plots showing, the initial design with high stress at the fillet (a), the over engineered design with increased fillet radius (b) and the altered topology design with rib added (c).](image)

At present, even if a truly optimal design is conceived, it will inevitably become compromised during the manufacturing stage due to the inherent limitations of conventional manufacturing technologies. Designers have always had to bear in mind the final production and assembly procedures that will be used to make the product – for example, design for injection moulding – and as a result, alter their designs to accommodate the selected processes. In the injection moulding example, this would
involve the introduction of draft angles and split lines, to allow the part to be removed from the tool; in addition to maintaining near constant wall thicknesses to aid the flow of the material; and minimising re-entrant features, such as undercuts, that would require expensive sliding cores, amongst others, [3]. In addition, further design considerations are required for assemblies to allow multiple parts to be assembled correctly, increasingly compromising a potentially optimal design. This scenario has led to the concepts of Design for Manufacture (DFM) and Design for Assembly (DFA) [4], both of which not only restrict designers’ freedom of creation but also dictate the design, inhibiting the production of truly optimised design solutions. Rapid Manufacturing (RM), due to its additive, layer-by-layer build nature, offers a notable alternative approach which will now be discussed in greater detail.

1.2 Rapid Manufacturing

RM has evolved from the layered manufacturing technologies that were originally conceived for prototype production, known collectively as Rapid Prototyping (RP) [5]. Although many commercial RP systems are available, it is not the purpose of this report to describe them; if more information is required there are numerous published works available, see [5-8]. In the same way that RP has had many alternative names, such as Solid Freeform Fabrication (SFF) [6] and Automated Fabrication (AF) [9], so too does RM, with other examples being, Advanced Digital Manufacturing (ADM), Direct Manufacturing (DM) and Constructive Manufacturing (CM), [10]. The names may be different but the principle is the same throughout, which is best explained by defining RM. There are many variations on the definition of RM, however the widely accepted definition is “the use of additive manufacturing techniques to make end use parts”, [10]. This definition helps to highlight the main differences between RP and RM. Firstly, whilst both RP and RM can be considered additive, RM is not expected to remain solely layer-based, unlike current RP systems [10]. Secondly, and more importantly, parts built via RM can be considered finished products or components that are fit for purpose compared to the prototype parts built via RP. However, it should also be noted that it is currently accepted within the RM community that parts created via RM may be subject to post processing in some way such as plating, bead-blasting, painting or infiltration [10], though this is consistent with conventional manufacturing.
RM benefits from all of the advantages it has inherited from RP, with the most important being the ability to produce highly complex and intricate parts of virtually any geometry that could not be made using conventional methods. All of this is without any tooling requirements [11]. RM also offers the potential for the stock free storage of products, with parts built when they are needed, enabling true just-in-time production; all that is required is the appropriate three dimensional (3D) data, raw material and an RM machine [12]. The advent of RM also gives rise to the possibility of decentralising production, with parts built where they are needed, possibly by the end user, thus eliminating the need for large scale manufacturing plants and subsequent shipping [12]. There are currently a variety of materials available for RM including plastics, ceramics and metals, in addition to the potential use of mixed or functionally graded materials (FGM), [10].

The fact that RM enables the elimination of all tooling also has significant time and cost savings within manufacturing, [13]. Tooling is an expensive but necessary investment of conventional manufacturing which therefore carries a substantial amount of risk associated with it. By removing the tooling requirements of manufacturing, RM enables radical unproven designs to be produced without the associated risk of costly tooling mistakes. The expected impact of RM on the manufacturing world is predicted to be so great it is being described as a "new industrial revolution", [14].

However, without tooling, the creation of the necessary Computer Aided Design (CAD) data is now the bottleneck in the design process. With the advent of RM, it is now arguably easier to build parts than it is to design them. The dilemma is that existing CAD systems have been developed to suit the needs of traditional manufacturing techniques and not RM. For example, the self intersecting geometry of a Klein bottle, shown in Figure 1.3a, is impossible to create as a single surface using existing CAD systems but is possible to fabricate via RM, as the Stereolithography (SLA) model of Figure 1.3b shows.
Unlike RM technologies, current CAD systems are also not able to consider graded material structures within designs or detailed textured surfaces on components. Instead a single material is considered throughout a solid CAD geometry, with surface textures represented by flat images wrapped around the solid geometry, [16]. Due to the advancements of RM, the development of appropriate CAD systems which address these shortcomings amongst others is a significant area of research, [16-19].

1.2.1 Applications

At present there are few commercial RM machines in existence. The current applications of RM are being realised using RP technologies such as SLA, Selective Laser Sintering (SLS) and 3 Dimensional Printing (3DP), [13]. In reality, the RP systems being used for RM were never intended for manufacturing as they were made with prototyping in mind and, as a result, the poor dimensional accuracy and surface roughness of parts coupled with the poor repeatability of the processes are not to the high standards expected of conventional manufacturing systems. In addition, the current RP machine and material costs are relatively high and there is, at present, a limited range of materials available, [20]. The current machine and material limitations are the focus of much ongoing research and development in these areas, [20-23]. Despite these obvious inhibitors to the uptake of RM, applications exist across many industry sectors where the benefits gained from adopting an additive fabrication approach outweigh the disadvantages of using current RP systems for manufacturing.

There are now many examples of where additive systems have been used for the manufacture of end-use parts, with the main technique adopted being SLS. For example, several hundred glass-filled nylon SLS parts have been made by Rocketdyne...
for the International Space Station and the entire space shuttle fleet, [24]. The low volume of the entire order made the use of RM financially feasible. Figure 1.4a shows a finished electrical capacitor box, whilst Figure 1.4b shows a support bracket.

![Figure 1.4 - Glass filled nylon SLS parts, electrical capacitor box (a) and support bracket (b). [24.]](image)

The Formula 1 industry sector has also been quick to make the leap from RP to RM, enabling parts to be fitted directly to cars, whilst also allowing quicker modifications to be made at a greatly reduced cost. A good example of this is a cooling duct designed to improve the cooling of the electrical system, by Renault Formula 1, shown in Figure 1.5a. The packaging problems of a modern F1 car result in the geometry of the cooling duct being too complex to manufacture in one piece using traditional manufacturing methods. Using SLS and nylon-12, Renault F1 were able to fabricate many one-piece on-car components that could be used in every race, Figure 1.5b, [25]. Further automotive applications have more recently been seen outside of Formula 1, with examples including the creation of bright-ware by Bentley Motors using Direct Metal Laser Sintering (DMLS) [26] and SLS plastic spring clips by MG Rover [27].

![Figure 1.5 - Renault Formula 1 cooling duct (a) and the complete car (b). [25.]](image)
An application of RM in the retail industry is given by the Netherlands based company Freedom of Creation (FOC), who partnered with Belgium based firm Materialsie. MGX to create many unique lampshade designs using RM [27], several of which are illustrated in Figure 1.6. Despite the expense of the technology and materials used, FOC now have over 30 retail outlets worldwide. In order to make production more economical, SLA lampshade designs incorporate a self supporting geometry to reduce the need for supports resulting in less waste of expensive resin. Likewise with SLS lampshades, designs were chosen that enabled nesting of geometries so that more of the entire build volume could be utilised [28].

![Lamp shade designs created by SLA and SLS](image)

FOC have not stopped at lampshade design, they have also made the leap of using additive fabrication to manufacture handbags, wristwatch bands and clothing, often involving complex textile designs [27]. The chain-mail like textiles enable the design of seamless clothing incorporating zips and other fasteners already assembled. This technique of textile creation has since been investigated by Bingham et al. where the collapsing of link data to reduce build cost and the automatic lofting of links over complex surfaces was also considered [30]. A similar chain-mail like approach has been adopted by AB Particular of Sweden in the manufacture of pre-assembled 18 karat gold chain jewellery via DMLS, [31, 32].

By combining RM production with reverse engineering equipment, individual custom fitting components can be created, allowing items to be tailored exactly to an individual. This possibility has far reaching implications, with potential applications across several areas including sports, medical, safety and the military [27]. There is also the opportunity to produce smart products, with sensors or computers embedded within custom-fitting items. Specific examples could include custom-fitting seats, footwear, helmets, glasses, protective clothing and items with hand grips, [33]. An example of
custom fitting a product to an individual is the World’s first bespoke, high performance football boot by Prior 2 Lever named the Assassin, shown in Figure 1.7. The athlete’s feet are digitally scanned to ensure the finished boots will be a perfect fit, therefore increasing comfort and reducing the risk of injury. Laser sintering technology is used to manufacture each individual custom outsole, [34].

Another application of RM combined with 3D scanning equipment is the creation of bespoke custom-fitting in-the-ear hearing aid shells by both Phonak and Siemens, shown in Figure 1.8. The use of RM coupled with reverse engineering technology has allowed for the customisation of each individual hearing aid shell for each user, with a better fit resulting in increased comfort and performance. Although Phonak use SLS technology to create polyamide shells whilst Siemens use acrylic based SLA resins, both companies have transformed what was traditionally a manual production process into a completely digital one, [27, 35, 36].
Another medical application is that of Align Technology, where customised SLA machines (double speed) are used to create patterns for sets of clear orthodontic aligners from patient specific data. The sets of orthodontic aligners are designed to progressively alter tooth alignment over a period of time and can be seen in Figure 1.9. Although not strictly an application of RM, more Rapid Tooling (RT), it is a good example of how an RP machine can be adapted and combined with scanning equipment to suit a custom-fit manufacturing process, [37].

Figures 1.9 - Align Technology’s orthodontic aligners, [37].

In addition to the applications already mentioned other examples of RM also exist. Examples include the creation of porous ceramic filters with complex internal structures using a variation of 3DP by Specific Surfaces Inc [27]. Also, the FBI have been using additive technologies to enable them to quickly hide surveillance equipment in everyday items such as smoke detectors [27]. Numerous freeform sculptures have been created from steel, later infiltrated with bronze, on a ProMetal machine by artist Bathsheba Grossman [38], examples can be seen below in Figure 1.10.

Figures 1.10 - Metal sculpture designs by Bathsheba Grossman, approximate size 100 mm, [38].
Additionally, the U.S. Army is considering a mobile parts hospital that could be dropped into the field of operation to allow broken tank parts to be made as and when needed via the Laser Engineered Net Shape (LENS) process, [27]. Furthermore, Imagen, an Ex One company, has produced gold dental copings using 3DP technology. Binder is jetted onto fine, gold-alloy powder to produce gold copings which are later infiltrated to remove porosity before being coated in porcelain to match teeth [32].

1.2.2 Design Freedom

RM’s ability to create virtually any geometry, irrespective of intricacy or complexity, has immense implications on the area of design with vastly increased design freedom. This point is demonstrated by the highly intricate and complex lightweight lattice structures illustrated in Figure 1.11, produced in Titanium and Stainless Steel via Selective Laser Melting (SLM) on an MCP Realizer machine [39].

![Figure 1.11 – Examples of highly intricate and complex lightweight metal lattice structures [39].](image)

RM offers increased design freedom by removing many of the restrictions previously imposed by the DFM and DFA considerations of conventional processes that effectively stifle design, [40]. However, it should be noted that all RP/RM systems still have inherent process limitations which will impact upon design freedom. For example, in the SLM process, support structures are often required and a metal substrate is needed (as seen in Figure 1.11) to anchor the part down. Despite the RP/RM process limitations, overall design freedom is vastly increased compared to conventional manufacturing techniques. Hague suggests the main design restrictions for RM will be the limitations of the designer’s imagination [41]. RM’s additional design freedom will have a profound affect in several key areas of design, particularly in the areas of part consolidation and design optimisation which will now be considered further.
Part Consolidation

Some layered manufacturing technologies allow parts to be built ready assembled, removing the need for many post manufacture assembly operations. This feature enables designers to ignore the conventional DFA considerations by consolidating many assembly parts into fewer pieces [40]. An industrial application of RM illustrating the consolidation of assembly parts is the redesign for RM of a complex ducting channel assembly from the aerospace sector, detailed in Figure 1.12. Exploded and assembled views of the original design are shown in Figure 1.12a and Figure 1.12b respectively. The conventional design was fabricated from vacuum formed plastic pieces, glued and bolted together, consisting of 16 parts in total. This assembly was subsequently modified by consolidating the separate assembly parts into a single piece for fabrication via RM, as shown in Figure 1.12c, [10].

![Air duct designs showing, exploded view of original assembly (a), assembled view of original assembly (b), and the consolidated RM design (c), [40].](image)

In fact, the automotive cooling duct, shown earlier in Figure 1.5, was designed and created by Renault F1 by adopting the concept of part consolidation. A further industrial example of part consolidation is that of an electric motor ring cover for the U.S. Navy Trident submarine. The assembly originally consisted of 25 parts that were largely welded together; however, using the ProMetal process by Ex One, a single piece cover was built in 45 hours [32].
Design Optimisation

Creating an optimal product often proves impossible to make conventionally, due to the DFM criteria. However, the increased design freedom afforded by RM can enable the realisation of increasingly complex designs that are fully optimised for their intended function [40], thus propagating the design for function design mentality [42]. An example of RM design optimisation of a current product is given using a Frontplate by Delphi Diesel Systems. The Frontplate completes the flow circuit of the fuel pump, accommodating several off the shelf components including couplings, sensors and a regulator, as shown by Figure 1.13a. Internal, gun drilled flow channels connect the various components together, as seen in Figure 1.13b. These channels have been optimised to reduce the resistance to flow, as shown by Figure 1.13c. The RM design has then been built up around these sweeping internal channels, whilst also being optimised for minimal weight [43], as illustrated by Figure 1.13d.

In the conventional Frontplate, the design of the internal flow channels was secondary to the DFM and DFA considerations of the product. In the redesign for RM, the approach taken was to consider the design and associated function of the flow channels first and foremost. This was coupled with a 35% reduction in volume and a slightly...
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

reduced envelope space due to some part consolidation, resulting in a more optimal design only made possible by the design freedom of RM. However, the unavailability of a suitable material and process currently prohibit the production line realisation of this RM design.

Another RM enabled example is that of Tsopanos et al. in the manufacture of heat transfer devices via SLM technology [44]. This investigation considered several novel heat sink designs, as seen in Figure 1.14, that although were not optimal, demonstrated the potential performance enhancements that could be achieved from creative designs.

![Figure 1.14 - Novel heat sinks created by SLM, (a) pin fin, (b) diamond and (c) V shape, [45].](image)

In summary, the increased design freedom afforded by RM will become the enabling technology that will break many of the DFM and DFA shackles [46]. As a result, optimal designs fabricated via RM will have fewer design compromises than before, allowing designers to adopt the methodology of manufacture for design without many manufacturing or assembly restrictions for the very first time [47]. These requirements are necessary to facilitate the production of truly optimal product structures. Hague suggests the advent of RM, as a valid fabrication alternative to traditional processes, will require new design systems in order to exploit the increased design freedom [43].

1.3 Research Aim

Assuming that RM solves the issue of being able to build optimal structures that are not possible via conventional manufacturing, then all that is needed is a method for generating the optimal designs in the first place. Ultimately, the aim of this research is to investigate a new design methodology capable of creating truly optimal designs, thereby unlocking the increased design freedom afforded by RM.
CHAPTER TWO

BACKGROUND & LITERATURE REVIEW

2.1 Introduction

Chapter I highlighted that current product components are not optimal, in that they are designed to meet design specifications and are often over engineered. Designs are then often compromised further in terms of optimality by the inherent limitations of conventional manufacturing processes. RM was shown to offer substantial advantages over conventional manufacturing processes, including significant increases in design freedom. RM can therefore be considered as the enabling technology for fabricating optimal products structures, which will require new design systems and methodologies in order to exploit the increased design freedom available.

This Chapter conducts a detailed review of past and current literature that is relevant to achieving the research aim (identified in Section 1.3.1), starting with the optimisation found in the Natural World, as this has often proved to be an abundant source of inspirational ideas for mankind, but also because it is the ultimate source of optimum design with countless natural designs perfected over many years of evolution. This is followed by a background of basic optimisation principles and more advanced numerical methods capable of simulating the design rules that Nature has followed when creating her optimal structures, as these are not constrained by mankind's DFMA criteria. The topics of structural optimisation, topology optimisation, genetic algorithms and their applications are then reviewed in depth as these are shown to be particularly relevant to exploiting RM's design freedom.
2.2 Optimisation in the Natural World

2.2.1 Darwin’s Theory of Evolution
Since life began on Earth it has been constantly adapting to best suit its environment in the daily challenge of struggling to survive. Charles Darwin’s theory of evolution explains how a competitive edge gained through genetic mutation is passed on to future generations allowing them to out-compete their contemporary rivals, [48]. The survival of the fittest is Nature’s way of guaranteeing that only the most optimal designs prevail with the others driven to extinction. Over a period of 3.8 billion years, this powerful optimisation process of natural selection has evolved the life-forms we can now see covering the planet.

During this vast amount of research and development time, Nature has regularly solved similar design problems that engineers and designers are facing today. The evolved design solutions found in the natural world are all efficient, novel, environmentally friendly and often elegant, and it is no wonder therefore that mankind is now turning to the natural world for inspiration when trying to produce innovative solutions to similar design problems.

Nature has been applying engineering principles for billions of years, perfecting her ingenious solutions every step of the way. As far as mankind is concerned the natural world can be classed as an abundant source of highly developed inspirational ideas. Over time, some of these natural solutions have been discovered by man – often by accident – and as a result man has learnt that it is possible to tap into Nature’s vast wealth of research and development. Indeed, using design ideas from the natural world is not a new concept – the Chinese wanted to make artificial silk over 2000 years ago, Leonardo da Vinci wanted to fly 400 years ago – and successful exploitation of these natural solutions has often benefited mankind, with examples including flight, Velcro and more recently Gecko tape and drag resistant swimwear.

D’Arcy Wentworth Thompson
The concept of using design solutions from the natural world is acceptable today but was considered a radical way of thinking in the early 1900s. At that time,
morphologists, studying plant and animal form and structure, were very apprehensive of reducing living things to mathematics and physics. Controversially, Scottish naturalist and mathematician D'Arcy Wentworth Thompson did exactly this in his book *On Growth and Form*, first published in 1917, [49]. Thompson criticised his contemporaries by saying "The zoologist or morphologist has been slow...... to invoke the aid of the physical or mathematical sciences..." and went on to say, "When the zoologist meets with a simple geometrical construction, for instance in the honeycomb......, he is deeply reluctant ...... to explain by geometry or mechanics the things which have their part in the mystery of life."

He also shocked his peers by stating, "Cell and tissue, shell and bone, leaf and flower, are so many portions of matter, and it is in obedience to the laws of physics that their particles have been moved, moulded and conformed......Their problems of form are in the first instance mathematical problems, their problems of growth are essentially physical problems, and the morphologist is, ipso facto, a student of physical science."

An example of his work saw Thompson describe four legged animal skeletons in mechanical terms and directly compare these to manmade bridges, stating that the animal’s legs were merely pillars with the skeletal spine representing the bridge’s span.

### 2.2.2 Biomimetics

The transfer of technology by mimicking creations from the natural world is now a growing area of research called biomimetics. This emerging field uses ideas and designs from nature and implements them into another field such as engineering, design or computing. However, straight technology transfer from the natural world by simply copying designs is not usually useful. This is largely due to the fact that the two scenarios are fundamentally different but share common features. Nature’s complex designs are often multifunctional and therefore man must identify and transfer the key design principles that are relevant to each individual case to serve the desired function. A good example are birds feathers, which have hollow quills enabling them to be both light and strong whilst at the same time providing thermal insulation for the bird. Clearly the need for thermal insulation of this kind is unnecessary in aircraft designs by man, and so simply copying the structure of a feather would prove unhelpful.
Nature's solutions are also predominantly made from composite, non-metals, constructed under ambient conditions in terms of temperature and pressure. Man is not restricted to these construction conditions or material limitations and should therefore be able to outperform nature's designs. However, it is easier to find evidence of the opposite, where time and again nature shows that we have a long way to go to even get close. One such example would be a solar cell compared to a plant undertaking photosynthesis. Each is essentially performing the same task, but the two efficiencies are not equal, with the plant being far superior, [50].

Early aviation attempts by man, including designs by Leonardo da Vinci, were all based on closely copying birds' feathers, wings and the way they were used, [51]. Flight was to be achieved by frantically flapping large manmade wing structures up and down, however, these early designs failed due to their size, (see Figure 2.1a). These large winged structures required a great amount of flapping power in order to achieve flight, far more than could be provided by man's arms and legs. In the natural world only smaller birds achieve flight by flapping their wings up and down regularly, larger birds tend to glide and soar on the winds' thermals, [51]. It was only later that this limitation of size was grasped and that the problem facing nature was more clearly defined and understood, enabling the useful elements from the biological design to be adopted correctly in the technology transfer. For larger flying machines, the wings needed to be fixed and controlled; this was the main reason the Wright brothers made such progress in aviation history (Figure 2.1b).

Figure 2.1 – Leonardo da Vinci's flying machine design (a) [52], and the Wright brothers successful 1903 glider (b) [53].
In addition to flight, the natural world has inspired the creation of many of man's designs, including bridges, buildings, materials, textiles and robots amongst others. Specific examples include drag reducing swim-wear inspired by shark skin [54, 55], stealth aircraft from the profiles of owl wings and feathers[56], miniature air vehicles from flying insects [56], an anti-reflective, anti-glare film based on the structure of Moth eyes and fuel cells designed from animal lung structures [56]. Four examples of biomimetics including Velcro™ [57], Gecko Tape [58], the Crystal Palace [59] and the Eiffel Tower [49] are discussed in detail in Appendix A.

2.2.3 Self-Optimising Structures

Structures in the natural world have been shown to be optimal solutions to the design problems that nature faces. Indeed, if they were not optimal solutions to their surroundings, then they would simply not have survived. These solutions have evolved over millions of years by the process of natural selection. Although survival of the fittest guarantees that the optimal solutions prevail over a long period of time, it does not allow for short term flexibility of design within the lifetime of a single generation. Natural solutions continually adapt themselves to best suit their environment, by the process of natural selection in the long term and adaptive growth in the short term. However, if a natural solution is to survive it must be able to respond quickly enough to its environmental changes; failure to adequately do so could result in extinction.

This ability to adapt to a changing environment, means that natural world solutions can be considered as self-optimising structures, something that is not currently possible to achieve within product design. Examples of self-optimising structures from the natural world include bones and trees, which will be discussed in greater detail later in Section 2.2.4 and Section 2.2.5 respectively.

Adaptive Growth

Natural selection effectively yields the blueprints of previously successful optimal designs to a set of previous environmental conditions, but it does not allow for short term corrections to shape or structure to best suit differing loading conditions. The flexibility of a design to react to new or altered loading conditions is achieved via adaptive growth, allowing life long adaptation of shape and structure, by either adding
more material where it is needed, adding no material where it is not needed, or even by removing material where it is no longer needed. Adaptive growth allows the load to be evenly spread throughout the structure, this situation gives rise to the *axiom of uniform stress*, [60]. This is the single most important design rule that nature follows when creating optimal structures – on average, over a period of time, stress acts uniformly over the surface enabling the load to be evenly distributed, thus ensuring there are no weak points in the structure.

### 2.2.4 Bones

Scientists have studied the various structures and microstructures of both human and animal bones for hundreds of years. Bones provide us with a great example of self-optimising structures from the natural world, being both lightweight and very strong. If bones were not optimised in terms of strength and weight they would face a similar scenario to the over dimensioned bracket component that was mentioned previously in Section 1.1. Excessive weight would expend additional energy during movement, not something an animal could easily afford to waste. An unnecessarily heavy animal would also be slower to move when compared to others around it, a potential death sentence in many cases. Nature’s survival of the fittest mentality is brutally efficient with no room for less than optimal designs. By enabling bones to self-optimise using adaptive growth, nature has allowed animals to only carry around the weight they need to from their skeletons. In many cases bones have hollow sections filled with cancellous tissue (marrow) making them lighter still – essential if energy is to be conserved for either hunting prey or fleeing predators alike. This is often the case in longer limb bones, particularly in legs, where this design has the advantage of being generally stronger and stiffer in bending than a similar solid sectioned bone.

**Wolff’s Law**

After studying the head of a human femur together, Von Meyer and Cullman proposed their *trajectorial theory of trabecular bone structure* [61]. This theory basically said that the internal ordered latticework of tiny ridges of bone, known as *trabeculae*, were orientated along the lines of principle stress. This concept was later used to influence the structure of the Eiffel tower (for further information see Appendix A). A German anatomist named Julius Wolff, who was a contemporary of Von Meyer and Cullman,
developed their theory even further. Wolff suggested that in addition to the trabeculae being orientated along the lines of principle stress, that the trabeculae could alter their orientation to accommodate changes in direction of stress. Wolff was the first to state that the structure of bone was capable of reacting to changes in stress, and went on to say that bones did this optimally so that they were able to endure their loads with the minimum amount of material. This general theory of bone adaptation was named after him simply as Wolff's law, [61]:

"Every change in the form and the function of a bone or of their function alone is followed by certain definite changes in their internal architecture, and equally definite secondary alterations in their external confirmation, in accordance with mathematical laws”.

Since Wolff's law, the successive studies of bone adaptation and remodelling have shown that bone density is highest in areas subjected to the highest shear stresses, this explains why many bones have hollow sections of cancellous tissue [62]. Later it was shown that specific cells were locally responsible for the addition of material (osteoblasts) and for the removal of material (osteoclasts) and that these cells can respond directly to mechanical strain [63]. The adaptive behaviour of bone growth has continued to be studied, leading some researchers to propose their own optimisation processes that mimic this behaviour [62, 63].

2.2.5 Botanical Trees

Every so often the weather systems in the British isles serve up a storm capable of ripping the roofs off houses and yet most trees can survive such storms, evidence that they are indeed optimum structures evolved to best suit their environment. To gain a competitive edge over other plants, trees grow taller, sending out their branches in all directions to present their food producing leaves to sunlight, thus driving the process of photosynthesis. There are literally millions of different types of trees with thousands of different species, however all share a common aim: to absorb as much light as possible using the least amount of material. The main function of any tree is to use its leaves to trap light energy and gather carbon dioxide, its root system to collect water and
nutrients and by the process of photosynthesis convert the mixture into a sugary starch food needed for growth, as shown by Figure 2.2.

Figure 2.2 – The process of Photosynthesis.

It follows therefore that a tree wishing to collect more light will try to spread more leaves over a greater area. In order to do this the tree will require longer branches to reach greater distances and thicker branches in order to physically support the added weight of having longer branches with more leaves. In turn this will require a longer, thicker trunk to support such branches and a more developed, larger root system to collect greater amounts of water and nutrients and to oppose the bending moment caused by the tree crown during winds, therefore stopping it from falling over.

The factors that dictate the extent of tree growth can be split into two distinct groups; those factors required for effective growth and those that inhibit such growth. The factors required for effective growth include light, water, minerals, nutrients and carbon dioxide gas, but it is the availability of these factors combined that ultimately dictates growth. There could be limited availability of these requirements due to a natural shortage, as with a lack of water in drought areas, or by competition for these requirements, such as light, due to high forest densities or where there is another tree species that is faster growing and therefore overshadowing. Rainforests are a good example of this being an environment in which trees thrive, but where young saplings only grow in locations where sunlight reaches the forest floor, created by gaps in the forest canopy by older trees falling down.
Factors that inhibit tree growth include the weather, soil type, pollution, abundance of predators that will eat leaves as well as the geography of the surrounding landscape. The weather dictates the amount of rainfall, the strengths and frequencies of wind and other external forces that act upon trees – such as the added weight of snowfall. The soil type governs the availability of nutrients and minerals, and determines the stranglehold the root system has in the ground. Pollution directly affects the water quality available for use in photosynthesis and this has already had a devastating effect on Scandinavian forests due to acid rain caused by high pollution levels. The abundance of predators that eat the leaves of a tree also inhibits growth, although many species have developed their own natural defence systems by growing thorns or using toxins. Finally the geography of the surrounding landscape is responsible for determining the levels of exposure to the elements and by limiting the factors required for effective growth as described above.

Optimal Behaviour of Trees

The greatest stresses that all trees experience daily are those induced by gravity and by bending moments caused by exposure to the wind. In order to survive, trees must develop structures strong enough to withstand these forces and achieve this by striving to minimise the impact of these external bending loads upon their structures. Throughout their lifetime, trees are constantly exhibiting optimal behaviour and are capable of doing this in a number of different ways, including growing upwards (negative geotropism), trunk tapering, having aerodynamic sections and by shedding leaves, [2].

If a tree grows predominantly vertically, opposing the direction of gravity, it keeps the centre of gravity of its crown above the centre of its root system. This behaviour will result in reducing the effects of gravity on the bending moment which would be caused by an off-centre crown as well as ensuring a taller tree structure that is more likely to out compete its rival trees in the competition for sunlight [64]. A tree that begins to lean to the side for one reason or another is often able to change its direction of primary growth to once again grow directly upwards in the future, as seen in Figure 2.3.
Exposure to the wind causes a considerable bending moment on a tree that is at its greatest at the base of the trunk. To compensate for this bending moment, the tree grows more wood around this base area than at the top, tapering the trunk. The more exposed a tree is to the wind, and the stronger the winds, the more tapered the trunk will become. Conversely, trees protected from the wind, such as those growing in a dense forest, usually exhibit very little noticeable taper. The direction of prevailing winds will also cause a tree to adopt an oval cross section, particularly around its trunk and largest branches, as shown schematically by Figure 2.4. This more aerodynamic shape will help to reduce the bending moment acting at the base of the tree that is induced by wind exposure, but only when the winds acting upon the tree are blowing in the same direction as the prevailing wind direction. The more dominant the direction and the strength of the prevailing wind dictates the degree of sectional shape optimisation. A tree in a location not subjected to any prevailing wind direction would have a circular cross section.
The shedding of leaves by deciduous trees in Autumn before the Winter months that bring stronger winds and less hours of daylight is also an example of trees exhibiting optimal behaviour for their environment. By shedding leaves the tree is able to reduce drag and therefore minimise the bending moment of the tree crown swaying in the stronger Winter winds. But not all trees shed their leaves, evergreens as the name suggests remain green all year round. They cope with stronger Winter winds by having smaller, finer leaves, (in some cases needles) that are inherently more wind resistant. As a result, it is not necessary for them to shed their leaves.

During the lifetime of any tree it is likely that its structure will be submitted to additional stresses induced by either encountering other objects during growth or by cracks developing during particularly windy conditions. Trees that encounter other objects during their growth, such as other trees, fences, walls, rocks etc. are immediately submitted to highly localised stress distributions. A tree's natural response to this situation is to grow extra wood at the point of contact of the foreign object to cushion the contact stresses, as seen in the examples of Figure 2.5.

![Figure 2.5 - Trees encountering objects whilst growing, (a) rock, (b) fence and (c) tombstone, [66].](image)

Over time, the enlarged contact area helps to spread the contact stresses more evenly until the axiom of uniform stress is achieved once again, [2]. A schematic example of this can be seen in Figure 2.6, where a tree trunk encounters a rock. If the foreign object is another tree, in some cases it is possible that the two trees can permanently join together.
Strong gusting winds are usually responsible for causing small cracks on the bark of a tree, these cracks induce notch stresses that unbalance the uniformity of stress distribution throughout the structure and threaten the very survival of the tree. A similar situation arises when part of the bark of a tree is removed, possibly by an animal biting or scratching at it. Underneath the bark of a tree is a growth layer of reaction wood called cambium. A tree subjected to notch stresses will immediately begin to react by laying down new wood over the wound, adapting its shape as fast as it can by reducing notch stresses. The tree tries to optimise its shape to one where the stress is uniformly distributed in order that it can survive the winds of the next storm, however, if a tree has insufficient time to react to notch stresses then crack propagation is likely to occur [2].

2.2.6 Review of Natural Optimal Structures

In comparison to bones, trees do not have hollow cross-sections, they obey a kind of Wolff’s law in that they are self-optimising by adding material where it is needed in a direct response to altered loading conditions and therefore different stresses. Although trees are capable of performing shape and structural optimisation via adaptive growth, they are not capable of removing material where it is no longer needed, this only happens naturally in trees by the process of decay. This difference in adaptive behaviour between bones and trees can largely be put down to the requirements of a tree being fundamentally different to those of an animal. Trees stand in one location for potentially hundreds of years, whereas animals have to move around in search of food and water, all the time carrying their weight around. It is much more important therefore for animals to be able to optimise their skeletal structures in terms of weight through subtractive means than it is for trees. A possible reason for trees to have solid sections is that they use their trunks and branches to transport the water they require to their leaves.
Now that optimisation in the natural world has been explored, a review of how mankind currently engineers product designs and implements optimisation into their structures is required. This is now presented in the following section.

2.3 Optimisation in Engineering

From an engineering point of view, products are designed to meet a set of design specifications identified early in the design process. These design specifications are ultimately the measures by which the finished product will be evaluated when determining the successfulness of the design. However, a successful design may not necessarily be an optimal design. As mentioned in Section 1.1, components are often over engineered in an attempt to ensure fitness for purpose within acceptable parameters [2]. This common practice causes an inefficient use of material with designs often having an excess of weight, a far from optimal scenario. In addition, components usually display a non-uniform stress distribution, which includes regions of low stress that are not fully loaded, and regions of high stress that are excessively loaded and therefore close to potential failure. This situation suggests that good mechanical design will result in an optimal structure, i.e. one in which there are no under loaded regions (excessive weight) and no overloaded regions (highly stressed) [2]. It follows therefore, that in order to achieve an optimal mechanical design, the axiom of uniform stress, Nature’s single most important design rule, must be obeyed. Hence, only material that is required is present and that material is located exactly where it is needed within the structure, without there ever being an excess.

Although there are many different types of design optimisation including, minimum weight, maximum strength, maximum stiffness, minimum resistance to flow, thermal conductance and acoustic transmission, they all require an optimum geometry and topology in order to achieve their goal (in addition to the appropriate material being selected). Before discussing structural optimisation, it is necessary to have a clear understanding of geometry and topology and the differences between them; hence these will now be discussed.
2.3.1 Geometry & Topology

The differences between geometry and topology are more easily explained using a simple example. Consider a cube; there are three types of geometric elements needed to define the cube, which are points, lines and planes. Eight points define twelve lines that in turn define six planes. By contrast, the topological definition of the cube is a region of space bounded by six planes bounded by their intersecting edges and connected to each other at their edges. The edges are derived from the intersecting faces. The edges are bounded by vertices and connected to each other at their common vertices. While the geometric elements of the cube consist of points, lines and planes, the topological elements consist of faces bounded by derived edges and vertices and how those faces, edges and vertices relate together. Therefore, geometry studies concentrate on the area, angle and other geometric properties of the cube whilst topological studies try to identify the relationship, or connectivity between the vertices on different faces [67].

The topological properties of an object do not necessarily change in respect to associated changes of the object’s geometric properties, with parametric changes in geometry often resulting in an unchanged topology. This phenomenon is demonstrated in Figure 2.7 by considering the simple cube example again, where various parametric changes to the geometry of the cube do not alter the resulting topology from that of the original cube. This is shown in Figure 2.7 by the 1st and 5th numbered vertices (in red) remaining connected to one another by the same edge (highlighted in yellow), despite parametric changes in geometry. Hence the four different geometries of Figure 2.7 are said to be topologically equivalent with each another [67].

Figure 2.7 - Constant topology despite parametric changes in geometry.
2.3.2 Structural Optimisation

Many optimisation problems within engineering design rely heavily on FEA software to identify areas of weakness in the design or areas of excessive material. One such area is the field of structural optimisation. The use of FEA programs within optimisation allows for hundreds of design variables and hundreds of constraints to be solved simultaneously. The field of structural optimisation encompasses numerous problem types, but all of them can be placed into one of three sub-categories namely, sizing optimisation, shape optimisation and topology optimisation, [68]. A brief overview of each of these three topics will now be presented in turn.

Sizing Optimisation

Sizing optimisation is typically applied to a truss type structure to obtain the optimal cross-sectional areas of beams [68]. In this case the sizing design variable would be the beam’s cross-sectional area, but frequently the material thickness of a plate or sheet is also used [69-71]. The approach has been successfully applied to the structural design optimisation of wind turbine towers by Negm & Maalawi, where the cross-sectional areas of tower segments were used as sizing design variables [72]. A schematic example of sizing optimisation applied to a truss structure can be seen in Figure 2.8.

![Schematic of sizing optimisation](image)

Figure 2.8 – Schematic of sizing optimisation, [73].

It is often desirable to limit the design variables to standard available sizes, resulting therefore, in the use of discrete variables to represent the standard sizes available [74]. Sizing optimisation problems are considered to be relatively straightforward as they do not require a change in the FE model of a structure as it is modified, (the FE mesh is unchanged and therefore there is no requirement to re-mesh).

Shape Optimisation

Shape optimisation is more advanced than sizing optimisation in that it determines the optimal boundaries of a structure for a given fixed topology. Design variables are typically geometric parameters, such as spline control points [75], defining the shape of
a structure in either 2D or 3D [68]. The shape optimisation technique has been applied by Rispler et al. in the design of adhesive fillets [76], by Waldman et al. in the design of shoulder fillets in flat plate plates [77], and by Jones et al. in the design of holes in plates for the consideration of fracture strength [78]. In each of these cases, spline control points were used as design variables in order to alter each shape boundary. A schematic example of shape optimisation is shown by Figure 2.9.

![Figure 2.9 - Schematic of shape optimisation.][73]

Unlike sizing optimisation, shape optimisation changes the FE model and therefore requires changes to the FE mesh. This will increase computational work and associated time due to difficulties in integrating the mesh regeneration and subsequent FEA into the optimisation method [79]. The combined size and shape optimisation of various truss structures for bridge design applications was conducted by Gil & Andreu [80]. In this study, strut cross-sectional areas were used for sizing design variables, whilst nodal co-ordinates corresponding to strut locations were used as shape design variables.

**Topology Optimisation**

Both sizing optimisation and shape optimisation, however, suffer from the disadvantage of being completely dependent on the initial structure and are therefore unable to introduce additional holes within the structure for the purpose of reducing weight. Topology optimisation was developed to overcome this deficiency and is consequently a much more powerful design tool, [68]. Topology optimisation is able to offer the best topology in addition to sizes and shapes, independent of the initial starting design structure. This enables these methods to be used earlier in the design process (at the conceptual stage) when compared to either sizing or shape optimisation. As with shape optimisation, topology optimisation causes changes to the FE model and therefore requires increased computation. A schematic of topology optimisation applied to a simple beam can be seen in Figure 2.10.
Out of the three structural optimisation sub-categories, topology optimisation offers the greatest opportunity for creating novel designs as a result of it being completely independent of any initial design, unlike both sizing and shape optimisation which rely on an initial design as a starting point. Consequently, the field of topology optimisation is the most relevant area of structural optimisation for this research, as it has the greatest potential for exploiting the increased design freedom afforded by RM. Topology optimisation will therefore need to be inspected in depth. However, before doing so, a greater understanding of the basic principles of optimisation and some of the more appropriate techniques developed by mankind is required. These topics are now presented.

2.4 Optimisation Principles & Techniques

Optimisation can be thought of as a process of improvement, applied in order to make existing ideas even better. Engineers create initial design ideas or concepts and use the process of optimisation to improve those ideas until an optimal solution is obtained [81]. For there to be an optimum or best solution implies that there are in fact numerous possible solutions to the problem and that these solutions are not of equal value or quality. Clearly some solutions are useful whilst others are not. Of all of the possible solutions to a design problem, many solutions could simply be unacceptable or infeasible due to zero or negative parameter values or because defined design constraints are broken and hence one or more design requirements are not met. Solutions that meet all of the requirements are called feasible (acceptable) solutions, however, these feasible solutions still vary in quality or value, [82]. Within this feasible group, solutions range from the worst to the best, or optimal solution.

Some simple problems have exact answers or distinct roots and therefore the best solution has a clearly defined answer. Other more complicated problems have various minimum or maximum solutions known as optimal points or extrema. Sometimes the
best solution maybe a relative definition, based on the subjective view of an individual user, hence the optimal solution could depend on the perception of the person formulating the problem.

2.4.1 Objective Function

Where there are many feasible design solutions for a system and some are better than others, a measure is required that can be used to compare and evaluate the differences between the various solutions on offer. The measure used is a scalar function of all of the various design variables needed to describe the problem analytically. This measure is called the objective function for the optimum design problem (sometimes called the cost or goal function) and is usually represented by \( f \) or \( f(x) \) to emphasize its dependence on a design variable, vector \( x \), [82]. As the optimisation process progresses, it is the objective function that will either be maximised or minimised.

Selection of a suitable objective function is an important decision in the design process as this identifies the subject of the optimisation process. Common objective functions include, minimise cost, maximise profit, minimise weight, minimise energy expenditure etc. In many situations an obvious objective function can be easily identified. For example, it is always desirable to minimise the manufacturing cost of a design, or to maximise the return on an investment.

There are many optimisation methods that exist and these can be simply divided into derivative (gradient based) and non-derivative (stochastic) methods. Some optimisation methods require the derivative of the objective function to be found in order to calculate the optimum solution (or even the 2\(^{nd} \) derivative in some cases). Other methods only require the objective function itself in order to calculate the optimum solution. Often the derivative of the objective function simply does not exist or is very difficult to find, particularly in engineering design problems, hence non-derivative methods are generally more suitable to these types of problems, therefore making them more relevant to this research [81]. An advantage of non-derivative methods is that they are more likely to find the global optimum and not get stuck or stagnate on local optimums as gradient based (derivative) methods often do. In addition, non-derivative methods
are generally able to handle discrete variables as well as continuous variables, whereas derivative methods can only handle continuous variables.

2.4.2 Derivative Methods

There are many variants of derivative or gradient based methods, (also called deterministic or hill climbing), but all start with a single solution to the design problem, represented by a single point in the search space and search for a better solution in the direction of the gradient of the objective function. If the new point has a better value of the objective function, it becomes the current point and the process is repeated.

To help explain gradient based optimisation methods, the schematic 2D example depicted in Figure 2.11 should be studied. The search starts at a single point on the 2D curve indicated by the green cross. The algorithm determines the initial search direction by calculating the local gradient (derivative of the objective function) around the starting point and travelling in the direction of the steepest descent for a minimisation problem. The algorithm moves in the direction of the red arrows to the next cross on the curve where the gradient is inspected again and the process repeated. After several iterations, the search position reaches a minimum location, where the gradient of the curve is zero. At this point the search terminates.

![Figure 2.11 - Schematic example of a gradient based optimisation process.](image)

The process in 3D is analogous to being on a hilly surface with the objective of finding either the maximum or minimum position on the surface. The contours of the hills
represent the design parameters and design constraints can be considered as fences which cannot be crossed. The algorithm can be considered as being a hiker, roaming the hilly surface as it searches for the required optimum position.

Gradient based methods are considered efficient, because they require just a few evaluations of potential solutions. However, because they are unable to search the entire search space they are often unable to find the global optimum that is sought. The solution obtained is entirely dependent on the initial starting location and it’s associated gradient. This is illustrated in Figure 2.11 by the two possible starting positions numbered 1 and 2. The first starting position would yield the desired global minimum but the second starting position would yield an inferior local (sub-optimal) minimum solution. The local minimum traps the algorithm, terminating the search prematurely before the global minimum can be found. To improve results, the search computation is usually repeated for a number of different starting points within the search space, in the hope that one of them is initially close to the global optimum.

2.4.3 Non-Derivative Methods

Non-derivative methods are also known as stochastic or probabilistic methods as they introduce randomness or chaos through the use of probabilities into the optimisation process. This chaotic behaviour enables non-derivative methods to exhibit a random search direction, unlike derivative techniques which follow predictable and logical search directions based on the local gradient. The ability to perform a random search direction, in contrast to what might be expected, gives non-derivative techniques the ability to escape local optima by effectively ‘jumping away’ from them in the search space.

Numerous non-derivative techniques exist, with many being inspired by observations from the natural world, therefore providing further examples of biomimetics. Specific techniques include Simulated Annealing (SA), Ant Colony Optimisation (ACO), Genetic Algorithms (GAs) and Particle Swarm Optimisation (PSO). These techniques will now be discussed in turn along with their potential suitability for this research.
Simulated Annealing

Based on initial work by Metropolis et al. in the 1950s [83], the method of SA was introduced by Kirkpatrick et al. in the early 1980s [84] and is derived from the annealing process of metal. In the annealing process, a metal is heated above its melting temperature and then gradually cooled. Throughout the cooling process, the atoms within the metal rearrange themselves until they produce a crystalline lattice, (composed of millions of atoms perfectly aligned). The creation of the lattice, a structure of increased order, results in lowering the energy level of the metal.

The SA optimisation method simulates the annealing process of metals, by minimising the energy level (corresponding to the objective function) of the system. The design variables are equivalent to the possible states for an equilibrium temperature. The process begins with a random guess of the objective function variable values; if a change of state leads to a lower energy level (improved objective function), it is immediately accepted. However, it is also possible for the method to accept a solution which produces a worse value of the objective function. The probability of accepting a worse solution with a higher energy level is small but does exist and is governed by the temperature of the system. As the annealing process is simulated, the temperature is gradually lowered, thus allowing the acceptance of worse solutions with greater probability at the beginning of the process and with smaller probability later on. The ability of the process to accept a higher energy level helps to prevent the system from getting trapped at a local energy minimum and therefore provides a means of finding the global optimum.

From a practical point of view, SA has a good chance of finding the global optimum and the solution obtained does not depend on the starting point for the search. In terms of engineering applications SA has been used for tackling a wide variety of problems including some in the area of structural optimisation, however, these tend to be limited to either sizing or shape optimisation, or a combination of both, and are seldom applied to topology optimisation. An example is given by Shea & Cagan in their Shape Annealing approach to the design of novel roof trusses [85] where SA has been used for both sizing and shape optimisation.
Ant Colony Optimisation

As ants walk they deposit a chemical pheromone on the ground, leaving a trail behind them wherever they go. Other ants are able to home in and follow these pheromone trails, leading them to food sources that have been found by previous ants along the shortest possible routes, [86].

Consider the case where there are three possible paths to a food source with one path shorter than the other two. Initially, the ants choose each path with an equal probability, and in doing so pheromone is deposited along all three paths. Ants that use the shortest path to travel to and from the food source will cause it to have the greatest amount of pheromone in the least amount of time. Since concentrated pheromone is better at attracting ants, more and more ants will choose the shortest path, as this path has the strongest pheromone trail. The increased number of ants on the shortest path has the effect of reinforcing the pheromone level on that path even further and therefore increases the probability of future ants choosing that path. As time goes on, the pheromone trails begin to degrade and disappear and, eventually, the only path that will have any trail remaining will be the shortest one, the one that is being constantly replenished by more ants travelling along it. In the end all the ants will follow the shortest path to the food source [87].

ACO was formed by Dorigo & Gambardella in 1997 [88] to simulate this behaviour in ants and is illustrated in Figure 2.12. Initial attempts at constructing an ACO algorithm were not very successful. They often converged prematurely on a sub-optimal solution as a result of too much virtual pheromone being laid too quickly [89]. To combat this inefficiency, a pheromone evaporation system was implemented, so that after a period of time the pheromone associated with a solution begins to disappear, thus allowing the algorithm to converge on a globally optimal solution (the shortest path). If the evaporation rate of the virtual pheromone is not fast enough, the algorithm will unavoidably converge on an inferior solution [89].
Figure 2.12 – Ant Colony Optimisation process with pheromone levels represented by lines. Four stages as follows; (a) ants arrive at decision point, (b) ants randomly choose both paths, (c) ants begin to choose shorter path over longer path and (d) pheromone accumulates quicker on shorter path, [88].

The first ACO algorithms were designed to solve travelling salesman type problems [90] (described further in Section 2.6.3), as these problems closely resemble finding the shortest path (solution) to a food source [88]. Travelling salesman type problems form the main area of ACO applications, though other examples can be found in literature [91]. However, all ACO applications involve the use of discrete variables [92], resulting in there being few applications in the field of structural optimisation. One of the few examples found in literature is presented by Serra & Venini, in their recent study of ACO applied to plane truss structures [93]. Here, truss sectional areas were considered at various discrete values, thereby enabling the most optimal sized truss section to be selected from those available.

Genetic Algorithms

Genetic Algorithms (GAs) are general search and optimisation routines based on the mechanics of natural selection and the Darwinian theory of evolution. GAs were initially developed by John Holland in the late 1960s and early 1970s at the University of Michigan. In 1975, Holland published *Adaptation in Natural and Artificial Systems*, [94], which sparked the beginning of the GA movement. The popularity of the technique rose following the successful application of a GA to a difficult gas pipeline transmission problem by Goldberg, a student of Holland, in 1989 [95].
GAs are different from many other search and optimisation methods in that they maintain a population of solutions throughout the entire process. Instead of processing a single solution in the available search space, they perform a multidirectional search, therefore increasing their chances of finding the global optimum solution by exploring more of the entire search space when compared to other methods [81].

GAs assume that the genetic pool of a given population potentially contains the optimum solution or an improved solution to the original. The operation of a typical GA is made up of three phases: initialisation, fitness determination and reproduction. A GA begins by creating an initial random population of solutions. The individual subjects that make up the population are called Phenotypes. Each phenotype is represented by an encoded (usually binary) string of ‘genes’ or ‘chromosome’, known as a Genotype. All of the individuals within the population are evaluated in terms of fitness, measured by a predefined fitness function, with the highest fitness values assigned to the best individuals. The GA then selects pairs of individuals, based on fitness values, from the population which will be allowed to reproduce (become parents) and applies genetic operators to them creating a new population of individuals (offspring). Following the initialisation phase, the fitness determination and reproduction phases repeat cyclically until some predefined termination criteria is satisfied and an optimum solution is obtained [96].

Each iteration of the process is called a generation. Each new generation will, as a result, contain less poor genetic material in its makeup, filtered out by what could be considered as selective breeding [97]. This increases the overall fitness of each new generation when compared to its parent generation and is an artificial version of natural selection, Darwin’s theory of survival of the fittest.

Most GAs are variations of the Simple Genetic Algorithm proposed by Goldberg in 1989 [95]. During the reproduction stage of Goldberg’s simple GA, the genetic operators of crossover and mutation are applied to the encoded strings of the selected parent phenotypes based on probabilities. Crossover essentially recombines the two parent chromosome strings, whereas mutation can introduce random changes to the strings themselves. Mutation enables the GA to escape local optima by introducing new
potentially useful genetic material into the existing genetic pool, hence ensuring population diversity is maintained [95].

Depending on how the GA is constructed, it may be possible for it to generate individual solutions that do not meet some or all of the problem’s constraints, (infeasible individuals). Where this is the case it is fairly common to introduce a penalty system that penalises individuals depending on how much they disobey the constraints [81, 98]. Selection of parents is then based on a weighted sum of fitness and penalty values. GAs can also often differ in the way that they create a new population. The most common method is to create a new population with the same number of individuals as the previous population purely by applying genetic operators to selected individuals. However, some GAs add new individuals to the current population, thus extending it in size, and then delete the least fit individuals to create the new population. There are even some GAs that apply a continuous replacement approach to individuals within a population and therefore do not use generations at all [81].

GAs have become extremely popular in many areas of maths, computing, science and engineering. This popularity is largely due to their robustness when solving a wide range of problems, in addition to their ability to handle either discrete or continuous variables [81, 95, 96]. Although not guaranteed to yield the globally optimum solution, a GA is often likely to return highly acceptable results. GAs are often applied to structural optimisation problems, however, most applications in this field tend to start with an existing design and use its geometrical parameters as the design variables to optimise. Such an application is considered to be either sizing or shape optimisation as it is completely dependent on the initial structure’s topology. Consequently, this method is unable to offer novel or original design solutions [99]. An example of this type of application is presented by Lampinen, [100], where the profile shape of an existing CAM component was successfully optimised using a GA. Some applications of GAs within the field of topology optimisation are present within literature [99, 101-103], indicating that GAs have good potential for application to this research.
Particle Swarm Optimisation

Initial simulations of bird flocking behaviour by both Reynolds [104] and Heppner & Grenander [105], inspired Kennedy & Eberhart to propose the PSO technique in 1995, based on animal group behaviour including fish schooling and bird flocking [106]. Reynolds studied the aesthetics of bird flocking choreography in his simulations, whereas Heppner & Grenander tried to discover the underlying rules that enabled large numbers of birds to flock synchronously, taking into account sudden changes in direction, scattering and regrouping. In the PSO technique, Kennedy & Eberhart suggested that the "social sharing of information among conspeciates offers an evolutionary advantage". Experimental analysis led Kennedy & Eberhart to develop this principle and realise that rather than being solely a social simulation, they had discovered a powerful new search algorithm which was capable of optimising a wide range of multidimensional problems [106].

The PSO method considers a population of potential solutions (referred to as particles), that fly around in a multidimensional search space. Throughout flight, each member of the population continually adjusts its position and velocity according to its own experience and to the experiences of other particles within the population. The PSO system therefore enables individual solutions to benefit from the knowledge and experiences of all of its members, with the resulting appearance of the population seeming to move together, despite each particle having a randomness associated with its behaviour. If we consider the example of a flock of birds, as shown in Figure 2.13, and apply the PSO approach to it, one member of the flock landing at a food source will result in other flock members modifying their flight paths in order that they descend (converge) on the successful individual, thus increasing their own individual chances of success and that of the combined flock.

Figure 2.13 - A flock of European Starlings in flight. [107].
The PSO approach is similar to that of GAs in that it maintains a population of individuals. However, unlike GAs, the swarm population is fixed in size and individuals belonging to the swarm (the particles) cannot be replaced. As the PSO approach is based on animal group behaviour and not on the process of evolution, there is no need for evolutionary operators such as crossover and mutation. In PSO, the particles deal directly with the problem variable values themselves and are not encoded representations as with GAs. The PSO approach has proved itself popular, particularly within the field of computer science. This has lead to the development of several PSO variations where specific behaviour has been observed from various natural swarms. Techniques include the Mosquito Attack Optimisation process by Bandyopadhyay et al. based on the method used by female mosquitoes to identify and attack their targets [108], and the Honey-Bees Mating Optimisation algorithm by Haddad et al. which is based on the mating process of honey bees [109]. The PSO approach is still relatively new in comparison to GAs, with far fewer applications found in literature, particularly in the field of structural optimisation. However, a recent study of PSO applications by Poli [110] has shown that the uptake of this method has grown exponentially in recent years, a trend that is anticipated to continue.

2.4.4 Multi-Objective Optimisation

The optimisation problems considered so far have all been of the form where, although many parameters might be being optimised in parallel, the quality of any particular solution can be measured by a single objective function. This approach works well for many problems, but not all problems are able to reduce to a single objective function. There are times when several criteria are present simultaneously, where there may appear to be two or more objective functions and it is either not possible, or wise to combine these into a single objective function, [95]. For example, we may wish to minimise the stress or deflection at a specific point in a structure and at the same time minimise the overall weight of the structure. When this is the case, these problems are said to be multi-objective or multi-criteria design optimisation problems, and have regularly attracted the attention of many researchers, [111-115].
Pareto Optimal

In multi-objective optimisation problems the concept of optimality is usually unclear as the optimisation process is required to perform a simultaneous improvement of all of the objective functions. The objectives are often conflicting so that an optimal solution in the conventional sense does not exist, as there is usually no single solution that is optimum with respect to all of the objectives. Trade-offs must be made based on the relative importance of one objective against another. This concept is known as *Pareto Optimality* [116]. From a multi-objective optimisation problem there will exist a set of optimal solutions, known as Pareto optimal (P-optimal) solutions or the Pareto Set. The goal of any multi-objective optimisation therefore, is to find as many of these P-optimal solutions as possible. Without additional information, all of these P-optimal solutions are equally acceptable, but the final selection of a particular solution from the Pareto Set is left for the user to decide based on valued judgement and experience.

A solution is considered P-optimal if no other solution dominates that solution with respect to the objective functions. A solution is called non-dominated if it is better in at least one objective than any other solution and no solution can be found that dominates it, it is therefore a member of the Pareto Set [95]. This definition is best explained with a simple diagram. Figure 2.14 shows all of the feasible solutions to a multi-objective optimisation problem that has two clear objective functions that require minimisation.

![Diagram of Pareto Optimality](image)

*Figure 2.14 – Feasible solutions to a multi-objective optimisation problem, showing the Pareto Set and the Pareto Front formed by the non-dominated solutions, [117].*
Solutions that are closest to the Y-axis are the most optimal solutions to the first objective function, however, some of these solutions (towards the top left) are not very optimal in terms of the second objective function. Similarly, the same can be said of points closest to the X-axis, they are optimal for the second objective function, but not all are optimal (towards the bottom right) in terms of the first objective function. The blue points represent Pareto optimal solutions to the multi-objective optimisation problem. These points form the Pareto Set with the line that connects these points being the Pareto Front. Solutions, or points that do not belong to the Pareto Set are dominated by the Pareto optimal solutions. It is from the Pareto Set that the user must choose the most appropriate solution.

2.4.5 Algorithm Review

This section aims to identify the most suitable algorithm for the intended research based on the information presented previously. In Section 2.4.1, non-derivative methods were already acknowledged as being preferable for this research compared to derivative methods, due to their ability to avoid local optima as a result of exhibiting stochastic search direction behaviour. Of the non-derivative methods inspected, SA is the only technique which performs a single search, a clear disadvantage compared to the other three methods. Equally, ACO is the only technique that exhibits difficulty in handling continuous variables. Whilst at this stage it is not known whether the variables will be either discrete or continuous, it is clearly not beneficial to select a technique that directly limits this choice. Furthermore, the conducted review of literature has highlighted the lack of applications of both SA and ACO to the field of structural topology optimisation.

Of the two remaining techniques, both GAs and PSO conduct the more favourable multi directional search, and both are capable of handling either discrete or continuous variables, hence either technique could be used in this research. However, due to the relative infancy of PSO compared to GAs, far fewer PSO applications can be found in the field of structural optimisation, let alone topology optimisation. Furthermore, GAs have proven themselves to be highly robust at solving a wide range of problems and have benefited from greater development due to their relative maturity, including their application to multi-objective problems. Hence, GAs were selected for use in this
research and will be explored in greater depth in Section 2.6. However, before doing so, it is now necessary to investigate the topic of topology optimisation further, as this was previously identified in Section 2.3.2 as being the most relevant area of structural optimisation to this research.

2.5 Topology Optimisation

Compared to sizing and shape optimisation, structural topology optimisation is a relatively new area of research which is also often referred to as configuration design [118]. Despite this fact, however, there is evidence that mankind recognised the need for an optimal structural layout (ideal topology) over 100 years ago in the work by Australian mathematician Michell in 1904 [119]. Michell’s work considered the theoretical optimal layout of minimum weight, planar truss structures that transmit a specified load without exceeding limits on the axial stresses in the bars. He devised conditions of optimality for simply loaded pin-jointed structures, including; “A more general class of optimal frames...consist of those whose bars,...form curves of orthogonal systems”. Commonplace within Michell’s theoretically ideal solutions are bars crossing over one another perpendicularly, in much the same way that the trabeculae latticework was observed to be arranged within the head of the human femur (see Section 2.2.4 and Appendix A for further information). An example of a Michell optimal layout solution structure can be seen below in Figure 2.15.

Later, in separate studies around the same time, both Hemp [121] and Prager [122] showed that Michell’s optimality conditions applied to only limited types of discrete structures where simple load cases and constraints were considered. Furthermore, Michell structures were shown to be impractical because they typically contained an
infinite number of bars. However, despite these limitations, Michell-type problems and solution structures are still widely seen within structural topology optimisation literature, as they are often used as a benchmark, examples include [2, 73, 120, 123].

Topology optimisation can be broadly classified into either discrete or continuum configuration problems. In discrete topology optimisation methods, the structure is modelled using discrete truss/beam/bar/strut/column elements, whereas in continuum topology optimisation methods, the structure is modelled as a continuum of distributed material. Both discrete and continuum topology optimisation methods have undergone extensive research and development in the past two decades, due in no small part to the dramatic growth in computing technologies. For a recent state of the art review, see Kicinger et al. [124]. Discrete and continuum structural topology optimisation methods will now be discussed in the following sections along with their potential suitability for this research.

2.5.1 Discrete Topology Optimisation

The origins of discrete topology optimisation approaches date back to the work by Michell in 1904 [119]. The previously mentioned limitations of Michell’s work were overcome by Dorn et al. by introducing a ground structure approach [125]. A ground structure consists of a grid of points (nodes) that represent joints, supports and loading locations, that are connected together by a finite number of structural members (trusses/beams/bars/struts/columns). Example ground structures exhibiting sparse and high connectivity of nodes can be seen illustrated in Figure 2.16.

![Figure 2.16 - 2D ground structures displaying (a) sparse connectivity and (b) high connectivity, [123].](image-url)
In the approach, all nodal locations remain fixed whilst unnecessary members are identified and subsequently removed from the ground structure [126-128]. Removal of members alters the connectivity of the structure, hence changing the overriding topology. Often a secondary sizing optimisation process is run on the resulting topology to determine the optimum cross-sectional areas for the remaining members.

Discrete topology optimisation using the ground structure approach has witnessed considerable research and development over several decades, with applications considering stiffness, displacement, stress, buckling and frequency problems. For reviews, see [123, 126-130]. These applications exist in spite of the many acknowledged limitations of the ground structure approach [131], which include:

- The optimal topology generated strongly depends on the initial ground structure.
- Optimal solutions are often obtained that are unrealistic.
- Ideally, a highly connected ground structure is needed which means too many members and nodes are present.
- If too many members are removed, the resulting truss becomes unstable.
- The introduction of additional nodes and/or members is extremely difficult.

A recent example of a discrete topology optimisation approach combined with sizing optimisation is presented by Wang [132]. Wang’s initial ground structure was based on a repeating Octet Truss unit cell proposed by Deshpande et al. [133]. A single octet truss unit cell is illustrated by the red geometry in Figure 2.17a whilst Figure 2.17b shows a 3×3×3 array of repeated unit cells. In Wang’s approach, cylindrical strut members were analysed for buckling failure, where strut diameters were represented as continuous variables, optimised using the PSO algorithm. Strut members were removed from the octet truss ground structure when individual strut diameters approached zero. Wang considered stiffness and compliance applications with varying success including a 2D cantilever beam as seen in Figure 2.17c, a 2D wing profile and a 3D hip bone scaffold [134, 135].
2.5.2 Continuum Topology Optimisation

Continuum topology optimisation is a numerical simulation technique that generates the optimal topological shape of a structure for a given mechanical load. The basic method solves the common engineering problem of determining the optimum arrangement of material distribution, with a limited amount of structurally isotropic material, within a given design space. Generating a structure’s topology that makes the best use of the material available introduces the concept of fully stressed design where, ideally, the stress in every part of a structure should be equal. This concept directly equates to the axiom of uniform stress [60] that was witnessed in natural optimal structures (see Section 2.2.3) and to Wolff’s Law, the general theory of bone adaptation [61]. Material that is lowly stressed is assumed to be underloaded and should therefore be removed, whereas regions of the design that are highly stressed are overloaded, requiring the addition of more material. By gradually removing inefficient material (underloaded), and lowering the stresses at overloaded regions, the stress level in the developing structure becomes increasingly uniform, and results in the structure’s topology evolving towards an optimum. Typically, this iterative process is implemented by either addition or removal of elements from an FE model.

When compared to discrete topology optimisation methods, continuum topology optimisation techniques offer greater topological freedom, due to them not being reliant on an initial ground structure. As a result, continuum topology optimisation methods have the greatest potential for application to this research and will now be considered in greater depth.
Homogenization Method

Whilst optimising the spatial thickness distributions of plate structures, Cheng & Olhoff recognised that regions of zero thickness were essentially holes in the plate structures [137]. Following this discovery, Kohn & Strang [138] first proposed using spatial distributions of design variables to change and optimise the topology of material distributions in a structure. Hence, the basic idea for continuum topology design was conceived which was later demonstrated as a computer based approach in 1988 by Bendsøe & Kikuchi, called the Homogenization Method, [139] which was intended for the minimum weight design of structural components. This method was developed for the creation of composite materials by introducing various microstructures within a material, based on a variable density approach. Instead of elements representing purely material being present (1) or a void (0), elements are allowed to have various densities (ranging from 0 to 1) throughout the optimisation process, yielding a solution in the form of a perforated composite material with a distributed microstructure.

From a conventional manufacturing point of view, the perforated regions within the composite structure representing the regions of intermediate density have little practical value as they are impossible to manufacture. This is a widely acknowledged limitation of the homogenization method [118, 132, 140]. Consequently, intermediate densities are often forced towards either a 1 or a 0 by the implementation of some form of penalty function, in order to define a final topology that is manufacturable. The most widely used approach is the Solid Isotropic Material with Penalisation (SIMP) technique which implements a power law penalty to intermediate material densities [141, 142].

Bendsøe later described the homogenization method as a material distribution problem [141]. Since its conception, considerable progress and development has been achieved with numerous variations to the original approach including the SIMP technique. For reviews, see [123, 143-145]. Notable works include the extension of the basic approach to 3D design domains by Mlejnek & Schirmacher [146], the solving of single and multiple eigenvalue vibration problems and the extension of the method to anisotropic plate optimisation problems by Tenek & Hagiwara [147, 148].
The redistribution of material within the design space is governed by some form of rejection criteria that is specific to each individual method and usually based on elemental stress/strain values. In the default methodology, a given amount of structural mass (the material volume fraction of the design area), is used to optimise a desired property of the structure. Frequently, the objective function is to maximise the structure’s stiffness for a given load case, or to minimise the structure’s weight for a given allowable displacement.

The design’s topological shape is generated within a predefined design space often called the design domain. In order for the material distribution and therefore the mass and associated structural behaviour to be represented adequately, the design domain is divided into many small discrete elements. The FE method is used to discretise the design domain, forming a mesh. The user then defines the loading and boundary conditions for the specific case and applies these to the necessary elements within the mesh, thus completing the FE model. Without any further guidance or decision making required from the user, the optimisation process systematically and iteratively eliminates and redistributes material throughout the design domain until a suitable structure is obtained (with regards to the objective function and the design constraints), thus providing the user with an initial idea of an efficient geometry, [123].

One of the problems associated with representing a structure’s topology by a collection of discrete elements is that there is a lack of boundary smoothness resulting in a structure with jagged edges; this is rarely acceptable in engineering applications. An example solution structure exhibiting jagged edges can be seen below in Figure 2.18.

![Figure 2.18 - 3D cantilever beam problem and jagged solution using the SIMP technique, [149].](image-url)
Using an increased number of smaller elements (mesh refinement) may help to overcome this problem, but will require additional computation and therefore increase solution time. Often it is prudent to begin with a coarse mesh to generate design structures quickly and then apply a higher fidelity (finer) mesh to improve the quality (smoothness) of a solution. Alternatively, a secondary smoothing step is sometimes performed as a shape optimisation process, once the optimal topology has been defined.

As mentioned earlier, topology optimisation is still a relatively young area of research and although several papers reporting industrial applications of topology optimisation have been published, the method itself, despite its attractiveness, is not widely known outside of the topology optimisation community. Tcherniak & Sigmund, [150], suggest that there are two main reasons for this. Firstly, commercially available topology optimisation software (e.g. Optistruct by Altair Engineering) is expensive and requires substantial training, making it unaffordable to many small companies. Secondly, as the topology optimisation method is still in its infancy, it is seldom mentioned in most modern text books on optimisation and is rarely taught on both undergraduate and graduate degree courses. As a result, many design engineers and students are oblivious to the method.

**TopOpt Website**

In an attempt to spread the concept and ideas of topology optimisation to an increased user group of designers in various fields of engineering, Tcherniak & Sigmund have created a topology optimisation program called *TopOpt*, which is publicly available over the Internet as an interactive design tool, [151]. This website allows users to access software that uses the material distribution topology optimisation method to solve linear elastic topology optimisation problems of statically loaded mechanical structures. Users are able to use the program directly through the web browser interface for 2D problems, but are required to download the 3D solver.

The program, described in detail by Tcherniak & Sigmund in a recent paper [150], is capable of solving standard compliance optimisation problems with up to three separate load cases. For more advanced optimisation problems or a greater number of load cases, the website directs the user towards a commercial topology optimisation software
The user defines the size and shape of the design domain (pale blue rectangle), and can specify any areas within that domain that are off limits with respect to material distribution (white rectangles), or any areas where material must remain present (blue rectangles). The user then defines the load cases and supports that are acting on the design domain and sets the volume fraction for the optimisation. The example shown in Figure 2.19 is that of a simple 2D bridge problem, where a uniformly distributed load is acting across a suspended road. The optimisation problem must be submitted in order to obtain results. In doing this, a sequence of images appear on the screen that show each iteration of the optimisation. These sets of images can be downloaded from the website as an animation once the program has completed the results. Figure 2.20 shows the results window that appears after a problem is submitted, in this case the final results for the 2D bridge problem defined earlier in Figure 2.19.
The results seen in Figure 2.20 show the optimum topological structure for the 2D bridge problem and therefore how the limited amount of material should be distributed within the design domain in order for the road to be fully suspended and capable of withstanding the uniformly distributed load. The results to the same bridge problem in 3D, after filtering of intermediate densities by penalisation, can be seen in Figure 2.21.

**Biological Growth**

A topology optimisation method based on the biological growth of trees was introduced by Mattheck and colleagues [2, 152-156]. In his *Soft Kill Option* (SKO), [155], Mattheck identifies the areas within a design that are not bearing much of the load and progressively removes material where it is not required until a rough shape develops.
He does this by altering the value of Young’s modulus of elasticity, $E$, across the component, so that it is highest at regions of high stress and lowest at regions of low stress. Each iteration involves changing the value of $E$ so that the underloaded material becomes less stiff and the highly loaded material becomes stiffer, identifying new material to be removed each time. A flow diagram of the SKO method can be seen in Figure 2.22. This technique gradually yields a shape with a more uniform stress distribution that is to some degree optimised in terms of weight.

![Flow diagram of the SKO method](image)

**Figure 2.22 – Flow diagram of the SKO method, [155].**

Mattheck also developed a shape optimisation technique called the *Computer-Aided shape Optimisation* (CAO) method, [152], that used the final design solution of the SKO as its starting block. The CAO technique works on a similar principle to the way that trees are able to use their growth layer of cambium to react to notch stresses when healing wounds. Mattheck artificially simulated the cambium layer in his finite element model of the SKO initial design by increasing the value of the coefficient of thermal expansion in this cambium layer, whilst keeping the coefficient constant throughout the remainder of the part. By substituting the mechanical stresses with similar thermal stresses of the same magnitude, the model can be analysed by performing thermal analysis. This method only allows the cambium layer to increase in volume during the thermal analysis and, as such, simulates the adaptive growth in trees, resulting in a near constant surface stress.
A cantilever beam example can be seen in Figure 2.23 which starts by defining the rectangular design area, boundary and loading conditions Figure 2.23a, then shows the optimal topology following SKO Figure 2.23b, followed by final solution after the secondary CAO shape optimisation step, Figure 2.23c.

Mattheck has demonstrated these methods on many examples from the natural world and has also implemented them successfully on some industrial components [2]. More recently, Mattheck’s biological growth methods have been used as “high tech sculpting tools” by artist Joris Laarman to create pieces of bone furniture for exhibition [157].

**Evolutionary Structural Optimisation**

Similar topology optimisation work has been carried out by Xie & Steven and colleagues in their work entitled *Evolutionary Structural Optimisation* (ESO), [120, 158-161]. The ESO approach is similar to Mattheck’s in that it uses FEA to identify inefficient material in a design that requires either moving or deleting, although it does not work through thermal analysis and requires a greater number of iterations. In its original form, the efficiency of material is determined by stress levels within elements, based on the fully stressed design approach.

The ESO method starts with all elements and nodes within the finite design domain being present (a ground mesh) then selectively removes elements and nodes where they are considered to be inefficient and therefore not needed. The material removal is governed by a rejection criteria that is stress based and is a gradual process requiring a number of iterations. Example ESO solution structures to the Michell-type bridge problem previously depicted in Figure 2.15 can be seen in Figure 2.24.
The ESO results displayed in Figure 2.24 share many similarities with the Michell solution previously shown in Figure 2.15. For example, both solution structures consist of an arch and several spokes between the load and the top of the arch, where the two legs of the arch are at a 45 degree angle to the horizontal baseline. However, there are notable differences between the two solution structures, with the ESO result suggesting it is desirable for the arch section to be thicker than for the spokes, whereas the Michell solution is restricted to having all members of equal cross-sectional area. In addition to various simple elasticity problems, the ESO method has been applied to heat conduction problems of printed circuit board substrates [158], the structural and spatial connection of multiple components [159], and to multi-criteria structural and thermal problems [160, 161].

A shortcoming of the ESO method is its inability to add material to a solution structure, which can be considered to be the reverse situation that was witnessed in the adaptive behaviour of biological trees (see Section 2.2.5). This behavioural limitation has been shown to sometimes result in a local optimum solution being generated, if elements are removed prematurely that are subsequently needed [162]. This shortcoming was later overcome by Querin et al. in their development of the ESO method, the Bi-directional Evolutionary Structural Optimisation (BESO) method, which permitted material addition as well as removal from the design domain, [163]. Furthermore, in comparison the BESO technique was shown to be approximately twice as fast as the original ESO approach [164].

**Metamorphic Development**

A comparable continuum topology optimisation method to ESO, named *Metamorphic Development* (MD) has been developed by Liu et al., [165-167]. MD, like ESO, requires numerous iterations of FEA to determine an optimal design topology, but
unlike ESO, MD does not require an initial ground mesh meaning that the design domain can be either finite or infinite. MD is capable of using both triangular and rectangular elements and, as a result, tends to generate structures with smoother boundaries when compared to those of ESO which is restricted to rectangular elements only. Both ESO and MD allow the removal of unwanted elements but MD, much like BESO, also permits elements to be added, even ones that have previously been removed, enabling the method to potentially avoid premature convergence on a local optimum. MD has been successfully applied to the optimisation of cylindrical nozzles for pressure vessel [166], and to the optimisation of turbine disk profiles [167]. More recently, Ngim et al. have used MD when considering the manufacturability of axisymmetric structures subjected to coupled pressure and thermal loads [168].

2.5.3 Applications

Several applications of discrete and continuum topology optimisation techniques have already been mentioned in Sections 2.5.1 and 2.5.2 respectively. Applications include the structural design of automotive and aeroplane components, buckling problems, dynamics problems, basic pressure load cases, thermal conduction problems and multi criteria structural or thermal problems, [123]. As mentioned earlier, the basic continuum topology optimisation method has been extended to various 3D problems where the design domain is essentially a volume of voxel elements. Several examples of 3D topology optimisation solutions can be seen in Figure 2.25.

![Figure 2.25 - 3D topology optimisation examples, structures include (from left to right), a cantilever beam, a table and a bridge design, [169].](image)

Topology optimisation methods are generally used to solve structural elasticity problems where the given loads are fixed. However, this application does not encompass all possible loading scenarios. Consequently, Chen & Kikuchi outlined a
continuum system for generating a topologically optimised linear elastic structure that is sensitive to design dependent loads [170]. In the paper, Chen & Kikuchi use hydrostatic pressure loading problems as a means to study structures which can resist varying loads optimally. This study shares similarities within Nature, as trees of the same species will grow differently in regions of high winds compared to trees that grow in more sheltered environments as detailed earlier in Section 2.2.5.

Continuum topology optimisation has also been used to design periodic materials that are tailored for a specific need. For example, Sigmund has used a homogenisation technique in the design of smart composite materials with desired performance characteristics, [171-175]. Example applications include the design of a negative Poisson's ratio material, and periodic materials with optimised thermal expansion coefficients or optimised piezoelectric coefficients. In Sigmund's approach, a single unit cell is optimised using either single or multiple materials. This unit cell is then repeatedly arrayed to form a larger periodic structure, as shown in Figure 2.26.

![Figure 2.26](image)

**Figure 2.26** – Optimised 2D unit cell with multiple materials (a), the arrayed 2D periodic structure (b), optimised 3D solid-void unit cell (c), the arrayed 3D periodic structure (d), [175].

Bruns & Sigmund have used a similar continuum topology optimisation technique to design structural micro mechanisms that exhibit snap-through behaviour [176]. These mechanisms employ controlled compliance to translate input motion into desired output motion in terms of magnitude and direction.

An interesting and more recent application of the continuum topology optimisation approach has been in solving Stokes flow problems by Borrvall & Petersson, [177]. In this application, the design domain has elements that represent the topology of a
structure and elements that represent a fluid moving either through or around that structure. The problems involve the minimisation of the fluid flow resistance of a structure. This work has since been extended by Gersborg-Hansen et al. [178] to encompass steady, incompressible, laminar, viscous flows at low to moderate Reynolds numbers, making the flow problems non-linear. In addition, inertia effects were introduced allowing new classes of optimisation problems including velocity-driven switches to be studied. Both this study and that of Borrvall & Petersson were limited to relatively simple 2D flow problems.

Now that topology optimisation has been investigated more thoroughly, a more in depth review of GAs is required, as these were shown to have the greatest potential for this research from all of the methods discussed earlier in Section 2.4.3.

2.6 Genetic Algorithms

GAs were previously introduced in Section 2.4.3, where they were shown to be multidirectional, probabilistic, non-derivative optimisation routines based on the mechanics of natural selection. Goldberg’s simple GA [95] describes the most basic form, the mechanism of which can be seen illustrated by the flowchart in Figure 2.27.
The probability of any individual surviving long enough to reproduce is determined by its fitness. Through the evolution process, the fitter individuals are likely to reproduce whereas the less fit ones will die off. The fitness function must indicate how well a solution fulfils the requirements of the given problem, hence, often the objective function can be used. The fitness evaluation step can be performed by specialist external analysis software such as FEA or CFD, which is particularly relevant for structural optimisation problems. In such cases fitness evaluation is usually slow, taking on average 60% to 90% of the overall computation time, which can significantly increase the time required for the evolution process to complete [99].

Only selected individuals are allowed to become parents and have offspring and selection is usually based entirely on fitness. Individuals with a higher fitness value will therefore have a higher probability of being selected and contributing to one or more offspring in the next generation. Individuals with low fitness values that are not selected for reproduction effectively die off. Several different selection techniques exist including:

- fitness proportional selection
- ranked selection
- tournament selection

The most common is fitness proportional selection, also known as roulette wheel selection, which uses reproduction probabilities based on weighted fitness values in order to select pairs of parents which will be allowed to reproduce. For additional information on selection techniques see [81, 96].

2.6.1 Genetic Operators

During the creation of a new population, the GA applies genetic operators to selected individuals from the current population. The three main genetic operators that are used in the creation of the offspring’s genotype are reproduction, crossover and mutation, [95]. These will now be discussed in greater detail.
Reproduction

The reproduction phase of a GA is responsible for the selection of sufficient suitable parents that will enable the creation of enough offspring to complete a new population. Not all of the individuals that form a new population need to be created through reproduction; some selected individuals can be simply copied unchanged from the previous population. This method ensures the survival of already developed fit solutions and is known as elite replacement [81]. It is the aim of reproduction that all offspring are slightly better solutions than their parents were, as they can benefit from inheriting good genetic material from each of their parent solutions.

Crossover

The recombination of the parents’ genotypes is achieved through crossover, inspired by the crossing over of chromosomes that occurs in the reproduction of biological organisms, combining some elements from each of the parents in the genetic makeup of the offspring’s genotype. One or more crossover points are selected, usually at random by the GA, within the chromosomes of the selected parents. The crossover points occur at the same position in each parent’s chromosome, splitting the chromosomes into sections defined by the crossover point locations. The sections of the two parent chromosomes are then interchanged between the parents, to form the offspring’s genotype [96]. Figure 2.28 shows a simple representation of one point crossover with the crossover point illustrated by the red dotted line.

A similar representation, this time of two point crossover, can be seen in Figure 2.29. The crossover genetic operator has a significant impact on the overall performance of the GA. As the synthesised evolution process progresses, and therefore the solutions converge, it is increasingly more desirable to keep developed fitter individuals intact, only making minor changes to their genetic makeup through crossover. To achieve this, an adaptively changing crossover rate must be applied which is governed by
probabilities that decrease throughout the optimisation process, with higher rates in earlier phases and a lower rate at the end.

![Figure 2.29 - Schematic representation of two point crossover, [99].](image)

Both Figure 2.28 and Figure 2.29 are examples of **complimentary crossover**, where the two selected parents produce two offspring by using up all of the genetic material from both parents. It is also possible to implement **non-complimentary crossover** into a GA, where the two parents produce a single offspring and therefore some of the genetic material is discarded. This is what happens in Nature.

**Mutation**

The **mutation** operator is inspired by the role of genetic mutation of an organism’s genes in natural evolution. The GA uses mutation probabilities to periodically make random changes or mutations in one or more members of the current population, yielding a new candidate solution (which may be better or worse than existing population subjects). As with crossover it is wise to control the rate of mutation, limiting its use towards the end of the evolution process when solutions are increasingly similar due to convergence. This is achieved by decreasing the mutation probability throughout the optimisation process [99]. When the mutation operator is activated, it effectively changes a ‘1’ to a ‘0’, or a ‘0’ to a ‘1’ in the binary encoded chromosome of an individual, as shown by Figure 2.30.

![Initial chromosome](image) ![Mutated chromosome](image)

*Figure 2.30 – Schematic representation of mutation operator occurring, [99].*

Mutation only occurs occasionally, (typical probabilities are 0.150 to 0.001 for optimisation problems), but is used by the GA to allow the emergence of new genetic
configurations which, by widening the genetic pool improve the chances of finding the optimum solution. The introduction of some potentially useful genetic material which would otherwise be permanently lost during some stage of reproduction and crossover, or which never existed inside the initial genetic pool, ensures that the GA does not stagnate by converging on a local optimum [81].

2.6.2 Advanced Genetic Algorithms

The two most common problems that occur with the use of standard GAs are those associated with premature convergence on a local optimum and their difficulty in coping with multiple optimisation criteria. The first problem would result in the population converging on a sub-optimal solution, an indication that the defined parameters are not ideally suited to the problem. If the GA is allowed to repeat its search, it is highly likely that it would yield different results from the first operation as it would converge on a different local optimum. The second problem is a widely acknowledged limitation of simple GAs. To address these issues and increase efficiency, the development of GAs has continued from their first introduction. As a result, distinct types of advanced GAs have been developed, some of these will now be considered.

Multi-Objective Genetic Algorithms

As mentioned previously, simple GAs have difficulty in coping with multiple optimisation criteria for complex problems. Although simple GAs can, and often do work with multi-objective problems by combining weighted objective values into one fitness value, this does not always yield optimum results [81]. Instead it would be necessary to use a Multi-Objective GA (MOGA), for optimisation problems with multiple criteria. For an overview of GAs in multi-objective optimisation see Fonseca & Fleming [179]. Additionally, literature surveys and comparative studies of MOGAs are given in the following texts [180, 181]. MOGAs work with multiple fitness values simultaneously and use Pareto optimality (see Section 2.4.4) to identify the better solutions – hence they are often referred to as Pareto GAs. The big advantage of GAs over most other search methods is that a GA manipulates a population of individuals. A MOGA therefore will ideally return a population with many members that belong to the Pareto Set in a single optimisation run, [182].
Schaffer developed the first MOGA known as the Vector Evaluating GA (VEGA) [183]. VEGA manipulates the selection mechanism of the GA to produce non-dominated individuals by designating each individual objective as the selection criteria for a portion of the population. Fourman [184] presents a tournament selection MOGA, which randomly chooses one objective to decide each tournament. Kurasawe [185] developed this strategy further by allowing the objective selection to be either random, fixed by the user, or allowed to evolve with the optimisation process. It should be noted however, that these MOGA approaches by Schaffer, Fourman and Kurasawe all tend to leave large portions of the Pareto Front undefined, with the Pareto solutions obtained being crowded together in clusters.

In order to establish the entire Pareto Front sufficiently, diversity within the population must be maintained. Kurasawe introduced crowding and dominance techniques in order to attempt to maintain diversity in the population with reasonable success. Additionally, maintaining diversity will tend to improve the robustness of the results in multi-objective problems by ensuring that there is sufficient genetic variety for reproduction mechanisms to operate with [186]. Goldberg introduced non-dominated sorting to rank a search population according to Pareto optimality. Goldberg also discussed using niching methods, sharing and speciation to promote diversity within the population so that the entire Pareto Front could be defined [95].

In the MOGA presented by Fonseca & Fleming [187-190] each individual within the population is ranked according to their degree of Pareto dominance. The more population members that dominate an individual, the higher ranking value awarded. In their MOGA, Fonseca & Fleming use mating restrictions and Goldberg’s sharing methods in order to maintain diversity within the population.

Goldberg’s initial thoughts on niching were later implemented by Srinivas & Deb in their Non-dominated Sorting GA (NSGA) [115]. In NSGA, the non-dominated individuals that form the Pareto set are identified and given a rank score of 1 before being removed from the population. The new Pareto set is identified from the remaining individuals within the population, given a rank score of 2 and then removed from the population. The process is repeated until the entire population is ranked.
Individual scores are then assigned to each individual within the same rank, ensuring that individuals belonging to rank 1 all score higher than individuals from rank 2. Individual scores are used to form selection probabilities between successive generations.

A slightly different approach, which is Pareto based but does not use ranking methods, is the Niched Pareto GA (NPGA) presented by Horn et al., [191]. This technique uses Pareto domination tournaments in order to select individuals as parents which can then reproduce to create the next generation. Non-dominated individuals are superior in the tournaments, however, if both or neither are dominated, selection then becomes based on the niche count of each candidate individual. This concept is best explained using the diagram shown in Figure 2.31 where two objectives are to be minimised.

![Diagram of Niched Pareto GA](image.png)

**Figure 2.31 - Diagram of Niched Pareto GA, [191].**

Of the two candidate solutions highlighted, a niche count is calculated for each based on the number of Pareto optimal solutions that exist within a specified distance or spread from the candidate solutions (defined by the dashed circles each at radius Sigma). The candidate solution with the lowest niche count will then be selected by the GA in preference to the candidate solution with the higher niche count in the hope that subsequently generated solutions will occupy this currently lowly populated area of the search space. In doing so, the GA helps to maintain population diversity whilst striving to define more clearly the overall Pareto Front by identifying further Pareto optimal solutions.
Parallel Genetic Algorithms

Much of the success of GAs is due to the fact that they are able to explore more of the available search space by performing a multidirectional search owing to the fact that they maintain a population of solutions. However, in many cases, the evaluation of the fitness function of each individual within the population can be very time consuming, particularly if the fitness evaluation is to be performed by external analysis software (for example FEA). A logical way to improve the fitness evaluation time is to evaluate the individuals in parallel rather than sequentially as in the traditional approach. Such an approach is implemented in Parallel GAs [81]. In some cases, this may be the only way for the GA to achieve acceptable results in a reasonable time.

There are several ways of implementing the simultaneous fitness evaluation by a GA, hence there are several different types of parallel GAs. Considerable work has been conducted on analysing the different ways to parallelise a GA [192-194], with the simplest method being to adopt a Master-Slave approach. Here, one processor (the master) stores a single population, whilst other processors (slaves) simultaneously evaluate the fitness of different individuals in parallel. When this master-slave approach is adopted, the solution of the Parallel GA will be identical to the solution obtained by a standard sequential GA, except that it will be obtained significantly faster. This parallel implementation is also referred to as a Global GA [96]. Cantu-Paz & Goldberg presented work estimating the time savings of a master-slave approach for various sizes of parallel processor arrangements, based on the total execution time for an entire generation to be evaluated for fitness [195]. Their study showed that increases in evaluation speed were related to the ratio of the time to compute the fitness, relative to the communication time between the master and slave processors, and that greatest speed increases were witnessed where fitness evaluation required significant computation, and when larger population sizes were used.

Alternative approaches to parallelising a GA involve the creation and management of multiple subpopulations. This group of techniques can be subdivided into Island GAs (also called Migration GAs) and Neighbourhood GAs (also called Diffusion or Fine-Grained GAs) [96]. In island GAs, the subpopulations are allowed to evolve separately from one another (using separate processors) in an attempt to mimic the geographical
separation of subpopulations witnessed in the natural world. Hence, each subpopulation is able to establish highly fit individuals, which ensures the overall process does not focus around a single optimal solution. The occasional switching of individuals between subpopulations is then permitted through migration; thereby ensuring population diversity is maintained [81]. Some island GAs switch random individuals between subpopulations, however, the study by Cantu-Paz recommends the switching of the most-fit individuals between separated islands [196]. Neighbourhood GAs are similar to island GAs in that they also employ subpopulations. However, the geographical barriers used in the migration approach of island GAs are replaced with geographical distances, where individuals are only permitted to reproduce with their closest neighbours [197].

2.6.3 Applications

The use of GAs is gaining a growing following in many scientific, engineering, business and social science disciplines. They are being used to help understand some of the processes and dynamics of natural evolution, to construct advanced adaptive systems and as a way to help solve a range of difficult modelling problems. GAs have become an attractive means for solving a number of optimisation problems, generally due to continuing improvements in computing performance and price. However, they still do tend to be computationally expensive. Successful applications include control [198], scheduling [199], route and network planning [200], layout design [201], component optimisation [100], optimisation of product families [202], robotics [203], spacecraft trajectories [204], music composition [205] and image processing [206].

A recent structural example of an application of GAs in the construction industry is given by the design of the Beijing national stadium for the 2008 Olympics in China, designed by ARUP Advanced Technology Group (see Figure 2.32). Nicknamed the Birds Nest, the stadium seats 80,000 spectators and is believed to be the world’s largest enclosed space, with no towers or cable nets forming the support. The irregular nature of the structure was inspired by randomness in Nature and a local style of pottery, and required new methods of designing structural steel sections in order to minimise the weight, [207].
Travelling Salesman Problem

The travelling salesman problem (TSP) is a common application of both GAs and ACO, with the objective being to find the shortest possible route (or tour) between a series of cities by an imaginary salesman. Typically the rules (constraints) state that no city can be visited more than once by the salesman and that the salesman can only travel in a straight line between cities [96].

The optimal solution to the TSP is found when the travel route minimises the total distance travelled by the salesman. Finding an optimal solution becomes more challenging as the number of cities involved increases. If we consider a simple case with only five cities there are 120 possible different tours. All 120 tour lengths could be calculated by a computer in a fraction of a second, easily identifying the shortest route.

If we then consider the TSP for ten cities, the computational time required would be significantly greater as there are now 3,628,800 possible tours. In the last two decades, enormous progress has been made with respect to solving larger travelling salesman problems, with the current World record yielding an optimal route for travelling through 3038 cities, which required a total of one and a half years of computational time. This progress has only partly been enabled due to the increasing hardware power of computers. Ultimately, it was the development of mathematical theory and therefore of more efficient algorithms that made this feat possible [81].

Many practical applications can be modelled as a TSP or as variants of it, including important applications in science and engineering. For example, in the manufacture of a printed circuit board, it is important to determine the best order in which the laser drills
the thousands of holes, as an efficient solution to this problem will reduce the manufacturer’s production costs. Similarly, the real-world routing of buses, airlines, delivery trucks and postal carriers can all be calculated, in addition to the simpler task of deciding pen movement of a plotter [208].

**Creative Design**

For many years there has been a general view that computers cannot be creative, and that creativity is a purely human attribute. Recently, however, so-called *creative evolutionary systems* have been developed using GAs which either aid human creativity, or solve problems only creative people could solve, or both combined, [209]. In systems that aid human creativity, selection is guided solely by the user. When the user is asked to rank the individuals within a population, they perform the role of the fitness evaluator. This has the advantage of avoiding premature convergence as the user has the opportunity and ability to enforce alternatives by steering the system, there is also no need to define a fitness function. Unfortunately, because of the requirement for the user to rank each individual sequentially, these systems can be very slow and there is the potential hazard of the user judging the same individual differently at different stages. As a result, these systems tend to have small populations to make them more manageable for the user [210].

Such an evolutionary system that aids human creativity was developed by Graham at Loughborough University. This system integrated a simple GA with a solid modelling CAD package to enable evolutionary product design. The resulting system was called *Evolutionary Form Design*, (EFD) [210]. Basic solid modelling features were controlled by the GA, including choice of primitive creation, Boolean operations (unite, intersect, subtract) and simple blends. The user was required to evaluate the fitness of each individual within the population, from an aesthetics viewpoint. Typical population sizes were between 10 and 14, with approximately 20 to 25 iterations of solutions before convergence. Example product solutions of the EFD system can be seen in Figure 2.33.
Furthermore, Graham looked briefly at what he termed *internal optimisation*, i.e. where the individual fitness values were determined by the GA and not by the user. This application was very limited, only considering simple geometric properties such as volume and surface area etc, [210].

**Application to Topology Optimisation**

As stated previously in Section 2.4.3, most applications of GAs to engineering design problems are in the form of either sizing or shape optimisation, where existing design parameters are often used as design variables. However, there are also some examples of GAs being applied to the field of topology optimisation, which will now be presented.

Hanna & Mahdavi have used a GA to define strut connectivity and nodal positions within a discrete topology optimisation system, [211-213]. The GA was used to evolve an optimal unit cell to suit a particular loading case, which was then repeatedly arrayed into a larger periodic microstructure. A secondary sizing optimisation step was then performed on the overall microstructure, before using SLA technology for final fabrication. An example optimised microstructure can be seen below in Figure 2.34.

![Figure 2.34](image-url)

*Figure 2.34 – Side view of optimised microstructure after both topology and sizing optimisation steps (left), and a fabricated SLA model of the final 3D microstructure (right), [213].*
Fadel and coworkers have used a GA to consider the optimal material distribution of multiple materials within a fixed shape and outer topology to solve multi-criteria problems including both structural and thermal FEA. Applications include the axisymmetric cross-section of a brake disc rotor, the optimal design of a 3D heterogeneous connector, both using Steel and Aluminium, and heterogeneous flywheel optimisation using Tungsten-carbide and Cobalt [214-216]. In similar work, Fadel used a gradient based technique to design the size and position of cooling channels within 2D injection mould tools, followed by a secondary material distribution step where multiple metals and ceramics were distributed by a GA to further improve upon cooling times [217]. In all of this work, the overall shape and outer topology remained fixed, meaning that additional voids could not be added. Fabrication was considered to be made possible via the LENS process, the manufacturability constraints of which were also considered by Fadel [214].

As was shown earlier with topology optimisation (see Section 2.5.2), it is possible to represent the topological shape of any object by dividing the predefined design domain into many small elements or cells. For a 2D design domain these elements could be considered *pixels*, similarly with a 3D design domain the elements could be considered to be *voxels*. This kind of cellular representation can be directly implemented into a GA by assigning binary values (0 and 1) to all of the elements within the design domain depending on whether material or a void is present, and mapping them into 2D or 3D chromosome representations accordingly, [99]. These types of GAs are sometimes called Cell GAs, and are capable of performing continuum topology optimisation, independent of any initial topology, therefore creating novel solutions. The application of a GA to topology optimisation offers the greatest opportunity for exploiting RM's design freedom of all of the methods seen within this literature review, as such it requires greater investigation. This type of GA application has been implemented by Jakiela et al. to study basic 2D cantilever beams and a Michell-type bridge problem [218]. Figure 2.35 shows a 2D chromosome representation of a simple cantilever beam topology.
As with standard topology optimisation problems, the poor smoothness of boundaries may not be acceptable. However, increasing the resolution of the cellular representation will improve smoothness but will also incur significant additional computation requirements [219]. In addition, when using a cell GA, small unwanted holes may appear in the final topological shape, [102].

One consequence of implementing topology optimisation into a GA in this manner, using 2D or 3D array chromosomes, is that crossover becomes more complicated in 2D and even more so in 3D. More advanced crossover operators must be used so that either 2D or 3D crossover can be achieved. Although there are several crossover operators that exist for cellular chromosome representations, [102], one of the simplest is a direct generalisation of standard one point complimentary crossover within a 1D bit-string. A crossover point is selected at random in the middle of the chromosome array (the same position in both parents), then complementary parts of the two parent’s chromosomes are interchanged diagonally. The crossover occurs in blocks, with block sizes determined by the location of the selected crossover point. A schematic diagram showing one point diagonal complimentary crossover for a 2D array can be seen in Figure 2.36.

Another disadvantage of having 2D or 3D genetic representations, is that fitness evaluation is considerably more complicated and therefore time consuming when compared to simple linear genetic codes (1D bit-strings). In an attempt to overcome this difficulty, Chapman et al, [103], devised a simple method for mapping a 2D
chromosome representation into a 1D bit-string, enabling the GA to evaluate fitness values quickly and apply 1D crossover operators.

![Figure 2.36 - One point diagonal crossover for 2D arrays, [99].](image)

An advantage of using 2D or 3D chromosome representations is that structures with more than one material can be constructed by using integer values (one for each material) instead of a purely binary representation. An example of this is depicted in Figure 2.37 where the 0’s represent voids, the 1’s represent material A and the 2’s represent material B.

![Figure 2.37 - Encoded ID integer string for multiple materials, [99].](image)

As seen previously it is possible to combine a GA within a continuum topology optimisation process. This has been achieved within a freeware, beta version software currently called DesignLab by DevDept, [220]. This software combines a GA and FE analysis with topology optimisation to create 3D components and is capable of considering either a single material or multiple materials. Within this software, a basic GA is used that equates individual solution fitnesses by using a weighted sum of several objectives – weight, deflection, Factor of Safety (FOS) – and introduces a penalty factor that reduces the combined fitness depending on how much an individual solution violates an objective. The fitness and penalty values are combined for each individual solution to calculate a cost or objective function. The capabilities and limitations of this software are currently unknown but are of considerable interest to this research. As such it is recommended that a thorough investigation of this software is undertaken in the subsequent Chapter.
Chapter Three

DesignLab Investigation

3.1 Introduction

The literature review conducted in Chapter 2 highlighted the need to thoroughly investigate the capabilities and limitations of existing GA based topology optimisation software. One such software identified was DesignLab, which is comprehensively investigated in this Chapter using simple cantilever beam problems, where both single and multiple materials are considered. Following this investigation a hypothesis is formulated. A new GA based topology optimisation system is then proposed that is capable of addressing the shortcomings of existing methods, whilst also being able to physically realise solutions using RM technologies. The proposed methodology of using various unit cell structures at differing cell densities is presented, followed by relevant research questions that aim to test the hypothesis. Finally, the remaining thesis structure is summarised.

3.2 DesignLab Investigation

3.2.1 Experimental Methodology

For this investigation, a simple cantilever beam problem was considered where the design domain was divided by 20 cells in X, 10 cells in Y and 1 cell in Z, with each cell having a length of side of 10 mm. The design domain for this study was 3D but can be considered as 2½D, being a simple extrusion in the Z-axis. The design domain was fully constrained down the left hand side with a downward point load applied to the top right hand corner, as can be seen in Figure 3.1.
The parameters used to control the behaviour of the GA throughout this investigation were the default software parameters, namely:

- initial population size of 120 individuals with a population of 60 individuals maintained thereafter,
- initial mutation probability rate of 0.015 dropping by 0.0025 for every 10 generations where results do not improve,
- desired FOS of 2.25 with a penalty of 25 applied to individuals that violate this,
- a minimum hole size of 1 cell,
- mating by single point uniform crossover,
- parent pairs selected by a weighted ranking of fitness values.

These parameters were entered into the initial problem definition but were not altered at any stage of the investigation to ensure consistency. DesignLab has been created in such a way that it supports parallelisation of the optimisation problem. This feature was used extensively throughout this investigation by the parallel networking of 12 Personal Computers (PCs) to enable considerably faster solution times.
Initially only a single material was selected (Steel), allowing the GA to distribute solid material cells and void cells throughout the design domain much like the work of Jakiela et al, [218]. This first scenario was repeated five times (trials A to E), to inspect repeatability. The five beam structures of the first scenario were analysed to see if any cells were solid or void in all of the five trials. This was performed by superimposing their structures on top of one another and adding them together. Cells that remained 0 were void in all five trials, cells that totalled 5 were Steel in all five trials and cells that were either 1, 2, 3 or 4 were Steel in some trials and void in others. The cells that remained solid or void throughout trials A to E were used to further constrain the design domain, thus steering the optimisation problem towards more desirable solutions based on the previous experience of the first scenario in an attempt to improve subsequent results. The additional constraints were added to the cantilever beam problem definition creating a second scenario, which was also run five times (trials F to J) to inspect repeatability.

The effects on the overall stress distribution of introducing another material into the problem were particularly of interest as this is beyond the scope of the work conducted in previous cell GA based topology optimisation studies [103, 218, 219], so the original cantilever beam problem previously defined in Figure 3.1 was repeated with two materials, Steel and Aluminium. This third scenario was also run five times (trials I to M).

### 3.2.2 Results

The results of the five trials of the first scenario (A to E), where the single material Steel was considered, and the combined structure of all five trials superimposed on one another is illustrated by Figure 3.2.
The results from the first scenario combined structures were used to create the constrained cantilever beam definition in DesignLab for the second scenario and can be seen in Figure 3.3, with blue areas indicating permanent void positions and purple areas indicating permanent Steel.
The five trials of the second scenario (F to J) were analysed in the same manner as those from the first, by superimposing the five beam structures on top of each other and adding them together to form the combined structure. This combined structure could be used to further constrain the beam problem design domain in additional scenarios. The results of the second scenario (trials F to J) can be seen in Figure 3.4 along with their superimposed combined structure.

Figure 3.4 – Results for second scenario, trials F to J and their combined structure.

The five beam structures of the third scenario, where two materials were considered, were also analysed in the same manner as before. However, this time, it was to see if any cells remained void, Steel or Aluminium in all of the five trials. The third scenario solutions and the combined structure are shown in Figure 3.5.
The analysis was performed by altering the values of the cells for the different materials by a factor of ten so that Steel cells were represented by a 10, Aluminium cells were represented by a 1 and void cells were represented by a 0 as before. When the five structures were added together, cells that totalled 50 were Steel in all five trials, cells that totalled 5 were Aluminium in all five trials and cells that remained 0 were void in all five trials as before. Cells that were anything other than 50, 5 or 0 in the combined structure fluctuated between Aluminium, Steel and void in the five trials. The combined structure depicted in Figure 3.5 can be used to increase the constraints on the two material problem definition.

The individual results of a single trial from the first scenario are shown in Figure 3.6a, with both solid Steel cells and void areas present. The stress distribution plot of Figure 3.6b shows that there are several areas of localised high and low stress; these have been circled in white. This demonstrates that this solution could therefore be improved upon further to remove these localised Stress Concentrations (SCs) and create a more uniform stress distribution.
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Figure 3.6 – Cantilever beam result for a single material, showing material distribution plot (a) and stress distribution plot (b) with areas of local high and low stress circled.

Similarly, the individual results of a single trial from the third scenario can be seen in Figure 3.7a, with solid Steel cells (purple), solid Aluminium cells (light grey) and void areas present. The stress distribution plot of Figure 3.7b shows that there are fewer areas of localised high and low stresses when compared to the stress plot of the single material problem in Figure 3.6. This suggests that the stress distribution is more uniform and therefore the structure is more optimal than the single material structure in Figure 3.6.

Figure 3.7 – Cantilever beam results for two materials, showing material distribution plot (a) and stress distribution plot (b) with areas of localised stress circled.

The numerical results and average values of the five trials for each of the three scenarios can be found in Table 3.1 with full detailed results presented in Appendix B.
Table 3.1 – Cantilever beam results and average values for DesignLab trials A to O.

<table>
<thead>
<tr>
<th></th>
<th>Number of</th>
<th>Cost Function</th>
<th>Max FOS</th>
<th>Min FOS</th>
<th>FOS Range</th>
<th>Number of Generations</th>
<th>Max Displacement (mm)</th>
<th>Weight (kg)</th>
<th>Volume Fraction Total</th>
<th>Volume of Steel</th>
<th>Volume of Aluminium</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Trial A</strong></td>
<td>1</td>
<td>27678</td>
<td>5.08</td>
<td>1.22</td>
<td>3.86</td>
<td>270</td>
<td>0.688</td>
<td>0.785</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial B</strong></td>
<td>1</td>
<td>22603</td>
<td>5.45</td>
<td>1.19</td>
<td>4.25</td>
<td>335</td>
<td>0.683</td>
<td>0.730</td>
<td>0.47</td>
<td>0.47</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial C</strong></td>
<td>1</td>
<td>25282</td>
<td>5.52</td>
<td>1.22</td>
<td>4.29</td>
<td>179</td>
<td>0.727</td>
<td>0.809</td>
<td>0.52</td>
<td>0.52</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial D</strong></td>
<td>1</td>
<td>21510</td>
<td>5.36</td>
<td>1.45</td>
<td>3.91</td>
<td>222</td>
<td>0.611</td>
<td>0.706</td>
<td>0.45</td>
<td>0.45</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial E</strong></td>
<td>1</td>
<td>26841</td>
<td>7.19</td>
<td>1.41</td>
<td>5.78</td>
<td>242</td>
<td>0.606</td>
<td>0.785</td>
<td>0.50</td>
<td>0.50</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Average A to E</strong></td>
<td>1</td>
<td>24783</td>
<td>5.72</td>
<td>1.30</td>
<td>4.42</td>
<td>250</td>
<td>0.663</td>
<td>0.763</td>
<td>0.49</td>
<td>0.49</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial F</strong></td>
<td>1</td>
<td>24297</td>
<td>4.58</td>
<td>1.21</td>
<td>3.37</td>
<td>215</td>
<td>0.619</td>
<td>0.746</td>
<td>0.48</td>
<td>0.48</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial G</strong></td>
<td>1</td>
<td>21434</td>
<td>4.58</td>
<td>1.34</td>
<td>3.23</td>
<td>205</td>
<td>0.585</td>
<td>0.691</td>
<td>0.44</td>
<td>0.44</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial H</strong></td>
<td>1</td>
<td>22711</td>
<td>4.93</td>
<td>1.23</td>
<td>3.70</td>
<td>257</td>
<td>0.564</td>
<td>0.706</td>
<td>0.45</td>
<td>0.45</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial I</strong></td>
<td>1</td>
<td>22660</td>
<td>6.63</td>
<td>1.41</td>
<td>5.22</td>
<td>194</td>
<td>0.575</td>
<td>0.714</td>
<td>0.46</td>
<td>0.46</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial J</strong></td>
<td>1</td>
<td>23197</td>
<td>5.12</td>
<td>1.18</td>
<td>3.94</td>
<td>220</td>
<td>0.642</td>
<td>0.722</td>
<td>0.47</td>
<td>0.47</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Average F to J</strong></td>
<td>1</td>
<td>22860</td>
<td>5.17</td>
<td>1.27</td>
<td>3.89</td>
<td>218</td>
<td>0.597</td>
<td>0.716</td>
<td>0.46</td>
<td>0.46</td>
<td>0.00</td>
</tr>
<tr>
<td><strong>Trial K</strong></td>
<td>2</td>
<td>20589</td>
<td>5.13</td>
<td>1.28</td>
<td>3.85</td>
<td>309</td>
<td>1.706</td>
<td>0.578</td>
<td>0.53</td>
<td>0.28</td>
<td>0.25</td>
</tr>
<tr>
<td><strong>Trial L</strong></td>
<td>2</td>
<td>17957</td>
<td>5.46</td>
<td>1.33</td>
<td>4.13</td>
<td>394</td>
<td>1.926</td>
<td>0.630</td>
<td>0.62</td>
<td>0.29</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>Trial M</strong></td>
<td>2</td>
<td>18272</td>
<td>4.07</td>
<td>1.21</td>
<td>2.66</td>
<td>429</td>
<td>2.058</td>
<td>0.503</td>
<td>0.55</td>
<td>0.20</td>
<td>0.36</td>
</tr>
<tr>
<td><strong>Trial N</strong></td>
<td>2</td>
<td>14620</td>
<td>4.88</td>
<td>1.63</td>
<td>3.24</td>
<td>379</td>
<td>1.774</td>
<td>0.663</td>
<td>0.60</td>
<td>0.33</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Trial O</strong></td>
<td>2</td>
<td>17596</td>
<td>4.86</td>
<td>1.51</td>
<td>3.36</td>
<td>296</td>
<td>1.542</td>
<td>0.561</td>
<td>0.54</td>
<td>0.26</td>
<td>0.28</td>
</tr>
<tr>
<td><strong>Average K to O</strong></td>
<td>2</td>
<td>17807</td>
<td>4.88</td>
<td>1.39</td>
<td>3.49</td>
<td>361</td>
<td>1.801</td>
<td>0.587</td>
<td>0.57</td>
<td>0.27</td>
<td>0.30</td>
</tr>
</tbody>
</table>

3.2.3 Discussion & Conclusions

From Table 3.1 it can be seen that there is a reduction in the average number of generations from the first scenario to the second scenario. This can be explained by the presence of additional constraints imposed on the second scenario based on the analysed results of the first scenario. These extra constraints effectively remove many structural permutations from the vast possibilities of the original problem, increasing the efficiency of the GA by preventing it from wasting time calculating already known poor solutions. Likewise, the average number of generations for the third scenario (two materials) is much greater than either of the single material scenarios due to the increased number of permutations possible for the structure. The GA now has to consider three options for each cell, namely void, Steel or Aluminium, as opposed to just the two of solid or void. By imposing additional constraints gathered from the analysis of the third scenario combined structure, it should be possible to reduce this average number of generations in subsequent scenarios. The introduction of additional
constraints would once again reduce the number of permutations the GA may consider, thus increasing the efficiency of the GA in reaching a solution in a shorter time.

The overall objective for these preliminary optimisation problems was to minimise the cost function. The cost function value is calculated by a weighted sum which is thought to include the weight of the solution structure, the maximum displacement (directly related to the stiffness or compliance of the structure) and the range of the FOS across the structure which bears a direct relationship to the stress distribution. Unfortunately, in this beta software version, the user is unable to inspect or alter the weightings of the weighted sum. The average cost value from the first scenario to the second scenario has improved (Table 3.1) and there is further improvement between the second and third scenarios. In each case the average FOS range has reduced resulting in a more uniform stress distribution across the structure, as was witnessed from Figure 3.6b to Figure 3.7b. The average weight also reduced through all three scenarios, however, the maximum displacement reduced in the single material scenarios then increased in the two material scenario. This is due to the inclusion of a less stiff material within the structure (Aluminium) which will deflect more readily than the stiffer Steel structures of the previous two scenarios. The maximum displacement values may not be included in the weighted sum of objectives that form the cost function or, if they are included, do not have a large weighting as this increase does not seem to have a noticeable effect on the cost values.

The most important observation in the results of Table 3.1 that should be noted is that the third scenario (two materials) had an improved average cost value and reduced FOS range when compared to either of the single material scenarios. This includes the second scenario where the results from the first scenario were used to further constrain the design domain, helping to steer the optimisation process by preventing the GA considering known poor permutations of structure. The FOS range results in Table 3.1 and the stress plots of the structures in Figure 3.6 and Figure 3.7 show that the stress distribution is more uniform where multiple materials are considered compared to a single material with a homogeneous structure. These results agree with the findings of Huang & Fadel [216], where their heterogeneous flywheel design was shown to exhibit
a more uniform distribution of Von Mises stresses compared to a single material, homogeneous solution when using a hybrid genetic and gradient based algorithm.

In conclusion, the results from this investigation of DesignLab show that the stress distribution was more uniform where multiple materials were considered when compared to a single material only. The two material scenario considered materials of significantly different densities, namely Steel and Aluminium. In effect, the Aluminium cells could be considered to be Steel cells of intermediary density. This suggests that future systems should either consider multiple materials, or multiple densities of a single material, in order to achieve a uniform stress distribution.

It should be noted that the use of multiple materials within RM to increase part functionality has been considered by many researchers in the form of Functionally Graded Materials (FGMs), [221-223]. There are, however, many fabrication issues to be addressed in such cases in addition to the dilemma of recycling components fabricated of multiple materials [214, 224]. Hence, due to the fabrication and recycling problems associated with mixed materials (FGMs), it is proposed that only a single material should be considered in future topology optimisation systems, with increased functionality obtained through intelligent design.

3.3 Hypothesis

It is suggested that in order to achieve a more uniform stress distribution, a stochastic structural topology optimisation system, which is capable of considering varying densities of a single material throughout the solution, is needed. It is thought that the different densities required could potentially be simulated by unit cell structures. Given that such a system does not currently exist, the effects on the stress distribution of such an approach are currently unknown and consequently represent a significant gap in the existing body of knowledge. When created, the proposed system will exploit RM's design freedom by fabricating optimal solutions without compromise, thereby forming a major contribution to scientific research.
3.4 Methodology

In order to potentially evade local optima, it is suggested that a system is created which combines a GA with topology optimisation, where FEA is used to determine fitness evaluation of designs. Consequently, the entire process will benefit from the stochastic search behaviour of the GA and, if using a MOGA, will have the added ability of being able to consider multiple objectives simultaneously.

Variable densities of a single material could be simulated by using unit cell structures, where the density of a single cell is considered to be the volume fraction of solid material within the overall cell volume. This approach has not been seen in the literature and potentially allows for a combination of multiple cell structures to be considered simultaneously, which could have advantageous structural properties. Such a system would only be able to physically realise its solutions by adopting RM as the fabrication route.

To test the hypothesis, a GA based topology optimisation system needs to be created that can consider multiple cell structures at various volume fractions. Once created, a single cell structure can be introduced into a simple solid and void problem, to see if the stress distribution is improved. Furthermore, greater variation in volume fractions can be investigated in addition to the mixing of different cell structures together, each time to see what effect these parameters have on the resulting stress distribution.

3.4.1 Ideal Scenario

It should be noted at this stage that although a single material system is under consideration, the results from the first and second DesignLab scenarios (both single material only) suggest that a uniform stress distribution will not be achieved if cells are purely either 100% solid or 100% void. This phenomena is solely dependent on the size of the unit cells used compared to the size of the design domain.

In order for any system to reach a truly globally optimal design solution, the geometry and topology should be completely free to ‘evolve’ into an ideal solution. However, this could only be achieved if the design domain is discretised into many small cells.
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

(voxels) resulting in an extremely high fidelity mesh. Kim & de Weck state that this would have a considerably negative effect on the required computation power, particularly if using a GA as a direct result of increasing the length of the bit string (chromosome) and the associated number of possible permutations, [219]. If such a high fidelity mesh was used then the rough surface appearance common in topology optimisation solutions would be much less of a problem due to the small size of the voxels representing curved geometries more accurately, this is demonstrated in Figure 3.8.

Despite the advances over recent decades in computing power, coupled with the decreasing cost of such equipment, the requirements of the idealised solution are still far beyond the resources of an average design engineer. This situation gives rise to the need for a compromise of the ideal scenario to enable designers using today’s computing resources to create optimal geometries which can then be fabricated via RM. Such a system is now proposed.

3.4.2 Proposed System

It is proposed that various densities of a single material are incorporated into a MOGA based topology optimisation system to allow for the creation of functionally graded structures. The required densities will be simulated by using unit cell structures at different volume fractions, which can be easily created by controlling the feature
parameters that define the unit cell structure. For example, Figure 3.9 shows a CAD model of a simple unit cell structure at various volume fractions, achieved by altering the parameters of the three hole features within the unit cell, which can be readily fabricated by RM.

Based on the work by Sigmund [171-175], it is clear that the unit cell structure depicted in Figure 3.9 will not be suited to all possible applications. Hence, a range of different cell structures should be considered by the proposed system, thereby ensuring that some cell structures are more appropriately suited to the particular loading condition in question. These various cell structures will exhibit differing mechanical responses depending on the loading scenario, due to their topological differences, even when they are comprised of identical amounts of material (equal volume fractions). In order for one structure’s mechanical response to be directly comparable with another, they must have equal volume fractions. This will identify which structure has the best distribution of material within its cell for the particular loading environment. Examples of various cell structures with a 50% volume fraction are illustrated in Figure 3.10.
A topology optimisation process that uses a GA to create components consisting of a single cell structure in varying densities could potentially create a cantilever beam similar to that shown in Figure 3.11a. The same approach if considering multiple cell structures may create a cantilever beam component similar to that shown in Figure 3.11b.

Such a system will be able to exploit the increased design freedom that RM technologies offer, enabling optimal design solutions with uniform stress distributions as seen in the natural world to be created that could not be manufactured via conventional techniques. Obvious application areas of this research include the design of optimal lightweight structures for the automotive and aerospace sectors in addition to possibilities within the medical sector related to the creation of bone scaffolds.

3.4.3 Research Questions

Stemming from the hypothesis and methodology, it is now possible to formulate several research questions. These are listed below.

1. Does the introduction of a 50% volume fraction cell structure into a simple solid void problem help to create a more uniform stress distribution compared to simple solid and void only problems?

2. Can the use of various multiple volume fractions of a single cellular structure improve the stress distribution uniformity compared to either the simple solid and void case, or the solid, void and 50% cell structure case?
3. Can using multiple cellular structures simultaneously within a design domain be more beneficial to satisfying the design objectives compared to multiple volume fractions of a single cellular structure?

4. A potential outcome of the proposed methodology is that the cell structures introduce localised SCs into the solution geometries, is this the case? If so, what effect do these SCs have on the overall optimisation process?

5. Is the FEA model a realistic representation of the physical RM model? This question can also be asked of existing topology optimisation software such as DesignLab.

3.4.4 Remaining Thesis Structure

In the methodology, a GA based topology optimisation system which is capable of testing the hypothesis was proposed. The remaining research will commence with an investigation into the suitability of the various unit cell structures proposed when subjected to various mechanical loadings. This investigation of cell structure mechanical responses is presented in Chapter 4.

The design, construction and development of the proposed GA based topology optimisation system, that is capable of considering unit cell structures that vary in form and volume fraction, is detailed in Chapter 5. A series of preliminary experiments are then conducted to allow for direct comparison with the earlier DesignLab results. These experiments and their results are presented in Chapter 6. An experimental roadmap was then outlined, with the aim of identifying which experiments and parameters were feasible for testing the hypothesis. These advanced experiments and their results are presented in Chapter 7.

All experimental results are analysed and discussed in Chapter 8 and the answers to the research questions are presented. Finally, Chapter 9 concludes the findings of this research and makes recommendations for further work.
CHAPTER FOUR

MECHANICAL RESPONSES

4.1 Introduction

To gain an initial understanding of the suitability of various cell structures to different loading conditions, an investigation of their mechanical responses, using FEA, is conducted and presented in this Chapter. The cell structures proposed in Figure 3.10, at several predefined volume fractions are arrayed into assemblies and then subjected to tension, compression, bending and torsional loads respectively. The behavioural responses of the assembled cell structures subjected to these common loading scenarios is observed and discussed in each case. However, before these behavioural responses are conducted it is worth briefly describing the principles of the FEA process.

4.2 Brief Introduction to FEA

FEA is a numerical method used for solving complex engineering problems accurately. Simple problems can be solved by hand calculations, however real world problems are usually too complex to be solved by hand, requiring numerous calculations that lend themselves more readily to computer based application. The technique uses a mathematical representation to approximate the real world problem. This is achieved by taking a representative model (a CAD model is often used) and discretising it into many small pieces known as elements. This process is known as meshing. Material properties are then applied to the mesh elements along with boundary and loading conditions that simulate the real world problem. The mechanical behaviour of any single element by itself is very well understood. FEA can then predict the behaviour of real world scenarios by combining all the behaviours of elements within a mesh. It does
this by solving simultaneous equations that calculate stiffnesses between connecting mesh points called nodes, [226].

The FEA procedure can be sub-divided into three distinct stages as follows:

1. **Pre-processing** – creation of mathematical model that is representative of the real world problem including application of appropriate loads and boundary conditions, mesh creation and application of material properties to mesh elements.

2. **Processing** – also known as solving, where the simultaneous equations are solved. Elemental stresses and strains are calculated in addition to nodal displacement values.

3. **Post-processing** – the viewing and interpretation of the FE results.

The meshing step is critical to the whole FE process including the accuracy of the solution. The higher the mesh fidelity, the more accurate the mathematical representation is of the real world problem. However, this also greatly increases the number of elements, which therefore involves more calculations to be solved requiring increased computation. Numerous element shapes exist for different applications. These include 2D lines for simple trusses, sheets and shells for more complicated 2D problems and 3D surface problems. In addition to these, brick shaped hexahedrons (HEX) and pyramid shaped tetrahedrons (TET) can be used for 3D solid problems, as illustrated in Figure 4.1.

![Figure 4.1 - 8 noded hexahedron and 4 noded tetrahedron 3D element shapes, [226].](image)

When using 3D solid elements, increased accuracy can be achieved by the introduction of additional nodes to the existing elements at the expense of increased computation, examples include 20 noded HEX elements and 10 noded TETs. HEX elements are generally regarded as being more reliable than TET elements at producing accurate results, however, HEX mesh generation is frequently more complicated and in some cases proves impossible, [226]. TET meshes on the other hand can often represent
complex geometries with ease and are frequently used. Whichever elements are selected for mesh generation, increased solution accuracy can also be achieved by using an adaptive mesh. In such cases, the FE process is repeated a number of times using a progressively higher fidelity mesh around areas of high stress until convergence errors are reduced to acceptable limits, [226].

4.3 Cell Structure Mechanical Responses

4.3.1 Experimental Methodology

The commercial 3D CAD package, Unigraphics NX2 [227], was used to create the eight cellular structures previously depicted in Figure 3.10, these were named ‘A’ to ‘H’ from left to right respectively. Each of these eight cell structures were taken in turn and by adjustment of the CAD model parameters, as demonstrated in Figure 3.9, cell geometries were created at 25%, 50% and 75% volume fractions, accurate to three decimal places. The eight cell structures at the three volume fractions can be seen in Figure 4.2.

![Figure 4.2 - Cell structures ‘A’ to ‘H’ at 25%, 50% and 75% volume fractions.](image)

The 24 possible cell geometries illustrated in Figure 4.2 were each taken in turn and scaled to fit exactly inside a 10 mm cube and then arrayed into a larger assembly consisting of 8 cells in X, 2 cells in Y and 2 cells in Z. In order to allow the same FE model to be used in all of the loading scenarios, a circular disc was added to one end of the assembled array of cells, which was necessary in order to apply the torque for the torsion scenario. One of the 24 arrayed cell assemblies with disc added can be seen in Figure 4.3.
With all 24 CAD models created, Unigraphics NX2 was then used as the pre-processing FE software to complete the FE models. This involved mesh generation, assigning material properties and the application of appropriate loading and boundary conditions to each CAD model individually. In each cell structure case, an automatic mesh was created using 10 noded tetrahedral elements (TET-10) as these were able to closely represent all of the complex cell structures considered. An arbitrary material of Steel was chosen for these FE tests, the material properties of which were applied to the mesh elements in all cell structure cases for all loading scenarios. The boundary conditions involved completely fixing all six degrees of freedom (translation and rotation about the X, Y and Z axis) of the end faces of the array (the ones furthest away from the disc). A static load of 5000 N was applied to the front circular face of the disc, with the load vector direction altering depending on the loading scenario (tension +X, compression –X and bending –Z). For the torsion scenario a 5000 Nmm torque was applied to the outside of the disc’s cylindrical face. A completed FE model displaying a TET-10 mesh, fixed boundary conditions and bending load can be seen in Figure 4.4.
With all the FE models completed in the manner described, they were passed one at a time to the associated FE solver for linear elastic analysis, in this case MSC Nastran,[228]. After completion of processing, each set of FE results was post processed using Unigraphics NX2 where displacement and Von Mises stress plots were inspected. The Von Mises stress plot to the FE model in Figure 4.4 is displayed in Figure 4.5.

![Von Mises stress plot for an 8 x 2 x 2 array of F25 cell structures subject to bending.](image)

The maximum and minimum elemental values of Von Mises stress ($\sigma_{vm}$) were recorded for each cell structure and load case combination in addition to the maximum nodal displacement values ($\delta$).

4.3.2 Results & Discussion

Normally, FEA results would be validated to ensure fitness for purpose in real world applications, as it is understood that the FEA technique is an approximation, which implies inherent errors are present. However, this is not a real world application (the material and applied loads are arbitrary) but a general investigation into the mechanical responses of the cell structures depicted in Figure 4.2, when subjected to different loading scenarios. As such, validation was considered unnecessary as the results were only intended to broadly identify an initial starting point for future experiments by
allowing the direct comparison of mechanical response results. This was achieved by consistently applying the FEA approach throughout these experiments (with a convergence error of less than 4%), hence, any errors in the results were considered to be consistent throughout and therefore irrelevant.

**Tension & Compression**

The maximum and minimum Von Mises stresses and the maximum nodal displacement values for the tension and compression load cases are displayed in Tables 4.1 and 4.2 respectively.

**Table 4.1 – FEA results of 8 × 2 × 2 arrays subjected to tension.**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>Max σv (MPa)</th>
<th>Min σv (MPa)</th>
<th>Max δ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.39 × 10⁻³</td>
<td>2.09 × 10⁻³</td>
<td>6.29 × 10⁻³</td>
<td>1.92 × 10⁻³</td>
<td>2.47 × 10⁻³</td>
<td>7.89 × 10⁻³</td>
<td>6.89 × 10⁻³</td>
<td>3.83 × 10⁻³</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.05 × 10⁻³</td>
<td>3.97 × 10⁻³</td>
<td>2.57 × 10⁻³</td>
<td>1.34 × 10⁻³</td>
<td>1.09 × 10⁻³</td>
<td>1.62 × 10⁻³</td>
<td>4.66 × 10⁻³</td>
<td>6.01 × 10⁻³</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.16 × 10⁻³</td>
<td>4.56 × 10⁻³</td>
<td>1.38 × 10⁻³</td>
<td>4.49 × 10⁻³</td>
<td>4.24 × 10⁻³</td>
<td>2.75 × 10⁻³</td>
<td>1.20 × 10⁻³</td>
<td>9.11 × 10⁻³</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.65 × 10⁻²</td>
<td>1.09 × 10⁻²</td>
<td>2.19 × 10⁻²</td>
<td>9.77 × 10⁻³</td>
<td>1.24 × 10⁻²</td>
<td>1.99 × 10⁻³</td>
<td>1.80 × 10⁻³</td>
<td>1.42 × 10⁻³</td>
<td>Max σv (MPa)</td>
<td>Min σv (MPa)</td>
<td>Max δ (mm)</td>
</tr>
<tr>
<td>6.51 × 10⁻¹</td>
<td>3.92 × 10⁻¹</td>
<td>6.75 × 10⁻¹</td>
<td>1.75 × 10⁻¹</td>
<td>1.83 × 10⁻¹</td>
<td>1.76 × 10⁻¹</td>
<td>3.73 × 10⁻¹</td>
<td>3.98 × 10⁻¹</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.82 × 10⁻²</td>
<td>1.96 × 10⁻²</td>
<td>2.62 × 10⁻²</td>
<td>1.88 × 10⁻²</td>
<td>1.80 × 10⁻²</td>
<td>3.96 × 10⁻²</td>
<td>2.49 × 10⁻²</td>
<td>2.74 × 10⁻²</td>
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<td>50%</td>
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<td>8.66 × 10⁻¹</td>
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<td>6.75 × 10⁻¹</td>
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<td>6.61 × 10⁻¹</td>
<td>Max σv (MPa)</td>
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<td>Max δ (mm)</td>
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<tr>
<td>9.31 × 10⁻¹</td>
<td>1.05 × 10⁻¹</td>
<td>9.60 × 10⁻²</td>
<td>4.06 × 10⁻¹</td>
<td>1.83 × 10⁻¹</td>
<td>4.39 × 10⁻¹</td>
<td>5.41 × 10⁻¹</td>
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**Table 4.2 – FEA results of 8 × 2 × 2 arrays subjected to compression.**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>Max σv (MPa)</th>
<th>Min σv (MPa)</th>
<th>Max δ (mm)</th>
</tr>
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<tbody>
<tr>
<td>4.39 × 10⁻³</td>
<td>2.09 × 10⁻³</td>
<td>6.29 × 10⁻³</td>
<td>1.92 × 10⁻³</td>
<td>2.47 × 10⁻³</td>
<td>7.89 × 10⁻³</td>
<td>6.89 × 10⁻³</td>
<td>3.83 × 10⁻³</td>
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<tr>
<td>5.05 × 10⁻³</td>
<td>3.97 × 10⁻³</td>
<td>2.57 × 10⁻³</td>
<td>1.34 × 10⁻³</td>
<td>1.09 × 10⁻³</td>
<td>1.62 × 10⁻³</td>
<td>4.66 × 10⁻³</td>
<td>6.01 × 10⁻³</td>
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<tr>
<td>5.16 × 10⁻³</td>
<td>4.56 × 10⁻³</td>
<td>1.38 × 10⁻³</td>
<td>4.49 × 10⁻³</td>
<td>4.24 × 10⁻³</td>
<td>2.75 × 10⁻³</td>
<td>1.20 × 10⁻³</td>
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<td>25%</td>
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<td>1.65 × 10⁻²</td>
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<td>9.77 × 10⁻³</td>
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<td>6.51 × 10⁻¹</td>
<td>3.92 × 10⁻¹</td>
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<td>1.75 × 10⁻¹</td>
<td>1.83 × 10⁻¹</td>
<td>1.76 × 10⁻¹</td>
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<td>1.80 × 10⁻²</td>
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The maximum stress values and nodal displacements from these experiments are identical in both tension and compression, as would be expected of a linear elastic material obeying Hooke's law [229], hence they have been combined into a single plot, shown in Figure 4.6. The minimum stress values have not been plotted as the presence of the circular disc in the model has caused these results to become unreliable (the disc was often the location of the minimum stress).
From Figure 4.6 it can clearly be seen that the various cell structures respond quite differently to one another when subjected to either tension or compression. The D, B and E structures exhibit the lowest stress and displacement values, whilst F, G and C exhibit the worst stress and displacement responses to these loads. The A and H structures can be considered to demonstrate average stress and displacement responses compared to the other cell structures.

Bending
The maximum and minimum Von Mises stresses and the maximum nodal displacement values for the bending load case are displayed in Table 4.3.
As before the maximum stress and displacements from this group of experiments has been plotted and can be seen in Figure 4.7.

**Figure 4.7** – Maximum Von Mises stress and maximum displacement for all 24 cell structure arrays subjected to bending.

Figure 4.7 shows that once again, the various cell structures respond quite differently to one another, this time when they are subjected to a bending load. The response behaviours to the bending load are quite similar to those of tension and compression. The D, B and E structures exhibit the lowest stress values, whilst G, F and C exhibit the worst stress values. However, this time the A structures display low displacement whilst the H structure exhibits similar displacement to those of G. In terms of stress, once again A and H can be considered to exhibit average responses compared to the other cell structures.

**Torsion**

The maximum and minimum Von Mises stresses and the maximum nodal displacement values for the torsion load case are displayed in Table 4.4.
Once again the maximum stress and displacements values have been plotted, these are shown by Figure 4.8.

Figure 4.8 shows the various cell structures respond quite differently to one another and to the previous experiments, this time when they are subjected to torsion. The structures that demonstrate the lowest stresses and displacements are C and F, whilst B, E, A, D and G all exhibit the high stress values, of which B, E, D and A also display high displacements. The H structure again appears to exhibit average stress and displacement responses compared to the other cell structures.
4.3.3 Conclusions

General observations from these experiments suggest that cell structures that are good at withstanding tension, compression or bending appear to be the worst in torsion and vice versa. Also, the H structure does not strictly excel or falter in any of the loading environments, being quite average in all cases compared to the other structures. This suggests that the H cells are possibly a good ‘all round’ cell structure. The highest stresses of all of the tests appear in the bending load case (two orders of magnitude higher than in any of the other experiments). This suggests that a bending type of problem will be the most challenging of the four load scenarios considered, when trying to achieve a geometry that exhibits a uniform stress distribution. As such, a bending problem in the form of a simply loaded cantilever beam is a justified choice as the focus of this research.

A further observation can be made when inspecting the experimental results displayed by Figures 4.6, 4.7 and 4.8. For all of the cell type structures, under all four loading scenarios, the greatest variance in both stress and displacement values was witnessed for the lowest volume fraction cells considered (25%). Additionally, it can be shown that as the volume fraction increases to 50%, the variance in both stress and displacement values reduces between different cell structures. This trend can be seen to continue as the volume fraction increases further to 75%, the stress and displacement values for all the different cell structures become similar to one another. This observation suggests that as the volume fractions of the various cell structures approaches 100%, the mechanical responses will become the same for all cell structures considered. This observed behaviour is in fact caused by the parabolic relationship that exists between the density (volume fraction) and Young’s Modulus (stress) values of microstructures [123]. This parabolic relationship between density and stress suggests that cell structures at several low volume fractions should have been used throughout the remainder of this research.
CHAPTER FIVE
SYSTEM DESIGN & DEVELOPMENT

5.1 Introduction

The previous Chapters have helped to identify several research questions, as outlined in Section 3.4.3. However, before these questions can be answered, a GA based topology optimisation system capable of considering unit cell structures that vary in form and volume fraction must first be constructed. This Chapter provides an outline of the methodology and modelling undertaken in the creation and development of such a system using a Knowledge Based Engineering (KBE) approach. Initially a brief introduction to KBE is given with emphasis towards the particular problem at hand, namely designing the required unit cell structure GA based topology optimisation system.

5.2 Knowledge Based Engineering

KBE is fundamentally about reuse, but the concept is much broader. The aim of KBE is to take advantage of any relevant information, past experience and expertise that can be applied to each phase of the engineering lifecycle of an end-user product [230]. KBE systems strive to capture product and process information to allow businesses to model an engineering process and then use the model to automate all or part of the process.

A KBE product model represents the engineering intent behind the whole design lifecycle of the product from conceptualisation through to manufacture, essentially recording the what, why and how of the design, [231]. The product model is a computer representation of the entire product design process and usually contains information on
both the product and processes that are used to create the final part. However, a product model can also draw upon information from outside its own environment, from so-called knowledge bases, [232]. Knowledge bases can exist in many forms including spreadsheets, databases, handbooks, engineering formulas, proprietary software, human judgement and heuristics, legacy programs and costing models, [231]. The ability to create and reference such knowledge bases whilst making their informational content readily available to aid the engineering process constitutes KBE, [230].

Traditional CAD systems typically allow parts, assemblies, and drawings to be altered in a parametric manner using the geometry and dimensional relationships as the driving force, thereby providing design automation capabilities. However, the capabilities of such systems in permitting extensive changes to the product configuration are often restricted. Consequently, CAD systems are incapable of answering many questions related to a design, such as: Have any design constraints been violated? How much will this product cost? Can this part be manufactured? Is the design optimal? [230]. In contrast, KBE has the ability to answer such questions, with knowledge as the central driving force the geometry is driven by configuration and engineering rules. KBE systems provide far more power and flexibility in the development of design automation systems compared to CAD systems. As such, KBE represents the merging of CAD, artificial intelligence and object-oriented programming, [231].

The intended use of KBE is to automate the creation of engineering solutions. This implies there is a repetitive engineering problem which can be automated. The KBE approach is particularly beneficial when the solution requires a combination of configuration, engineering and geometry elements. Configuration refers to the selection and assembling of individual components that combine together to form a complete coherent product. Here, the term engineering refers to the actual decision making process of the system in terms of the validity and applicability of the components, whilst geometry refers to physical organisation of all of the components, [230].

The proposed GA based topology optimisation approach using cellular structures, involves the repeated evaluation of numerous topologies via the solving of FEA and can
therefore be considered as a KBE system. In this instance, the configuration element describes the action of the GA in selecting cell structures to occupy positions within the design domain. The engineering element directly relates to the evaluation of the system objectives using FEA and the geometry element corresponds to the creation of the individual cellular structures and of the overall cellular beam topology.

5.2.1 TechnoSoft’s Adaptive Modelling Language

In the early 1990’s, a Cincinnati based company, TechnoSoft Inc., created their own Adaptive Modelling Language (AML) for KBE applications [231]. AML is described as a KBE modelling framework that enables multidisciplinary modelling and integration of the entire product and process development cycle [231]. The concept of KBE, the reuse of information, is captured within the modelling language of AML resulting in the ability to create complex system rules that are vastly more powerful than simple parametric changes.

AML is based on the concept of object-orientated programming, where the building blocks of user developed applications are objects not procedures or functions as in CAD based design automation systems. These objects are created as instances of predefined AML classes, and as such inherit their property information from these classes, [233]. TechnoSoft have built a robust mathematical modeller into AML that provides access to logical operators, many mathematical functions and matrix manipulation in addition to common looping constructs. All computation in AML is essentially demand-driven whereby only that which is required is calculated. This is enabled by the automatic dependency tracking between objects and properties within AML applications. Consequently, AML is able to make effective use of the computational resources available [231].

Using AML, common software programs can be linked together in the KBE product or process model including CAD models, Excel files, text files, executable programs and external functions. For example the dimensions and parameters of a ProENGINEER CAD model can be driven by FORTRAN code functions, generating a text file output whilst also linking to data and macros contained within an Excel costing model, [234].
AML can facilitate the automated use of industry standard FE pre and post processing and solving software, including mesh generation, through software specific interface modules and AML's mesh database, [233]. FE interface modules already exist for Patran, Nastran, ANSYS, LS-DYNA and MARC amongst others. In addition to these, TechnoSoft have used AML to create several ‘black-box’ modules that users can readily incorporate into their own applications. Modules relevant to this research include an optimisation module containing a GA and a distributed computing module which can enable remote and parallel computation across distributed workstations.

AML has been applied to numerous industrial KBE applications and has repeatedly proven itself useful to solving problems involving structural design and optimisation where FEA is combined with optimisation routines. Several applications exist within the fields of automotive and aeronautical engineering [235, 236], including projects at NASA's Langley Research Centre [237]. As such, AML was chosen to create the cellular structure GA based topology optimisation system required in order to answer the research questions identified in Chapter 3.

5.3 Basic System Design & Methodology

Having identified the software to be used in the creation of the required system, the next stage is to outline the proposed system design and suggest the methodology by which this will be achieved. The proposed system will combine topology optimisation with a GA which utilises FEA to evaluate the fitness of each solution. This is a similar approach to that of the DesignLab software investigated in Chapter 3. However, the proposed system will expand upon this capability by allowing the discretised design domain to be filled with single material, cellular structures which vary in form and volume fraction. The architecture of the proposed system is outlined in the flowchart seen in Figure 5.1.
The GA will essentially call the cell structures from a knowledge base (internal library), until the whole design domain is populated. A simple brick mesh like that used in DesignLab will then be applied to the whole design domain, created internally within AML, so that a single 8 noded HEX element represents each cell structure. With each cellular structure represented by an 8 noded HEX element, their differing behavioural responses to mechanical loads could be simulated by adjusting the equivalent material properties of each HEX element. With the material properties assigned to the model accordingly, a linear elastic FEA will then be performed on the solution topology. The FE results will be used for feedback into a GA to determine fitness evaluation against the specified optimisation objectives. The GA will use this objective data when selecting parent pairs from which to evolve new beam topologies in subsequent generations.

The methodology adopted in developing the required system, as proposed in Figure 5.1, can be broadly divided into the following steps:

- Identification of system objectives.
- Creation of cell structure library/database.
• Creation of problem definition environment.
• Integration of FEA and verification of results.
• Integration of MOGA.
• Addition of KBE design rules.

Using the methodology outlined above, the required system was created by the author using AML (the full code for which can be found in Appendix C). The focus of this research will be the study of cellular structures on a simple cantilever beam problem. This problem type was chosen for several reasons. Firstly, the cell structure mechanical response tests within Chapter 4 indicated that a bending load caused the highest stress values to be exhibited within the overall structure. This suggested therefore that a bending scenario would prove the most challenging load case to optimise in terms of creating a more uniform stress distribution. Secondly, a cantilever beam problem is a widely understood relatively simple engineering problem that has frequently been the focus of other topology optimisation systems, and can therefore be used to benchmark the results against. Thirdly, a cantilever beam was chosen as the study of the DesignLab investigation, allowing a direct comparison to be made between two GA based topology optimisation systems.

A brief overview of the work undertaken in completing each stage of the methodology outlined above is now given, however, additional detail can be found in Appendix D.

5.3.1 System Objectives

The DesignLab investigation in Chapter 3 showed how a weighted sum of objectives could be applied to the topology optimisation of a cantilever beam. In this instance the weightings of the weighted sum were unknown to the user, as were the precise objectives that were combined together, although FOS was definitely considered. Consequently, the relationships between the weighted sum objective components were not able to be inspected within the DesignLab investigation and are therefore not completely appreciated at this time.

If a weighted sum of objectives is to be implemented within this basic system design, the relationships, in terms of trade-offs, between the individual objectives must first be
fully understood. For this reason, the three system objectives chosen for this research will remain separated from each other, allowing for the possible creation of a combined weighted sum at a later date. The three system objectives are defined as follows:

1. Minimise the mass of the beam geometry (kg).
2. Minimise the maximum nodal displacement of the beam geometry (mm).
3. Minimise the maximum minus the minimum elemental Von Mises stress values (MPa).

The first objective is unconnected to the FE results, being purely dependent on the specific beam geometry. This objective should drive the removal of material from the design domain, either by choosing a lower volume fraction cell type or a void cell. The second objective is essentially concerned with increasing the beam's stiffness (or reducing compliance) and is based directly on the FE results. The impact of this objective is expected to be the addition of material to the design domain through the preference of higher volume fraction cell types. The third objective is also dependent on the FE results and will seek to reduce the stress range across the beam structure, hence creating a more uniform stress distribution as desired and frequently witnessed in natural world structures. The individual effects of this objective are expected to be a mixture of addition of material in some cases and the removal of material in others in order to reduce the Von Mises stress-range exhibited.

A MOGA will be used to evaluate these three objectives for each beam topology, allowing the creation of a 3D Pareto Set of equally optimal solutions. The Pareto Set should clarify the relationships between these multiple objectives when they are implemented together within the same optimisation problem, therefore showing the trade-offs between their conflicting agendas.

These three objectives have been defined using AML. However, in order to create the beam topologies made of cellular structures for the assessment of these objectives, the cellular structures themselves must first be created using AML. This process will now be outlined.
5.3.2 Creation of Cell Structures

The eight different cellular structures named A to H that were previously created using Unigraphics NX2 in Chapter 3, all had to be recreated within AML. The approach taken to accomplish this task was to define a generic class for each of the eight different cell structures using basic geometric objects and Boolean constructs. The three volume fractions required of each cell structure (25%, 50% and 75%) could then be created as an instance of the generic class tailored to reflect the specific volume fraction desired. This approach is more easily understood with an example, as illustrated by Figure 5.2.

![Diagram showing the creation of A-class cell structures at different volume fractions](image)

Figure 5.2 - Creation stages of A-class cell structures at different volume fractions.

Figure 5.2 shows how the generic A-class (more descriptively known as box-with-holes-class) was defined using three cylinder-objects and a box-object positioned and orientated as shown. These geometric objects were then combined within a difference-object (Boolean subtraction), resulting in the generic A-class structure in the centre of the Figure. Figure 5.2 also shows the A75, A50 and A25 classes that inherit their structures from the previously defined generic A-class. This approach of inheriting
information from previously defined classes is more efficient than defining all of the classes separately as the only differences between the A75, A50 and A25 classes are the diameters of the cylinder-objects used and the associated changes in volume fractions.

All of the cell structures previously considered in Chapter 4 were defined in AML in a similar manner (as seen in Appendix C), with the only differences being the number of cylinder-objects used, their positions and orientations, and the type of Boolean construct implemented. The three volume fractions of each cell structure class were created in exactly the same way in each case, with the structure inherited from the predefined generic class. All of the 24 cell type class definitions (8 structures × 3 volume fractions) along with a solid-class (solid cube) and void-class (empty cube) were defined within AML effectively creating a cell structure library or database. These cell type options could then be called upon to fill positions within the design domain, hence creating a unique cellular structure based topology.

5.3.3 Default Problem Definition

A 3D design domain has been defined using AML allowing cell structures to occupy positions within a Cartesian coordinate environment. This design domain has been constructed in such a way as to define a cantilever beam type problem by default, including load and boundary constraints, as previously discussed in Section 5.3. The default problem definition consists of a design domain in the form of a 10×5×1 array of 10 mm cells (side length of cube). The load and boundary condition locations are created automatically by intersections with predefined fixed geometry, consisting of a sheet for the constraints and a line for the load. The default cantilever beam problem definition is illustrated in Figure 5.3, with the cyan coloured points indicating constrained nodes and the red coloured points indicating loaded nodes. The red arrows indicate the direction of the default load. The sheet and line geometry are parametrically linked to the overall size of the design domain, allowing changes to the design domain size to be reflected in the size of the sheet and line geometry.
The default problem definition specifies initial input values for the system parameters which can easily be modified by the user to suit a particular application. These inputs include the load magnitude (1000 N), the load direction (negative Y axis), the degrees of freedom of constrained nodes (all 3 rotation and all 3 translation), the chosen material (Aluminium), and which cell types will initially be considered (void-class solid-class A50-class). The default cell type options are randomly called from their AML library to take up the 50 positions within the design domain, resulting in the creation of a cellular beam structure as demonstrated by the example shown in Figure 5.4.

Each cellular beam geometry has a corresponding chromosome representation in the form of a 3D integer array, with each of the possible positions within the design domain corresponding to a discrete variable. The discrete options for each variable are the integer values assigned to the available cell types, determined by the order in which the available cell types appear in a list counting up from zero. For the default problem definition therefore, the integer values are a ‘0’ for void-class, a ‘1’ for solid-class and a
‘2’ for A50-class for all 50 discrete variables. Figure 5.5 shows the chromosome string representation of the example beam geometry from Figure 5.4 highlighted in blue, with the default cell types available highlighted in green.

The likelihood that one cell type will be selected over another for any of the discrete variables is governed by a probability of selection. This probability is calculated by how many times the cell type is entered in the green highlighted list of Figure 5.5. Therefore, for the previous example using the default problem parameters, the probabilities of selection for the three available cell types (void-class, solid-class and A50-class) are all equal at exactly one third as all of them appear once in the list. If the available cell types list was then changed to (list ‘void-class ‘solid-class ‘solid-class ‘A50-class), this would mean that void-class would be assigned the integer value of ‘0’, solid-class would be assigned ‘1’ and ‘2’, and A50-class would be assigned ‘3’. Hence the probabilities of selection for void-class, solid-class and A50-class would therefore change to 0.25, 0.50 and 0.25 respectively.

The default problem definition was created to allow the automated fast construction of cantilever beam type problems. However, the default settings can be modified to enable problem variations to be considered, with examples of possible modifications including hollow and I-section problems, are given in Appendix D. Furthermore, the user can alter the locations of the applied load and boundary conditions by either moving or rotating the associated sheet or line geometry.
5.3.4 Integration of FEA & Verification of Results

With the basic problem definition created as previously described, the next stage was to fully integrate the FEA process within the basic system design. To accomplish this, AML was used to create the required simple brick mesh internally, consisting of a single 8 noded HEX element to represent each cellular structure within the design domain. AML's Nastran interface module was then incorporated into the basic system to perform the processing stage of the FE process.

With Nastran now integrated into the topology optimisation system to solve the FE jobs, the results obtained needed to be verified to provide confidence in the system. Verification was achieved by substituting the chromosome array for a solution previously generated by DesignLab (obtained in Chapter 3) into the AML system, matching the problem definition previously used. The problem was then analysed using AML integrated with Nastran, and the FE results post processed using Patran so that the interpretation of the results was temporarily completely independent of AML.

Figure 5.6a shows the Von Mises elemental stress plot for the chosen beam viewed using Patran. This plot was then compared to the original DesignLab stress plot for the same beam geometry, shown in Figure 5.6b. These two stress plots show an excellent correlation indicating that the FE calculations were indeed being calculated consistently, suggesting that the various data input from AML was being entered and interpreted correctly. Subsequently, the post processing of the FE data was automated within AML to be plotted upon the user's request.
5.3.5 Integration of MOGA

With the FEA process fully integrated into the basic system, and the generated results verified, the GA required to perform the optimisation process can now be included. As stated in Section 5.3.1, there will be three separate system objectives. To allow the simultaneous evaluation of these multiple objectives, a MOGA is required. Literature suggests that in order to achieve high performance and increased efficiency, a GA should ideally be tailored specifically to a particular application. However, this approach is based on years of experience in solving similar types of problems and average results should still be achieved using any generic GA or MOGA according to the *No Free Lunch* theorem, [238]. Therefore, to gain an initial understanding of the effectiveness of a GA based cellular structure approach to topology optimisation, the MOGA incorporated within AML’s optimisation module, AMOpt, was used for the remainder of this research. This MOGA has been written in AML by TechnoSoft and is referred to as TSMOGA, which is based on the *Niched Pareto GA* by Horn et al. [191], explained previously in Section 2.6.2.

In addition to the standard GA parameters of population size and number of generations, several other parameters exist within the TSMOGA including; noise power, spread (sigma), extreme value and probability of mutation. The parameters that govern the behaviour of the TSMOGA can all be seen in Figure 5.7. Where relevant to this research, these parameters will be explained in greater detail.

![Figure 5.7 – TSMOGA parameters, [234].](image-url)
Noise Power – this parameter is used to control mutation rates for problems containing continuous variables. Consequently it is of little interest to this research where discrete variables are used. However, further information can be found in [239].

Spread (Sigma) – this parameter is used to control the quality of the Pareto Front that is computed for a multi-objective solution. It is a measure of spread of the solution and the larger the value, the more spread out will be the optimal solutions on the Pareto Front. This allows the final Pareto solution to search and spread out over larger areas of the objective search space. Sigma is used during the GA selection step when choosing parent pairs for reproduction based on niche counts as described earlier, [191, 239].

Extreme Value – this parameter is not directly used by TSMOGA during operation. It is an absolute value that is assigned to an objective function in case the objective function fails to evaluate, hence acting as an error recovery mechanism. This is typically set to be a very high number for minimisation problems therefore highly penalising solutions to which it is assigned. The extreme value is returned instead of the actual objective value when the objective function fails, [239].

Probability of Mutation – this parameter is used to control mutation rates for problems containing discrete variables. It is the probability that a particular discrete variable will be mutated through a random change to a different value and must be set to between 0.0 and 1.0, [239].

The Restart from File? and Save for Restart? options allow the user to divide an optimisation run into multiple segments that seamlessly follow on from one another as if the entire run had occurred as a single continuous operation, [239].

The Interrupt Property allows the creation of user defined termination criteria for the optimisation process. Termination criteria can be set based on convergence of subsequent solutions, however, where multiple objectives are considered this can be difficult if not impossible to implement effectively so it is not set by default. If not implemented the termination criteria of TSMOGA will be the completion of the specified number of generations, [239].
5.3.6 Basic System Process Flowchart

The proposed basic system, previously depicted in Figure 5.1, has been created using AML as described in the previous Sections and Appendix D. The completed basic system design is illustrated in the form of a flowchart in Figure 5.8, where all of the various stages and interactions of the entire optimisation process can be seen. All actions performed by AML are coloured in pale blue with steps performed by Nastran in pink.

![Figure 5.8 - Basic system process flowchart.](image)

The whole process is started by defining the particular cantilever beam problem and MOGA parameters using a basic GUI. The MOGA commences by creating an initial random population of beam geometries made up of cellular structures. Each geometry is considered in turn and evaluated with respect to the three separate system objectives.
The first objective (minimise mass) can be derived directly from the geometry, however, the remaining two objectives can only be evaluated after performing FEA. As these two objective values are demanded by the MOGA, the creation of the simple brick mesh created internally within AML executes. The mesh data is then combined with the material properties, load and boundary conditions using AML’s Mesh Database (Mesh.db) to complete the FE pre-processing stage. This results in the creation of a *.bdf file, the required input deck file for Nastran to solve. The Nastran job is then completed creating a *.f06 results file. This results file is parsed (searched through) by AML to find the appropriate displacement and Von Mises stress data which is then fed back into the MOGA to allow the remaining two objectives to be evaluated.

This process is repeated for all of the individual beam members of the current population. When the existing population has been evaluated the process continues with the selection of parent beams by the MOGA for subsequent reproduction via crossover and mutation, which in turn generates new geometry parameters (new topologies). The process continues in this manner until eventually the termination criteria of the MOGA have been satisfied or the maximum number of iterations reached, hence stopping the optimisation run.

This basic system was developed further to incorporate some KBE design rules that help control the behaviour of the optimisation process and some file export options to enable selected beam geometries to be inspected outside of the system. These include a Microsoft Excel spreadsheet, a 3D Parasolid file and a triangulated STereoLithography (STL) file. The KBE design rules will now be discussed in the following sections respectively, with the file export options detailed in Appendix D.

5.3.7 Addition of KBE Design Rules

In order for the basic system to operate as intended, three KBE design rules were implemented into the system. These three rules will now be explained further.

Rule 1 – Loaded Cells Cannot be Void

The cell positions within the design domain that are loaded must always be prevented from becoming void cells throughout the entire optimisation process. Failure to do so
would result in the optimisation process terminating almost immediately as a consequence of the GA misinterpreting the FE results. An example beam topology that would give rise to this situation is depicted in Figure 5.9a. The FE results returned from such a topology would show the beam to have a maximum displacement of 0.0 mm and a Von Mises stress-range of 0.0 MPa, as shown in Figure 5.9b.

![Figure 5.9 - Beam geometry with a void at the loaded cell position (a) and the resulting stress plot (b).](image)

When this data is fed back into the GA for fitness evaluation purposes, the GA is effectively fooled into thinking that this unacceptable solution satisfies the second and third objectives perfectly. Hence the GA assumes the geometry to be far superior to any real solution where actual displacement and stress results are obtained. As a result, the GA readily selects the unacceptable geometry for reproduction and after two or three subsequent generations (depending upon population size) the entire population will consist of variations of this geometry. This results in the optimisation process swiftly being artificially steered towards a known flawed 'optimal' solution. Consequently, a rule was introduced to prohibit this behaviour by removing the associated void integer value (usually a 0) from the list of discrete variables for any loaded cells. The loaded cells are still able to change freely between the non-void cell types available until the most suitable is found for the specific application.

**Rule 2 – Removal of Invalid Geometries**

The FE result files (*.f06 format) of some beam geometries contain fatal error messages that warn the user about excessive pivot ratios causing the FE run to terminate. An example *.f06 Nastran results file displaying this error message is shown in Figure 5.10 with the error highlighted in red.
Geometries that cause excessive pivot ratios are considered to be invalid solutions and can be categorised into two distinct groups as demonstrated by Figure 5.11.

The first group consists of geometries where the loaded cells are completely disconnected from the constrained cells. Figure 5.11 shows two examples of this load-constraint disconnection in beams (a) and (b), where the disconnection is highlighted by
the yellow dashed lines. The second group consists of geometries that have cells joined to one another by single diagonal connections only. Figure 5.11 also shows two examples of this diagonal only connection, in beams (c) and (d), the invalid regions are highlighted by the yellow dashed circles.

A rule was introduced to prevent the occurrence of invalid geometries from influencing the optimisation run. This rule was implemented by allowing AML to search through each *.f06 results file for the USER FATAL MESSAGE string when the results are inspected for feedback into the GA. If a results file is found to contain this error message, it is immediately thrown out of the optimisation run, effectively removing the particular beam solution from the GA population. If the results file does not contain this message string, the stress and displacement values are returned for the objectives instead. Figure 5.12 shows the relevant section of AML source code that the topology optimisation system uses to identify invalid geometries.

![AML Source Code](image)

**Figure 5.12** – Section of AML source code that searches for the USER FATAL MESSAGE.

**Rule 3 – Cell Type Valid Neighbours**

This valid neighbours rule was needed to help guide the user when selecting cell types that could be used together for a particular problem. Some cell type combinations would not be beneficial, for example B25 and A25 would not share any contact surfaces...
if they were neighbours (adjacent to one another). As a result they would be unable to transmit any load and associated stress from one to the other, not much use for structural scenarios. In order to establish what cell type combinations were valid neighbours, all of the possible neighbouring combinations were subjected to a Boolean unite feature within Unigraphics NX2. All combinations that did not result in an error were considered valid. The results to these valid neighbours tests are shown in Table 5.1.

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</tbody>
</table>

Cell structure combinations that produce invalid neighbours are represented in Table 5.1 by red crosses, with green ticks indicating valid neighbour combinations. The information contained within Table 5.1 was entered into the basic topology optimisation system using AML. This rule was implemented by the creation of a pop-up interactive
form that helps guide the user when selecting cell types for a particular problem. As the user selects options from the form, the form display updates, greying-out options that are no longer available due to previous selection choices. The selection order is therefore important and will also determine the associated integer values to the selected cell types. Figure 5.13 shows the form before and after cell type selections have been made. The right hand form shows selections made highlighted in green and the associated greyed-out cell types highlighted in red as a consequence of these selections.

Figure 5.13 – Valid neighbours form before (left) and after (right) cell type selections have been made.

5.4 Requirement for a More Advanced System

The idea behind the basic system was to use a single 8 noded HEX element to represent each cellular structure within the design domain, such that the overall beam geometry
could be meshed and solved quickly using FEA; an example of this equivalent representation is shown in Figure 5.14. The HEX elements would have the physical behaviours of the cellular structures mapped to them in the form of equivalent material properties. In order to accomplish this, the physical responses of the various cellular structures, at multiple volume fractions and cell sizes, would need to be determined through further investigation (i.e. numerous FEA runs and physical tests).

![Figure 5.14 - Equivalent FEA representation of cellular structures in the basic system.](image)

Whilst this simplified approach would be quick to mesh and solve in terms of FEA, initial investigations suggested that the derivation of the equivalent mechanical properties of each cellular structure was not a trivial matter. FEA was performed on a number of standard tensile test dogbone samples incorporating cellular structures, such as that depicted in Figure 5.15.

![Figure 5.15 - FE model of a standard tensile test specimen incorporating cellular structures.](image)
The FE results indicated that the size, volume fraction and spatial arrangement with regards to adjacent cells, all had a complex effect on the equivalent material properties required. One of the main issues with this simplification, is that the FEA assumes there is material connectivity between HEX elements, which is not necessarily present in all of the cellular structures. This issue is highlighted by the unconnected diagonally positioned B50 cells seen in Figure 5.14, whereas the simplified HEX representation (also pictured) displays diagonal connectivity.

Based on these initial findings, the concept of representing each cellular structure by a single HEX element is over simplified and therefore unrealistic. The basic system is fine for solving brick mesh problems where the material properties are known like those previously solved using DesignLab, where no complex cellular structures were considered. However, a significantly more accurate geometric representation is required for each cellular structure in order to generate meaningful FE results within the optimisation process. This will remove the need for multiple equivalent material properties, allowing a single material to be defined and used.

### 5.5 Advanced System Design

The previous section highlighted the need for an advanced system if a cellular structure approach to topological design is to be followed. The advanced system must be able to represent the actual cellular structures themselves within the FE steps to a reasonable level of accuracy, and unlike the basic system, must not rely on any kind of material property mapping that would be critical to the overall success of the system.

The cell structures that are being considered have already been successfully represented to an acceptable level of accuracy during the Cell Structure Mechanical Responses tests in Chapter 4. Here a TET mesh was generated for each cell structure array in preparation for FEA to be performed. A similar procedure should therefore be followed for implementation within the advanced system using AML.

The existing HEX mesh generation in the basic system is performed internally by AML. However the automatic creation of a complex TET mesh in a similar manner by AML, is a far from trivial process. TechnoSoft have avoided this issue altogether by creating
an interface module that links directly with the industry standard pre-processing software, Patran [240]. This Patran interface module has been incorporated within the basic system in AML to allow the automatic generation of complex TET meshes, as required for the advanced system. This is detailed in the AML code of Appendix C.

Figure 5.16 illustrates both the basic HEX and the advanced TET meshes generated for the same beam geometry when a Von Mises stress plot is produced.

![HEX mesh and TET mesh](image)

Figure 5.16 – Basic HEX and advanced TET mesh solutions to the same beam geometry.

It is clear that a considerably more realistic representation of the actual cellular structures is achieved using the TET mesh generated in Patran compared to the simple HEX mesh via AML. However, the number of elements and nodes within such a TET mesh is significantly higher than in the simplified HEX mesh, resulting in a substantial increase in meshing time required. Consequently, the larger mesh file also causes a slower processing time due to the increased FE calculations that have to be solved.

Over the duration of an optimisation run it is likely that thousands of meshes will need to be generated, one for each topology considered. If a single TET mesh generation and its subsequent processing are slow procedures, the consequences will be significantly
amplified over the entire optimisation run. It is for this reason that 4 noded TET elements have been used in the automatic Patran meshes, as opposed to the 10 noded TET elements used in Chapter 4, in an attempt to reduce computational demands.

The TET meshes created using Patran are not adaptive in as much as they are only generated once to keep computation requirements manageable. Nevertheless, the meshes are able to refine themselves when describing more intricate cellular structures by using smaller sized TET elements where they are required. This mesh refinement is demonstrated in Figure 5.17 using a $5 \times 5 \times 1$ array of void, solid, A25, C50 and H25 cell types. This feature of the advanced system has the benefit of representing detailed structures accurately without having an excessively high fidelity mesh throughout.

![Figure 5.17 - A 5x5x1 array and its automatic TET mesh exhibiting refinement by describing finer cell structure geometries using smaller TET elements.](image)

The resolution of the TET meshes generated by Patran can be refined by the user by adjustment of a maximum element size scale factor which is a fraction of the cell size. This parameter will be inspected further in Chapter 7 with respect to its effects on meshing times, processing times and solution accuracies for various beam geometries.

### 5.5.1 Advanced System Process Flowchart

The advanced system design process flowchart with its automatic TET mesh generation step added via Patran can be seen in Figure 5.18. As before, all actions performed by AML are coloured in pale blue with steps performed by Nastran in pink and steps by Patran in green.
With the Patran step now incorporated into the system to allow the creation of a TET mesh, there is now a possible need for increased computation power due to the complexity and size of the meshes created via this method. Consequently the effects that the mesh resolution will have on mesh generation times and subsequent FE solving times will be of particular interest to the remainder of the research as any impact will have severe implications being multiplied thousands of times over the entire optimisation process.

To allow the effects of computational power on TET mesh generation and subsequent FE solving to be inspected, a scheduling software application for running remote jobs...
was incorporated into the advanced system. This arrangement has been implemented using AML's Distributed Computing Interface (DCI) module, which enables the execution of executable programs over remote workstations, [241]. The DCI module is able to communicate with a program called TechnoSoft Computing Resources Manager (TSCRM) which in turn, assigns jobs to a TechnoSoft Execution Manager (TSXM) program. This arrangement is illustrated in Figure 5.19, with the red arrow indicating two way file transfer between the DCI module and the TSCRM program. The blue dashed line shows the basic minimum setup required where a single instance of the TSCRM and TSXM programs are installed on the same workstation. This basic setup has been followed when incorporated into the advanced system design, the full AML code of which can be found in Appendix C.

It should be noted that further TSXM installations can be added to the single TSCRM as shown in Figure 5.19, thereby increasing the executable resources available to the TSCRM. This can be achieved by the addition of further workstations to form a cluster of TSXM installations. In this form the TSCRM can enable the remote parallel execution of jobs over the clustered TSXM workstations, [241].

**5.5.3 Advanced Remote System Process Flowchart**

The remote job scheduling software has been implemented into the advanced system design for both Patran and Nastran jobs as detailed in the flowchart in Figure 5.20. As before, all actions performed by AML are pale blue with steps by Nastran in pink and steps by Patran in green. All steps performed remotely are highlighted in yellow.
All of the options for both the basic and advanced systems including mesh types and all remote jobs have been combined into a simple GUI using AML, further details of which can be found in Appendix D.
CHAPTER SIX

PRELIMINARY EXPERIMENTS & RESULTS

6.1 Introduction

The previous Chapter detailed the design and development of the required GA based topology optimisation system using cellular structures. Now that this system has been created using AML, some initial areas of interest can be inspected in this Chapter of preliminary experiments.

These preliminary topology optimisation experiments will allow a direct comparison to be drawn with the DesignLab investigation conducted in Chapter 3. To enable this comparison, the basic AML system (HEX mesh) will be utilised along with void and solid cells of two different materials (Steel and Aluminium). In addition to the comparison with DesignLab, these preliminary experiments will help to determine suitable parameter values for subsequently more advanced experiments and provide insight into the relationships between the three system objectives and their trade-offs by systematically suppressing combinations of the objectives.

6.2 DesignLab Equivalent Experiments

6.2.1 Experimental Method

The two material DesignLab experiments conducted in Chapter 3 were recreated within the basic AML system. The default problem definition previously described in Section 5.3.3 had to be modified to allow the design domain to take the form of a 20×10×1 array of 10 mm cells. The load magnitude was altered to match that used in
the earlier investigation as was the probability of mutation. The material properties used in these experiments were taken from those given by DesignLab to ensure consistency with the earlier investigation and can be seen in Appendix E. The experimental parameters used for these DesignLab equivalent experiments are summarised in Table 6.1, including the integer values assigned to each cell type and the probabilities of selection for each cell type by the GA.

<table>
<thead>
<tr>
<th>Experiment Numbers 1-8</th>
<th>Experiment Names</th>
<th>DesignLab Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 – Minimise Mass, (kg)</td>
<td>Number of Variables = (20x10x1) = 200</td>
<td></td>
</tr>
<tr>
<td>2 – Minimise Displacement, (mm)</td>
<td>Population Size = 50</td>
<td></td>
</tr>
<tr>
<td>3 – Minimise Stress-Range, (MPa)</td>
<td>Number of Generations = 200</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Probability of Mutation = 0.015</td>
<td></td>
</tr>
<tr>
<td>Cell Types</td>
<td>Integer Value</td>
<td>Probability of Selection</td>
</tr>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.33333</td>
</tr>
<tr>
<td>Solid-class</td>
<td>1</td>
<td>0.33333</td>
</tr>
<tr>
<td>Solid2-class</td>
<td>2</td>
<td>0.33333</td>
</tr>
</tbody>
</table>

Mesh Type = HEX
Mesh Maximum Scale Factor = N/A

In total eight experiments were run in this investigation as detailed below. Initially all three objectives were used followed by two alternating objectives at a time and finally by each single objective individually. The final experiment was conducted as a continuation of the previous single objective run to investigate the effect of increasing the number of iterations on the chosen objective value.

1. All three objectives (mass, displacement and stress-range)
2. Two objectives (mass and displacement)
3. Two objectives (displacement and stress-range)
4. Two objectives (mass and stress-range)
5. Single objective (mass only)
6. Single objective (displacement only)
7. Single objective (stress-range only)
8. Single objective (stress-range only) continuation from experiment 7.
Each of the eight experiments listed above were run sequentially using the basic AML topology optimisation system. The results and discussion for each of these experiments is now presented in turn.

6.2.2 Results & Discussion

Experiment 1

Experiment 1 consisted of running the DesignLab equivalent experiment with all three system objectives present. As such, a three dimensional Pareto Set was generated (one dimension for each objective), which can be seen plotted in Figure 6.1.

The 3D Pareto plot shown in Figure 6.1 is difficult to interpret thereby making the relationships in terms of trade-offs between the three system objectives unclear. Consequently, the same Pareto Set data has been plotted as three separate 2D graphs in Figure 6.2, each displaying two of the three system objectives in an attempt to overcome this visualisation problem.
Figure 6.2 shows that there are no clearly visible trends in the data generated by experiment 1 and that the data is not evenly distributed across the design space with many solutions being clustered together in groups with some scattering also present. The entire Pareto Set of equally optimal solutions for experiment 1 can be seen in full in Appendix F. The extreme solutions have been identified from the full Pareto Set as those solutions which best satisfy each objective individually. These are displayed in Table 6.2.
Initial observations of the results of experiment I are that the Pareto Set size was 259 and the number of invalid beam geometries was 232 or 2.32% of the total 10000 iterations considered by the MOGA. Additionally, the minimal mass obtained was 0.470 kg, the minimal displacement was 0.391 mm and the minimal stress-range was 102.457 MPa for this experiment. A further observation is that the Pareto solutions seem to favour Aluminium cells (grey) over Steel (brown) or void cells despite them all having equal probabilities of selection of exactly one third. This is possibly due to Aluminium being considered as a satisfactory compromise in fulfilling all three system objectives simultaneously.

After this initial experiment, it is difficult to determine the relationships in terms of trade-offs between the three system objectives. Also it is unknown whether any of the values obtained so far are considered to be high or low or what influences them. However, through execution of the next seven experiments and observing their results, a greater understanding of the whole process will be achieved. The next three experiments will each consider two objectives only, with the third objective (alternating each time) remaining suppressed.
Experiment 2

In this experiment the mass and displacement objectives were considered with the stress-range objective suppressed. With just two system objectives used this time, only a 2D Pareto Set was generated, which is seen plotted in Figure 6.3.

Figure 6.3 shows that the two objectives used in this experiment, namely mass and displacement, appear to trade-off with one another in a clear and predictable manner as expected in Section 5.3.1. The minimise mass objective tends to subtract material from the design domain whereas the minimise displacement objective adds material. The plotted data is fairly evenly distributed across the design space thereby defining a clear Pareto Front; however, clusters of solutions are still evident in some areas. The entire Pareto Set of equally optimal solutions for experiment 2 can be seen in full in Appendix G. Upon closer inspection of this full Pareto Set, it can be seen that an invalid beam solution, as defined by KBE rule 2 in Section 5.3.7, has managed to enter and remain within the Pareto Set. The beam in question (number 39) is displayed in Figure 6.4 with the cause of the solution’s invalidity highlighted by the red circle.
As explained earlier in Section 5.3.5 of the previous Chapter, the TSMOGA will assign the *Extreme Value* (in this case 10000000) to any objective function that fails to execute and would otherwise return an error. In this example the *displacement* objective requires information from the FE results file, however, the invalid geometry causes the FE step to fail resulting in the assignment of the extreme value by TSMOGA to this objective function. Invalid solution geometries are usually of little interest to the user in these types of optimisation problems and are often ignored by the algorithm. This is true except when the remaining objective value, in this case *mass*, is considered low compared to the other solutions. As a consequence of this, the algorithm adds the invalid solution to the Pareto Set. When included within the Pareto Set these invalid geometries are considered *freak* solutions, indicating that they are an undesired product of the optimisation process.

The extreme solutions have been identified from the full Pareto Set as before, ignoring the freak solution, and are displayed in Table 6.3.

<table>
<thead>
<tr>
<th>Experiment 2</th>
<th>Experiment Name – DesignLab Equivalent Exp 2</th>
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</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 72</td>
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<tr>
<td><strong>Minimal Mass Beam Geometry:</strong></td>
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</tr>
<tr>
<td>Beam Number</td>
<td>= 55</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.329 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 2.463 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= N/A MPa</td>
</tr>
<tr>
<td><strong>Minimal Displacement Beam Geometry:</strong></td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 25</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.563 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.380 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= N/A MPa</td>
</tr>
</tbody>
</table>

Initial observations of the results generated by experiment 2 are that the Pareto Set size is smaller at 72 compared to experiment 1 at 259, and that the number of invalid solutions considered in experiment 2 were 568 or 5.68% of the total 10000 iterations; this is more than double the number considered in experiment 1. The Pareto Set size reduction can be explained by the use of two system objectives in this experiment compared to the three that were used in experiment 1, resulting in the Pareto Set solutions being scattered across a 2D plane in this case compared to a 3D volume in experiment 1. Hence the overall trade-offs required for this experiment were less due to
there being one less objective function to satisfy (i.e. the creation of a 2D Pareto Front requires fewer points compared to a 3D Pareto Front). A further observation of the results is that, as with experiment 1, the Pareto solutions for experiment 2 seem to favour Aluminium cells over Steel cells, even in the extreme minimal displacement beam geometry where Steel would be expected. The cause of this behaviour is believed to be due to Aluminium meeting the combined requirements of both system objectives that were used simultaneously. For example, Aluminium is stiffer than Void but also lighter than Steel. This behaviour will be investigated further in the remaining experiments.

If the actual result values of the objective functions for the extreme solutions from experiments 1 and 2 are compared with one another, it can be observed that the minimal displacement values reduce by 2.87% from 0.391 mm in experiment 1 to 0.380 mm in experiment 2. Similarly, the minimal mass values can be seen to reduce by 30.06% from 0.470 kg in experiment 1 to 0.329 kg in experiment 2. Both of these reductions (considered as improvements, as the intention is to minimise the objective functions) can be explained by the suppression of the third system objective function, thereby permitting the MOGA to concentrate on the remaining two objectives.

The concentration of the MOGA on the two remaining system objectives is also believed to account for the considerable increase in the number of invalid geometries from experiment 1 to experiment 2. It is thought that the minimise mass objective is the most influential cause of invalid geometries due to its desire to remove material from the design domain and that by allowing the MOGA to concentrate on this objective, a higher number of invalid geometries will be considered. To test this theory the occurrence of this phenomenon will be observed more closely throughout the remaining experiments where the minimise mass objective is active.

Experiment 3

In this experiment the displacement and stress-range objectives were considered with the mass objective suppressed. As with experiment 2, two objectives were considered, resulting in a 2D Pareto Set being generated which is seen plotted in Figure 6.5.
Figure 6.5 shows that the two objectives used in this experiment, namely displacement and stress-range, appear to trade-off with one another, but with a degree of unpredictability when compared to experiment 2. The plotted data consists of clusters of points that form a reasonably well defined Pareto Front but with noticeable gaps. The entire Pareto Set of equally optimal solutions for experiment 3 can be seen in full in Appendix H, which does not include any freak solutions. As before, the extreme solutions have been identified from the full Pareto Set, and are displayed in Table 6.4.

**Table 6.4 – Experiment 3 Pareto Set extreme solutions.**

<table>
<thead>
<tr>
<th>Experiment 3</th>
<th>Experiment Name – DesignLab Equivalent Exp 3</th>
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<tbody>
<tr>
<td>Pareto Set Size</td>
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<tr>
<td>Minimal Displacement Beam Geometry:</td>
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<tr>
<td>Beam Number</td>
<td>= 24</td>
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<tr>
<td>Obj 1 - Mass</td>
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<td>Obj 2 - Displacement</td>
<td>= 0.376 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 125.093 MPa</td>
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<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 75</td>
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<tr>
<td>Obj 1 - Mass</td>
<td>= N/A kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.518 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 98.397 MPa</td>
</tr>
</tbody>
</table>

Initial observations are that the Pareto Set size is similarly small to experiment 2 at 79. This was expected as there are again two system objectives to trade-off between. The number of invalid solutions for experiment 3 were 307 or 3.07% of the total 10000 iterations. This is significantly lower than in experiment 2 and is expected to be as a
result of the influential minimal mass objective being suppressed for this experiment. A further observation of the results is that there is an increased use of Steel in the Pareto solutions compared to the previous experiments; this is to help satisfy both the minimise stress-range objective and the minimise displacement objective.

If the actual objective function values for this experiment are compared with those obtained in experiment 1, it can be witnessed that the minimal displacement values reduce by 3.85% from 0.391 mm in experiment 1 to 0.376 mm in experiment 3. Similarly, the minimal stress-range values can be seen to reduce by 3.96% from 102.457 MPa in experiment 1 to 98.340 MPa in experiment 3. Both of these reductions can again be explained by the suppression of the third system objective function, which allows the MOGA to concentrate on the remaining objectives.

Experiment 4

In this experiment the mass and stress-range objectives were considered with the displacement objective suppressed this time. As with experiments 2 and 3, due to there being two objectives used, a 2D Pareto Set was generated which is plotted in Figure 6.6.

Figure 6.6 – 2D Pareto plot generated by experiment 4, Mass v Stress-Range.

Figure 6.6 shows that the two objectives used in this experiment, namely mass and stress-range, appear to trade-off with one another with increased unpredictability compared to the previous experiments. The plotted data does not clearly define a Pareto
Front as many gaps appear due to there being few data points and clustering present. The entire Pareto Set of equally optimal solutions for experiment 4 can be seen in full in Appendix I. Upon closer inspection of this full Pareto Set a freak solution (beam number 13) can be found. From the full Pareto Set, the extreme solutions have been identified ignoring the freak solution, and are displayed in Table 6.5.

**Table 6.5 – Experiment 4 Pareto Set extreme solutions.**

<table>
<thead>
<tr>
<th>Experiment 4</th>
<th>Experiment Name – DesignLab Equivalent Exp 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 30</td>
</tr>
<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 15</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.283 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= N/A mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 420.944 MPa</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 6</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.447 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= N/A mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 97.519 MPa</td>
</tr>
</tbody>
</table>

Initial observations of the results of this experiment are that there was a much smaller Pareto Set of 30 equally optimal solutions and that the number of invalid solutions at 2629 or 26.29% of the total 10000 iterations was significantly higher than all the previous experiments. This experiment was conducted using two system objectives as with experiments 2 and 3. As two objectives were used the Pareto Set size was expected to be, and indeed was found to be, smaller than that of experiment 1 where three objectives were used. However, based on the Pareto Set sizes obtained in experiments 2 and 3, at 72 and 79 respectively, the size of 30 obtained for this experiment is considerably smaller.

The significantly higher number of invalid solutions can be explained by the combination of the active system objectives for this experiment, namely the influential minimal mass objective which strives to remove material from the design domain, and the minimal stress-range objective which attempts to remove inefficient material. As a consequence of these two objectives working together to simultaneously remove material, many invalid beam geometries were considered by the MOGA. It is thought that this behaviour is responsible for the Pareto Set size being smaller than expected based on previous experiments.
As with experiments 1 and 2, Aluminium appears to have been more favourable than Steel in the Pareto solutions of experiment 4, once again believed to be as a result of Aluminium being a good compromise material in fulfilling both of the system objectives. A further observation of the Pareto Set solutions is that there is evidence of the checkerboard phenomenon in many of the solution geometries, (rapid oscillations of diagonally positioned solid and void elements) a common occurrence within many continuum topology optimisation techniques detailed in literature and now a research topic in its own right, [242-246].

If the actual objective function values for experiment 4 are compared with those obtained earlier in experiment 1, it can be seen that the minimal stress-range values reduce by 4.82% from 102.457 MPa in experiment 1 to 97.519 MPa in experiment 4. Similarly, the minimal mass values can be seen to reduce by 39.88% from 0.470 kg in experiment 1 to 0.283 kg in experiment 4. In fact, these are the lowest stress-range and mass values obtained so far in all of the preliminary experiments completed. Both of these reductions can again be partially explained by the suppression of the third system objective function, thereby enabling the MOGA to concentrate on the remaining two objectives, but their improvement is also as a result of these particular system objectives working together slightly more, compared to the previous combinations considered, towards a common goal. This fact was mentioned earlier in terms of removal of material from the design domain by both the minimal mass objective and by the minimal stress-range objective, suggesting that there is less of a trade-off between these two system objectives.

The remaining preliminary experiments will each consider just a single system objective to inspect the suitability and efficiency of the MOGA in solving these types of topology optimisation problems.

Experiment 5
In this experiment only the mass objective was considered with the displacement and stress-range objectives suppressed. As detailed earlier in Section 5.3.1, the only system objective that is completely unconnected to the FE results is the minimise mass objective which is entirely dependent on the specific solution geometry. Consequently,
if the only objective being considered is the minimise mass objective as is the case here, there is no requirement for the solution to form any kind of beam structure whatsoever as it is not required to support the applied load. As a result, there are no invalid or freak solutions for this experiment, which makes this experiment substantially different from everything else considered. Also, as only a single objective was considered, the experiment did not produce a Pareto Set just a single minimal mass solution which can be seen in Table 6.6.

<table>
<thead>
<tr>
<th>Experiment 5</th>
<th>Experiment Name – DesignLab Equivalent Exp 5</th>
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<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 1</td>
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<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
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<tr>
<td>Beam Number</td>
<td>= N/A</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.057 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= N/A mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= N/A MPa</td>
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</tbody>
</table>

An initial observation of this experiment is that it ran considerably quicker to the previous experiments at less than 1 second per iteration compared to approximately 6 to 7 seconds per iteration for all of the previous experiments. This fact was due to this experiment not requiring any FE calculations to be performed as the only objective value used was completely independent of any FE.

For this experiment, where the only objective was to minimise the mass of material within the design domain, a single known ideal solution exists. This ideal solution consists of the design domain being entirely filled with void cells with the exception of the loaded cell (due to the 1st KBE design rule loaded cells cannot be void, see Section 5.3.7) which should be Aluminium as this is lighter than Steel. The minimal mass solution shown in Table 6.6 indicates that this experiment did not manage to reach the known ideal solution in the 10000 iterations considered. However, the optimum solution obtained was certainly close to the ideal with many void cells present, suggesting that if permitted to consider further iterations, the MOGA would indeed reach this known optimum solution.
Experiment 6

In this experiment only the displacement objective was considered with the mass and stress-range objectives suppressed. As with experiment 5 where a single objective was considered, instead of a Pareto Set, a single minimal displacement solution was obtained which is displayed in Table 6.7.

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<tr>
<th>Experiment 6</th>
<th>Experiment Name – DesignLab Equivalent Exp 6</th>
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<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 1</td>
</tr>
<tr>
<td>Minimal Displacement Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= N/A</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= N/A kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.376 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= N/A MPa</td>
</tr>
</tbody>
</table>

As with the previous experiment, a single known ideal solution exists for this experiment where the only objective is to minimise the displacement. The ideal solution this time would be an entirely solid Steel design domain, as this would achieve the greatest stiffness possible therefore minimising the displacement. The minimal displacement solution shown in Table 6.7 shows that this experiment, much like experiment 5, did not manage to reach the known ideal solution in the 10000 iterations considered. However, much like the previous experiment the optimum solution obtained was on its way to the ideal, being entirely free from voids as desired, and containing a higher proportion of steel cells compared to all of the other solutions achieved so far. Hence, a 3.90% reduction in the displacement values was achieved from experiment 1 (0.391 mm) to experiment 6 (0.376 mm). As before, it could be suggested that the solution could be improved upon further until the ideal solution is achieved if the MOGA was permitted to consider further iterations.

The number of invalid geometries considered by the MOGA during this experiment was noticeably lower than in any of the previous experiments at 119 or 1.19% of the total 10000. This was due to the fact that the only objective being considered for this experiment was the minimal displacement objective, which strives to fill the design domain with material (ideally higher stiffness material). Consequently, as the MOGA nears the ideal optimum, invalid geometries are less likely to occur.
Experiment 7

In this experiment, only the stress-range objective was used, with the mass and displacement objectives remaining suppressed. As a single objective was considered a single minimal stress-range solution was obtained which is displayed in Table 6.8.

<table>
<thead>
<tr>
<th>Experiment 7</th>
<th>Experiment Name – DesignLab Equivalent Exp 7</th>
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<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 1</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= N/A</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= N/A kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= N/A mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 94.726 MPa</td>
</tr>
</tbody>
</table>

Initial observations of this experimental result are that a good mix of Steel, Aluminium and void cells are present throughout the solution structure. In addition to this, there is evidence of mild checkerboarding but also some clearly defined structural holes consisting of several void cells that have been grouped together. Comparing the actual objective function value obtained here with that of experiment 1 gives a reduction of 7.55% in stress-range from 102.457 MPa in experiment 1 to 94.726 MPa in experiment 7. The solution structure for experiment 7 is now the lowest stress-range solution obtained so far in all of the preliminary experiments conducted. The number of invalid geometries for this experiment was 442 or 4.42% of the total 10000 considered.

As with the previous two experiments, where a single system objective was considered in isolation, a single ideal solution must exist. However, in this case it is not readily known, being difficult to determine with any certainty. Consequently, without knowing the ideal solution structure, it is difficult to judge how the MOGA has performed in its quest to seek out this ideal optimum compared to experiments 5 and 6.

In an attempt to help establish the ideal stress-range solution for this experiment, the TopOpt software, [151] was utilised to define and solve the simple cantilever beam problem using the homogenisation method, with the results displayed in Figure 6.7. As mentioned earlier in Chapter 2, TopOpt is an example of a topology optimisation technique that does not force intermediate density cells to be either 1 or 0 but allows them to remain in between, as demonstrated by the solution window on the right of
Figure 6.7. The TopOpt solution to the cantilever beam problem yields a structure with a uniform stress distribution by permitting the density of the material to alter across the structure of the beam.

Using the TopOpt solution as an indication of what the ideal minimal stress-range beam solution should look like, it is apparent that the achieved experimental result does not resemble this assumed ideal. Obvious material differences occur between the two techniques with TopOpt allowing continuous material densities and the basic AML topology optimisation system permitting discrete material choices with predefined densities. However, even with these material differences taken into account the assumed ideal solution proposed by TopOpt should still serve as a good guide for an ideal minimal stress-range solution. As the result obtained in experiment 7 does not resemble this assumed ideal, the experiment will be extended for a further 10000 beam iterations in the following experiment, taking the total number of iterations considered to 20000. Continuing this experiment for a further 10000 iterations will help determine whether or not the MOGA can achieve a minimal stress-range solution resembling the assumed ideal.

**Experiment 8**

This experiment, as just mentioned, is a continuation of experiment 7 where only the stress-range objective was considered and the mass and displacement objectives remained suppressed. The starting point for experiment 8 was the final solution obtained in experiment 7. Experiment 8 was run for 10000 iterations yielding a single minimal stress-range solution which can be seen displayed in Table 6.9.
Table 6.9 - Experiment 8 minimal stress-range solution.

<table>
<thead>
<tr>
<th>Experiment 8</th>
<th>Experiment Name – DesignLab Equivalent Exp 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 1</td>
</tr>
</tbody>
</table>

Minimal Stress-Range Beam Geometry:

- Beam Number = N/A
- Obj 1 - Mass = N/A kg
- Obj 2 - Displacement = N/A mm
- Obj 3 - Stress-Range = 93.727 MPa

As in experiment 7, the minimum stress-range solution achieved here has a good mix of Steel, Aluminium and void cells throughout the structure, again with mild checkerboarding also present. As with the previous experiment, the assumed ideal solution proposed by TopOpt was not reached despite the increased number of iterations. This suggests that either the number of iterations is still not high enough or that the MOGA is unable to locate the optimum solution, possibly because it has become trapped at a local sub-optimum solution.

The MOGA’s progress at minimising the stress-range objective function throughout the combined 20000 iterations of experiments 7 and 8 can be seen in Figure 6.8. This Figure, in addition to the results from experiments 7 and 8, demonstrates that a marginal improvement of the objective function was achieved through the course of experiment 8’s 10000 iterations (1.05%). This improvement, however, was at the expense of considerable additional computation time (approximately a further 20 hours).

![Figure 6.8](image-url)

*Figure 6.8 – Combined stress-range results for experiments 7 and 8 plotted continuing from one another.*
If the minimum stress-range value for experiment 8 is compared with the extreme solution obtained in experiment 1, it can be seen to reduce by 8.52% from 102.457 MPa in experiment 1 to 93.727 MPa in experiment 8, a new lowest stress-range value obtained by the preliminary experiments. This reduction is illustrated in Figure 6.9 where the Von Mises stress plots are displayed for experiments 1 and 8 using the same scale for comparison, along with the corresponding material distributions.

Figure 6.9 shows that the stress distribution is more uniform, and therefore more optimal, for the beam structure generated by experiment 8 compared to the extreme solution of experiment 1. This is indicated by there being an increased number of cells which appear green in the stress plot for experiment 8, whereas there are a higher number of both underloaded cells (coloured blue), and overloaded cells (coloured red-orange) in the stress plot for experiment 1. This observation restates the 8.52% improvement shown from the actual stress-range values.

Figure 6.9 also highlights areas within both of the solution structures from experiments 1 and 8 where specific cells could be changed to achieve a more uniform stress distribution overall. Examples include the loaded cells in both cases which should be Steel and not Aluminium and the overloaded cells to the bottom left of each solution which should also be Steel and not Aluminium. Additionally, the underloaded blue
region to the middle left of Figure 6.9d should be made up of either void cells or Aluminium cells, or a combination of the two instead of the predominantly Steel cells used. These observations indicate the MOGA has not managed to find the ideal minimal stress-range solution for experiment 8, but also suggest that the MOGA may have stagnated at a local optimum, otherwise these seemingly obvious changes to the beam structures would not be evident within Figure 6.9.

6.2.3 DesignLab Results Comparison

Now that some initial results have been generated using the basic AML system a brief comparison with DesignLab can be performed. In order to directly compare the results of both systems, the DesignLab beam topologies obtained as solutions in Chapter 3 have been entered into the basic AML system. This has allowed each beam structure to be analysed by the three separate system objectives within the AML basic system, (i.e. not by the DesignLab weighted sum). The results obtained are displayed in Table 6.10.

Table 6.10 – AML topology optimisation system results for the DesignLab beam solutions.

<table>
<thead>
<tr>
<th>DesignLab Trial K Solution:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj 1 - Mass</td>
<td>0.578 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>2.275 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>261.118 MPa</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DesignLab Trial L Solution:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj 1 - Mass</td>
<td>0.633 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>2.422 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>209.290 MPa</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DesignLab Trial M Solution:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj 1 - Mass</td>
<td>0.503 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>2.450 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>271.920 MPa</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DesignLab Trial N Solution:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj 1 - Mass</td>
<td>0.663 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>2.420 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>217.131 MPa</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DesignLab Trial O Solution:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Obj 1 - Mass</td>
<td>0.561 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>2.090 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>255.377 MPa</td>
<td></td>
</tr>
</tbody>
</table>

Upon initial observation, the DesignLab results displayed in Table 6.10 appear to be inferior to those obtained via the basic AML topology optimisation system, as the stress-range values are significantly higher for the DesignLab solutions. However, it should be noted that the DesignLab experiments had a different goal, namely to achieve
a predefined FOS of 2.25 for the Von Mises stress, whereas the preliminary experiments strived to minimise the stress-range. Taking this fact into account and comparing the stress-range values once more, it can be seen that the stress-ranges are similar for both the DesignLab solutions and those obtained by the basic AML topology optimisation system. Furthermore, the minimal mass values are also comparable for beam solutions created by both systems. This outcome is somewhat unexpected when the material distribution plots are inspected as the DesignLab solutions show a higher percentage of Steel compared to the basic system solutions with perhaps the exception of the solutions from experiments 7 and 8. This suggests therefore, that the DesignLab solutions may exhibit higher mass values, however, due to the high number of void cells within the DesignLab solutions, the mass values remain similar in both systems.

The increased use of Steel in the DesignLab solutions would suggest that these beam structures might be stiffer than those generated by the basic AML system, but surprisingly this is not the case. All of the DesignLab solutions are more compliant structures displacing a minimum of 2 mm or more; in comparison, the beam solutions generated by the basic AML topology optimisation system have a typical maximum displacement of less than 0.6 mm. This observation reconfirms the assumption made earlier in Chapter 3, where it was suggested that maximum displacement may not be included in the weighted sum objective used by DesignLab, or that if it was included, it does not have a high weighting in the combined weighted sum objective.

The number of iterations considered by the DesignLab investigation and in these preliminary experiments differed greatly. In the DesignLab experiments, each one terminated when the cost function converged within an acceptable tolerance and remained constant for ten successive generations. The number of generations taken to achieve this varied from 296 to 429 with an average of 361 (from Table 3.1), resulting in an average of 21660 iterations. In contrast, the preliminary experiments were run for a predefined, fixed number of iterations as termination criteria could not be defined for simultaneous multiple objectives. This value was the product of the population size (50) and the number of generations (200), which resulted in 10000 for each case, with the exception of the solution to experiment 8 which was obtained after a total of 20000 iterations due to experiment 8 being a continuation of experiment 7.
CHAPTER SEVEN

ADVANCED EXPERIMENTS & RESULTS

7.1 Introduction

In Chapter 6, a series of eight preliminary experiments were conducted using the basic topology optimisation system created in AML, the construction of which was detailed in Chapter 5. This resulted in an initial understanding of the basic system, in addition to the TSMOGA's suitability to these types of cellular topology optimisation problems being gained. Finally, a comparison was made with the DesignLab results that were obtained previously in Chapter 3.

This Chapter details a further series of ten experiments, of subsequently increasing complexity, using the more advanced AML topology optimisation system, in order to answer the research questions identified earlier in Chapter 3. These advanced topology optimisation experiments implement the novel approach of distributing various single material unit cell structures throughout a design domain by a MOGA to create optimal beam topologies. This approach is enabled by the implementation of the more advanced TET meshes, which are able to represent the complex unit cell geometries reasonably realistically. However, before these advanced experiments can be conducted, the effects of TET mesh fidelity on meshing and FE analysis solve times will be investigated, by altering the TET mesh maximum element size scale factor parameter for various beam geometries of increasing complexity, for different local and remote computing configurations.
7.2 TET Mesh Time Trials

These trials involved the collection of mesh and solve times, at different mesh resolutions, for four beam geometries of increasing complexity and is detailed in full in Appendix J. The mesh fidelity was altered in each case by altering the TET mesh maximum element size scale factor parameter, as illustrated by Figure 7.1.

![Figure 7.1 - TET mesh fidelity as a scale factor of cell size, 0.5, 0.35, 0.25 and 0.15 from left to right.](image)

Mesh and FE analysis solve times were calculated for various local and remote computational arrangements. These included two local arrangements, one on a laptop PC and one on a desktop PC, in addition to two remote arrangements. Both of the remote configurations used the aforementioned laptop PC to run AML locally, sending meshing jobs remotely to Patran and analysis jobs remotely to Nastran via the TSCR and TSXM applications, as previously detailed in Section 5.5.2.

The investigation of TET mesh fidelity effects on computational times required for the meshing and analysis of executable jobs, detailed in Appendix J, showed that the fastest result was achieved using the local desktop PC computational configuration. In addition, these trials showed that for more complex beam geometries, it is not always possible to create a solid TET mesh for a given mesh size. The main finding of these trials was that the time required to mesh and solve the sample beam geometries considered, rose exponentially with respect to higher fidelity mesh sizes. Based on these results, a coarse TET mesh is recommended for the advanced experiments to reduce computation.
7.3 Advanced Experiments

7.3.1 Experimental Roadmap

With the effects of TET mesh fidelity investigated, an experimental roadmap was established for the advanced topology optimisation experiments. This experimental roadmap will consider which experiments and parameters are feasible and from those, which are useful for this research. The valid neighbours results, seen earlier in Table 5.1, indicate which cell type combinations may be considered together within the same problem design domain. This data, in combination with the successful and failed creations of a solid TET mesh for an array of each cell structure, as displayed in Table 7.1, highlight the experimental parameters that should be considered.

Table 7.1 - Successful and failed solid TET mesh creations for all cell structures at various TET mesh element maximum size scale factors.

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<tr>
<td>0.20</td>
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<tr>
<td>0.15</td>
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</tbody>
</table>

To help determine which cellular structures should be used for the advanced experiments, the simulated mechanical responses of each structure subjected to bending loads, conducted earlier in Chapter 4, must be considered. This mechanical response to bending data was used in conjunction with the valid neighbours and TET mesh creation results to establish the rough experimental plan. Based on this information, it was decided that the experimental plan should commence by initially considering simple solid and void problems, with subsequent advanced experiments building in complexity (i.e. introducing a single cell structure, having multiple volume fractions of a cell and mixing different cell structures together). The specific cell types chosen in each case are detailed in the description for each experiment.
As suggested in Section 3.3, due to the fabrication and recycling issues associated with FGMs, only a single material was used throughout the advanced experiments with Aluminium arbitrarily selected. Increased functionality was obtained through intelligent design by considering cellular structures. The default cantilever beam problem definition load of 1000 N was used in all the advanced experiments, distributed evenly across all of the loaded nodes for each particular geometry.

Results from the preliminary experiments suggested that either the number of iterations should be increased or the number of variables reduced if the MOGA is to achieve a final minimal stress-range solution close to the proposed ideal optimum. Now that a more sophisticated mesh is to be used for these advanced experiments, compared with the simple HEX mesh used by the preliminary experiments, additional computational time is required to mesh and solve each beam topology, as evidenced by the results of the TET mesh time trials (Appendix J). Consequently, the decision was made to use the default cantilever beam problem definition, resulting in a smaller design domain (therefore fewer variables) comprising of $10 \times 5 \times 1$ cells that are 10 mm$^3$, in an attempt to make the necessary computation more manageable. In addition to this and based on the results of the TET mesh time trials, the decision was made to use a low fidelity mesh (maximum element size scale factor of 0.50) throughout the advanced experiments to minimise computational demands.

The number of iterations considered by the advanced experiments also required adjustment from the preliminary experiments. Due to complications arising from a memory leak, which will be explained further in Chapter 8, the advanced experiments were not run in a single continuous operation. Instead, they were each run in multiple segments, taking advantage of the Restart from File? and Save for Restart? options available to the TSMOGA as explained in Section 5.3.5. However, as a consequence of this, keeping track of the number of invalid beam geometries considered throughout the numerous experimental segments proved almost impossible. Therefore, to ensure consistency, each advanced experiment was run until 10000 valid beam geometries had been considered by the MOGA, with the population size remaining at a constant 50 individuals. With the experimental roadmap established and key parameters determined, the advanced experiments are now each presented in turn.
7.3.2 Simple Solid & Void Problems

As suggested by the experimental plan, the advanced experiments were to consider simple solid and void problems initially. It is useful at this stage therefore, to consider the behaviour of a completely solid Aluminium beam. Such a beam was created within the advanced AML topology optimisation system using the default cantilever beam problem definition and analysed using a TET mesh maximum element size scale factor of 0.50. The Von Mises stress plot for the entirely solid Aluminium beam is shown below in Figure 7.2.

![Von Mises stress plot for a completely solid 10×5×1 Aluminium beam using a TET mesh element maximum size scale factor of 0.50.](image)

Figure 7.2 highlights the areas within the solid design domain that are inefficiently loaded, with much of the beam structure exhibiting a blue colour in the Von Mises stress plot indicating that it is underloaded. The mass of the beam in Figure 7.2 was 0.136 kg, the maximum nodal displacement was 0.062 mm and the stress-range across the structure was 55.662 MPa. These results are effectively the benchmark results that all the solid and void advanced experiments should be compared against.

**Experiment 9**

Experiment 9 consisted of running a simple solid void problem using the advanced AML topology optimisation system with all three system objectives active. The parameters that were used in this experiment are displayed in Table 7.2. The resulting 2D and 3D Pareto plots for experiment 9 are displayed in Figure 7.3.
Table 7.2 - Parameters used in experiment 9.

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Experiment Name</th>
<th>Solid Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Objectives:

1 - Minimise Mass, (kg)  
2 - Minimise Displacement, (mm)  
3 - Minimise Stress-Range, (MPa)

<table>
<thead>
<tr>
<th>Number of Variables</th>
<th>Population Size</th>
<th>Number of Generations</th>
<th>Probability of Mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>((10 \times 5 \times 1) = 50)</td>
<td>50</td>
<td>200</td>
<td>0.015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cell Types</th>
<th>Integer Value</th>
<th>Probability of Selection</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.5</td>
<td>Void</td>
</tr>
<tr>
<td>Solid-class</td>
<td>1</td>
<td>0.5</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>Mesh Maximum Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>TET</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Figure 7.3 shows the distribution of the Pareto optimal solutions for experiment 9 exhibit a noticeably increased level of order compared to those of experiment 1 (Figure 6.2). The well defined Pareto Front in Figure 7.3 indicates good trade-offs between the objectives.
The entire Pareto Set of equally optimal solutions for experiment 9 can be seen in full in Appendix K. Upon closer inspection of this full Pareto Set, a single freak solution (beam number 23) was found. From the full Pareto Set, the extreme solutions have been identified (ignoring the freak solution), and are displayed in Table 7.3.

### Table 7.3 – Experiment 9 Pareto Set extreme solutions.

<table>
<thead>
<tr>
<th>Experiment 9</th>
<th>Experiment Name – Solid Void Pareto Set Size = 93</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 46</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.055 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 1.763 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 220.969 MPa</td>
</tr>
<tr>
<td>Minimal Displacement Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 39</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.136 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.064 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 55.980 MPa</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 24</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.108 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.096 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 37.258 MPa</td>
</tr>
</tbody>
</table>

An initial observation of the results to experiment 9 are that the Pareto Set size was 93, which is significantly smaller than experiment 1 at 259. This is thought to be due to there being fewer design variables and fewer cell type options for experiment 9, resulting in fewer permutations that are possible. Consequently, the MOGA has a smaller design space to search and is therefore more likely to find solutions that dominate others, forming a clearly defined Pareto Front, as was seen in Figure 7.3.

If the actual objective function values of the minimal stress-range solution for experiment 9 are compared with the values of the benchmark solid beam, it can be shown that there is a 33.06% reduction in stress-range, from 55.662 MPa in the solid beam to 37.258 MPa in experiment 9. Similarly there is a 20.45% reduction in mass, from 0.136 kg in the solid beam to 0.108 kg in experiment 9. These substantial improvements in both mass and stress-range reductions are both at the expense of increasing the maximum displacement of the beam. The entirely solid beam is in fact the ideal structure for a minimal displacement solution.
It was decided beneficial to repeat this experiment with just the minimise stress-range system objective used. This repeated experiment would therefore be similar to experiment 7, which was unable to find a solution structure that resembled the assumed optimum of Figure 6.7.

**Experiment 10**

As suggested above, experiment 10 was a repeat of experiment 9 but with only the stress-range objective considered, the mass and displacement objectives were suppressed. The parameters that were used in experiment 10 are displayed in Table 7.4.

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Experiment Name</th>
<th>Solid Void Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.4 – Parameters used in experiment 10.**

<table>
<thead>
<tr>
<th>Objectives:</th>
<th>Solid Void Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td></td>
</tr>
<tr>
<td>Number of Variables</td>
<td>(10 x 5 x 1) = 50</td>
</tr>
<tr>
<td>Population Size</td>
<td>50</td>
</tr>
<tr>
<td>Number of Generations</td>
<td>200</td>
</tr>
<tr>
<td>Probability of Mutation</td>
<td>0.015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cell Types</th>
<th>Integer Value</th>
<th>Probability of Selection</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.5</td>
<td>Void</td>
</tr>
<tr>
<td>Solid-class</td>
<td>1</td>
<td>0.5</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

Mesh Type = TET

As with experiments 5, 6, 7 and 8 where only a single objective was used a Pareto Set cannot be created, instead a single optimum solution is generated. Table 7.5 shows the minimal stress-range solution for experiment 10.

<table>
<thead>
<tr>
<th>Experiment 10</th>
<th>Experiment Name</th>
<th>Solid Void Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pareto Set Size</td>
<td></td>
</tr>
<tr>
<td>= 1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 7.5 – Experiment 10 minimal stress-range solution.**

Minimal Stress-Range Beam Geometry:

- Beam Number = N/A
- Obj 1 - Mass = N/A kg
- Obj 2 - Displacement = N/A mm
- Obj 3 - Stress-Range = 41.069 MPa
It can be seen from Table 7.5 that the minimal stress-range solution for experiment 10 has a value of 41.069 MPa. This result, much like that of experiment 9, is a significant improvement on the benchmark solid beam in terms of stress-range (26.22%) and mass reductions at the expense of displacement. However, this stress-range value is 10.23% higher than that of the extreme solution generated by experiment 9, despite the MOGA only considering a single objective in this experiment compared to the three objectives used in experiment 9.

In the previous Chapter, it was shown that improved results were obtained when the algorithm had fewer objectives to consider. This outcome was believed to be as a result of the algorithm being able to concentrate on fewer objectives combined with searching a smaller design space. The stress-range results of experiments 9 and 10 contradict this observation from the earlier preliminary experiments. One likely reason for this behaviour is that experiment 10 may have become trapped at a local sub-optimal solution. Evidence of this potential outcome can be seen when comparing the solution structure generated by experiment 10 with the assumed ideal solution proposed by Topopt (Figure 6.7). It can be seen that there are noticeable differences between the two structures indicating that experiment 10 has not reached the assumed optimal. A further repeat of this experiment could be used to determine whether experiment 10 had indeed been trapped at a local optimum.

Experiment 11

Experiment 11, as just mentioned, is a repeat of experiment 10 but with two main differences to help overcome the MOGA's premature convergence on a sub optimal solution. Firstly, the probability of mutation has been increased from 0.015 to 0.150 to help reduce the risk of remaining trapped at a local optimum by effectively increasing the MOGA's ability to 'jump out' of such regions of the search space. Secondly, the optimisation process was steered in a known good direction by constraining some of the design variables based on previous results. The removal of design variables from the design domain will also reduce the number of permutations that are possible for the MOGA to consider. Consequently, a good solution should be achieved more quickly resulting in more iterations remaining to search for the global optimum solution.
Figure 7.4 shows the constraints that were applied to the discrete variables of the design domain for experiment 11, with a ‘1’ representing Solid, ‘0’ representing Void and a ‘?’ indicating the discrete variable remains unconstrained.

![Figure 7.4 - The constraints applied to the design domain for experiment 11.](image)

The constraints that were applied to the design domain as illustrated in Figure 7.4 were derived from the ten lowest stress-range solutions from the Pareto Set of experiment 9. These ten solutions were superimposed upon one another and constraints were generated by cells remaining either solid or void throughout all of the structures, in much the same way that the DesignLab trials were analysed earlier in Chapter 3. The parameters that were used in experiment 11 can be seen below in Table 7.6.

![Table 7.6 - Parameters used in experiment 11.](image)

As with the previous experiment where a single objective was used a single optimum solution was generated. The minimal stress-range solution for experiment 11 is shown in Table 7.7.
A Genetic Algorithm Based Topology Optimisation Approach 
for Exploiting Rapid Manufacturing’s Design Freedom

Table 7.7 – Experiment 11 minimal stress-range solution.

<table>
<thead>
<tr>
<th>Experiment 11</th>
<th>Experiment Name – Solid Void Stress Repeat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 1</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= N/A</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= N/A kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= N/A mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 36.182 MPa</td>
</tr>
</tbody>
</table>

The minimal stress-range solution displayed in Table 7.7 has a value of 36.182 MPa. As with experiments 9 and 10, this result is a major improvement on the benchmark solid beam in terms of stress-range and mass reductions at the expense of displacement. This value is a slight improvement of 2.89% on experiment 9, but more importantly, 11.90% lower than the result generated by experiment 10 (from 41.069 MPa to 36.182 MPa). This substantial stress-range improvement over the result achieved in experiment 10, coupled with the fact that this solution structure shares notable similarities with the assumed ideal solution proposed by TopOpt, suggests that experiment 11 has managed to find the global optimal solution for the given input parameters. In addition, the result of experiment 11 confirms that experiment 10 did indeed converge on a local sub-optimal solution. The MOGA’s progress at minimising the stress-range objective function throughout the 10000 iterations of both experiments 10 and 11 can be seen below in Figure 7.5.

Figure 7.5 – Stress-range results for the 10000 iterations of experiments 10 and 11.
Figure 7.5 shows experiment 11 (red data) achieved a superior final solution, and at a faster rate compared to experiment 10 (blue data) which exhibited a more gradual improvement over the course of the 10000 iterations. This behaviour is as expected due to the constraints that were added to the design domain for experiment 11. Additionally, the red data of experiment 11 shows greater variability compared to the blue data of experiment 10. This is most likely as a result of having a higher probability of mutation, permitting the MOGA to exhibit greater randomness in its search for an improved solution. The Von Mises stress plots for the final solution structures from experiments 10 and 11, using the same scale for comparison, can be seen in Figure 7.6.

Both of the Von Mises stress plots displayed in Figure 7.6 highlight regions of TET elements that are either underloaded (blue) or overloaded (red-orange), in addition to regions that are optimally loaded (green). However, the overloaded regions of Figure 7.6b are not as overloaded as those of Figure 7.6a, and there are also fewer of them. Both beam structures in Figure 7.6 illustrate the considerable improvement that has been made by experiments 10 and 11 compared to the benchmark solid beam, the Von Mises stress plot of which was seen earlier in Figure 7.2.
The simple solid and void problems considered in experiments 9, 10 and 11 have all proved useful in determining the input parameters for the remaining experiments. The results of experiments 9 and 10 showed how the earlier observation from the preliminary experiments; namely, improved results are obtained when the MOGA has fewer objectives to consider, was not always the case, and that when the stress-range objective was used in isolation, the MOGA was susceptible to being trapped at local optima. This result was confirmed by experiment 11, which additionally showed that the higher probability of mutation rate of 0.150 is more favourable than 0.015 and so this higher value will be used for all of the remaining experiments.

These experiments have demonstrated that it can be advantageous to consider all three system objectives, as this enables the MOGA to move around the search space more when trying to generate a Pareto Set, thus reducing the likelihood of being trapped by local optima. Hence, from this point forward, the remaining advanced experiments will consider all three system objectives.

7.3.3 Addition of a Single Cell Structure

Up to this point, the advanced experiments conducted have all considered simple solid and void problems. Now, to test the hypothesis, cellular structures will be introduced as cell type options within the design domain. A sensible place to begin would be to add a 50% volume fraction cell structure into the existing solid void problem as this is exactly halfway between the two existing options (solid 100% and void 0%), to see what effect this has on the results. This experiment will yield results capable of answering one of the research questions raised in Section 3.4.3. The question of which cell structure to use, can be answered by referring back to Table 4.3 and Figure 4.7, which clearly show B and D were the best structures in bending. However, out of these two cell type options, only the D structure can connect to neighbouring cells in the same manner as solid cells can, i.e. horizontally, vertically and diagonally, therefore the D50 structure will be added to the simple solid void problem for experiment 12.

Experiment 12

This experiment considered the solid void problem, with the addition of D50 cell types, using all three objectives. Table 7.8 contains the experimental parameters used.
Table 7.8 – Parameters used in experiment 12.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>12</th>
<th>Experiment Name</th>
<th>Solid Void D50 Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives:</td>
<td></td>
<td>Number of Variables</td>
<td>= (10x5x1) = 50</td>
</tr>
<tr>
<td>1 - Minimise Mass, (kg)</td>
<td></td>
<td>Population Size</td>
<td>= 50</td>
</tr>
<tr>
<td>2 - Minimise Displacement, (mm)</td>
<td></td>
<td>Number of Generations</td>
<td>= 200</td>
</tr>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td></td>
<td>Probability of Mutation</td>
<td>= 0.150</td>
</tr>
<tr>
<td>Cell Types</td>
<td></td>
<td>Integer Value</td>
<td>0</td>
</tr>
<tr>
<td>Void-class</td>
<td></td>
<td>Probability of Selection</td>
<td>0.33333</td>
</tr>
<tr>
<td>Solid-class</td>
<td></td>
<td>Material</td>
<td>Void</td>
</tr>
<tr>
<td>D50-class</td>
<td></td>
<td></td>
<td>Aluminium</td>
</tr>
<tr>
<td>Mesh Type</td>
<td>TET</td>
<td>Mesh Maximum Scale Factor</td>
<td>= 0.50</td>
</tr>
</tbody>
</table>

The resulting 2D and 3D Pareto plots generated by experiment 12 are displayed below in Figure 7.7.

Figure 7.7 – Experiment 12 Pareto Set data plotted as three separate 2D plots and a combined 3D plot.
Figure 7.7 shows the distribution of the Pareto Set for experiment 12 across the 3D search space is reasonably ordered but exhibits more scatter compared to the Pareto Set of experiment 9 (Figure 7.3). The increased scattering indicates that the trade-offs between objectives for this experiment, were more haphazard than in experiment 9.

The entire Pareto Set of equally optimal solutions for experiment 12 can be seen in full in Appendix L. Upon closer inspection of this Pareto Set, a single freak solution (beam number 103) was found. From the full Pareto Set, the extreme solutions have been identified as before, again ignoring the freak solution, and are displayed in Table 7.9.

<table>
<thead>
<tr>
<th>Experiment 12</th>
<th>Experiment Name - Solid Void D50 Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 120</td>
</tr>
<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 2</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.040 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 9.016 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 1879.226 MPa</td>
</tr>
<tr>
<td>Minimal Displacement Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 68</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.136 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.062 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 55.662 MPa</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 22</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.122 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.073 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 39.517 MPa</td>
</tr>
</tbody>
</table>

Initial observations of the results for experiment 12 include the Pareto Set size being 120, higher than experiment 9 at 93, as a result of the increased permutations possible due to the additional cell type option of D50. In terms of the structures generated, the minimal mass beam displayed in Table 7.9 is constructed entirely from D50 cell structures and voids, making good use of the less dense D50 cells. Similarly, the minimal displacement beam achieved was the known ideal solution for these input parameters, of an entirely solid beam. However, the minimal stress-range solution achieved, was not at all as expected, being constructed from nearly all solid cells with five voids and only two D50s. Additionally, the minimal stress-range structure does not resemble the assumed ideal proposed by TopOpt in any way whatsoever.
In fact, all of the Pareto optimal low stress-range solutions within Appendix L exhibit similar unexpected topologies consisting predominantly of solid cells. These results suggest that the inclusion of the D50 cells appear to introduce SCs into the overall solution structures. Consequently, the MOGA tends to avoid the D50 cell type option in the construction of low stress-range solutions, choosing in preference the solid cells, despite them both having equal probabilities of selection. Hence, the low stress-range solutions generated by this experiment steer clear of the D50 cells and the SCs associated with them.

Further indication of this behaviour is evident when searching through the Pareto optimal solutions, found in Appendix L, as several exist which display topologies more closely resembling the assumed ideal proposed by TopOpt and the solution generated by experiment 11. Examples of such beam topologies taken from the Pareto Set can be seen in Figure 7.8. The higher stress-range values of these beams demonstrate the inclusion of D50 cells introduces SCs into the overall beam structures.

It is not a trivial matter to reduce or remove the SCs associated with the cellular structures, although this issue will be discussed in greater detail in the subsequent Chapter. For now, the important thing is to try to limit the influence of SCs in terms of steering the behaviour of the optimisation process.

### 7.3.4 Removing the Solid Cells

One method of preventing the MOGA from selecting solid cells without SCs, over cell structures with SCs, would be to completely remove the solid cells. Therefore, every cell type option remaining would introduce a SC, thus stopping their inclusion from steering the optimisation process. In order to evaluate the results of the remaining advanced experiments, a new benchmark beam is required that contains entirely cell structures in place of the solid cells. The stiffer D structure of D75 was chosen for this
purpose, the mass of such a beam was 0.104 kg, the maximum nodal displacement was 0.114 mm and the stress-range across the structure was 162.680 MPa.

**Experiment 13**

Experiment 13 considered a simply loaded, D75 and void problem to compare against the new benchmark beam. Table 7.10 contains the experimental parameters used.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>Experiment Name</th>
<th>D75 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Objectives:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - Minimise Mass, (kg)</td>
<td>Number of Variables = (10x5x1) = 50</td>
<td></td>
</tr>
<tr>
<td>2 - Minimise Displacement, (mm)</td>
<td>Population Size = 50</td>
<td></td>
</tr>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td>Number of Generations = 200</td>
<td></td>
</tr>
<tr>
<td><strong>Probability of Mutation = 0.150</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Cell Types</strong></td>
<td><strong>Integer Value</strong></td>
<td><strong>Probability of Selection</strong></td>
</tr>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>D75-class</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Mesh Type = TET</strong></td>
<td><strong>Mesh Maximum Scale Factor = 0.50</strong></td>
<td></td>
</tr>
</tbody>
</table>

The entire Pareto Set of equally optimal solutions for experiment 13 can be seen in full in Appendix M in addition to the 2D and 3D Pareto plots. Once again, a single freak solution (beam number 7) was found in the Pareto Set. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.11.

<table>
<thead>
<tr>
<th>Experiment 13</th>
<th>Experiment Name – D75 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pareto Set Size</strong> = 42</td>
<td></td>
</tr>
<tr>
<td><strong>Minimal Mass Beam Geometry:</strong></td>
<td></td>
</tr>
<tr>
<td>Beam Number = 22</td>
<td></td>
</tr>
<tr>
<td>Obj 1 - Mass = 0.050 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement = 1.378 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range = 492.119 MPa</td>
<td></td>
</tr>
<tr>
<td><strong>Minimal Displacement Beam Geometry:</strong></td>
<td></td>
</tr>
<tr>
<td>Beam Number = 41</td>
<td></td>
</tr>
<tr>
<td>Obj 1 - Mass = 0.087 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement = 0.139 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range = 161.990 MPa</td>
<td></td>
</tr>
<tr>
<td><strong>Minimal Stress-Range Beam Geometry:</strong></td>
<td></td>
</tr>
<tr>
<td>Beam Number = 19</td>
<td></td>
</tr>
<tr>
<td>Obj 1 - Mass = 0.083 kg</td>
<td></td>
</tr>
<tr>
<td>Obj 2 - Displacement = 0.171 mm</td>
<td></td>
</tr>
<tr>
<td>Obj 3 - Stress-Range = 154.993 MPa</td>
<td></td>
</tr>
</tbody>
</table>
An observation of the results presented in Table 7.11 is that the Pareto Set size is small at 42. Compared to experiment 12, there are fewer permutations possible due to there being fewer cell type options available, so this value is expected to be smaller than it was for experiment 12. However, the Pareto Set size obtained in this experiment is also significantly less than that of experiment 9, despite both experiments having an identical number of permutations. The reason for this outcome is currently unknown, but could be investigated further by repeating both experiments.

It should also be noted that all of the stress-range values obtained in this experiment are substantially higher compared to those of experiments 9–12, due to the absence of any solid cells. Compared to the new benchmark beam, the extreme minimal stress-range solution made a modest improvement of 4.73%, from 162.680 MPa to 154.993 MPa, in addition to a 19.99% reduction in mass, from 0.104 kg to 0.083 kg, both at the expense of increased nodal displacement. It is hoped that further improvements can be made, particularly in stress-range reduction, by the introduction of additional volume fractions of the D cell structure.

### 7.3.5 Varying Volume Fractions

In order to answer another of the research questions, the effects on the results of introducing differing cell structure volume fractions must be investigated. Experiment 14 was performed for this purpose, using a mixture of D class volume fractions.

**Experiment 14**

Experiment 14 considered a simply loaded problem with all three system objectives, using D75, D50, D25 and void cells. Table 7.12 contains the experimental parameters that were used. The resulting 2D and 3D Pareto plots generated by experiment 14 are displayed in Figure 7.9.

The Pareto plots depicted in Figure 7.9 show the most clearly defined Pareto Front of all of the experiments conducted, illustrating the benefit of having multiple volume fractions. They also highlight for the first time, the fact that the stress-range and displacement objectives are directly proportional to one another, a contradiction to that which was seen earlier in Figure 6.5 of the preliminary experiments.
Table 7.12 - Parameters used in experiment 14.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>Experiment Name</th>
<th>D75</th>
<th>D50</th>
<th>D25</th>
<th>Void</th>
<th>Pareto</th>
</tr>
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<tbody>
<tr>
<td>Objectives:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - Minimise Mass, (kg)</td>
<td>Number of Variables</td>
<td>(10x5x1) = 50</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 - Minimise Displacement, (mm)</td>
<td>Population Size</td>
<td>= 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td>Number of Generations</td>
<td>= 200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Mutation</td>
<td>= 0.150</td>
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</tr>
</tbody>
</table>

<table>
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<tr>
<th>Cell Types</th>
<th>Integer Value</th>
<th>Probability of Selection</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.25</td>
<td>Void</td>
</tr>
<tr>
<td>D25-class</td>
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<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>D50-class</td>
<td>2</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>D75-class</td>
<td>3</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>Mesh Maximum Scale Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>TET</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Figure 7.9 – Experiment 14 Pareto Set data plotted as three separate 2D plots and a combined 3D plot.
The entire Pareto Set of equally optimal solutions for experiment 14 can be seen in full in Appendix N. Once again, a single freak solution (beam number 187) was found in the Pareto Set. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.13.

| Experiment Name | D75 | D50 | D25 | Void Pareto
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Experiment 14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pareto Set Size</td>
<td>242</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Minimal Mass Beam Geometry:
- Beam Number: 125
- Obj 1 - Mass: 0.030 kg
- Obj 2 - Displacement: 2.699 mm
- Obj 3 - Stress-Range: 1189.292 MPa

### Minimal Displacement Beam Geometry:
- Beam Number: 162
- Obj 1 - Mass: 0.080 kg
- Obj 2 - Displacement: 0.181 mm
- Obj 3 - Stress-Range: 183.647 MPa

### Minimal Stress-Range Beam Geometry:
- Beam Number: 234
- Obj 1 - Mass: 0.066 kg
- Obj 2 - Displacement: 0.279 mm
- Obj 3 - Stress-Range: 158.955 MPa

The results in Table 7.13 show the Pareto Set size for this experiment was large at 242, due to the increased number of cell type options available resulting in higher permutations that were possible. Compared to the new benchmark beam, the extreme minimal stress-range solution made a slender improvement of 2.29%, from 162.680 MPa to 158.955 MPa, but this was slightly worse than experiment 13, (2.56% higher). However, this slight increase in stress-range was at the benefit of a 20.83% reduction in mass, from 0.083 kg to 0.066 kg.

### 7.3.6 Changing the Cell Structure Type

The introduction of additional volume fractions of the D cell structure in experiment 14 did not result in the desired outcome of an improvement in stress-range reduction from the results yielded in experiment 13. Therefore, in order to further investigate the effects of considering multiple volume fractions of a single cell structure, experiments 13 and 14 will be repeated in the following two experiments (15 and 16), using the alternative B cell structure in place of the previously considered D cells.
Experiment 15

Experiment 15 was intended to be conducted as a direct comparison to experiment 13 and consisted of a simply loaded problem with all three system objectives, using B75 and void cells. However, before this experiment could be run, the associated line geometry that defines the default applied load had to be modified, as the B cell structures do not have any material present along the unit cube edges. The modifications are clearly illustrated in Figure 7.10, with the loaded nodes displayed in red.

Figure 7.10a shows how the B cell structure could not intersect with the original line geometry position. Figure 7.10b shows the first modification that was made, which was to lower the line geometry by half a cell width. However, meshing tolerances often caused the intended nodes not to intersect with the lowered line geometry, as shown in Figure 7.10b by the single unsymmetrical loaded node, highlighted red. Consequently, all of the intended nodes could not be guaranteed to be selected via this arrangement. Figure 7.10c shows the final modification that was made, which was to replace the lowered line geometry with a sheet, thereby enabling the repeated capture of symmetrical nodes. With the loading modifications made, the experiment was run. The parameters that were used for this are summarised in Table 7.14.

As with the previous experiment, various 2D Pareto plots were generated in addition to a combined 3D plot and are displayed in Appendix O. Care should be taken when comparing these plots with those of the previous experiments, as the results of this experiment are not directly comparable due to the modification made to the applied load. However, it should be noted that similar general trends can still be seen.
entire Pareto Set of equally optimal solutions for this experiment can also be seen in Appendix O. Once again, a single freak solution (beam number 32) was found. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.15.

Table 7.14 – Parameters used in experiment 15.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>Objectives:</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>1 - Minimise Mass, (kg)</td>
</tr>
<tr>
<td></td>
<td>2 - Minimise Displacement, (mm)</td>
</tr>
<tr>
<td></td>
<td>3 - Minimise Stress-Range, (MPa)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Objective</th>
<th>Number of Variables</th>
<th>Population Size</th>
<th>Number of Generations</th>
<th>Probability of Mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>(10x5x1) = 50</td>
<td>50</td>
<td>200</td>
<td>0.150</td>
</tr>
<tr>
<td>Displacement</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress-Range</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cell Types</th>
<th>Integer Value</th>
<th>Probability of Selection</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.5</td>
<td>Void</td>
</tr>
<tr>
<td>B75-class</td>
<td>1</td>
<td>0.5</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

Mesh Type = TET, Mesh Maximum Scale Factor = 0.50

Table 7.15 – Experiment 15 Pareto Set extreme solutions.

<table>
<thead>
<tr>
<th>Experiment 15</th>
<th>Pareto Set Size</th>
<th>Minimal Mass Beam Geometry:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>= 75</td>
<td>Beam Number = 57</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obj 1 - Mass = 0.027 kg</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obj 2 - Displacement = 17.157 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Obj 3 - Stress-Range = 2024.654 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimal Displacement Beam Geometry:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Number = 11</td>
</tr>
<tr>
<td>Obj 1 - Mass = 0.102 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement = 0.108 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range = 91.896 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Minimal Stress-Range Beam Geometry:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Number = 73</td>
</tr>
<tr>
<td>Obj 1 - Mass = 0.098 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement = 0.113 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range = 80.815 MPa</td>
</tr>
</tbody>
</table>

The Pareto Set size of 75 for experiment 15 is between the values obtained in experiments 9 and 13, where the number of cell type options, and therefore the permutations possible, were identical. A notable observation of the results depicted in Table 7.15 is that the extreme minimal displacement and minimal stress-range beams are similar to one another, being almost entirely made from B75 cells, indicating that
the displacement and stress-range objectives are not necessarily working against each other in terms of trade-offs. Additionally, it should be noted that the minimal mass solution in Table 7.15 is the closest resemblance to a straight beam achieved so far. Although not necessarily an ideal solution in terms of the remaining two system objectives, it almost directly connects the constrained and loaded faces together. This solution has been made possible by the inability of B75 cells to connect diagonally to one another, being forced instead to form horizontal and vertical connections only. In comparison to experiments 9 and 13, where similar cell type options were considered, the minimal mass solutions repeatedly exhibited alternating diagonal connections of cell structure and void cells, resulting in the checkerboard phenomenon.

The extreme minimal stress-range solution for this experiment had a value of 80.815 MPa, which is almost half the value obtained in experiment 13. Similarly, there was a sizeable reduction of approximately 34% in displacement. Both reductions were at the cost of an 18% increase in mass. However, as stated previously, these results should be compared with previous results with caution due to the modified loading condition applied.

Experiment 16
Experiment 16 consisted of a simply loaded problem with all three system objectives, using B75, B50, B25 and void cells. As with experiment 15, the applied load was modified as detailed in Figure 7.10c, hence these results can be compared with those from experiment 15 to inspect the effects of considering multiple volume fractions of a single cell structure. With the loading modifications made, the experiment was run. The parameters that were used are summarised in Table 7.16.

As with the previous experiment, various 2D Pareto plots were generated in addition to a combined 3D plot and are displayed in Appendix P. These plots are somewhat distorted by several high values of stress-range and maximum nodal displacement although similar general trends can still be seen. The entire Pareto Set of equally optimal solutions for this experiment can also be seen in Appendix P. Once again, a single freak solution (beam number 198) was found. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.17.
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

Table 7.16 – Parameters used in experiment 16.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>16</th>
<th>Experiment Name</th>
<th>B75 B50 B25 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 - Minimise Mass, (kg)</td>
<td></td>
<td>Number of Variables</td>
<td>(10x5x1) = 50</td>
</tr>
<tr>
<td>2 - Minimise Displacement, (mm)</td>
<td></td>
<td>Population Size</td>
<td>= 50</td>
</tr>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td></td>
<td>Number of Generations</td>
<td>= 200</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Probability of Mutation</td>
<td>= 0.150</td>
</tr>
<tr>
<td>Cell Types</td>
<td></td>
<td>Integer Value</td>
<td>Probability of Selection</td>
</tr>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.25</td>
<td>Void</td>
</tr>
<tr>
<td>B25-class</td>
<td>1</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>B50-class</td>
<td>2</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>B75-class</td>
<td>3</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>Mesh Type</td>
<td>TET</td>
<td>Mesh Maximum Scale Factor</td>
<td>= 0.50</td>
</tr>
</tbody>
</table>

Table 7.17 – Experiment 16 Pareto Set extreme solutions.

<table>
<thead>
<tr>
<th>Experiment 16</th>
<th>Experiment Name – B75 B50 B25 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 234</td>
</tr>
</tbody>
</table>

Minimal Mass Beam Geometry:
- Beam Number = 54
- Obj 1 - Mass = 0.031 kg
- Obj 2 - Displacement = 85.234 mm
- Obj 3 - Stress-Range = 10975.919 MPa

Minimal Displacement Beam Geometry:
- Beam Number = 67
- Obj 1 - Mass = 0.084 kg
- Obj 2 - Displacement = 0.170 mm
- Obj 3 - Stress-Range = 125.619 MPa

Minimal Stress-Range Beam Geometry:
- Beam Number = 67
- Obj 1 - Mass = 0.084 kg
- Obj 2 - Displacement = 0.170 mm
- Obj 3 - Stress-Range = 125.619 MPa

The Pareto Set size of experiment 16 is comparable to that of experiment 14, due to them both having equal cell type options and therefore an equal number of possible permutations. A notable observation of the results depicted in Table 7.17 is that the extreme minimal displacement and minimal stress-range beams for experiment 16 are in fact the same solution (beam number 67) from the Pareto Set. This result indicates the displacement and stress-range objectives have not compromised each other in terms of trade-offs, but have in fact complemented each other in steering the optimisation.
process in the same direction, as was also witnessed in experiment 15. This evidence confirms the observation made earlier about Figure 7.9, which highlighted the stress-range and displacement objectives appeared to be directly proportional to one another.

The extreme minimal stress-range solution for this experiment had a value of 125.619 MPa, which is 55.44% higher than the 80.815 MPa value achieved in experiment 15. This result clearly indicates that the introduction of additional volume fractions of the B cell structure in experiment 16 did not result in the desired outcome of an improvement in stress-range reduction from the results yielded in experiment 15. In addition to the increase in stress-range from experiment 15 to experiment 16, an increase of 50.44% was witnessed in displacement, however, these increases were both coupled with a 14.29% reduction in mass. This experiment, along with experiment 14, has established that considering various volume fractions of the same cell type does not enable the minimal stress-range to be reduced. The next stage of investigation is to inspect the effects of considering multiple cell types within a single design domain.

### 7.3.7 Mixing Different Cell Structures

The effect of mixing different cell types together within the same design domain is of particular interest, as it is hoped that the resulting heterogeneous structure will benefit from the properties of its constituent parts. The remaining two experiments were conducted to investigate this topic, yielding results which answer another of the research questions. After considering different volume fractions of two separate cell structures in experiments 14 and 16 (D and B respectively), it is logical to combine these two structures in experiments 17 and 18, for comparison. One option would be to consider the three volume fractions of 25%, 50% and 75% for both D and B structures, in addition to void cells. However, if all three volume fractions for both D and B structures were used together, the number of cell type options would be seven in total (including void), compared to the four that were used in both experiment 14 and 16. This difference will have a direct impact on the number of permutations that are possible, thus creating a larger search space that the MOGA must explore in order to find the Pareto optimal solutions. Consequently, the MOGA would not be expected to perform as well in the same number of iterations if a larger search space is to be covered.
In addition to the increased permutations, this proposed problem option was found to encounter significant meshing difficulties for numerous individual trial beam topologies, and could not therefore be implemented. In order to overcome these meshing difficulties, a further KBE rule would have to be defined and added to the advanced AML topology optimisation system. This rule would be necessary to allow, for example, D50 and B50 cells to be considered within the same problem, by preventing them from occupying adjacent positions to each other within the design domain. The addition of this fourth KBE rule is anticipated to have a detrimental effect on the computation time required to run the experiment, as each of the beam topologies considered must be thoroughly inspected at every location within the design domain to see if they are in violation of this new rule.

Experiment 17

As a viable compromise to the existing conditions noted above, experiment 17 consisted of a repeat of experiment 14, with the exception of B50 cells added in place of the D50 cells, once again with the applied load modified as depicted by Figure 7.10c. Hence, a simply loaded problem, using D75, B50, D25 and void cells was considered. The parameters used in experiment 17 are summarised in Table 7.18. The resulting Pareto plots that were generated by experiment 17 are displayed in Figure 7.11.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>17</th>
</tr>
</thead>
</table>
| Objectives:  
1 - Minimise Mass, (kg)  
2 - Minimise Displacement, (mm)  
3 - Minimise Stress-Range, (MPa) |

<table>
<thead>
<tr>
<th>Experiment Name</th>
<th>D75</th>
<th>B50</th>
<th>D25</th>
<th>Void</th>
<th>Pareto</th>
</tr>
</thead>
<tbody>
<tr>
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<td>= 50</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population Size</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Generations</td>
<td>= 200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of Mutation</td>
<td>= 0.150</td>
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<td></td>
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<table>
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</thead>
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<td>0.25</td>
<td>Void</td>
</tr>
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<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>B50-class</td>
<td>2</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>D75-class</td>
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<td>0.25</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mesh Type</th>
<th>TET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh Maximum Scale Factor</td>
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</tbody>
</table>
The Pareto plots displayed in Figure 7.11 show experiment 17 follows similar trends to earlier experiments (12–16), but with increased scatter. The higher degree of scattering is particularly evident in the 2D plots of mass v stress-range and displacement v stress-range. The increased scatter causes the Pareto Front to be more unclear in its definition, taking on a more linear shape, with the exception of a few outlying data points, compared to the well defined curved Front depicted earlier in Figure 7.9. This change in shape is a strong indication that the system objectives are not able to trade-off as well with one another as in earlier experiments.

The entire Pareto Set of equally optimal solutions for experiment 17 can be seen in full in Appendix Q. Once again, a single freak solution (beam number 12) was found. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.19.
Table 7.19 - Experiment 17 Pareto Set extreme solutions.

<table>
<thead>
<tr>
<th>Experiment 17</th>
<th>Experiment Name – D75 B50 D25 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>= 170</td>
</tr>
<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 159</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.032 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 1.532 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 711.699 MPa</td>
</tr>
<tr>
<td>Minimal Displacement Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 165</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.067 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.247 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 141.348 MPa</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>= 141</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>= 0.066 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>= 0.250 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>= 138.426 MPa</td>
</tr>
</tbody>
</table>

Table 7.19 shows the Pareto Set size of experiment 17 is approximately 25% smaller than both experiments 14 and 16, despite all three experiments having an equal number of permutations. The reason for this is thought to be related to the more linear shape of the Pareto Front, as fewer data points are required to define a straight line compared to a curve. All three extreme solutions display a good mix of all the cell type options throughout their construction, as do the remaining Pareto solutions in Appendix Q.

The B50 cells were introduced as a replacement for the D50 cells, in an attempt to reduce the minimum stress-range, thereby creating a more optimal beam structure. Hence, the extreme minimal stress-range solution from this experiment should be compared with those obtained previously in experiments 15 and 16, where the loading conditions were identical. When comparing the extreme minimal stress-range solutions from experiments 16 and 17, it can be seen that both the displacement and stress-range values have increased by 47.06% and 10.19% respectively, whilst mass has reduced by 21.43%. This pattern continues when comparing the solutions from experiments 15 and 17, with both displacement and stress-range values significantly increasing by 121.24% and 71.29% respectively, with mass reducing by 32.66%. These results clearly indicate that this particular mix of cell structures considered by experiment 17 has not had the desired effect of reducing the minimum stress-range. The remaining experiment will consider a different mix of cell structures to inspect whether this outcome will be repeated.
Experiment 18

This experiment, as just explained, is essentially a repeat of experiment 17 but with an alternative mix of cell structures, with all D cells replaced by B cells and vice versa. As such, experiment 18 consisted of a simply loaded problem, where B75, D50, B25 and void cells were considered together. The parameters that were used in experiment 18 are summarised in Table 7.20. The resulting Pareto plots that were generated by experiment 18 are displayed in Figure 7.12.

<table>
<thead>
<tr>
<th>Experiment Numbers</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives:</td>
<td></td>
</tr>
<tr>
<td>1 - Minimise Mass, (kg)</td>
<td>Number of Variables = (10x5x1) = 50</td>
</tr>
<tr>
<td>2 - Minimise Displacement, (mm)</td>
<td>Population Size = 50</td>
</tr>
<tr>
<td>3 - Minimise Stress-Range, (MPa)</td>
<td>Number of Generations = 200</td>
</tr>
<tr>
<td>Probability of Mutation = 0.150</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Cell Types</th>
<th>Integer Value</th>
<th>Probability of Selection</th>
<th>Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void-class</td>
<td>0</td>
<td>0.25</td>
<td>Void</td>
</tr>
<tr>
<td>B25-class</td>
<td>1</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>D50-class</td>
<td>2</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
<tr>
<td>B75-class</td>
<td>3</td>
<td>0.25</td>
<td>Aluminium</td>
</tr>
</tbody>
</table>

Mesh Type = TET  
Mesh Maximum Scale Factor = 0.50

The Pareto plots displayed in Figure 7.12 show the most clearly defined Pareto Front of all of the experiments conducted, even superior to the Front depicted in Figure 7.9 by experiment 14. The well defined curved plots for experiment 18 also indicate that the system objectives were able to trade-off well with one another. These plots are in contrast to those obtained in experiment 17, which also considered mixed cell structures, where a more linear Pareto Front was generated with significant data scatter.

The entire Pareto Set of equally optimal solutions for experiment 18 can be seen in full in Appendix R. Once again, a single freak solution (beam number 102) was found. The extreme Pareto solutions were identified as before, ignoring the freak solution, and are displayed in Table 7.21.
Figure 7.12 – Experiment 18 Pareto Set data plotted as three separate 2D plots and a combined 3D plot.

Table 7.21 – Experiment 18 Pareto Set extreme solutions.

<table>
<thead>
<tr>
<th>Experiment 18</th>
<th>Experiment Name – B75 D50 B25 Void Pareto</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pareto Set Size</td>
<td>137</td>
</tr>
<tr>
<td>Minimal Mass Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>90</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>0.031 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>4.830 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>1687.044 MPa</td>
</tr>
<tr>
<td>Minimal Displacement Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>75</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>0.079 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>0.163 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>110.571 MPa</td>
</tr>
<tr>
<td>Minimal Stress-Range Beam Geometry:</td>
<td></td>
</tr>
<tr>
<td>Beam Number</td>
<td>130</td>
</tr>
<tr>
<td>Obj 1 - Mass</td>
<td>0.084 kg</td>
</tr>
<tr>
<td>Obj 2 - Displacement</td>
<td>0.164 mm</td>
</tr>
<tr>
<td>Obj 3 - Stress-Range</td>
<td>99.683 MPa</td>
</tr>
</tbody>
</table>
Table 7.21 shows the Pareto Set size of experiment 18 is approximately 20% lower than that of experiment 17, despite both experiments being similar to one another in terms of input parameters. Both experiments 17 and 18 yielded a significantly smaller Pareto Set size than experiments 14 and 16, which are comparable in terms of the number of cell type options considered. Previously, it was thought that the lower Pareto Set size of experiment 17 was due to the more linear shape of the Pareto plot (Figure 7.11). However, the fact that experiment 18 also yielded a comparatively small Pareto Set size, whilst displaying a curved Pareto plot (Figure 7.12) suggests that the plot shape is unrelated to the size of the Pareto Set. Instead, the reduced Pareto Set sizes witnessed in experiments 17 and 18 are thought to be as a direct consequence of using mixed cell structure types. Table 7.21 also shows that a good mix of the cell type options have been used in the three extreme solutions. This observation is also evident in the remaining Pareto solutions seen in Appendix R.

Due to the modified loading conditions made to all experiments after experiment 14, the extreme solutions from this experiment should only be compared with those obtained by experiments 15, 16 and 17. When comparing these extreme minimal values, it can be seen that the extreme minimal stress-range solution of experiment 18 has a lower stress-range value than that achieved in experiments 16 and 17, by 20.65% and 27.99% respectively. Whilst this result indicates that this mix of cells has indeed managed to create a more optimal structure than previous experiments, by lowering stress-range, it should also be noted that the stress-range value for experiment 18 is 23.35% higher than in experiment 15. Despite this last result not being favourable, it is clear that experiment 18 has performed considerably better than either experiment 16 or 17. To demonstrate this success, it can be shown that the extreme minimal stress-range solutions to this experiment and that of experiment 16 have an exactly equal mass of 0.084 kg. However, whilst using the same amount of material, experiment 18 managed to use it more efficiently, by reducing the stress-range by 20.65% in addition to a 3.53% reduction in displacement, a notable advancement. This improved outcome can only be as a direct consequence of the particular mix of cell structures considered by experiment 18, thereby suggesting that in some instances, using mixed cell structures can have beneficial results.
CHAPTER EIGHT

DISCUSSION

8.1 Introduction

This Chapter forms the main discussion of this thesis. It provides answers to the research questions that were raised previously in Section 3.4.3, based on the results of Chapter 7 and some validation regarding comparisons between the FE representations of various beam topologies and their corresponding RM models. In addition to answering the research questions, an assessment of the current AML topology optimisation system is made. This assessment investigates and discusses the performance, stability and computational demands of the system. Finally, a critique of the research that was undertaken is presented and reflected upon.

8.2 Research Question Answers

Based on the results of the advanced experiments in the previous Chapter, it is now possible to answer all of the research questions that were stated earlier in Section 3.4.3, with the exception of the final question which will require some additional investigation. Each of the research questions are presented in turn in the following sections, with their individual answers discussed in each case.

Question 1

*Does the introduction of a 50% volume fraction cell structure into a simple solid void problem help to create a more uniform stress distribution compared to simple solid and void only problems?*
In order to answer this research question, the stress-range results from experiment 9 where solid and void cells were considered, must be compared with the results obtained in experiment 12, where the inclusion of a 50% volume fraction cell structure was first considered. It should be noted that this question has effectively already been answered during Chapter 7, when these experimental results were first presented and discussed. The extreme minimal stress-range solution from experiment 9 had a value of 37.258 MPa. In comparison, the extreme minimal stress-range solution from experiment 12 had a value of 39.517 MPa, which equates to an increase of 6.06% in Von Mises stress-range. These results indicate that the inclusion of a 50% volume fraction cell structure, in this case the D50 cell, into a solid and void problem did not lower the overall stress-range. This outcome was not expected and is clearly not that which was desired, however, the cause will be discussed in the answering of research question 4.

**Question 2**

*Can the use of various multiple volume fractions of a single cellular structure improve the stress distribution uniformity compared to either, the simple solid and void case, or the solid, void and 50% cell structure case?*

In order to answer this research question, the results from two pairs of experiments should be compared with one another, namely experiment 13 with 14, and 15 with 16. The results from these four experiments were presented previously in Chapter 7. Experiment 13 considered a D75 and void problem, whereas experiment 15 considered a B75 and void problem. Additional volume fractions of 25% and 50% of the D and B cell type structures were added in experiments 14 and 16 respectively. The results from experiments 13 and 14 showed that the extreme minimal Von Mises stress-range value actually increased by the introduction of additional volume fractions of the D cell structure. This unexpected and undesirable behaviour was later confirmed by the results from experiments 15 and 16, where the Von Mises stress-range was again shown to increase, this time by the introduction of further volume fractions of the B cell structure. These results suggest that using multiple volume fractions of a single structure is not beneficial for lowering the overall stress-range. It should be noted however, that these increases in stress-range were also accompanied by significant reductions in mass.
Additionally, the inclusion of multiple volume fractions, despite the increased permutations, saw experiments 14 and 16 exhibit more clearly defined Pareto Fronts than either of experiments 13 and 15, suggesting improved trade-offs between system objectives.

**Question 3**

*Can using multiple cellular structures simultaneously within a design domain be more beneficial to satisfying the design objectives compared to multiple volume fractions of a single cellular structure?*

The results from the final three advanced experiments should be compared with one another in order to answer this research question. For ease of comparison, the extreme minimal mass, displacement and stress-range values from experiments 16 to 18 are presented in Table 8.1.

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extreme Mass</td>
<td>0.031 kg</td>
<td>0.032 kg</td>
<td>0.031 kg</td>
</tr>
<tr>
<td>Displacement</td>
<td>85.234 mm</td>
<td>1.532 mm</td>
<td>4.830 mm</td>
</tr>
<tr>
<td>Stress-Range</td>
<td>10975.92 MPa</td>
<td>711.699 MPa</td>
<td>1687.044 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extreme Mass</td>
<td>0.084 kg</td>
<td>0.067 kg</td>
<td>0.079 kg</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.170 mm</td>
<td>0.247 mm</td>
<td>0.163 mm</td>
</tr>
<tr>
<td>Stress-Range</td>
<td>125.619 MPa</td>
<td>141.348 MPa</td>
<td>110.571 MPa</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>16</th>
<th>17</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extreme Mass</td>
<td>0.084 kg</td>
<td>0.066 kg</td>
<td>0.084 kg</td>
</tr>
<tr>
<td>Displacement</td>
<td>0.170 mm</td>
<td>0.250 mm</td>
<td>0.164 mm</td>
</tr>
<tr>
<td>Stress-Range</td>
<td>125.619 MPa</td>
<td>138.426 MPa</td>
<td>99.683 MPa</td>
</tr>
</tbody>
</table>

The results in Table 8.1 demonstrate the varied effects of mixing different cell types together within the same problem. It can be shown from these results that experiment 17, where both D and B cells were considered, was unable to improve upon any of the extreme values obtained in experiment 16, where just B cells were considered. These results in isolation may lead to the assumption that it is not beneficial to mix different cell types together. However, when including the results from experiment 18 in the comparison, where an alternative mix of D and B cells was considered, it can be seen
that substantial improvements were witnessed in the extreme minimal objective values. The clearest comparison can be drawn between the extreme stress-range solutions of experiments 16 and 18 where, with an equal amount of material, experiment 18 was able to generate a solution which had a lower maximum displacement value in addition to a reduction in stress-range of approximately 20%. This achievement demonstrates material was used more efficiently in experiment 18 compared to experiment 16, resulting in a more optimal solution being created. Furthermore, the Pareto Front generated in experiment 18 was the most well defined of all of those produced, a clear indicator of improved trade-offs between the system objectives in this experiment, a direct result of mixing different cell structures together.

These further results indicate that the overall outcome of mixing different cell types together depends entirely upon the particular cell structures that are to be used in each case. In some instances this can be to detrimental effect, as in experiment 17, and in others to beneficial effect, as in experiment 18 where the overall structural properties could be considered blended like an alloy. However, one benefit of considering mixed cell structures together, which was demonstrated by both experiment 17 and 18, was the generation of a smaller Pareto Set size, clearly advantageous if a user is to manually choose a final solution.

**Question 4**

A potential outcome of the proposed methodology is that the cell structures introduce localised SCs into the solution geometries, is this the case? If so, what effect do these SCs have on the overall optimisation process?

This question has to some extent already been answered in Chapter 7, when the results obtained by the advanced experiments were first presented. These results have shown that the cell structures used in the proposed methodology do in fact introduce localised SCs into the solution geometries. In addition, the SCs appear to be higher for the lower volume fraction cells. This phenomenon is the cause of the undesirable outcomes as detailed in the answers to the previous research questions.
The occurrence of these cell induced SCs appears to steer the optimisation process, best evidenced by the results from experiment 12. Here the extreme minimal stress-range solution was found to include only a few of the D50 cell structures used, with the MOGA selecting the non SC inducing solid cells in preference. This behaviour was further evidenced by Figure 7.11, which showed Pareto optimal beam topologies from experiment 12, which more closely resembled the assumed TopOpt ideal, had higher stress-range values. The preferential selection of cells with lower SCs tended to result in low stress-range solutions being typically heavier, which in turn also resulted in lower displacement values. It follows therefore, that the displacement and stress-range objectives began to display a more common goal, as was witnessed in the Pareto plots, as opposed to conflicting with each other.

The SCs are thought to be induced by the sharp edges in the make up of the cell structures. This suggests therefore, that a possible way of reducing them and their impact upon the overall optimisation process, would be to replace the sharp edges with curved fillets. This proposed concept is illustrated in Figure 8.1 where the original B25 cell structure has been used as an example.

Figure 8.1 - Original B25 unit cell structure with sharp edges present (a) and modified B25 structure with curved fillets (highlighted red) added in place of sharp edges (b).

Figure 8.1a shows the original B25 unit cell structure with sharp edges present where the three cylinders intersect. These sharp edges are then removed by the introduction of curved fillets to the B25 structure, displayed in red in Figure 8.1b.
Question 5

Is the FEA model a realistic representation of the physical RM model? This question can also be asked of existing topology optimisation software such as DesignLab.

The previous experimental results are insufficient to answer this final question. In order to answer it, the FE and RM models of the same beam topology must be compared when loaded in bending. A sample beam geometry from Appendix J was chosen for this purpose. The selected beam topology was TET meshed and solved using FEA within the advanced AML system, the results of which can be seen below in Figure 8.2.

![Figure 8.2 - FE model representation of sample beam topology showing, Von Mises stress plot of entire beam (a), and close up view of diagonally connected cells within the beam (b).](image)

The same sample beam topology was then exported from the advanced AML system in STL file format, enabling a physical RM model to be fabricated using SLS technology. This RM model was then clamped in a vice and loaded in such a way as to roughly simulate the bending load and boundary conditions that were applied to the FE model. The clamped and loaded RM model of the beam topology can be seen in Figure 8.3.

![Figure 8.3 - Clamped and loaded RM model of selected sample beam topology showing, entire beam (a), and close up view of diagonally connected cells within the beam (b).](image)
Although the TET mesh used in the construction of the FE model is able to represent the cellular structures of the RM model fairly accurately, comparing Figure 8.2 with Figure 8.3 demonstrates how the behaviour of the FE and RM models of the same beam topology differ when subjected to a simple bending load. This is particularly evident when comparing the highlighted regions within Figures 8.2b and 8.3b. Figure 8.2b illustrates how diagonally connected cells within the FE model are permanently connected along the edge where nodes are shared by cells. When subjected to bending, the diagonally connected cells distort under the applied load thus transferring its affects through the FE structure. However, Figure 8.3b demonstrates how the same diagonally connected cells within the RM model, when subjected to a similar bending load, are able to slide past one another, due to them not being physically connected to each other.

This comparison suggests that the FE models are not realistic representations of the RM models. However, it can also be said that the RM model does not necessarily replicate the FE model, due to inaccuracies in build tolerance, which could be the cause of the disconnected diagonal cells within the RM model. The fabrication of further RM model beam topologies indicated that sometimes diagonally connected cells would fuse together along the shared edge, when using SLS technology. However, despite this situation yielding a closer match between the FE and RM models, the repeatability and reliability of diagonally connected fused cells could not be guaranteed.

The behavioural discrepancy between the FE and RM models used in this research answers the first part of the final research question. The remaining part of the question extends this query to other topology optimisation software, such as DesignLab. The earlier investigation of DesignLab demonstrated that the FE model representations of the beam geometries also considered diagonal cells to be permanently connected by the shared edge nodes of the HEX mesh. This FE model representation must now be compared to a corresponding RM model. One of the final DesignLab solutions from Chapter 3 was selected for this comparison. The chosen beam was directly exported in STL file format, enabling a physical RM model to be fabricated, this time using SLA technology. The resulting SLA model can be seen in Figure 8.4a. The requirement to load the RM model was unnecessary as closer inspection of diagonally positioned cells within the SLA structure showed daylight gaps were present, as seen in Figure 8.4b.
The presence of daylight gaps within the RM model indicates that the diagonally positioned cells were not connected to each other. Hence, any load applied to the overall beam structure would not be able to be transmitted through the disconnected diagonal cells. This result has demonstrated that the FE model used by DesignLab is not a realistic representation of the RM model constructed directly from the software. However, inaccuracies in the SLA build process could be responsible for the disconnected diagonal cells within the RM model. Based on this significant discrepancy between the behaviours of the FE and RM models of both the AML topology optimisation system and DesignLab, further numerical validation was therefore deemed unnecessary at this point.

Upon reflection, this comparison study could have been conducted much sooner, thereby enabling a potential solution to have been identified, implemented and tested in the remaining research. It should be noted however, that the assumed connection of diagonally positioned cells is common within the topology optimisation field. A direct consequence of this assumption is that checkerboarding is often present in results, leading some researchers to add post process filters to actively seek out and remove solutions that exhibit this phenomenon, [242-246]. In many areas of the topology optimisation field, solutions are purely theoretical, being impossible to manufacture. However, recent advances in additive fabrication techniques now enable many previously unmanufacturable solutions to be physically realised, which in turn highlights the significance of this behavioural discrepancy when going from a virtual solution directly to a physical model.
A possible means of reducing the discrepancy in behavioural differences of the FE and RM models, would be to introduce some form of sub-frame within the design domain. The sub-frame would guarantee that diagonally positioned cells within the design domain are connected to each other by some additional geometry. Figure 8.5 demonstrates two different sub-frame concepts, but either would connect the diagonal cells along their shared edge. A sub-frame would add extra material where the daylight gaps appeared in Figure 8.4, thereby ensuring the RM model more closely resembles the corresponding FE representation.

![Figure 8.5 - Potential sub-frame ideas using square columns (a) and spheres (b).](image)

The inclusion of a sub-frame into the design domain may also help to reduce the occurrence of SCs at diagonal cell connections, particularly if the sub-frame geometry extends along the entire shared edge, as in Figure 8.5a. If a sub-frame was to be implemented into the existing AML topology optimisation system, further numerical analysis would be required in order to validate how well the FE and RM models now compare to each other. However, several other differences are known to exist between the FE and RM models that are expected to cause disparity of any results. These differences include, for simplification over a long optimisation run, the FE models assuming linear elastic analysis of an arbitrary isotropic material (Aluminium), whereas in reality, RM materials usually display a degree of anisotropic properties and often exhibit non-linear behaviour.
8.3 Assessment of the Current System

The assessment of the current AML topology optimisation system can be divided into three separate categories, namely; system performance, system stability and computational demands. Each of these three topics will now be discussed in turn in the subsequent sections.

8.3.1 System Performance

The performance of the AML topology optimisation system can be judged by the quality of the results obtained in the preliminary and advanced experiments. Judging the quality of the results obtained, however, is often not a straightforward matter. The results to the preliminary experiments can be compared with those obtained via DesignLab, which was undertaken earlier in Section 6.2.3. The results from the advanced experiments are more difficult to quantify, due to the novelty of using cellular structures within a GA based topology optimisation system. As such, these results are essentially only comparable with other results from the advanced experiments.

Iterations & Permutations

The comparison between the results of the preliminary experiments and those of the DesignLab investigation highlighted that on average DesignLab considered approximately 21660 iterations per experiment before the GA converged, based on 361 generations of 60 individuals. This compares to the 10000 iterations considered by the preliminary experiments, based on 200 generations of 50 individuals. Considering therefore, that the preliminary experiments evaluated half as many iterations as the DesignLab experiments, it was encouraging to find that the TSMOGA achieved results of a similar quality to those from DesignLab.

A logical question arising from this observation regarding the discrepancy in iterations is therefore, were sufficient iterations considered by these experiments? This question is best answered with the aid of the preliminary experiments where only single system objectives were considered in isolation and where ideal solutions were known, namely experiments 5 and 6. These experiments showed that although the TSMOGA was close to obtaining the known optimum solution, it was unable to reach it within 10000
iterations. However, in order to assess the performance of TSMOGA in these experiments, the number of iterations must be directly compared to the number of permutations possible for each experiment. The total number of permutations ($N_p$) possible varies for each experiment based on the number of discrete design variables used and the number of cell type options available, as defined by Equation 8.1.

$$N_p = X^a \times Y^b$$

(Eqn. 8.1)

Where:

'X' is the total number of cell types available,

'Y' is the number of non-void cell types available,

'a' is the number of non-loaded discrete variables,

'b' is the number of loaded discrete variables,

and the sum of 'a' and 'b' equals the total number of discrete variables.

If experiment 1 is considered as an example (values taken from Table 6.1), using Equation 8.1, the total number of permutations possible are calculated as follows:

$$N_p = 3^{199} \times 2^1$$

$$= 1.77076 \times 10^{95}$$

The total number of permutations for all of the preliminary and advanced experiments has been calculated using Equation 8.1. This data is presented below in Table 8.2 along with the number of iterations considered by each experiment and the percentage of the design space that has been searched by TSMOGA.

The data in Table 8.2 clearly illustrates the vastness of the design search space for each experiment, based on the high number of permutations in each case. It also highlights that only a tiny fraction of the entire design space is searched by TSMOGA when 10000 iterations are considered. This is particularly evident for the preliminary experiments due to the higher number of design variables which cause greater permutations. This data reinforces the requirement for a population based stochastic optimisation algorithm, as each member of the population can simultaneously jump around the vast search space in search of the global optimum.
Table 8.2 - Numbers of iterations and permutations for all of the conducted experiments in addition to the percentage of design space searched by TSMOGA in each case.

<table>
<thead>
<tr>
<th>Experiment Number</th>
<th>Iterations Considered</th>
<th>Permutations Possible</th>
<th>% of Design Space Searched</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-8</td>
<td>10,000</td>
<td>1.77 x 10^65</td>
<td>5.65 x 10^-60</td>
</tr>
<tr>
<td>9</td>
<td>10,000</td>
<td>5.63 x 10^14</td>
<td>1.78 x 10^-69</td>
</tr>
<tr>
<td>10</td>
<td>10,000</td>
<td>5.63 x 10^14</td>
<td>1.78 x 10^-69</td>
</tr>
<tr>
<td>11</td>
<td>10,000</td>
<td>3.44 x 10^10</td>
<td>2.91 x 10^-65</td>
</tr>
<tr>
<td>12</td>
<td>10,000</td>
<td>4.79 x 10^23</td>
<td>2.09 x 10^-18</td>
</tr>
<tr>
<td>13</td>
<td>10,000</td>
<td>5.63 x 10^14</td>
<td>1.78 x 10^-69</td>
</tr>
<tr>
<td>14</td>
<td>10,000</td>
<td>9.51 x 10^29</td>
<td>1.05 x 10^-24</td>
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<tr>
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<td>10,000</td>
<td>5.63 x 10^14</td>
<td>1.78 x 10^-69</td>
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<tr>
<td>16</td>
<td>10,000</td>
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<td>17</td>
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<tr>
<td>18</td>
<td>10,000</td>
<td>9.51 x 10^29</td>
<td>1.05 x 10^-24</td>
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Based on the data in Table 8.2, it is now clear just how well TSMOGA performed during experiments 5 and 6 in obtaining a solution close to the known optimum within 10000 iterations. In addition, the results from experiments 10 and 11 confirmed that TSMOGA had indeed become trapped at a local sub-optimal solution during experiments 7 and 8. These results also demonstrated that this hazard could be reduced by increasing the probability of mutation or even sidestepped by ensuring all three system objectives are considered simultaneously, as was the case in experiment 9. The results from the more complex advanced experiments, where cellular structures have been used, have already been discussed when answering the research questions in Section 8.2. Of these results it can be said that overall, the current system underperformed, with immense computation power required that did not yield desirable results. This was due in no small part to the occurrence of SCs throughout the beam structures, induced by the cell structures themselves, and their capacity to steer the overall optimisation process.

With some assessment made as to the quality of the results obtained, it should also be noted that the overall system performance must also take into account the stability and computational demands of the system. These topics will now be discussed respectively in the following sections.
8.3.2 System Stability

The system stability is an important factor when assessing the current AML topology optimization system, the construction of which was detailed in Chapter 5 and Appendix D. There are three separate issues known to exist within the current system that require further investigation and development in order to improve system stability. These known issues include mesh creation problems, a memory leak and two other unresolved error messages. The mesh creation problem and memory leak issue are worthy of further explanation, as such these topics will be discussed in the following sections.

Mesh Generation Failed

On several occasions when using the advanced AML topology optimization system, an error message was produced when trying to create the required TET mesh. The error message read ‘Mesh generation failed’ and is displayed in Figure 8.6.

It is thought that this error occurred as a result of the automated TET mesh tolerance parameters being inappropriate for some of the particular beam topologies considered by the advanced experiments. The error typically occurred 2-5 times within each of the more complex advanced experiments, causing the entire experimental segment to fail each time. The values of the TET mesh tolerance parameters were varied in an attempt to overcome this error but unfortunately the efforts were fruitless. This suggests that either the tolerance parameters were still inappropriate, or the automated Patran meshing job request timed out, or the particular beam geometries concerned were unable to be meshed by a solid TET mesh representation.
Memory Leak

This next system stability problem, known as a memory leak, was mentioned briefly in Section 7.3.1. Throughout each iteration cycle of the advanced topology optimisation process, AML requires the specific beam geometry to be meshed using Patran and solved using Nastran. In order to complete these tasks, a portion of the PC’s Random Access Memory (RAM) is assigned for each job. When the job in question is complete the RAM that was allocated to it should be released back to the PC for use in other applications. However, the AML topology optimisation system that has been designed and developed in this research, does not release all of the RAM back to the PC when Patran and Nastran jobs complete. Instead some of the RAM (estimated to be a few KB each time) remains allocated by AML, rendering it unavailable for other applications.

If these jobs were isolated events, the leaked memory would be unmissed by the user and ultimately released when AML is closed. However, this phenomenon occurs repeatedly throughout an optimisation process, resulting in the gradual build up of leaked RAM, until either the optimisation process stops, or the finite amount of available physical memory (RAM) is exhausted. When the latter occurs, the PC begins to use a portion of designated hard disk space known as virtual memory to complete necessary tasks and run applications. However, read and write times to and from virtual memory (known as ‘paging’) are considerably slower than read and write times to and from physical memory (typically by 20 times), often resulting in the PC effectively grinding to a halt, with many applications failing to respond. As such, in preference to this undesirable situation, the optimisation process was stopped and restarted regularly throughout the experiments, with AML being closed and reopened each time, thereby releasing any previously ‘lost’ memory.

This releasing strategy is demonstrated in Figure 8.7, by viewing the Task Manager CPU Usage History and Page File Usage History, before (left) and after (right) AML is closed at the end of an AML topology optimisation run. The left hand image shows that at the end of the optimisation run, with AML still running, 906 MB of the available physical RAM (highlighted blue) is allocated by the PC operating system. The right hand image shows how the allocated RAM drops to 388 MB when AML is closed, indicated by the corresponding spike in CPU Usage History, highlighted red.
closing of AML therefore, has released 518 MB of RAM back to the PC’s operating system in this example.

This memory leak problem has been reported back to TechnoSoft who have successfully managed to reduce it’s effects to some extent, particularly for the basic AML system using the HEX mesh. Nevertheless, the memory leak still remains a major unresolved issue, causing all of the advanced experiments to be run in multiple segments, each one starting where the last ended, with AML being restarted at each interval in order to release ‘lost’ memory. As the advanced experiments increased in complexity, so too did the extent of the memory leak, as a direct result of dealing with larger mesh and analysis jobs.

8.3.3 Computational Demands

The experimental results from the previous two Chapters have demonstrated how highly computationally demanding this research has been. For example, each preliminary experiment using the basic HEX mesh and 200 design variables, required approximately 19.4 hours to complete, based on 10000 iterations at 7 seconds each. The investigation of TET mesh fidelity on computation time, conducted in Section 7.2 and Appendix J, indicated that the advanced AML topology optimisation system required significantly more computing power than the basic AML system and that this computational increase was exacerbated by mesh fidelity. Consequently, a coarse TET mesh was used throughout the advanced experiments, in addition to a reduction in the number of design variables from 200 to 50. Despite these changes, the advanced experiments still
required a substantial amount of computation. Based on 10000 iterations, with between 5 and 7 minutes required for meshing, solving and evaluating each iteration (where cellular structures were present), estimates for the duration of each advanced experiment range from 34.7 days to 48.6 days. However, these figures can be considered low estimates as they assume that each advanced experiment will run continuously for 24 hours per day, 7 days per week, which is currently impossible due to the existing memory leak problem detailed in Section 8.3.2. Additionally, these estimate figures do not take into account the requirement to repeat failed experiment segments due to the other system stability errors mentioned in Section 8.3.2. Equally, the computational requirements of considering invalid beam geometries were not accounted for in the duration estimates, nor any system downtime caused by power failures, all of which occurred when the advanced experiments were conducted.

The actual durations for the advanced experiments where cellular structures were used, typically ranged from 38 days to 56 days, which is clearly too long. Such excessive experiment durations would dramatically limit the uptake and application of such a system. It should be noted that the author spent a considerable amount of time trying to address the massive computational demands of the system by implementation of a Supercomputer arrangement available at Cambridge University. However, unfortunately these attempts proved unsuccessful (further information regarding this topic is given in Appendix J). In order to achieve the same results in a more acceptable period of time, either the necessary computation required would need to be reduced, or the computation power available increased or redistributed. Methods of achieving these outcomes will now be discussed in the following sections.

Simplified Cell Structures

The unit cell structures chosen for this research were all designed using a combination of cubes and curved geometry in order to benefit from the self-supporting builds permitted by many RP/RM processes (e.g. SLA). However, the resulting curved cellular geometries are more difficult to mesh accurately unless a high fidelity is used, which is computationally exhaustive. One method of reducing the computational demands of the current system therefore, would be to simplify the cell structures that are used, making them easier to mesh. A possible way of achieving this outcome would be
to replace the curved faces within the cell structures with faceted faces. This suggestion is demonstrated in Figure 8.8a, which shows a faceted B25 cell structure, where the cylinders have been replaced by extruded octagons. Taking this concept one stage further, the proposed curved fillets that were previously added to the B25 structure in Figure 8.1b could be replaced by edge chamfers in order to reduce SCs, as shown below in Figure 8.8b. A further simplification to the faceted B25 cell structure, illustrated in Figure 8.8, which should radically reduce computation would be to replace the octagon cross-sections with square cross-sections.

![Figure 8.8 - Simplified B25 unit cell structure (a), and the same faceted geometry with edge chamfers added to reduce SCs (b).](image)

This method of simplifying the cell structures would obviously make them less likely to self-support using certain RP/RM technologies. However, this is less of a problem for many powder based technologies involving polymers, where surrounding powder is used to help support parts throughout fabrication. In addition, this proposed modification to the cell structures would enable greater similarity between the RM model of a specific geometry and the FE representation used in the optimisation process. A further simplification of the cell structures used that would significantly reduce computation would be to only use 2D cell structures. This proposal would obviously limit any experiments to 2D, in addition to prohibiting complex RM models from being created, with the only remaining option available being 2½D extrusions.

**Removal of Invalid & Repeated Geometries**

In addition to simplifying the cell structures used, further computational savings could be made by the addition of two new rules within the AML topology optimisation...
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system. The first rule would be introduced in order to prevent invalid beam geometries, as defined by Figure 5.11, from being considered. Invalid beam geometries are of no interest to this research, but are currently permitted to be considered by the MOGA, thereby causing unnecessary computation. This usually results in one of the invalid geometries entering the Pareto Set in the form of a freak solution, as defined in Section 6.2.2, which is also clearly undesirable. This situation is demonstrated by Table 8.3, where the freak solutions from all of the conducted experiments are displayed, with the exception of Experiment 2, shown previously in Figure 6.4.

Table 8.3 – Pareto Set freak solutions generated by the preliminary and advanced experiments.

<table>
<thead>
<tr>
<th>Experiment 4 Freak</th>
<th>Experiment 9 Freak</th>
<th>Experiment 12 Freak</th>
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<tr>
<td>Experiment 13 Freak</td>
<td>Experiment 14 Freak</td>
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<td>Experiment 16 Freak</td>
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<td>Experiment 18 Freak</td>
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The addition of a rule to check if a proposed beam topology is valid or invalid could potentially, in the case of the advanced experiments, save hours or even days of unnecessary computation time. In order to achieve this saving, the validity check would need to be performed prior to the computationally demanding tasks of creating a TET mesh and solving the FE equations. However, the validity of a beam topology can only be determined based on the FE results at present. Therefore, one method of implementing this new rule into the advanced AML topology optimisation system would be to run a basic FE analysis (i.e. using the simple HEX mesh) on each beam first. The outcome of the basic FE analysis would determine the beam’s validity, therefore establishing whether advanced analysis (i.e. using a TET mesh) was required.

This proposed method of implementation obviously has time implications, as the expected savings are at the expense of performing a basic FE analysis for each beam.
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topology considered. The estimated typical time costs of such a system modification are expected to be approximately 19.4 hours for each advanced experiment, based on 10000 iterations at 7 seconds for each HEX mesh creation and evaluation. The estimated typical time savings could range from an expected minimum of 25 hours, based on 250 invalid geometries at 6 minutes per iteration, rising to a possible 20.8 days, based on 5000 invalid geometries at 6 minutes each. Obviously the actual time savings are inherently determined by the number of invalid beam geometries encountered, which in Section 7.3.1 was previously deemed to be difficult to measure due to the advanced experiments having to be run in multiple segments. However, a somewhat unreliable technique for counting the number of valid geometries emerged during the course of conducting the advanced experiments (based on counting Patran session files). This technique suggested that the number of invalid beam geometries typically considered throughout the advanced experiments was between 4000 and 13000 depending on the number of cell type options available, suggesting therefore, that the time savings of this rule implementation would far outweigh the costs incurred.

Previously in Section 8.3.1, it was shown that the number of iterations considered by these experiments is miniscule compared to the total number of possible permutations. Therefore, if a beam topology is considered more than once in an experiment, valuable computational resources would be wasted. Hence, a second rule could be added to the AML topology optimisation system to prevent the MOGA from repeatedly considering the same beam topologies, thus saving computational resources. In order to achieve this, some kind of database file would be required that would maintain an up to date record of all of the previously considered beam topology chromosome representations. For each new iteration of the optimisation process, prior to any meshing and analysis steps, this file would be cross referenced to see if an identical beam topology had previously existed. If the database file revealed that a particular beam topology had already existed, the beam would be discarded immediately, thus ensuring that the computationally intensive meshing and analysis steps would not be repeated.

Obviously if this rule was implemented, the time required to check the database file would take longer as each experiment progresses, as it would contain a greater number of entries to cross reference. There is a possibility therefore, that this check may
become a further drain on computational resources and may also lead to additional problems if experiments are to be conducted in multiple segments. The exact implementation of this rule would therefore require careful consideration and testing.

Parallel Arrangement

In order to reduce the overall advanced experiment durations, in addition to reducing the necessary computation the system could be reconfigured into a parallel arrangement, thereby permitting the experiment to be distributed and solved simultaneously. This arrangement is made possible due to GAs being naturally suited to parallel configurations, as discussed earlier in Section 2.6.2, due to them maintaining a population of individuals. The fitness evaluation of individuals can be performed simultaneously in a parallel system, therefore increasing the speed at which a single population can be evaluated and hence reducing the overall optimisation time. Before possible parallel arrangements are discussed it is worth noting the basic sequential computational arrangement of the current system, which is illustrated in Figure 8.9.

![Diagram of the sequential computational arrangement](image)

*Figure 8.9 – Simplified sequential computational arrangement of the current system.*

The first method of parallelising the AML topology optimisation system would be to run the whole sequential system in parallel, with both Patran and Nastran running simultaneous jobs each using a separate instance of AML. This arrangement would be
based around the classic master-slave approach, as discussed earlier in Section 2.6.2. It would therefore require a designated install of AML to manage the distributed jobs and perform the necessary MOGA functions based on the information passed to it from the distributed AML slaves, as illustrated by Figure 8.10.

The parallel computational arrangement in this case would be in the form of a cluster of networked workstations or PCs, with the network speed being critical to the overall experiment duration. This parallel arrangement would prove costly in terms of software licenses as each member of the cluster would need a separate software license for AML, Patran and Nastran. It should also be noted that substantial rework of the AML topology optimisation code (see Appendix C) would be required to convert the current sequential system into this parallel form. The required rework would mainly be to reconfigure the naming, management and access permissions of created files and folders in addition to how the MOGA creates and manages populations.

The slow stages within the creation and evaluation of a single individual using the current sequential system are known to be the meshing jobs via Patran and the FE solving jobs via Nastran, as was evidenced within Section 7.2. Therefore, a more ideal parallel computational arrangement would be to only have a single instance of AML.
running which distributes all the computationally exhaustive Patran and Nastran jobs across a parallel network, as illustrated in Figure 8.11. The parallel computational arrangement in this case could either be in the form of a cluster of networked workstations/PCs as before, or to multiple processors of a supercomputer. Once again network speed is critical to the overall experiment duration, however, data shared between multiple supercomputer processors is expected to be faster than between networked workstations/PCs. This parallel arrangement is also cheaper to setup in terms of software licenses as only a single AML license is required.

Unfortunately, this ideal parallel system is phenomenally difficult to create due to the meshing and analysis jobs not flowing directly into one another, with data being passed from Patran back into AML before being reconfigured for entry into Nastran. The simultaneous handling of numerous mesh files and results data combined within the TSMOGA optimisation is an enormous task that is not possible at this time. TechnoSoft have suggested this is a multi million dollar (U.S.) project that could easily span several years of development. The task would be significantly easier to implement if AML was able to perform the meshing and FE solving tasks itself internally, without the need to communicate data between other pieces of software. This way, it could be
8.4 Critique of Research Undertaken

The AML topology optimisation system that has been designed, developed and investigated during this research has enabled enough experiments of sufficient complexity to be completed, in order to answer four of the research questions. The ability of the system to export a 3D STL file for fabrication via RM enabled the final research question to be answered sufficiently. With regards to the experiments themselves, the question of whether the number of iterations was high enough has already been asked and answered. Based on the experimental results obtained, the suitability of the TSMOGA has also been determined and justified for this form of discrete variable topology optimisation. In terms of the experimental configurations that could have been investigated, the unexpected occurrence of meshing incompatibilities caused numerous limitations as to what was possible to conduct. Equally, Section 8.3.3 demonstrated how highly computationally demanding this research has been, particularly when progressing to the more realistic TET mesh.

As a consequence of the meshing incompatibilities and high computational requirements which were underestimated, a lack of truly advanced experiments were conducted in this research. It was originally hoped that both hollow and I-section beam problems would be investigated, with both simple and offset loadings applied, in addition to studying Michell-type bridge problems [119]. All of these problem types are possible to create within the existing AML topology optimisation system, however, in terms of answering the research questions, all were deemed unnecessary and outside the focus of the current research scope.

In the desire to generate 3D solutions that could later be fabricated via RM, a simpler 2D system that would have been significantly faster to mesh and solve was overlooked. Although now no longer necessary as the 3D system would supersede it, a 2D system would have proved itself useful in identifying earlier the problems now known to exist within the 3D system, namely the existence of SCs. A 2D system would also not have been able to construct the more advanced experiments that were originally anticipated,
namely hollow or I-section beams, and would not have been able to consider offset loadings. However, as mentioned previously, the experiments conducted in this research were not as advanced as these examples, to the point where a 2D system could probably have been used to answer most of the research questions.

From a critical point of view, the AML topology optimisation system created for this research has been shown to be slow at both meshing and solving. In addition, the experiments involving cellular structures were shown to yield solutions which contained SCs, induced by the cellular structures themselves, and that where possible in the case of minimising stress-range, the MOGA would preferentially select topologies that contained fewer cell structures. Hence, the occurrence of cell structures and their associated SCs tended to steer the optimisation process, although this fact does ultimately demonstrate that the system works. It is vital therefore, that this issue of SCs occurring within the cells is addressed. This issue may be resolved by simply modifying the existing cell structures, by way of replacing the sharp internal edges with curved fillets, as demonstrated in Figure 8.1. Alternatively, some form of transition cells may be required to alleviate the undesired SCs by allowing cell structures to somehow merge into their direct neighbours. This possible approach may be particularly advantageous when considering advanced experiments with mixed cell structure types. However it should be recognised that, if implemented, such an approach has the potential to greatly increase computation, although this may be an acceptable consequence if stress-range results improve significantly.

The final point concerns the use of discrete variables, necessary if different unit cell structures are to be considered simultaneously within the design domain. The use of discrete variables for all of the different cell types and volume fractions makes the current system directionless when searching for the global optimum topology. To clarify, if a beam was found to be overloaded at a specific location and therefore required the cell in question to be replaced with a higher volume fraction, the current system is just as likely to replace the cell with a lower volume fraction as it is a higher one. This is because the system does not permit local knowledge to be reused in the form of heuristics, instead being solely reliant on the MOGA's stochastic capabilities. This situation would be made worse if more volume fractions were considered, as the
corresponding increase in permutations would make the MOGA’s task significantly more difficult and eventually unachievable.

On the positive side, the AML topology optimisation system created does enable true structural topology optimisation to be performed, using various unit cell structures at differing volume fractions of a single material – a highly novel idea unseen in existing literature. It is also capable of generating 3D STL files of chosen solutions for direct fabrication via RM technologies. As such, this novel system is capable of exploiting much more of RM’s design freedom in comparison to other systems where only a single unit cell structure was considered, e.g. Wang’s octet-truss, PSO method [132], or where an optimised unit cell is repeatedly arranged, [171-175, 211-213]. Upon reflection, although the work completed is highly novel, being a significant advancement on existing works, the overall quality of many experimental results and the existence of numerous unresolved issues has been disappointing. Furthermore, the computation required for the advanced experiments is clearly unacceptable at present. However, despite these notable difficulties, the results to date have shown that there is sufficient merit in the system to warrant further research and development.
CHAPTER NINE

CONCLUSIONS & RECOMMENDATIONS

9.1 Introduction

This Chapter provides conclusions that have been drawn from the research in addition to recommendations for areas of further work based upon the research issues reported in this thesis.

9.2 Conclusions

The conclusions that have been drawn from this research are as follows:

1. The DesignLab investigation showed that a more uniform stress distribution, as is desirable for an optimal structure, was achieved when two dissimilar materials were considered together, compared to just a single material which exhibited areas of localised high and low stresses. These results corroborate the findings of similar studies found in literature, where single and multiple materials have been considered together.

2. The FEA results of the mechanical response experiments suggested that cell structures that were good at withstanding tension, compression or bending appeared to be the worst in torsion and vice versa. In addition, the highest stresses of all of the tests appeared in the bending load case, (two orders of magnitude higher), suggesting that this was the most challenging loading condition considered when trying to achieve a uniform stress distribution.
3. The comparative study showed that, on average, the DesignLab software considered roughly twice as many iterations as the preliminary experiments. Despite this, both sets of results obtained were found to be of a similar quality. Additionally, experiments 5 and 6 demonstrated the system was working as intended, as the solutions obtained were close to the known ideals.

4. Early advanced experimental results indicated that when only a single objective was considered, the system was susceptible to becoming trapped by local optima. This risk was later mitigated by increasing the probability of mutation and by simultaneously considering multiple objectives.

5. Advanced experimental results indicated that undesirable localised SCs were present within the solution topologies, which were introduced by the cellular structures themselves. The occurrence of these cell induced SCs tended to steer the overall optimisation process, best evidenced by the results from experiment 12. The preferential selection of cells with lower SCs by the MOGA tended to result in low stress-range solutions being typically heavier than expected, which in turn also resulted in lower displacement values. As a consequence of this behaviour, the displacement and stress-range objectives began to share a more common goal by conflicting less with one another.

6. Advanced experimental results showed that considering multiple volume fractions of a single cellular structure did not result in lowering the Von Mises stress-range value. This unexpected and undesirable behaviour was thought to be as a direct consequence of lower volume fraction cell structures introducing higher localised SCs throughout the solution topology. However, the inclusion of additional volume fractions, despite the increased permutations, did result in improved objective trade-offs, evidenced by the improved Pareto Fronts generated by experiments 14 and 16 and by the significant reductions in mass that accompanied the increases in stress-range.

7. The advanced experiments which considered mixing different cell types together within the same problem indicated that the overall outcome was entirely dependent upon the particular cell structures used in each case. In some instances this was to detrimental effect, as in experiment 17, and in others to beneficial
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effect, as in experiment 18. However, one unanimous benefit of considering mixed cell structures together was the generation of a smaller Pareto Set size, clearly advantageous if a user is to manually choose a final solution.

8. A fundamental behavioural difference was observed when comparing the FE and RM models of sample beam topologies subjected to bending. This discrepancy was noticeable where diagonally positioned cells occurred, as these were assumed to be permanently connected via shared nodes in the FE model, yet were disconnected and able to slide over one another in the RM model. This suggested that the FE models were not realistic representations of the RM models. Furthermore, other differences are known to exist between the FE and RM models that would cause numerical disparity of results.

9. The behavioural discrepancy between the FE and RM models of the created system was also shown to exist within the DesignLab software, where diagonally positioned cells were again assumed to be connected through shared nodes in the FE model. This inconsistency is also anticipated to be found within other topology optimisation systems, as it is common for their FE models to be based on a grid like representation, which often results in checkerboarding.

10. The generation of experimental results during this research has been massively computationally demanding. Typical durations ranged from approximately 19 hours for preliminary experiments and up to 56 days for advanced experiments. Clearly, these experimental durations are unfavourable and would dramatically limit the uptake and application of such a system, irrespective of any improvements in design. Therefore, this system still requires further development in order to achieve results in a shorter time. Simplification of the existing cell structures should enable reductions in computational demands.

11. For each experiment, the number of iterations considered was compared to the total number of possible permutations. This highlighted that only a tiny fraction of the entire design space was actually searched by the MOGA during 10000 iterations, due to the high number of permutations in each case which resulted in a vast design search space. In light of these observations, it is now clear just how
difficult a job the MOGA had and yet how well it performed, this was particularly evident in the results from experiments 5 and 6.

9.2.1 Contribution to Academic & Industrial Community

During the course of this research, a novel GA based topology optimisation system, that uses various unit cell structures that vary in form and volume fraction, which is capable of exploiting the increased design freedom afforded by RM technologies, has been designed, created, tested and evaluated. At present, this research has generated a conference paper for the 17th Solid Freeform Fabrication Symposium in Austin, Texas, [247], which was widely well received, in addition to a journal paper which is currently under submission.

In its current form, the AML topology optimisation system created is of more interest within the academic community rather than within industry. This is partly due to the technique being highly computationally demanding, resulting in the slow generation of results, but is also due to the fact that the results that are being generated are not of sufficient quality to warrant interest from industry at this stage. The current system therefore, requires further academic development before industry may benefit, with possible areas recommended for development discussed in Section 9.3. However, when developed further, it is anticipated that potential industrial application areas will be found in the aerospace and automotive sectors, with possible applications also extending to the medical sector.

Applications within the aerospace and automotive sectors are expected to include lightweight structures, where the efficient use of material is of high importance, such as aircraft wings, as small weight reductions will result in significant fuel savings over the lifetime of the vehicle. Equally, at a time where the European Union recycling targets of an automotive vehicle are increasing, the efficient use of a single material through intelligent design will help to meet the costs of recycling. Possible application areas within the medical sector are likely to be in the area of bone scaffolds on medical implants, where irregular porous topologies can benefit bone growth.
9.3 Recommendations for Further Work

As a result of the research conducted, the author recognises and recommends the following areas for further work, which can be subdivided into two separate areas. These areas span practical improvements to the current system, in addition to expanding the research scope and will be discussed in turn in the following sections.

9.3.1 Improving the Current System

The most obvious improvements that are required to the current AML topology optimisation system fall into four main areas. Firstly, the cell structures that are used must be modified so as to reduce the SCs that currently occur, thus mitigating the steering effects on the overall optimisation process. Possible methods of achieving this outcome have previously been presented including, replacing the sharp internal edges with curved fillets and the use of transition cells to allow the structures of neighbouring cells to somehow merge together.

Secondly, the known system stability issues that currently exist, as detailed in Section 8.3.2, need to be fixed. These issues include a TET meshing error, a memory leak and two unresolved error messages. Thirdly, the performance of the current system can be improved in practical terms, by reducing the necessary computation and/or by reconfiguring the system to allow the required computation to be distributed across a parallel arrangement, as discussed earlier in Section 8.3.3. Possible methods of reducing the necessary computation include, simplifying the cell structures to make them less curved and therefore easier to mesh, and introducing additional rules that prevent repeated or invalid beam geometries from being considered in the computationally exhaustive FE stages. The system could also be modified to recognise symmetry within problem definitions, thereby reducing computation by enabling mirrored half or quarter models to be utilised in the analysis.

Finally, the behavioural discrepancy between the current FE and RM models must be addressed, otherwise potential users will have little confidence in the approach. One possible method of resolving this issue would be to include some form of sub-frame structure into the design domain, detailed in Section 8.2. However, in order to increase
the validity of the FE calculations, several changes would need to be implemented to allow the FEA to be more realistic of the actual RP and RM processes. This would include the addition of actual RP/RM material properties being entered into the FE materials data file (Appendix E), in addition to considering anisotropic behaviour (to account for process build direction) and non-linear analysis where appropriate (e.g. Duraform).

Further improvements to the current system could be gained by altering the GA that the system uses. Different MOGAs should be investigated as to their suitability and efficiency for possible replacement of TSMOGA. Alternatively, TSMOGA could be customised further with the development of the AML ga.class (see Appendix C), which was previously added unsuccessfully in order to introduce termination criteria into the current system. However, for the termination criteria to function correctly, the three system objectives should be reconfigured into a single weighted sum of objectives. The system objectives themselves could be modified by the introduction of target goals which would limit the objectives in the form of constraints.

The system variables could also be modified to improve the performance of the current system. The different cell structures are permitted within the same design domain by using discrete variables and this should be maintained. However, it may be possible to control the volume fractions of the cell structures by using continuous variables rather than discrete. This should enable the system to employ heuristics making the overall search more directional and should also enable the possibility of, say a 29.4% volume fraction cell, if so desired by the algorithm to best satisfy the objective function(s). This approach may be possible by creating some form of hybrid stochastic and gradient based system. It should be noted however, that such a modification is anticipated to cause further TET meshing difficulties.

In order to assess the successfulness of these proposed modifications to the current system, it may prove beneficial to conduct a design of experiments study that will establish critical experimental parameters for modification and also identify which experiments are necessary.
9.3.2 Increasing the Research Scope

The research scope could initially be increased by using the AML topology optimisation system to conduct increasingly advanced experiments using a variety of unit cell structures. Suggestions include hollow and I-section beam problems with both simple and offset loadings, Michell-type bridge problems and even extending the study to multiple load cases.

The research scope could be increased further by modifying the system to allow seeded regions within the design domain to be defined, therefore favouring one cell type in preference to another when mixing different cell types together. This approach could prove beneficial in controlling the distribution of cells and their behavioural responses across a structure, or merely as a method of steering the optimisation process away from undesirable solutions.

Another possible way of expanding the research scope would be to investigate different unit cell shapes, other than cubic. For example, some form of 3D hexagons or octagons could be used, as long as they tessellate without voids (e.g. truncated octahedrons). Considering non-cubic unit cells allows the possibility of always having face-to-face contact between neighbouring cell positions within the design domain and not just edge or corner connections as is currently possible. This modification may assist in the transferral of applied loads through a structure and should also enable a closer comparison to be made between the FE and RM models, without the need for any form of sub-frame.

Assuming that the occurrence of SCs within the cell structures is resolved to some extent, Birefringent (double refraction) stress analysis of clear SLA structures could be used as a possible method of validating that a uniform stress distribution exists, or alternatively of identifying the locations of remaining SCs. If this analysis suggested that SCs were still present, the implementation of a secondary smoothing operation (shape optimisation) could be added to the system in order to reduce remaining SCs within a given topology. This modification would impact on the computational demands of the system but would also expand the existing scope of research.
Finally, a significant increase in research scope could be achieved by the integration of CFD analysis, thereby enabling the system to perform multi-disciplinary topology optimisation. Although this change is anticipated to exacerbate the high computational demands of the system, by having two highly computational stages to the fitness evaluation of each individual, it should also enable the method to have a wider appeal with potentially increased application areas. Ideally, to limit the rise in computation, such a system would benefit from the possibility of only meshing once, with the Patran mesh created for FE also being used for CFD analysis.
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APPENDIX A – Biomimetics Examples

Velcro™ – (hook and loop fastener)

The inspiration for the famous hook and loop fastener came to the attention of Swiss inventor George de Mestral whilst out walking his dog in 1948. When de Mestral and his dog returned from their walk he noticed that they were both covered in cockleburs (small plant seed pods). The cockleburs proved difficult to remove, particularly from the dog’s fur. De Mestral was curious as to how the cockleburs were attaching themselves so strongly and examined some under a microscope. He discovered that on each spike were a number of tiny hooks as shown in Figure A.1a.

![Figure A.1 - A close up of a Cocklebur (a) [A1], and the magnified structure of Velcro (b) [A2].](image)

De Mestral realised the importance of this application, and thought it would be useful in attaching two materials together. In 1955 De Mestral patented his hook and loop fastener, Velcro, [A3]. The name came from the French words velour for velvet and crochet for hook. Velcro consists of two pieces of material, one with hundreds of tiny plastic hooks (like the cockleburs) and the other made of felt like fibres (like dog fur) that catch the hooks, (see Figure A.1b). Nature had designed the cockleburs like this in order to allow passing animals to unwittingly distribute the seed pods along their journeys.

Gecko Tape

The Gecko lizard is one of nature’s best climbers. It is capable of running up polished vertical surfaces at speeds of over a metre per second and even capable of hanging upside down on a sheet of glass holding on by just a single toe. For many years scientists have wondered how this animal is capable of such feats. Many thought that it...
secreted some sort of adhesive glue or used suction cups but there are no residual traces left behind on the surface. It is also difficult to believe these lines of thought after seeing a Gecko run across sand before climbing a shear glass wall. It would appear that somehow the Gecko is able to keep it’s feet clean from surface debris, something impossible to achieve with any type of adhesive.

In 2000, a group of scientists in California finally solved the mystery of how Gecko’s are able to perform this amazing behaviour, [A4]. On the soles of their feet are billions of tiny hairs called setae as shown in Figure A.2, these hairs are so small (typically between 0.2 to 0.5 microns in diameter) that they are able to interact with the surface on which they are placed at a molecular level. The Van der Walls intermolecular forces are individually weak, but when billions of these are combined they are able to produce an attractive force that is far superior to any manmade adhesive.

A group of UK scientists led by Professor Andre Geim at Manchester University have recently mimicked the nanoscopic hairs and fabricated them onto a flexible substrate [A6]. This material, made of millions of artificial hairs, has been called Gecko Tape and is a manmade copy of the soles of a Gecko’s feet, inspired by nature. Each synthetic hair is made from a material called Kapton and measures 2.0 microns in height and 0.2 microns in diameter, see Figure A.3a.
Although Gecko Tape has many possible future applications, the current production methods mean that only small quantities are being made at a large expense. In addition to this the synthetic hairs are tending to bunch together after a couple of applications as shown in Figure A.3b, resulting in the tape only having a similar durability to existing adhesive tapes. Geim believes that these problems will soon be overcome and that the tape will be commercially available within the next ten years.

**The Crystal Palace**

In 1826 a botanist called Joseph Paxton was employed as a gardener on the Chatsworth Estate by the Duke of Devonshire. Paxton grew many exotic plants including a large water lily named Victoria Amazonica. The large flat leaves often grow to over a meter in diameter and are strong enough to support the weight of a man as shown in Figure A.4a. Paxton examined the underneath of the leaves and noticed they got their strength from their structure of stiff radial ribs and slender, flexible, cross-linked ribs as seen in Figure A.4b, which when combined produce a rigid support structure for a large area, [A7].

---

**Figure A.3** - Scanning electron micrographs of the Gecko Tape surface showing the ordered synthetic setae (a) [A6], and bunching of the artificial hairs (b) [A6].

---

**Figure A.4** - Victoria Amazonica lilies demonstrating their ability to support the weight of a man (a) [A8] and the underneath radial and cross-linked ribbed structure (b) [A8].
This structure inspired Paxton to successfully design and build a 300ft conservatory at Chatsworth House. The glass and iron roof was light but also stiff enough to span large gaps. Later Paxton entered a larger version of his greenhouse design into the competition to design the exhibition hall for the world’s first Great Exhibition of 1851. The structure was to be built at Hyde Park in London and was to reflect the splendour of the exhibition and the greatness of Victorian Britain. Despite Paxton having no architectural or engineering background his design was accepted over 245 other designs, [A7]. Paxton’s controversial lack of architectural knowledge caused prominent engineers and scientists to raise doubts about the structure’s safety. It was feared that resonance caused by large crowd movements within the Palace’s 18 acres would cause the structure to vibrate and possibly collapse, as had been the case with some contemporary bridge designs. A scale model was built in order to prove the safety and stability of the design after which the construction of the Crystal Palace was approved.

The finished Crystal Palace was a truly revolutionary structure of its time, a sketch of which can be seen in Figure A.5. It had over 293,000 panes of glass, each measuring 12” by 49”, which required 4500 tons of ironwork to support it. The Palace was 1848 feet long, 408 feet wide and 108 feet tall, providing approximately 18 acres of exhibition space, despite its enormous size it only took eight months to build by 2000 men, [A7].

![Figure A.5 – A drawing of the finished Crystal Palace at Hyde park, [A9].](image)

At the end of the Great Exhibition, Paxton moved the Crystal Palace to Sydenham, where it stood for over 80 years housing a variety of events until a fire caused it to collapse in 1936.
The Eiffel Tower

During the early 1850's a German Professor of anatomy called Hermann Von Meyer was studying the human thigh bone (femur) at the point where it inserts into the hip joint. He was interested in the way the femur head extended sideways into the hip socket allowing the body’s weight to be supported off-centre by the femur. Von Meyer began to inspect the bony internal structure of the femur and noticed that at its head was an ordered lattice arrangement of tiny ridges of bone, called trabeculae, (see Figure A.6a). Although Von Meyer described the latticework arrangement he was unable to explain why it was formed in the manner it was, [A10].

In 1866 the Swiss engineer, Karl Cullman, visited Von Meyer in his laboratory. Von Meyer showed Cullman a bone specimen and the engineer immediately noticed that the internal orientation of the trabeculae were along the lines of compressive and tensile stress within the structure, [A10] as shown in his diagram, Figure A.6b. Cullman showed that the trabeculae were essentially acting as a series of studs and braces and proved that the structure in the femur head was probably the most efficient arrangement for supporting off-centre loads.

The French bridge builder and structural engineer, Gustave Eiffel, used the basic principle of building along the lines of force in his design and construction of the Eiffel Tower in 1889. Eiffel used a stud and braces latticework arrangement to support the off-centre weight of his 312m high curved iron structure, as shown in Figure A.7.
A genetic algorithm based topology optimisation approach for exploiting rapid manufacturing's design freedom is capable of withstanding the tensile and compressive forces induced by the wind. The same principle has also been used to design other skyscrapers including the former World Trade Centre in New York.

Figure A.7 – The curved tower structure designed by Gustave Eiffel, [A11].

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APPENDIX B – DesignLab Results

Trial A

Figure B1 – Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial A.

Figure B2 – Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial B

Figure B3 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial B.

Figure B4 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutations rate against Generations plot (b).

Appendix B - DesignLab Results - B2 - Darren Watts - September 2008
Appendix B – DesignLab Results

Trials

Figure B5 – Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial C.

Figure B6 – Weight & Factor of Safety against Generations plots (a) and Cost & Mutation rate against Generations plot (b).
Trial D

Figure B7 – Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial D.

Figure B8 – Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial E

Figure B9 - Stress distribution plot (a) and Material distribution plot (b) for camilever beam trial E.

Figure B10 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).

Appendix B - DesignLab Results

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Trial F

Figure B1.1 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial F.

Figure B1.2 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial G

Figure B13 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial G.

Figure B14 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial H

Figure B1.5 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial H.

Figure B1.6 - Weight & Factor of Safety against Generations plot (a) and Coat & Mutation rate against Generations plot (b).
Trial I

Figure B17 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial 1.

Figure B18 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial J

Figure B10 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam Trial J.

Figure B20 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
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Trial K

Figure B21 – Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial K.

Figure B22 – Weight & Factor of Safety against generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial L

Figure B23 – Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial L.

Figure B24 – Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations Plot (b).
Trial M

Figure B25 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial M.

Figure B26 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
Trial N

Figure B27 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial N.

Figure B28 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
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APPENDIX C - AML Source Code (V-22)

Trial O

Figure B20 - Stress distribution plot (a) and Material distribution plot (b) for cantilever beam trial O.

Figure B20 - Weight & Factor of Safety against Generations plot (a) and Cost & Mutation rate against Generations plot (b).
APPENDIX C – AML Source Code (V-23)

cell-classes.aml

;: This code sample is provided as an example of how to perform a particular
;: kind of task using foundation software provided by TechnoSoft. It is
;: provided "as is" without warranty of any kind, expressed or implied.
;: Licensed users of the Adaptive Modeling Language are free to use and
;: modify this code as long as the the original credits and disclaimers are
;: maintained.

Author: Darren Watts & Jeremy Johnson
Created: Wed Oct 05 18:31:28 2005
Purpose: Advanced Training
(in-package :aml)

(define-class topology-cell-class
  :inherit-from (object)
  :properties
  (side-length (default 1.0))
  (volume-fraction (default 0.5))
  (origin (default '(0 0 0))
  (render 'shaded)
  (material-name (default 'steel))
  (id-tag (new-tag-id 3))
)

(define-method get-cell-volume topology-cell-class ()
  (* :volume-fraction
      (expt :side-length 3))
)

(define-method void-cell? topology-cell-class ()
  nil)

(define-method get-valid-neighbour-cell-types topology-cell-class ()
  nil)

(define-class solid-class
  :inherit-from (topology-cell-class box-object)
  :properties
  (color "grey")
  (height "side-length"
    width "side-length"
    depth "side-length"
    (ref-coord-sys :class 'coordinate-system-class
                   :origin "origin"
    )
    reference-coordinate-system 'ref-coord-sys
    orientation (list (translate (list (half "side-length")
                      (half "side-length"))

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volume-fraction 1.0
material-name (default 'steel)

(define-method get-valid-neighbour-cell-types solid-class ()
  ;; whole list
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class solid-2-class ;; changed from topology-cell-solid-2-class
  ;; inherit-from (topology-cell-class box-object) :properties
  (color "brown"
  height "side-length"
  width "side-length"
  depth "side-length"
  (ref-coord-sys :class 'coordinate-system-class
    origin "origin"
  )
  reference-coordinate-system 'ref-coord-sys
  orientation (list (translate (list (half "side-length"
    (half "side-length"
    (half "side-length")))
  )
  volume-fraction 1.0
  material-name (or (the superior material-name-2 (:error nil))
    (the superior material-name))
  )
)

(define-method get-valid-neighbour-cell-types solid-2-class ()
  ;; whole list
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class void-class ;; changed from topology-cell-void-class
  ;; inherit-from (topology-cell-class) :properties
  (volume-fraction 0.0
  geom nil
  material-name 'void
  )
)

(define-method tsi::get-geom void-class () ;; changed from topology-cell-void-class
  nil
)

(define-method void-cell? void-class () ;; changed from topology-cell-void-class
  t
)

(define-method get-valid-neighbour-cell-types void-class ()
  ;; whole list
  '(
  )
)
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

(define-class topology-cell-box-with-holes-class
  :inherit-from (topology-cell-class difference-object)
  :properties (
    ;; input properties
    cylinder-diameter (default 0.56721999999969415) ;; volume-fraction = 0.5
    color "green"
    ;; construction objects
    scaled-diameter (* "cylinder-diameter "side-length)
    (box :class 'box-object
      height "side-length
      width "side-length
      depth "side-length
    )
    (cyl-x :class 'cylinder-object
      height (* 1.5 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate 90 :y-axis))
    )
    (cyl-y :class 'cylinder-object
      height (* 1.5 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate 90 :x-axis))
    )
    (cyl-z :class 'cylinder-object
      height (* 1.5 "side-length"
      diameter "scaled-diameter"
    )
    (ref-coord-sys :class 'coordinate-system-class
      origin "Origin"
    )
    ;; difference-object properties
    reference-coordinate-system "ref-coord-sys"
    orientation (list (translate (list (half "side-length"
                                          (half "side-length"
                                          (half "side-length")))))
    object-list (list "box" "cyl-x" "cyl-y" "cyl-z"
    )
  )
)

(define-class A50-class
  :inherit-from (topology-cell-box-with-holes-class)
  :properties (
    volume-fraction 0.5
    cylinder-diameter 0.56721999999969415
    material-name (default 'aluminium)
  )
)

(define-method get-valid-neighbour-cell-types A50-class ()
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)
)

(define-class A25-class
  :inherit-from (topology-cell-box-with-holes-class)
  :properties (
    volume-fraction 0.5
    cylinder-diameter 0.56721999999969415
    material-name (default 'aluminium)
  )
)

(define-method get-valid-neighbour-cell-types A25-class ()
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)
)
A Genetic Algorithm Based Topology Optimisation Approach Loughborough for Exploiting Rapid Manufacturing’s Design Freedom

:inherit-from (topology-cell-box-with-holes-class)
:properties {
  volume-fraction 0.25
  cylinder-diameter 0.768769999999687
  material-name (default 'aluminium)
}

(define-method get-valid-neighbour-cell-types A25-class ()
  ;; whole list except B25 B50 B75 D75
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (A25-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
}

(define-class A75-class
  ;; changed from topology-cell-box-with-holes-75-class
  :inherit-from (topology-cell-box-with-holes-class)
  :properties {
    volume-fraction 0.75
    cylinder-diameter  0.3692000000000692
    material-name (default 'aluminium)
}

(define-method get-valid-neighbour-cell-types A75-class ()
  ;; whole list except B25
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
}

(define-class A12.5-class
  ;; whole list except B25 B50 B75
  :inherit-from (topology-cell-box-with-holes-class)
  :properties {
    volume-fraction 0.125
    cylinder-diameter 0.8967999999999964
    material-name (default 'aluminium)
}

(define-method get-valid-neighbour-cell-types A12.5-class ()
  ;; whole list except B25 B50 B75
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
}

(define-class A37.5-class
  ;; whole list except B25 B50 B75
  :inherit-from (topology-cell-box-with-holes-class)
  :properties {
    volume-fraction 0.375
    cylinder-diameter 0.6641299999999949
    material-name (default 'aluminium)
}

(define-method get-valid-neighbour-cell-types A37.5-class ()
  ;; whole list except B25 B50
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

Appendix C – AML Source Code (V-23)

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(define-class A62.5-class
  :inherit-from (topology-cell-box-with-holes-class)
  :properties
  (volume-fraction 0.625
  cylinder-diameter 0.471050000000001
  material-name (default 'aluminium))
)

(define-method get-valid-neighbour-cell-types A62.5-class ()
  ;; whole list except B25
  (void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
  class)(A62.5-class)(A75-class)(A87.5-class)(B25-class)(B75-class)(C25-class)(C50-
  class)(C75-class)(D25-class)(D50-class)(D75-class)(E25-class)(E50-class)(E75-
  class)(F25-class)(F50-class)(F75-class)(G25-class)(G50-class)(G75-class)(H25-class)(H50-class)(H75-
  class)
)

(define-class A87.5-class
  :inherit-from (topology-cell-box-with-holes-class)
  :properties
  (volume-fraction 0.875
  cylinder-diameter 0.2498100000000018
  material-name (default 'aluminium))
)

(define-method get-valid-neighbour-cell-types A87.5-class ()
  ;; whole list
  (void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
  class)(A62.5-class)(A75-class)(A87.5-class)(B25-class)(B75-class)(C25-class)(C50-
  class)(C75-class)(D25-class)(D50-class)(D75-class)(E25-class)(E50-class)(E75-
  class)(F25-class)(F50-class)(F75-class)(G25-class)(G50-class)(G75-class)(H25-class)(H50-class)(H75-
  class)
)

(define-class topology-cell-inverted-box-with-holes-class
  :inherit-from (topology-cell-class union-object)
  :properties
  ;; input properties
  side-length (default 1.0)
  cylinder-diameter (default 0.56721999999969415) ;; volume-fraction = 0.5
  color "orange"

  ;; construction objects
  scaled-diameter (* 'cylinder-diameter 'side-length)
  (cyl-x :class 'cylinder-object
    height **side-length
    diameter **scaled-diameter
    orientation (list (rotate 90 :y-axis)))
  (cyl-y :class 'cylinder-object
    height **side-length
    diameter **scaled-diameter
    orientation (list (rotate 90 :x-axis)))
  (cyl-z :class 'cylinder-object
    height **side-length
    diameter **scaled-diameter
  )
  (ref-coord-sys :class 'coordinate-system-class
    origin **origin)

  ;; union-object properties
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reference-coordinate-system ref-coord-sys
orientation (list (translate (list (half "side-length"
(half "side-length"
(half "side-length")))
object-list (list "cyl-x" "cyl-y" "cyl-z"
)
)

(define-class B50-class ;; changed from topology-cell-inverted-box-with-holes-50-class
:inherit-from (topology-cell-inverted-box-with-holes-class)
:properties {
  volume-fraction 0.5
  cylinder-diameter 0.56721999999969415
  material-name (default 'aluminium)
}
)

(define-method get-valid-neighbour-cell-types B50-class ()
  ;; whole list except A12.5 A25 A37.5 A50 C25 C50 F25 F50
  (void-class) (solid-class) (solid-2-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class B75-class ;; changed from topology-cell-inverted-box-with-holes-75-class
:inherit-from (topology-cell-inverted-box-with-holes-class)
:properties {
  volume-fraction 0.75
  cylinder-diameter 0.768769999999687
  material-name (default 'aluminium)
}
)

(define-method get-valid-neighbour-cell-types B75-class ()
  ;; whole list except A12.5 A25 C25 F25 F50
  (void-class) (solid-class) (solid-2-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class B25-class ;; changed from topology-cell-inverted-box-with-holes-25-class
:inherit-from (topology-cell-inverted-box-with-holes-class)
:properties {
  volume-fraction 0.25
  cylinder-diameter 0.3692000000000692
  material-name (default 'aluminium)
}
)

(define-method get-valid-neighbour-cell-types B25-class ()
  ;; whole list except A12.5 A25 A37.5 A50 A62.5 A75 C25 C50
  (C75 F25 F50 G75 G25)
  (void-class) (solid-class) (solid-2-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class topology-cell-box-with-half-holes-class
:inherit-from (topology-cell-class difference-object)
:properties {

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;; input properties
cylinder-diameter (default 0.4010700000000011) ;; volume-fraction = 0.5
color "blue"

;; difference-object properties
reference-coordinate-system "ref-coord-sys"
orientation (list (translate (list (half "side-length"
(half "side-length")))
object-list (list "box" "cyl-x1" "cyl-x2" "cyl-x3" "cyl-x4" "cyl-y1" "cyl-y2"
"cyl-y3" "cyl-y4" "cyl-z1" "cyl-z2" "cyl-z3" "cyl-z4")

;; construction objects
scaled-diameter (* "cylinder-diameter" "side-length")

(box :class 'box-object
  height "side-length"
  width "side-length"
  depth "side-length"
)

(cyl-x1 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :y-axis)
    (translate (list 0.0 0.0 0.0 (- (* 0.5 "side-length")))
    ))

(cyl-x2 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :y-axis)
    (translate (list 0.0 0.0 0.0 (* 0.5 "side-length"))))

(cyl-x3 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :y-axis)
    (translate (list 0.0 0.0 0.0 (* 0.5 "side-length"))))

(cyl-x4 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :y-axis)
    (translate (list 0.0 0.0 0.0 (- (* 0.5 "side-length"))))
)

(cyl-y1 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :x-axis)
    (translate (list 0.0 0.0 0.0 (- (* 0.5 "side-length"))))
)

(cyl-y2 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :x-axis)
    (translate (list (* 0.5 "side-length"))
    ))

(cyl-y3 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :x-axis)
    (translate (list 0.0 0.0 0.0 (* 0.5 "side-length"))))

(cyl-y4 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"

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orientation (list (rotate 90 :x-axis)
  (translate (list (- (* 0.5 ^^side-length)) 0.0
  0.0)))
}
)
(cyl-z1 :class 'cylinder-object
  height ("^"side-length)
diameter "^"scaled-diameter
  orientation (list (translate (list 0.0 (* 0.5 "^"side-length) 0.0))
  0.0))
}
(cyl-z2 :class 'cylinder-object
  height ("^"side-length)
diameter "^"scaled-diameter
  orientation (list (translate (* 0.5 -^side-length) 0.0
  0.0))
)
(cyl-z3 :class 'cylinder-object
  height ("^"side-length)
diameter "^"scaled-diameter
  orientation (list (translate (- (* 0.5 "side-length)) 0.0
  0.0))
)
(cyl-z4 :class 'cylinder-object
  height ("^"side-length)
diameter "^"scaled-diameter
  orientation (list (translate (- (* 0.5 "side-length)) 0.0
  0.0))
)
(ref-coord-sys :class 'coordinate-system-class
  origin "^"origin
 )
)

(define-class C50-class ;; changed from topology-cell-box-with-half-holes-50-class
  :inherit-from (topology-cell-box-with-half-holes-class)
  :properties ({
    volume-fraction 0.5
    cylinder-diameter 0.4010700000000011
    material-name (default 'aluminium)
  })
)

(define-method get-valid-neighbour-cell-types C50-class ()
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class)
  (A62.5-class) (A75-class) (A97.5-class) (B75-class) (C25-class) (C50-class) (C75-class)
  (D75-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class)
  (G75-class) (H25-class) (H50-class) (H75-class)
 )
)

(define-class C25-class ;; changed from topology-cell-box-with-half-holes-25-class
  :inherit-from (topology-cell-box-with-half-holes-class)
  :properties ({
    volume-fraction 0.25
    cylinder-diameter 0.5435899999999609
    material-name (default 'aluminium)
  })
)

(define-method get-valid-neighbour-cell-types C25-class ()
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class)
  (A62.5-class) (A75-class) (A97.5-class) (B75-class) (C25-class) (C50-class) (C75-class)
  (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class)
  (G75-class) (H25-class) (H50-class) (H75-class)
 )
)

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(define-class C75-class ;; changed from topology-cell-box-with-half-holes-75-class
  :inherit-from (topology-cell-box-with-half-holes-class)
  :properties (volume-fraction 0.75
cylinder-diameter 0.261060000000001106
material-name (default 'aluminium)
  )
)

(define-method get-valid-neighbour-cell-types C75-class ()
' ;; whole list except B25 D25
(void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
class)(A62.5-class)(A75-class)(A87.5-class)(B50-class)(B75-class)(C25-class)(C50-
class)(C75-class)(D50-class)(D75-class)(E25-class)(E50-class)(E75-class)(F25-class)(F50-
class)(F75-class)(G25-class)(G50-class)(G75-class)(H25-class)(H50-class)(H75-class)
)

(define-class topology-cell-inverted-box-with-half-holes-class
  :inherit-from (topology-cell-class difference-object)
  :properties (;
; input properties
cylinder-diameter (default 0.4010700000000011) ;; volume-fraction = 0.5
color "purple"

; ; difference-object properties
reference-coordinate-system `ref-coord-sys
orientation (list (translate (list (half `side-length)
  (half `side-length)
  (half `side-length))))
object-list (list "box "cylinder-difference"

; ; construction objects
scaled-diameter (* `cylinder-diameter `side-length)

cylinder-difference :class 'difference-object
  object-list (list "box "cyl-x1 "cyl-x2 "cyl-x3 "cyl-x4 "cyl-y1 "cyl-y2 "cyl-y3 "cyl-y4 "cyl-z1 "cyl-z2 "cyl-z3 "cyl-z4")
  (box :class 'box-object
    height `side-length
    width `side-length
    depth `side-length
  )
  (cyl-x1 :class 'cylinder-object
    height (^^side-length)
    diameter ^^scaled-diameter
    orientation (list (rotate 90 :y-axis)
     (translate (list 0.0 0.0 (- (* 0.5 "side-length").
    ))
  )
  (cyl-x2 :class 'cylinder-object
    height (^^side-length)
    diameter ^^scaled-diameter
    orientation (list (rotate 90 :y-axis)
     (translate (list 0.0 (* 0.5 "side-length") 0.0)
    )
  )
  (cyl-x3 :class 'cylinder-object
    height (^^side-length)
    diameter ^^scaled-diameter
    orientation (list (rotate 90 :y-axis)
     (translate (list 0.0 0.0 (* 0.5 "side-length") 0.0)
    )
  )
  (cyl-x4 :class 'cylinder-object
    height (^^side-length)
    diameter ^^scaled-diameter
    orientation (list (rotate 90 :y-axis)
(translate (list 0.0 (- (* 0.5 "side-length")))
  0.0))
)
(cyl-y1 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :x-axis)
    (translate (list 0.0 0.0 (- (* 0.5 "side-length")))))
)
(cyl-y2 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (rotate 90 :x-axis)
    (translate (list (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-y3 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list 0.0 (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-y4 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list 0.0 (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-z1 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list 0.0 (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-z2 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-z3 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list 0.0 (- (* 0.5 "side-length")) 0.0 0.0))
)
(cyl-z4 :class 'cylinder-object
  height ("side-length")
  diameter "scaled-diameter"
  orientation (list (translate (list (- (* 0.5 "side-length")) 0.0 0.0))
)
(ref-coord-sys :class 'coordinate-system-class
  origin "origin"
)
)

(define-class D50-class ;; changed from topology-cell-inverted-box-with-half-
holes-50-class :inherit-from (topology-cell-inverted-box-with-half-holes-class)
 :properties (
  volume-fraction 0.5
  cylinder-diameter 0.4010700000000011
  material-name (default 'aluminium)
)
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(define-class D25-class  ;; changed from topology-cell-inverted-box-with-half-holes-25-class
  :inherit-from (topology-cell-inverted-box-with-half-holes-class)
  :properties (
    volume-fraction 0.25
    cylinder-diameter 0.26106000000001106
    material-name (default 'aluminium)
  )
)

(define-class D75-class  ;; changed from topology-cell-inverted-box-with-half-holes-75-class
  :inherit-from (topology-cell-inverted-box-with-half-holes-class)
  :properties (
    volume-fraction 0.75
    cylinder-diameter 0.5435899999999609
    material-name (default 'aluminium)
  )
)

(define-method get-valid-neighbour-cell-types D25-class ()
  '(void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-method get-valid-neighbour-cell-types D75-class ()
  '(void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)

(define-class topology-cell-box-with-diagonal-holes-class
  :inherit-from (topology-cell-class difference-object)
  :properties (;
    ;; input properties
    cylinder-diameter (default 0.3716000000000116) ;; volume-fraction = 0.5
    color "magenta"
    ;; construction objects
    scaled-diameter (* cylinder-diameter 'side-length)
    (box :class 'box-object
         height 'side-length
         width 'side-length
         depth 'side-length)
    (cyl-1 :class 'cylinder-object
           height 0.5
           color "magenta"
           depth 0.1)
))
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(define-class E75-class ;; changed from topology-cell-box-with-diagonal-holes-75-class
  :inherit-from (topology-cell-box-with-diagonal-holes-class)
  :properties (
    volume-fraction 0.75
    cylinder-diameter 0.2423999999999878
    material-name (default 'aluminium)
  )
)

(define-method get-valid-neighbour-cell-types E75-class ()
  ;; whole list except F25
  (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class)
  (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class)
  (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class)
  (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)
)
)

(Program)

(define-class topology-cell-inverted-box-with-diagonal-holes-class
  :inherit-from (topology-cell-class intersection-object)
  :properties (
    ;; input properties
    cylinder-diameter (default 0.3716000000000116) ;; volume-fraction = 0.5
    color "cyan"
    ;; construction objects
    scaled-diameter (* 'cylinder-diameter 'side-length)
    (box :class 'box-object
      height "side-length"
      width "side-length"
      depth "side-length"
    )
    (cyl-1 :class 'cylinder-object
      height (* 2.0 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate 125.26438968275465431537700033002 :x-axis)
                     (rotate 45 :z-axis))
    )
    (cyl-2 :class 'cylinder-object
      height (* 2.0 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate -125.26438968275465431537700033002 :x-axis)
                       (rotate 45 :z-axis))
    )
    (cyl-3 :class 'cylinder-object
      height (* 2.0 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate 125.26438968275465431537700033002 :y-axis)
                       (rotate 45 :z-axis))
    )
    (cyl-4 :class 'cylinder-object
      height (* 2.0 "side-length"
      diameter "scaled-diameter"
      orientation (list (rotate -125.26438968275465431537700033002 :y-axis)
                         (rotate 45 :z-axis))
    )
    (cylinder-union :class 'union-object
      object-list (list "cyl-1 "cyl-2 "cyl-3 "cyl-4)
    )
    (ref-coord-sys :class 'coordinate-system-class
      origin "origin"
    )
  )
)

;; intersection-object properties
reference-coordinate-system "ref-coord-sys"
orientation (list (translate (list (half *side-length)
(half *side-length)
(half *side-length))))

object-list (list 'box 'cylinder-union)

(define-class F50-class ;; changed from topology-cell-inverted-box-with-diagonal-
holes-50-class :inherit-from (topology-cell-inverted-box-with-diagonal-holes-class)
:properties (volume-fraction 0.5
cylinder-diameter 0.37160000000000116
material-name (default 'aluminium)
)

(define-method get-valid-neighbour-cell-types F50-class () ;; whole list except B25 B50 B75 D25 D50 E25 E50 G25
(void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
class)(A62.5-class)(A75-class)(A87.5-class)(C25-class)(C50-class)(C75-class)(D25-
class)(D50-class)(F25-class)(F50-class)(F75-class)(G25-class)(G50-class)(G75-class)(H25-
class)(H50-class)(H75-class)
)

(define-class F25-class ;; changed from topology-cell-inverted-box-with-diagonal-
holes-25-class :inherit-from (topology-cell-inverted-box-with-diagonal-holes-class)
:properties (volume-fraction 0.25
cylinder-diameter 0.24243000000010775
material-name (default 'aluminium)
)

(define-method get-valid-neighbour-cell-types F25-class () ;; whole list except B25 B50 B75 D25 D50 E25 E50 E75
(void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
class)(A62.5-class)(A75-class)(A87.5-class)(C25-class)(C50-class)(C75-class)(D25-
class)(F25-class)(F50-class)(F75-class)(G25-class)(G50-class)(G75-class)(H25-class)(H50-class)(H75-class)
)

(define-class F75-class ;; changed from topology-cell-inverted-box-with-diagonal-
holes-75-class :inherit-from (topology-cell-inverted-box-with-diagonal-holes-class)
:properties (volume-fraction 0.75
cylinder-diameter 0.5012600000000442
material-name (default 'aluminium)
)

(define-method get-valid-neighbour-cell-types F75-class () ;; whole list except B25 D25 E25
(void-class)(solid-class)(solid-2-class)(A12.5-class)(A25-class)(A37.5-class)(A50-
class)(A62.5-class)(A75-class)(A87.5-class)(B50-class)(B75-class)(C25-class)(C50-
class)(C75-class)(D50-class)(D75-class)(E25-class)(E50-class)(E75-class)(F25-class)(F50-class)(F75-
class)(G25-class)(G50-class)(G75-class)(H25-class)(H50-class)(H75-class)
)

(define-class topology-cell-box-with-seven-holes-class :inherit-from (topology-cell-class difference-object)
:properties (;; input properties
cylinder-diameter (default 0.3135200000000085) ;; volume-fraction = 0.5
color "yellow"
)
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;; construction objects
scaled-diameter (* "cylinder-diameter "side-length)

(box :class 'box-object
  height "side-length
  width "side-length
  depth "side-length)

(cyl-1 :class 'cylinder-object
  height (* 2.0 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate 125.2643896827546543153777000033002 :x-axis) (rotate 45 :z-axis))

(cyl-2 :class 'cylinder-object
  height (* 2.0 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate -125.2643896827546543153777000033002 :x-axis) (rotate 45 :z-axis))

(cyl-3 :class 'cylinder-object
  height (* 2.0 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate 125.2643896827546543153777000033002 :y-axis) (rotate 45 :z-axis))

(cyl-4 :class 'cylinder-object
  height (* 2.0 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate -125.2643896827546543153777000033002 :y-axis) (rotate 45 :z-axis))

(cyl-x :class 'cylinder-object
  height (* 1.5 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate 90 :y-axis))

(cyl-y :class 'cylinder-object
  height (* 1.5 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate 90 :x-axis))

(cyl-z :class 'cylinder-object
  height (* 1.5 "side-length)
  diameter "scaled-diameter
  orientation (list (rotate 90 :z-axis))

(ref-coord-sys :class 'coordinate-system-class
  origin "origin)

;; difference-object properties
reference-coordinate-system 'ref-coord-sys
orientation (list (translate (list (half "side-length)
  (half "side-length)
  (half "side-length)))

object-list (list "box "cyl-1 "cyl-2 "cyl-3 "cyl-4 "cyl-x "cyl-y "cyl-z)


(define-class G50-class
  ;; changed from topology-cell-box-with-seven-holes-50-class
  :inherit-from (topology-cell-box-with-seven-holes-class)
  :properties {
    volume-fraction 0.5
    cylinder-diameter 0.31352
    material-name (default 'aluminium)
  }

(define-method get-valid-neighbour-cell-types G50-class ()
  ;; whole list except F25 H25 H50
  '(void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C45-class) (C62.5-class) (C75-class) (C87.5-class) (C100-class) (D25-class) (D50-class) (D75-class) (D100-class) (E25-class) (E50-class) (E75-class) (E100-class) (F25-class) (F50-class) (F75-class) (F100-class) (H25-class) (H50-class) (H75-class) (H100-class) (V25-class) (V50-class) (V75-class) (V100-class))

Appendix C - AML Source Code (V-23) - C15 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

Loughborough University

The code snippet defines various classes for topology optimization in rapid manufacturing, inheriting from different building blocks and setting properties such as volume fraction, cylinder diameter, and material name. For example:

```scheme
(define-class G25-class
  ;; changed from topology-cell-box-with-seven-holes-25-class
  :inherit-from (topology-cell-box-with-seven-holes-class)
  :properties
  (volume-fraction 0.25)
  (cylinder-diameter 0.42778000000000077)
  (material-name (default 'aluminium))
)
```

Each class specifies properties like volume fraction and cylinder diameter, and some methods are defined to get valid neighbour cell types.

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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

```lisp
(cyl-3 :class 'cylinder-object
  height (* 2.0 "side-length"
  diameter "scaled-diameter"
orientation (list (rotate 125.2643896827546543153773770033002 :y-axis))
)

(cyl-4 :class 'cylinder-object
  height (* 2.0 "side-length"
  diameter "scaled-diameter"
orientation (list (rotate -125.2643896827546543153773770033002 :y-axis))
)

(cyl-x :class 'cylinder-object
  height (* 1.5 "side-length"
  diameter "scaled-diameter"
orientation (list (rotate 90 :y-axis))
)

(cyl-y :class 'cylinder-object
  height (* 1.5 "side-length"
  diameter "scaled-diameter"
orientation (list (rotate 90 :x-axis))
)

(cyl-z :class 'cylinder-object
  height (* 1.5 "side-length"
  diameter "scaled-diameter"
)

(cylinder-union :class 'union-object
object-list (list 'cyl-1 'cyl-2 'cyl-3 'cyl-4 'cyl-x 'cyl-y 'cyl-z)
)

(ref-coord-sys :class 'coordinate-system-class
  origin "origin"
)

;; intersection-object properties
reference-coordinate-system "ref-coord-sys
orientation (list (translate (list (half "side-length"
  (half "side-length")
  (half "side-length")))
object-list (list "box "cylinder-union")
)
)

(define-class H50-class ;; changed from topology-cell-inverted-box-with-seven-
holes-50-class :inherit-from (topology-cell-inverted-box-with-seven-holes-class)
:properties (volume-fraction 0.5
cylinder-diameter 0.3135200000000135
material-name (default 'aluminium)
)
)

(define-method get-valid-neighbour-cell-types H50-class ()
  '(void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-
class) (A62.5-class) (A75-class) (B75-class) (B50-class) (B25-class) (B25-class) (C25-
class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-
class) (F25-class) (F50-class) (F75-class) (G75-class) (H25-class) (H50-class) (H75-class)
)
)

(define-class H25-class ;; changed from topology-cell-inverted-box-with-seven-
holes-25-class :inherit-from (topology-cell-inverted-box-with-seven-holes-class)
:properties (volume-fraction 0.25
cylinder-diameter 0.20348000000002348
material-name (default 'aluminium)
)
)

(define-method get-valid-neighbour-cell-types H25-class ()
```

A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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 Appendix C - AML Source Code (V-23) - C18 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

class) (C50-class) (C75-class) (D50-class) (D75-class) (E50-class) (E75-class) (F50-class) (F75-class) (G50-class) (G75-class) (H50-class) (H75-class)

('A62.5-class' ';; whole list except B25 (void-class) (solid-class) (solid-2-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('A75-class' ';; whole list except B25 (void-class) (solid-class) (solid-2-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('A87.5-class' ';; whole list (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('B25-class' ';; whole list except A12.5 A25 A37.5 A50 A62.5 A75 C25 C50 C75 F25 F50 (void-class) (solid-class) (solid-2-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('B50-class' ';; whole list except A12.5 A25 A37.5 A50 A25 C50 F25 F50 (void-class) (solid-class) (solid-2-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('B75-class' ';; whole list except A12.5 A25 C25 F25 F50 (void-class) (solid-class) (solid-2-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('C25-class' ';; whole list except B25 B50 B75 D25 D50 D75 E25 G25 (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('C50-class' ';; whole list except B25 B50 D25 D50 B25 (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('C75-class' ';; whole list except B25 D25 (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('D25-class' ';; whole list except C25 C50 C75 F25 F50 F75 (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('D50-class' ';; whole list except A50 C25 C50 F25 F50 (void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-class) (A37.5-class) (A50-class) (A75-class) (A87.5-class) (B25-class) (B50-class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-class) (E50-class) (E75-class) (G25-class) (G50-class) (G75-class) (H25-class) (H50-class) (H75-class)

('D75-class' ';; whole list except A25 C25 F25

A Genetic Algorithm Based Topology Optimisation Approach ILM Loughborough for Exploiting Rapid Manufacturing's Design Freedom

Appendix C - AML Source Code (V-23) - C20 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

(A75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-
class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G75-class) (H25-class) (H50-
class) (H75-class)

('H75-class' (whole list except G25)
(\void-class) (solid-class) (solid-2-class) (A12.5-class) (A25-
class) (A37.5-class) (A50-class) (A62.5-class) (A75-class) (A87.5-class) (B25-class) (B50-
class) (B75-class) (C25-class) (C50-class) (C75-class) (D25-class) (D50-class) (D75-class) (E25-
class) (E50-class) (E75-class) (F25-class) (F50-class) (F75-class) (G50-class) (G75-class) (H25-
class) (H50-class) (H75-class)

(t nil)
)
)}
cell-mesh-classes.aml

;;; ---

**(define-class solution-structure-mesh-class)
:inherit-from (object)
:properties (
  num-cells-x (default 10)
  num-cells-y (default 5)
  num-cells-z (default 1)
  cell-size (default nil)
  point-coords-list (default nil)
  cell-objects-list (default nil)
  cell-objects-grouped-list (default nil)
  constrained-node-id-list (default nil)
  loaded-node-id-list (default nil)
  load-vector (default '(0.0 -1000.0 0.0))
  constrained-nodes-interface-object (default nil)
  loaded-nodes-interface-object (default nil)

(meshdb :class 'meshdb-class)

(mesh-database-object ^meshdb
  (populate-meshdb :class 'smash-event
  before-smash (smash-value (the db-id (:from ^mesh-
  database-object)))

  (constrained-nodes-default :class 'mesh-query-nodes-from-interface-class
    interface-object (when ""populate-meshdb
    **constrained-nodes-interface-object)
    color "cyan"
    line-width 10
  )

  (loaded-nodes-default :class 'mesh-query-nodes-from-interface-class
    interface-object (when ""populate-meshdb ""loaded-
  nodes-interface-object)
    color "red"
    line-width 10
  )

:subobjects

(nodes-query :class 'mesh-nodes-query-class
  mesh-entities-list (when ""populate-meshdb
    (query-node-ids ""mesh-database-object))

Appendix C - AML Source Code (V-23) - C22 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach 
for Exploiting Rapid Manufacturing's Design Freedom

(all-elements-query :class 'mesh-elements-query-class 
  mesh-entities-list (when "populate-meshdb 
  database-object)) 
  (length (query-element-ids 'mesh-
  database-object)) 

(cell-queries :class 'series-object 
  class-expression 'mesh-elements-3d-query-class 
  quantity (length (rest 'cell-objects-grouped-list)) 
  init-form 'tagged-object-list (when "populate-meshdb 
  (nth 'index (rest "cell-
  objects-grouped-list))) 
  color (the color (:from (first "tagged-object-
  list) "white")) 
)

(constrained-nodes :class 'mesh-nodes-query-class 
  mesh-entities-list (when "populate-meshdb 
  constrained-node-id-list) 
  line-width 10 
  color "cyan" 
)

(loaded-nodes :class 'mesh-nodes-query-class 
  mesh-entities-list (when "populate-meshdb 
  loaded-node-id-list) 
  line-width 10 
  color "red" 
)

(load-vectors :class 'series-object 
  class-expression 'vector-class 
  quantity (length "loaded-node-id-list) 
  init-form 'base-point (nth 'index "cell-
  id-list) point-coords-list) 
  direction "load-vector 
  length (* 1.5 "cell-size) 
  color (the superior superior superior loaded-nodes 
  color 
  line-width 2 
)

(defun INDEX3D (x y z num-x num-y) 
  (+ (* z num-x num-y) (* y num-x) x))

(define-method populate-meshdb-for-structure solution-structure-mesh-class () 
  (let* ((db !meshdb) 
    (num-x !num-cells-x) 
    (num-y !num-cells-y) 
    (num-z !num-cells-z) 
    (num-x-points (1+ num-x)) 
    (num-y-points (1+ num-y)) 
    (points-list !point-coords-list) 
    (node-tag (new-tag-id 3)) 
    (adjusted-node-tag (get-adjusted-id-tag node-tag)) 
    (cells !cell-objects-list) 
    
  ) 
  (store-nodes-for-tagid db adjusted-node-tag :solid points-list) 
  (loop 
    with all-nodes = (query-node-ids db) 
    with tag-type = :solid 
    with element-type = :hexa 
    for z from 0 to (1- num-z) 
    do (loop for y from 0 to (1- num-y) 
      do (loop for x from 0 to (1- num-x) 
        for index = (+ x (* num-x y) (* z num-x num-y)) 
        for cell = (nth index cells) 
        for id = (first (get-id-tags cell)) 
        
  )

Appendix C - AML Source Code (V-23) - C23 - Darren Watts - September 2008
for cell-nodes = (list (nth (index3d x y z num-x-points) all-nodes) (nth (index3d (1+ x) y z num-x-points) all-nodes) (nth (index3d x (1+ y) z num-x-points) all-nodes) (nth (index3d (1+ x) (1+ y) z num-x-points) all-nodes) (nth (index3d (1+ x) y (1+ z) num-x-points) all-nodes) (nth (index3d x (1+ y) (1+ z) num-x-points) all-nodes) (nth (index3d (1+ x) (1+ y) (1+ z) num-x-points) all-nodes) (nth (index3d (1+ x) y (1+ z) num-x-points) all-nodes) (nth (index3d (1+ x) (1+ y) (1+ z) num-x-points) all-nodes) (nth (index3d (1+ x) y z num-x-points) all-nodes))
do (store-elements-for-tagid db id tag-type element-type 1 cell-nodes)
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Author: Darren Watts & Jeremy Johnson
Created: Fri Oct 07 22:41:12 2005

Purpose: Advanced Training

(in-package :aml)

(define-class topology-optimisation-objects-setup-class
  :inherit-from (object)
  :properties(
    topology-source-object (default nil)
  )
  :subobjects(
    (geometry-volume :class 'dcms-objective-class
      Design-property-object (the solution-geometry-volume self (:from ^topology-source-object :error nil :relation nil))
      minimize? t
    )
    (geometry-mass :class 'dcms-objective-class
      Design-property-object (the solution-geometry-mass self (:from ^topology-source-object :error nil :relation nil))
      minimize? t
    )
    (max-displacement :class 'dcms-objective-class
      Design-property-object (the max-displacement self (:from ^topology-source-object :error nil :relation nil))
      minimize? t
    )
    (stress-min-max-range :class 'dcms-objective-class
      Design-property-object (the stress-min-max-range self (:from ^topology-source-object :error nil :relation nil))
      minimize? t
    )
  ))

(define-class topology-optimisation-setup-class
  :inherit-from (dcms-exploration-object-manager-class)
  :properties(
    topology-source-object :class 'object-selection-property-class
    label "Topology Source Object"
    formula nil ;; instance of topology-allowed-classes-list '([topology-manager-class])
    cell-type-index-list (the cell-type-index-list (:from "topology-source-object")
    cell-type-list (let ((class-list (the cell-class-names-list (:from "topology-source-object")
    )))
  )
)

A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

(\textbf{C}26 \textbf{-} Darren Watts – September 2008)
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

property-classes.aml

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;;;; maintained.
;;;;Author: Darren Watts & Jeremy Johnson
;;;;Created: Wed Oct 05 18:31:28 2005
;;;;Purpose: Advanced Training
;;;;
;;;; (in-package :aml)

(define-class 2d-list-from-3d-list-property-class
  (inherit-from (data-matrix-property-class change-event)
    :properties(
      x-cells-quantity (default 2) ;; >=1
      y-cells-quantity (default 2) ;; >=1
      z-cells-quantity (default 1) ;; >=1
      z-index-selected 0
      source-property-object (default nil)
    ;; flat list of cells in all directions in the form '(x0y0z0 x1y0z0 ...
      x0y1z0 x1y1z0 ... x0y0z1 ...etc)
    ;; internal props
      start-index (+ "z-index-selected" "x-cells-quantity" "y-cells-quantity")
      end-index (+ "start-index" (- (* "x-cells-quantity" "y-cells-quantity") 1))
    formula (let (xq ! x-cells-quantity)
      (source-list (the (:from !source-property-object :error nil)))
      (sublist (when source-list (subseq source-list !start-index !end-index))))
    (loop for i from 0 to (- (length sublist) 1) by xq
      for j upfrom 1
      collect (subseq sublist i (* q j))))
  )
solution-structure-analysis-class.aml

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;;;;Phone: (513) 985-9877
;;;;Fax: (513) 985-0522

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maintained.

Author: Darren Watts & Jeremy Johnson
Created: Wed Oct 05 21:51:04 2005
Purpose: Advanced Training

(in-package :aml)

(define-class solution-structure-analysis-class
  (inhibit-from (analysis-model-class)
    (properties
      ;; input properties
      (run-remote? (default nil))
      (solution-structure-mesh-object (default nil))
      (load-vector (default '(0 -1000 0)))
      (dof-list (default '(1 2 3 4 5 6)))

      ;; internal properties
      (cell-queries (children (the cell-queries (:from "solution-structure-mesh-object"); cell-objects-list (append-list (rest (the cell-objects-grouped-list (:from "solution-structure-mesh-object")))
      (constrained-nodes (the constrained-nodes (:from "solution-structure-mesh-object")))
      (loaded-nodes (the loaded-nodes (:from "solution-structure-mesh-object")))

      ;; analysis model properties
      (analysis-type :linear-static)
      (load-case-objects-list (list "load-case-1")
      (materials-list (remove-duplicates (loop for ps in "property-set-objects-list" collect (the material-name (:from ps)))))
      (element-set-3d-objects-list (children "solid-element-sets")
      (property-set-objects-list (children "solid-property-sets")
      (material-catalog-object "material-catalog")
      (mesh-object (the mesh-database-object (:from "solution-structure-mesh-object")))

      ;; output properties
      (maximum node displacement magnitude (mm)
      (max-displacement (when "valid-nastran-run?" (nth 1 (the superior nastran-results displacements-node-table node-min-max))))
      (max Von Mises stress minus min Von Mises stress (N/mm2)
      (stress-min-max-range (when "valid-nastran-run?" (- (nth 1 (the superior nastran-results stresses-solid-element-table element-min-max))
      (nth 0 (the superior nastran-results stresses-solid-element-table element-min-max)))))
      (valid-nastran-run? (valid-nastran-run? (the nastran-analysis-results-results-file-path (:from "nastran-results")))

    ))
  )

A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

(material-catalog-file-name (system-resource :analysis-interface "data" "metric-cell-materials.txt")
  id-manager-object (default nil)
)
(solid-property-sets :class 'series-object
class-expression 'analysis-property-set-3d-type-1-class
quantity (length "cell-objects-list")
init-form '(material-name (the material-name (:from
(nth ^index "cell-objects-list"))))
)
(solid-element-sets :class 'series-object
class-expression 'analysis-element-set-3d-type-1-class
quantity (length "cell-queries")
init-form '(property-set-object (nth ^index (children
"solid-property-sets"))
"cell-queries"))
)
(fixed-nodes-constraint :class 'analysis-constraint-displacement-class
target-object "constrained-nodes
  tx (find 1 "dof-list")
  ty (find 2 "dof-list")
  tz (find 3 "dof-list")
  mx (find 4 "dof-list")
  my (find 5 "dof-list")
  mz (find 6 "dof-list")
)
(nodal-load :class 'analysis-load-force-nodal-class
target-object "loaded-nodes
  num-nodes (length (get-node-ids "target-object")
  load-vector (multiply-vector-by-scalar "load-vector (/ 1
  "num-nodes") ;;distribute load across nodes
)
(load-case-1 :class 'analysis-load-case-class
  load-objects-list (list "nodal-load")
  constraint-objects-list (list "fixed-nodes-constraint")
)
;; runs local or remote nastran depending on the value of run-remote?
(brick-nastran-interface-object :class 'remote-nastan-interface-class
  analysis-model-object "superior
  delete-output-files-before-run? t
)
(nastran-results :class 'analysis-post-processing-structural-linear-
  static-nastran-class
  mesh-database-object "mesh-object
  analysis-interface-nastran-object "brick-nastran-
  interface-object
  mesh-query-objects-list "cell-queries"
)
)
(defun valid-nastran-run? (nastran-results-file)
  (with-open-file (buffer nastran-results-file :direction :input :if-does-not-exist nil)
    (loop
      for line = (read-line buffer nil nil)
      while line
      if (find "USER FATAL MESSAGE" line)
      do (return nil)
      finally (return t)
    ))
)
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### topology-manager-class.aml

```aml
;:input properties
  cell-size (default 10.0)
  cell-class-names-list (default (list 'void-class
;'solid-class
;'solid-2-class
;'A25-class
;'A50-class
;'A75-class
;'A12.5-class
;'A37.5-class
;'A62.5-class
;'A87.5-class
;'B25-class
;'B50-class
;'B75-class
;'C25-class
;'C50-class
;'C75-class
;'D25-class
;'D50-class
;'D75-class
;'E25-class
;'E50-class
;'E75-class
;'F25-class
;'F50-class
;'F75-class
;'G25-class
;'G50-class
;'G75-class
;'H25-class
;'H50-class
;'H75-class))

num-cells-x (default 10)
num-cells-y (default 5)
num-cells-z (default 1)

out-geom-file-path (default (logical-path :optimisation-output (format nil "-a" (object-name :superior))))
```

---

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Author: Darren Watts & Jeremy Johnson
Created: Wed Oct 05 18:32:29 2005
Purpose: Advanced Training
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

Out-excel-file-template-path (default (logical-path :topology-optimisation-output "template.xls"))
out-excel-file-path (default (logical-path :topology-optimisation-output (format nil "-a.xls" (object-name :superior)))))
excel-start-cell-name (default "A1")

Constrained-node-id-list (default (the superior brick-structure-mesh constrained-nodes-default mesh-entities-list))
loaded-node-id-list (default (the superior brick-structure-mesh loaded-nodes-default mesh-entities-list))

ignore-cell-index-list (default nil) ;; (default '(0 1 2 3 4))
load-vector (default '(0 -1000 0))
dof-list (default '(1 2 3 4 5 6))
material-name (default 'aluminium)
material-name-2 (default 'steel)
mesh-type (default (nth 0 '(brick tet)))
max-element-size (default 0.5)
run-nastran-remote? (default nil)
run-patran-remote? (default nil)

;: internal properties
cell-class-available-types (default (list 'void-class 'solid-class 'solid-2-class 'A25-class 'A50-class 'A75-class 'A12.5-class 'A37.5-class 'A62.5-class 'A87.5-class 'B25-class 'B50-class 'B75-class 'C25-class 'C50-class 'C75-class 'D25-class 'D50-class 'D75-class 'E25-class 'E50-class 'E75-class 'F25-class 'F50-class 'F75-class 'G25-class 'G50-class 'G75-class 'H25-class 'H50-class 'H75-class 'I25-class 'I50-class 'I75-class 'J25-class 'J50-class 'J75-class 'K25-class 'K50-class 'K75-class 'L25-class 'L50-class 'L75-class 'M25-class 'M50-class 'M75-class 'N25-class 'N50-class 'N75-class 'O25-class 'O50-class 'O75-class 'P25-class 'P50-class 'P75-class 'Q25-class 'Q50-class 'Q75-class 'R25-class 'R50-class 'R75-class 'S25-class 'S50-class 'S75-class 'T25-class 'T50-class 'T75-class 'U25-class 'U50-class 'U75-class 'V25-class 'V50-class 'V75-class 'W25-class 'W50-class 'W75-class 'X25-class 'X50-class 'X75-class 'Y25-class 'Y50-class 'Y75-class 'Z25-class 'Z50-class 'Z75-class))
non-void-cell-index-list (loop for eid in (remove-duplicates mesh-database-object (from 'brick-structure-mesh)) for db = (the *loaded-node-id-list elements = (query-elements-on-nodeid db nid) for connected-elements)) collect (1- eid))

grouped-point-coords-list (loop with size = "cell-size" for z from 0 to "num-cells-z" collect (loop
for y from 0 to ^num-cells-y  
collect (loop   
  for x from 0 to ^num-cells-x   
  for coords = (list (* size x))   
  (* size y)   
  (* size z))   
  collect coords)   
)

;; flat list of all point coordinates '((x y z)(x y z)...)
point-coords-list (append-list (append-list 'grouped-point-coords-list))

cell-origin-coords-list (loop   
  with size = 'cell-size   
  for z from 0 to (1- ^num-cells-z)   
  append (loop   
    for y from 0 to (1- ^num-cells-y)   
    append (loop   
      for x from 0 to (1- ^num-cells-x)   
      for coords = (list (* size x))   
      (* size y)   
      (* size z))   
      collect coords)   
  )
)

;; flat list of indecies into cell-class-names-list (one for each cell)
;; optimisation variables come from this property

cell-type-index-list (loop   
  for i from 1 to (* ^num-cells-x ^num-cells-y ^num-cells-z)   
  collect (random (length ^cell-class-names-list))
)

(excel-client-out :class excel-client-class   
  file-name 'out-excel-file-template-path)

excel-sheet-names-list (list "Z Layer 1"   
  "Z Layer 2"   
  "Z Layer 3"   
  "Z Layer 4"   
  "Z Layer 5"   
  "Z Layer 6"   
  "Z Layer 7"   
  "Z Layer 8"   
  "Z Layer 9"   
  "Z Layer 10")

generate-excel-spreadsheet (write-cell-type-index-list-to-excel :superior)

cell-objects-list (children 'cells)
cell-objects-grouped-list (loop   
  for type in ^cell-class-names-list   
  collect (children 'cells :class type))
solution-structure-mesh-object (if (equal ^mesh-type 'tet)   
  'tet-structure-mesh   
  'brick-structure-mesh)

cell-points :class 'series-object   
  class-expression 'cell-point-object   
  quantity (length ^cell-objects-list)   
  init-form '(let* ((cell (nth ^index   
    ^cell-objects-list)))  
    (orig (the origin (:from cell)))   
    (offset (half (the side-   
      (list (+ (nth 0 orig) offset)   
        (+ (nth 1 orig) offset)   
        (+ (nth 2 orig) offset)))))
  )
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

Appendix C - AML Source Code (V-23) - C33 - Darren Watts - September 2008

```lisp
(objects-list) :error "black")
(nth 'index "cell-objects-list"))

(cells :class 'simple-sequence-class
  object-class-name-list (loop
    for index upfrom 1
    for type in "cell-type-index-list
    for class-name = (nth (floor type) "cell-class-names-list"
    for obj-name = (read-from-string (format nil "cell--all index")
    collect (list obj-name class-name)

  init-form 'origin (nth 'index "cell-origin-coords-list"
    side-length "cell-size"

material-catalog (the superior brick-structure-analysis material-catalog-

;; output properties
output-geom-file (when (equal 1 (save-object-list-geom (children "cells"
  "out-geom-file-path
  :scale-factor 0.001

;; we are modeling in mm, UG assumes meters for parasolid import
  (format nil "-a.xmt_txt" "out-geom-file-path")

write-stl-file (loop
  for child-object in (children "unioned-cells-stl"
  for count upfrom 1
  do (write-stl-file child-object (format nil "-a.stl"
    "out-stl-
    file-path count)

;; optimisation objectives
solution-geometry-volume (loop for cell in (children "cells"
  sum (get-cell-volume cell)

solution-geometry-mass (loop
  with cat = "material-catalog"
  for cell in (children "cells"
  for vol = (get-cell-volume cell)
  for mat-name = (the material-name (: from cell))
  for mat-obj = (get-material-object-from-catalog cat
t-obj
  for density = (tsi::get-material-property mat-obj
t-name "mass-density")
  for mass = (* vol density)
  sum mass

max-displacement (if (equal "mesh-type "tet")
  (the max-displacement (:from "tet-structure-
analysis"))

(stress-min-max-range (if (equal "mesh-type "tet")
  (the stress-min-max-range (:from "tet-
structure-analysis"))
```
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for Exploiting Rapid Manufacturing’s Design Freedom

Appendix C - AML Source Code (V-23) - C34 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

(index-list :cell-type-index-list) ;;(loop for i in :cell-type-index-list collect (floor i))) (loop for i in :cell-type-index-list collect (floor i))) ;; making sure they are integers, could be redundant when new AMOPT \(v > 2 \cdot 72\) is used

(start-cell :excel-start-cell-name)

(when xl
  (loop
    for z from 0 to (1- num-z)
    for sheet = (nth z sheet-list)
    for start-index = (* z num-x num-y)
    for end-index = (* (1+ z) num-x num-y)
    for table-vals = (subseq index-list start-index end-index)
    for table-vals-final = (loop for y from (1- num-y) downto 0
    append (subseq table-vals (* num-x y) (* num-x (1+ y))))
    ;; need to reverse row order
    for num-columns = num-x
    for l = (+ 2 (* num-x num-y))
    for val = (append (list 1 num-columns) table-vals-final)
    for start-cell-pos = (get-cell-position xl start-cell sheet)
    for end-cell = (cell-name-from-position (list (+ (first start-cell-pos) (1- num-
    x))))
    (+ (second start-cell-pos) (1- num-
    x)))))
    for cell-range = (format nil "-a:-a" start-cell end-cell)
    do
      (write-cell xl sheet cell-range val)
      (save-workbook xl :filename !out-excel-file-path)
      (out-excel-file-path)
    )
  )
)
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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Author: Darren Watts & Jeremy Johnson

Created: Thu Oct 06 21:57:31 2005

Purpose: Advanced Training

(in-package :aml)

(define-class cell-type-data-matrix-form-class
  :inherit-from (ui-data-matrix-form-class)
  :properties (
    ...
  )
)

(define-method post-apply-method cell-type-data-matrix-form-class ()
  (let ((prop-obj (the model-property-object))
        (source-prop-obj (the source-property-object (:from prop-obj :error nil)))
        (source-list (the (:from source-prop-obj :error nil))))
    (when source-prop-obj
      (loop for i from (the start-index (:from prop-obj)) to (the end-index (:from prop-obj))
            for v in (append-list (the (:from prop-obj)))
            do (setf (nth i source-list) v))
      (change-property-value (the self (:from source-prop-obj)) source-list t :force? t))
  )
)

(define-class topology-manager-data-model-class
  :inherit-from (data-model-node-mixin topology-manager-class)
  :properties (num-cells-x :class 'editable-data-property-class
                           formula :inherit-formula
                           label "Number Cells in X"
                        )

(num-cells-y :class 'editable-data-property-class
              formula :inherit-formula
              label "Number Cells in Y"
           )

(num-cells-z :class 'editable-data-property-class
              formula :inherit-formula
              label "Number Cells in Z"
            )

(cell-size :class 'editable-data-property-class
           formula :inherit-formula
           label "Cell Size (mm)"
        )

(material-name :class 'option-property-class
                formula :inherit-formula
                label "Material"
                mode 'menu
                options-list '(steel aluminium duraform 3d Systems-si40
                               huntsman-7560 huntsman-7580)
            )

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for Exploiting Rapid Manufacturing’s Design Freedom

Appendix C - AML Source Code (V-23) - C37 - Darren Watts - September 2008

A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing’s Design Freedom

Appendix C - AML Source Code (V-23)

Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

(list (the superior edit-data self) 'data-matrix-form-class 'cell-type-data-matrix-form-class))
(grid-object :class 'grid-from-points-coordinates-class
points-coordinates-list (the superior superior point-coords-list))

(define-method randomize-structure topology-manager-data-model-class ()
(smash-property-value (the cell-type-index-list self))
(regen))

(define-method select-constrained-nodes-from-grid topology-manager-data-model-class ()
(draw !grid-object)
(zoom :all)
(let* ((selected-nodes (loop
with all = !point-coords-list
for c in (tsi::select-points-by-point-on-grid)
collect (1+ (position c all :test 'roughly-same-vector)))
))
(when selected-nodes
(undraw !grid-object)
(change-property-value (the constrained-node-id-list self)
slected-nodes
:t :force? t)
)
)

(define-method select-loaded-nodes-from-grid topology-manager-data-model-class ()
(draw !grid-object)
(zoom :all)
(let* ((selected-nodes (loop
with all = !point-coords-list
for c in (tsi::select-points-by-point-on-grid)
collect (1+ (position c all :test 'roughly-same-vector)))
))
(when selected-nodes
(undraw !grid-object)
(change-property-value (the loaded-node-id-list self)
slected-nodes
:t :force? t)
)
)

(define-method select-ignored-cells topology-manager-data-model-class ()
(draw !cell-points :draw-subobjects? t)
(zoom :all)
(let* ((selected-cells (loop
for cell in (tsi::get-objects :class 'cell-point-object)
collect (the index (:from cell)))
))
(when selected-cells
(undraw !cell-points :subobjects? t)
(change-property-value (the ignore-cell-index-list self)
slected-cells
:t :force? t)
)
)

(define-method generate-geom-file topology-manager-data-model-class ()
:output-geom-file)

(define-method generate-stl-file topology-manager-data-model-class ()
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

(define-method generate-excel-file topology-manager-data-model-class ()
  !generate-excel-spreadsheet
)

(define-method show-excel-file topology-manager-data-model-class ()
  !generate-excel-spreadsheet
  (make-visible !excel-client-out t)
)
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing’s Design Freedom

---

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modify this code as long as the original credits and disclaimers are
maintained.

Author: Darren Watts & Robert Badawy
Created: Wed Apr 26 20:47:18 2006
Purpose: Advanced Training

(in-package :aml)

(define-class topology-multiga-optimisation-class
  :inherit-from (dcms-exploration-multiga-optimization-class)
  :properties {
    exploration-method-label "Topology Optimisation MOGA"
    ;; Data collected as the algorithm is running
    ;; (These properties are updated at the end of each generation)
    objective-values-current-generation nil
    objective-values-prev-generation nil
    objective-values-median-compounded nil
    ;;
    no-improvement? (not (topology-improvement? superior))
    interrupt-property-object (the superior no-improvement? self)
  }
)

;; Customised Inherited Methods

(define-method perform-after-evaluation topology-multiga-optimisation-class (population objective-values)
  ;; (when ! objective-values-current-generation
  ;;   (change-value ! objective-values-prev-generation ! objective-values-current-generation))
  (change-value ! objective-values-current-generation objective-values)
)

(define-method run-optimization topology-multiga-optimisation-class ()
  (ignore-current-dependency
   (change-value ! objective-values-current-generation nil)
   (change-value ! objective-values-prev-generation nil)
   (change-value ! objective-values-median-compounded nil)
  )
  ;; execute the inherited behavior
  (call-next-method)
)

;; New Methods

;; this method is used with the optimisation interrupt feature.
;; return nil if there has been no improvement, ie the optimisation should stop
(define-method topology-improvement? topology-multiga-optimisation-class ()
  (let ((current-median (let ((vlist (loop for obj-values in ! objective-values-current-generation
                                             collect (nth 2 obj-values)))))
    ;; Integrate the median of the objective values
    (median-value (median vlist))
    ;; Compare the median value with the best objective value
    (best-value (first superiors))
    (topology-improvement? median-value best-value))))

Appendix C - AML Source Code (V-23) - C41 - Darren Watts - September 2008
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(defun init-topology-amopt-gui ()
  (unless (property? (model-manager) 'amopt-exploration-class-list)
    (add-property (model-manager) 'amopt-exploration-class-list nil))
  ;; (change-value (the amopt-exploration-class-list (:frcTn (model-manager)))
  ;;   'topology-multiga-optimisation-class
dcms-exploration-multiga-optimization-class
  amopt-multiga-distributed-optimization-class
  amopt-doe-distributed-optimization-class
  )
)
choose-cell-types-form-class.aml

;;;;;; --
;;;;;; TechnoSoft Inc.
;;;;;; Copyright (c) 1993 - 2005
;;;;;;
;;;;;; 11180 Reed Hartman Hwy
;;;;;; Cincinnati, OH 45242
;;;;;; Phone: (513) 985-9877
;;;;;; Fax:  (513) 985-0522
;;;;;;
;;;;;; This code sample is provided as an example of how to perform a particular
;;;;;; kind of task using foundation software provided by TechnoSoft. It is
;;;;;; provided "as is" without warranty of any kind, expressed or implied.
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;;;;;; Licensed users of the Adaptive Modeling Language are free to use and
;;;;;; modify this code as long as the the original credits and disclaimers are
;;;;;; maintained.
;;;;;;
;;;;;; Author: Darren Watts & Jeremy Johnson
;;;;;; Created: Mon Apr 24 21:14:17 2006
;;;;;;
;;;;;; Purpose: Advanced Training
;;;;;;
;;;;;;
;;;;;; (in-package :aml)
;;;;;;
;;;; (define-class choose-cell-types-scrollable-form
;;;; ;inherit-from (ui-scrolled-window-class series-object)
;;;; :properties (
;;;;     ;; input properties
;;;;     cell-types-name-list (default nil)
;;;;     ;; internal properties
;;;;     scroll-range (list (- width 20)
;;;;             (+ (* "pixel-unit-height" quantity)
;;;;             (* 5 "quantity"))
;;;;     class-expression 'ui-toggle-button-class
;;;;     quantity (length "cell-types-name-list")
;;;;     init-form '(measurement 'pixels
;;;;             x-offset 10
;;;;             width 280
;;;;             y-offset (+ (* "height" index)
;;;;             (* (+ 1 "index") 5))
;;;;             height "pixel-unit-height"
;;;;             attachment-info-list '(top top right right)
;;;;             label (write-to-string (nth "index" "cell-types-name-list"))
;;;;             tooltip "label"
;;;;             label-align :left
;;;;             status nil
;;;;             gray? (is-valid-neighbour? "superior" read-from-string
;;;;             "label")
;;;;     )
;;;;     select-action '(update "superior"
;;;;     release-action '(update "superior"
;;;;     )
;;;;     valid-cell-types (loop
;;;;             with valid-list = "cell-types-name-list"
;;;;             for flag in (series-members "superior")
;;;;             for type in "cell-types-name-list"
;;;;             when (the status (:from flag))
;;;;             do (setf valid-list (intersection valid-list
;;;;             (get-valid-neighbour-
;;;;             cell-types-for-type type))
;;;;     )
;;;;     finally (return valid-list)
;;;;     selected-cell-types (loop
;;;;             for flag in (series-members "superior")
;;;;             for type in "cell-types-name-list"
;;;;             when (the status (:from flag))
;;;;             collect type
;;;;     )
;;;;)

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(define-method is-valid-neighbour? choose-cell-types-scrollable-form (cell-type)
  (not (member cell-type !valid-cell-types)))

(define-class choose-cell-types-scrollable-form-class
  :inherit-from (ui-active-form-class)
  :properties
    ;; input properties
    cell-types-name-list '(void-class
      solid-class
      solid-2-class
      A12.5-class
      A25-class
      A37.5-class
      A50-class
      A62.5-class
      A75-class
      A87.5-class
      B25-class
      B50-class
      B75-class
      C25-class
      C50-class
      C75-class
      D25-class
      D50-class
      D75-class
      E25-class
      E50-class
      E75-class
      F25-class
      F50-class
      F75-class
      G25-class
      G50-class
      G75-class
      H25-class
      H50-class
      H75-class)
    ;; internal properties
    x-offset 100
    y-offset 100
    width 300
    height 300
    pixel-unit-height 24
    pixel-unit-width 70
    ;; output properties
    selected-cell-types (the superior scrollable-form selected-cell-types)
  }
  :subobjects
    (scrollable-form :class 'choose-cell-types-scrollable-form
      measurement 'pixels
      x-offset 0
      y-offset 0
      width "width
      height (~"height
        (+ "pixel-unit-height 10))
        attachment-info-list '(top bottom left right)
      )
    (ok-button :class 'ui-action-button-class
      measurement 'pixels
      button1-action '(hide 'superior)
      x-offset (half (+ "width (* 2 width) 5))
      y-offset (+ (the superior superior scrollable-form height) 5)
      height "pixel-unit-height
      width "pixel-unit-width
    )
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

Appendix C - AML Source Code (V-23)  - C45 -  Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

remote-nastran-interface-class.aml

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;;; Author: Darren Watts & Jeremy Johnson

;;; Purpose: Advanced Training

(in-package :aml)

(define-class remote-nastran-interface-class
  (inherit-from (nastran-analysis-class)

  :properties (
    ;; input properties
    run-remote? (default t)

    ;; internal properties
    time-to-solve nil
    run-nastran@ (let* ((data-file (data-file)
                        (start-time (get-universal-time))
                        (time-to-solve (get-universal-time)
                        (start-time))
        (if run-remote?
            (progn
              (the run-nastran@ (:time 'nastran-remote-analysis))
              (change-value "time-to-solve" (- (get-universal-time)
                        start-time))
            )
        )
        let* (current-directory (pwd))
        (nastran-return-value (progn
                        "data-file"
                        "analysis-directory"
                        "delete-output-files"
                        (delete-output-files
                        (delete-file (format nil
                        (delete-file (format nil
                        (delete-file (format nil
                        (delete-file (format nil
                        (run-program "nastran-command"
                        ))

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(start-time))

(start-time)

:subobjects {
  (nastran-remote-analysis :class 'nastran-remote-analysis-class
    bdf-local-file-path "data-file
    bdf-remote-file-path (logical-path "working-
    directory "nastran-file-name)
    operating-system "windows" ;; this might change
    working-directory "c:|\temp|\tscrm|\nastran\"
  ;; this might change
  "nastran.bat") ;; this might change
  receive-file-list (list (logical-path "analysis-
    directory (format nil "-a.f06" "model-name"))
  receive-rfile-list (list (logical-path "working-
    directory (format nil "-a.f06" "model-name"))
    tscrm-name 'tscrm
    wait? t
  )

  )
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

simplified-union-inner-faces-removed-class.aml

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; Author: Darren Watts & Jeremy Johnson

; Created: Wed Apr 25 21:35:08 2006

; Purpose: Advanced Training

(in-package :aml)

(define-class simplified-union-inner-faces-removed-class
  :inherit-from (geom-object)
  :properties (
    source-object (default nil)
  )
)

(define-method ts*:create-geom simplified-union-inner-faces-removed-class ()
  (let* ((g (vgl:copy-geom (the geom (:from !source-object))))
         (faces (loop
                    for f in (vgl:k-sub-geoms g 2)
                    for s = (vgl:sup-geoms f)
                    when (> (length s) 1)
                    collect f))
         (vgl:delete-faces faces) ;;delete internal faces
         (vgl:simplify-geom g) ;; remove unnecessary edges
         g)
  )
)

(define-class tagged-simplified-union-inner-faces-removed-class
  :inherit-from (tagging-object simplified-union-inner-faces-removed-class)
)

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Author: Darren Watts & Jeremy Johnson

Created: Wed Apr 25 21:35:08 2006

Purpose: Advanced Training

Appendix C - AML Source Code (V-23) - C48 - Darren Watts - September 2008
solution-structure-patran-mesh-class.aml

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maintained.

Author: Darren Watts & Jeremy Johnson
Created: Thu Apr 27 19:31:28 2006
Purpose: Advanced Training

(in-package :aml)

(define-class solution-structure-patran-mesh-class
  :inherit-from (patran-mesh-interface-class)
  :properties {
    ;; input properties
    max-element-size (default 0.25)
    cell-objects-list (default nil)
    cell-size (default nil)
    constrained-nodes-interface-object (default nil)
    loaded-nodes-interface-object (default nil)
    run-remote? (default nil)
    ;; internal properties
    (meshdb :class 'meshdb-class)
    (cells-union :class 'tagged-union-object
      object-list "cell-objects-list
      simplify? nil
      overwrite-other-tags? t
      tag-attributes (list "max-element-size 0.0625 0 0.1 0 1 0 10.0
      point-coords-list (loop
        for n in (get-node-ids "loaded-nodes")
        collect (nth 2 (query-node-info "meshdb n)))
    )
    ;; remote execution properties
    remote-working-directory "c:\\temp\\tscrm\\patran\"
    operating-system "windows"
    tscrm-name tscrm
    patran-scripts-path (let* (local-path (system-resource :patran-meshdb-
      interface "bin" (platform-name) "")
      )
      (or (if *run-remote? 
        remote-working-directory
        (local-path)
        (mesh-error-message "Cannot find Patran Scripts
Path")
      )
      write-session-file? (if *run-remote? 
        (write-remote-session-file (the superior))
        (tsi::write-session-file (the superior))
      )
      write-attributes-file? (if *run-remote? 
        (write-remote-attributes-file (the superior))
        (tsi::write-attributes-file (the superior))
      )
    )
  }
)
A Genetic Algorithm Based Topology Optimisation Approach
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(time-to-mesh nil)

(mesh (let* ((obj-to-mesh "object-to-mesh")
             (start-time (get-universal-time))
             (mesh (if "run-remote?"
                     (generate-remote-patran-mesh (the superior))
                     (tsi::generate-patran-mesh (the superior))))
        (change-value "time-to-mesh" (- (get-universal-time) start-time))
        (or mesh
            (mesh-error-message "Mesh generation failed")))
    )

;; patran-mesh-interface properties
(mesh-database-object "meshdb"
  object-to-mesh "cells-simplified-union"
  solid-mesh? t
  solid-mesh-from-fem? t)

:subobjects
;; this unions and simplifies all of the cells, removing internal faces
(cells-simplified-union :class 'simplified-union-inner-faces-removed-class
  source-object "cells-union"
  )
(constrained-nodes :class 'mesh-query-nodes-from-interface-class
  mesh-object "superior"
  interface-object "constrained-nodes-interface-object"
  color "cyan"
  line-width 10
 )

(loadded-nodes :class 'mesh-query-nodes-from-interface-class
  mesh-object "superior"
  interface-object "loaded-nodes-interface-object"
  color "red"
  line-width 10
 )

(load-vectors :class 'series-object
  class-expression 'vector-class
  quantity (length "point-coords-list")
  init-form '(base-point (nth "index" "point-coords-list"
                 direction "load-vector"
                 length (* 1.5 "cell-size"
                 color (the superior superior superior loaded-nodes.color)
                 line-width 2
 )
 )
)

solution-structure-tet-analysis-class.aml

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;; maintained.
;; Purpose: Advanced Training
;;--------------------------------------

(in-package :aml)

(define-class solution-structure-tet-analysis-class
  (inherit-from (analysis-model-class)
    (properties ( ; input properties
                 run-remote? (default nil)
                 solution-structure-mesh-object (default nil)
                 material-name (default nil)
                 load-vector (default '(0 -1000 0))
                 dof-list (default '(1 2 3 4 5 6))

                 ; internal properties
                 mesh-database-object (the mesh-database-object (:from ^solution-structure-mesh-object))
                 cells-query-object (the solid-elements-query (:from ^solution-structure-mesh-object))
                 constrained-nodes (the constrained-nodes (:from ^solution-structure-mesh-object))
                 loaded-nodes (the loaded-nodes (:from ^solution-structure-mesh-object))

                 ; analysis model properties
                 analysis-type :linear-static
                 load-case-objects-list (list ^load-case-1)
                 materials-list (list ^material-name)
                 element-set-3d-objects-list (list ^solid-element-set)
                 property-set-objects-list (list ^solid-property-set)
                 material-catalog-object ^material-catalog
                 mesh-object ^mesh-database-object

                 ; output properties
                 ; maximum node displacement magnitude
                 max-displacement (when ^valid-nastran-run?
                   (nth 1 (the superior nastran-results displacements-node-table node-min-max)))

                 ; max Von Mises stress minus min Von Mises stress
                 stress-min-max-range (when ^valid-nastran-run?
                   (nth 0 (the superior nastran-results stresses-solid-element-table element-min-max)))

                 ; path (:from ^nastran-results))
    )

  )

;subobjects {
  (material-catalog :class 'analysis-material-catalog-class

A Genetic Algorithm Based Topology Optimisation Approach
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material-catalog-file-name (system-resource :analysis-interface "data" "metric-cell-materials.txt")
id-manager-object (default nil)
)
(solid-property-set :class 'analysis-property-set-3d-type-1-class
material-name "material-name"
)
(solid-element-set :class 'analysis-element-set-3d-type-1-class
property-set-object ^'solid-property-set
query-objects-list (list "cells-query-object")
)
(fixed-nodes-constraint :class 'analysis-constraint-displacement-class
target-object "constrained-nodes
tx (find 1 ^dof-list)
ty (find 2 ^dof-list)
tz (find 3 ^dof-list)
mx (find 4 ^dof-list)
my (find 5 ^dof-list)
mz (find 6 ^dof-list)
)
(nodal-load :class 'analysis-load-force-nodal-class
target-object "loaded-nodes
num-nodes (length (get-node-ids ^target-object))
load-vector (multiply-vector-by-scalar "load-vector (/ 1 ^num-nodes))
);; distribute load across nodes
)
(load-case-1 :class 'analysis-load-case-class
load-objects-list (list "nodal-load")
constraint-objects-list (list "fixed-nodes-constraint")
)
(tet-nastran-interface-object :class 'remote-nastran-interface-class
analysis-model-object "superior"
delete-output-files-before-run? t
)
(nastran-results :class 'analysis-post-processing-structural-linear-static-nastran-class
mesh-database-object "mesh-object"
analysis-interface-nastran-object "tet-nastran-interface-object"
mesh-query-objects-list (list "cells-query-object")
)
)

(defun valid-nastran-run? (nastran-results-file)
(with-open-file (buffer nastran-results-file :direction :input :if-does-not-exist nil)
(loop
  for line = (read-line buffer nil nil)
  while line
  if (find "USER FATAL MESSAGE" line)
    do (return nil)
  finally (return t)
})
)
APPENDIX D – Detailed System Design & Development

Editing the Default Problem Definition

Using AML an interactive window was created called the *Edit Z Layer Form*, to allow the user to inspect and edit a beam’s corresponding chromosome representation. Within this window the chromosome string is arranged as a 3D array, split into 2D arrays of different Z layers. The form displays the selected Z layer for editing, allowing the user to edit the beam structure by simply replacing any of the existing integer values with new ones corresponding to the assigned cell types. Figure D.1 shows the Edit Z Layer Form of the example beam geometry previously shown in Figure 5.4.

![Edit Z Layer Form](image)

**Figure D.1 – The ‘Edit Z Layer Form’ for the example beam geometry.**

The Edit Z Layer Form can be used to create geometries that are of interest to the user and can also help to enable the creation of more complex problems than the default definition. For example, certain regions of the structure within the design domain can be altered by using the Edit Z Layer Form and these cells then have the option of remaining the same throughout the entire optimisation process by removing the associated cell locations from the list of discrete variables. This procedure has been followed to allow the creation of the hollow and I-section beam geometries displayed in Figure D.2, where certain cells were changed to become voids before being removed from the list of discrete variables.
The default problem definition was created to allow the automated fast construction of cantilever beam type problems. However, the default settings can be redefined to enable problem variations to be considered. The user can easily alter the default problem parameters including the number of cells in X, Y and Z that form the design domain, the size of those cells, the cell types considered, the selected material, the load magnitude and direction, the degrees of freedom of the constrained nodes and the cell locations removed from the optimisation process. Figure D.3 shows examples of increasing the size of the design domain (a) and the number of cell types used (b).

In addition to the problem definition options stated previously, the user can also alter the locations of the applied load and boundary conditions by either moving or rotating the associated sheet or line geometry. This would allow the study of alternative problems to simple cantilever beams.
File Output Options

The system has been created in such a way as to allow selected beam geometries to be outputted in three different formats, namely a Microsoft Excel spreadsheet, a 3D Parasolid file and a triangulated STereoLithography (STL) file. Each of these file outputs will now be discussed in turn.

Microsoft Excel Workbook Output

The Microsoft Excel file output can help the user analyse the chromosome integer makeup of beam geometries in much the same way the DesignLab results were analysed earlier in Chapter 3. However, this file output option automatically creates the 3D chromosome array in a Microsoft Excel Workbook (with each 2D layer in the Z direction exported onto a separate sheet), unlike the analysis of the DesignLab results where chromosome arrays had to be entered manually into Excel. This information can be used to spot structural trends that can help the user construct additional constraints for subsequent optimisation runs. The introduction of constraints in this manner effectively steers the GA towards known good solutions or away from known poor solutions by influencing the topological permutations that are possible. The Microsoft Excel output file for the beam geometry previously shown in Figure 5.4 can be seen in Figure D.4.

![Figure D.4 – Microsoft Excel file output of example beam geometry.](image-url)
Parasolid File Output

Unlike the Microsoft Excel file output, the Parasolid file output option considers the actual cellular structures and not merely their integer representation. This output option is used to export the entire 3D solution geometry from the system, enabling it to be imported back into other CAE software for subsequent actions. For example, this could be to allow 3D manipulation within a CAD system or to enable further FEA to be performed or alternatively CFD analysis. The Parasolid file output for the example beam previously shown in Figure 5.4, after importing into Unigraphics NX2 can be seen in Figure D.5.

![Parasolid file output of example beam geometry viewed in Unigraphics NX2.](image)

STL File Output

The need to create an STL file in order to produce physical parts via RP/RM technology is a fundamental part of this research. Initially it was thought that an exported Parasolid file could be simply converted into an STL file using CAD software. However, this method proved unsuccessful as STL files created in this manner were considered to be of poor quality, often exhibiting numerous bad edges and duplicate faces as a result of the internal faces between adjacent cells, leading to potential RP/RM build problems. The STL file output option was created to overcome this problem by automatically creating a high quality STL file, with no internal faces or bad edges, directly from the AML topology optimisation system. The STL file output for the example beam
previously shown in Figure 5.4, can be seen in Figure D.6, viewed in the STL manipulation software Magics.

Graphical User Interface

All of the options for both the basic and advanced systems including mesh types and all remote jobs have been combined into a simple GUI using AML, which can be seen in Figure D.7.

The GUI consists of several options including basic controls for the design domain (number of cells, cell size, material), load and constraint options, remote job options for both Patran and Nastran jobs, meshing options, file output options and Edit Z Layer options. The implementation of the 'Valid Neighbours?' KBE rule is also present in the GUI along with a button to select cell locations within the design domain that should be removed from the optimisation variables list. The GUI is somewhat interactive, by automatically updating the remaining options available to the user depending on what has been previously selected.
Figure D.7 – Advanced remote system GUI.
### APPENDIX E - Materials Data File

*Ver 2.0

<table>
<thead>
<tr>
<th>Units 1</th>
<th>Property-names</th>
<th>Id</th>
<th>MaterialType</th>
<th>Unit</th>
<th>Ym</th>
<th>PR</th>
<th>MassDensity</th>
<th>ShearModulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-)</td>
<td>linear-elastic</td>
<td>(-)</td>
<td>unit-id</td>
<td>Youngs-Modulus</td>
<td>(-)</td>
<td>(/ N (mm 2))</td>
<td>Poissons-Ratio</td>
<td>Mass-Density</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus</th>
<th>Density</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>void</td>
<td>0.10000e1</td>
<td>0.00</td>
<td>0.00000</td>
</tr>
<tr>
<td>steel</td>
<td>200000.0</td>
<td>0.30</td>
<td>7.85e-6</td>
</tr>
<tr>
<td>aluminium</td>
<td>71000.0</td>
<td>0.33</td>
<td>2.77e-6</td>
</tr>
</tbody>
</table>

*820 degrees celsius

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's Modulus</th>
<th>Density</th>
<th>Shear Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>duraform</td>
<td>1.06000e7</td>
<td>0.33</td>
<td>0.098</td>
</tr>
<tr>
<td>3dsystems-si40</td>
<td>1.06000e7</td>
<td>0.33</td>
<td>0.098</td>
</tr>
<tr>
<td>huntsman-7560</td>
<td>1.06000e7</td>
<td>0.33</td>
<td>0.098</td>
</tr>
<tr>
<td>huntsman-7580</td>
<td>1.06000e7</td>
<td>0.33</td>
<td>0.098</td>
</tr>
</tbody>
</table>

NOTE - Data in red is incomplete and requires verification, entered as demonstration only.
APPENDIX F – Experiment 1 Pareto Set

Cell Types Used = Solid (Steel), Solid2 (Aluminium) & Void, Pareto Set Size = 259
<table>
<thead>
<tr>
<th></th>
<th>Mass</th>
<th>Displacement</th>
<th>Stress range</th>
</tr>
</thead>
<tbody>
<tr>
<td>1)</td>
<td>0.53139 kg</td>
<td>0.52567 mm</td>
<td>104.25483 MPa</td>
</tr>
<tr>
<td>2)</td>
<td>0.50738 kg</td>
<td>0.52009 mm</td>
<td>115.63172 MPa</td>
</tr>
<tr>
<td>3)</td>
<td>0.52169 kg</td>
<td>0.52296 mm</td>
<td>106.68178 MPa</td>
</tr>
<tr>
<td>4)</td>
<td>0.53508 kg</td>
<td>0.40539 mm</td>
<td>125.52929 MPa</td>
</tr>
<tr>
<td>5)</td>
<td>0.54062 kg</td>
<td>0.40298 mm</td>
<td>123.25881 MPa</td>
</tr>
<tr>
<td>6)</td>
<td>0.53508 kg</td>
<td>0.43438 mm</td>
<td>107.16540 MPa</td>
</tr>
<tr>
<td>7)</td>
<td>0.52862 kg</td>
<td>0.53667 mm</td>
<td>103.48791 MPa</td>
</tr>
<tr>
<td>8)</td>
<td>0.52077 kg</td>
<td>0.53181 mm</td>
<td>105.96655 MPa</td>
</tr>
<tr>
<td>9)</td>
<td>0.55724 kg</td>
<td>0.42208 mm</td>
<td>118.56896 MPa</td>
</tr>
<tr>
<td>10)</td>
<td>0.52631 kg</td>
<td>0.47039 mm</td>
<td>106.49032 MPa</td>
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<tr>
<td>11)</td>
<td>0.52585 kg</td>
<td>0.52993 mm</td>
<td>104.06136 MPa</td>
</tr>
</tbody>
</table>

Appendix F - Experiment 1 Pareto Set - F2 - Darren Watts - September 2008
<table>
<thead>
<tr>
<th>No.</th>
<th>Mass</th>
<th>Displacement</th>
<th>Stress range</th>
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<tr>
<td>12)</td>
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<td>0.53940 mm</td>
<td>104.15385 MPa</td>
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<tr>
<td>13)</td>
<td>0.53940 mm</td>
<td>0.55165 mm</td>
<td>105.96163 MPa</td>
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<tr>
<td>14)</td>
<td>0.51015 kg</td>
<td>0.55337 mm</td>
<td>106.40286 MPa</td>
</tr>
<tr>
<td>15)</td>
<td>0.50276 kg</td>
<td>0.54337 mm</td>
<td>106.29109 MPa</td>
</tr>
<tr>
<td>16)</td>
<td>0.52677 kg</td>
<td>0.44341 mm</td>
<td>122.75238 MPa</td>
</tr>
<tr>
<td>17)</td>
<td>0.53379 kg</td>
<td>0.40675 mm</td>
<td>122.82157 MPa</td>
</tr>
<tr>
<td>18)</td>
<td>0.52446 kg</td>
<td>0.41131 mm</td>
<td>123.43438 MPa</td>
</tr>
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<td>19)</td>
<td>0.53508 kg</td>
<td>0.40711 mm</td>
<td>123.16666 MPa</td>
</tr>
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<td>20)</td>
<td>0.54293 kg</td>
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<td>122.82157 MPa</td>
</tr>
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<td>21)</td>
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<td>22)</td>
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</tr>
<tr>
<td></td>
<td>Mass</td>
<td>Displacement</td>
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<td>---</td>
<td>------------</td>
<td>--------------</td>
<td>--------------------</td>
</tr>
<tr>
<td>23</td>
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<td>25</td>
<td>0.49353 kg</td>
<td>0.60343 mm</td>
<td>105.80208 MPa</td>
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<td>26</td>
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<td>0.55236 mm</td>
<td>103.53614 MPa</td>
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<td>0.53508 kg</td>
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<tr>
<td>28</td>
<td>0.55078 kg</td>
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<tr>
<td>29</td>
<td>0.48291 kg</td>
<td>0.57110 mm</td>
<td>124.93668 MPa</td>
</tr>
<tr>
<td>30</td>
<td>0.48199 kg</td>
<td>0.61686 mm</td>
<td>110.98093 MPa</td>
</tr>
<tr>
<td>31</td>
<td>0.52031 kg</td>
<td>0.54564 mm</td>
<td>103.69494 MPa</td>
</tr>
<tr>
<td>32</td>
<td>0.52169 kg</td>
<td>0.41148 mm</td>
<td>123.57242 MPa</td>
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Appendix F – Experiment 1 Pareto Set

Darren Watts – September 2008
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A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing’s Design Freedom

67) Mass = 0.51846 kg
       Displacement = 0.48382 mm
       Stress range = 106.76203 MPa

68) Mass = 0.52908 kg
       Displacement = 0.44244 mm
       Stress range = 105.45551 MPa

69) Mass = 0.54293 kg
       Displacement = 0.42652 mm
       Stress range = 107.16529 MPa

70) Mass = 0.48060 kg
       Displacement = 0.52366 mm
       Stress range = 131.58533 MPa

71) Mass = 0.51338 kg
       Displacement = 0.41968 mm
       Stress range = 123.00503 MPa

72) Mass = 0.48014 kg
       Displacement = 0.60893 mm
       Stress range = 111.58507 MPa

73) Mass = 0.52308 kg
       Displacement = 0.53227 mm
       Stress range = 103.62887 MPa

74) Mass = 0.51615 kg
       Displacement = 0.47871 mm
       Stress range = 107.01237 MPa

75) Mass = 0.54939 kg
       Displacement = 0.40058 mm
       Stress range = 119.19429 MPa

76) Mass = 0.55447 kg
       Displacement = 0.42281 mm
       Stress range = 118.56888 MPa

77) Mass = 0.50276 kg
       Displacement = 0.56215 mm
       Stress range = 105.68828 MPa
### Mass, Displacement, and Stress Range

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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

89) Mass = 0.48383 kg
Displacement = 0.50711 mm
Stress range = 130.37310 MPa

90) Mass = 0.49122 kg
Displacement = 0.52163 mm
Stress range = 123.44387 MPa

91) Mass = 0.53693 kg
Displacement = 0.44096 mm
Stress range = 106.95288 MPa

92) Mass = 0.56326 kg
Displacement = 0.56981 mm
Stress range = 102.45664 MPa

93) Mass = 0.55447 kg
Displacement = 0.39695 mm
Stress range = 119.17897 MPa

94) Mass = 0.52031 kg
Displacement = 0.54582 mm
Stress range = 103.26708 MPa

95) Mass = 0.49861 kg
Displacement = 0.58352 mm
Stress range = 104.75609 MPa

96) Mass = 0.51800 kg
Displacement = 0.55551 mm
Stress range = 102.97005 MPa

97) Mass = 0.54339 kg
Displacement = 0.39869 mm
Stress range = 123.25597 MPa

98) Mass = 0.50738 kg
Displacement = 0.55447 mm
Stress range = 105.97738 MPa

99) Mass = 0.49076 kg
Displacement = 0.58776 mm
Stress range = 106.87021 MPa

Appendix F – Experiment 1 Pareto Set
Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

177) Mass = 0.50230 kg
     Displacement = 0.46774 mm
     Stress range = 131.57239 MPa

178) Mass = 0.55909 kg
     Displacement = 0.44917 mm
     Stress range = 105.25950 MPa

179) Mass = 0.53093 kg
     Displacement = 0.54020 mm
     Stress range = 103.29770 MPa

180) Mass = 0.49399 kg
     Displacement = 0.48430 mm
     Stress range = 129.29271 MPa

181) Mass = 0.51200 kg
     Displacement = 0.55574 mm
     Stress range = 103.95497 MPa

182) Mass = 0.47552 kg
     Displacement = 0.52362 mm
     Stress range = 133.58785 MPa

183) Mass = 0.48014 kg
     Displacement = 0.57398 mm
     Stress range = 124.93488 MPa

184) Mass = 0.53785 kg
     Displacement = 0.40272 mm
     Stress range = 125.52765 MPa

185) Mass = 0.55447 kg
     Displacement = 0.42337 mm
     Stress range = 118.54088 MPa

186) Mass = 0.52723 kg
     Displacement = 0.40577 mm
     Stress range = 125.52920 MPa

187) Mass = 0.47968 kg
     Displacement = 0.61488 mm
     Stress range = 110.98231 MPa

Appendix F – Experiment 1 Pareto Set

Darren Watts – September 2008
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A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

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254) Mass = 0.54801 kg  
                 Displacement = 0.43146 mm  
                 Stress range = 106.69013 MPa

255) Mass = 0.49030 kg  
                 Displacement = 0.58537 mm  
                 Stress range = 112.49682 MPa

256) Mass = 0.51384 kg  
                 Displacement = 0.41581 mm  
                 Stress range = 123.07004 MPa

257) Mass = 0.51754 kg  
                 Displacement = 0.54951 mm  
                 Stress range = 103.54336 MPa

258) Mass = 0.49630 kg  
                 Displacement = 0.58212 mm  
                 Stress range = 105.29463 MPa

259) Mass = 0.49630 kg  
                 Displacement = 0.55931 mm  
                 Stress range = 106.43583 MPa
APPENDIX G – Experiment 2 Pareto Set

Cell Types Used = Solid (Steel), Solid2 (Aluminium) & Void, Pareto Set Size = 72
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

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A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

12) Mass = 0.49722 kg
Displacement = 0.42681 mm

13) Mass = 0.46260 kg
Displacement = 0.59934 mm

14) Mass = 0.40489 kg
Displacement = 0.69042 mm

15) Mass = 0.48337 kg
Displacement = 0.46182 mm

16) Mass = 0.52864 kg
Displacement = 0.39228 mm

17) Mass = 0.53600 kg
Displacement = 0.38893 mm

18) Mass = 0.49999 kg
Displacement = 0.41998 mm

19) Mass = 0.50599 kg
Displacement = 0.40927 mm

20) Mass = 0.53877 kg
Displacement = 0.38727 mm

21) Mass = 0.51476 kg
Displacement = 0.40225 mm

22) Mass = 0.51753 kg
Displacement = 0.40018 mm
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

23) Mass = 0.35826 kg
    Displacement = 1.10443 mm

24) Mass = 0.38642 kg
    Displacement = 0.76436 mm

25) Mass = 0.56324 kg
    Displacement = 0.37974 mm

26) Mass = 0.42890 kg
    Displacement = 0.62478 mm

27) Mass = 0.34995 kg
    Displacement = 1.33672 mm

28) Mass = 0.52307 kg
    Displacement = 0.39774 mm

29) Mass = 0.53323 kg
    Displacement = 0.38962 mm

30) Mass = 0.38088 kg
    Displacement = 0.82277 mm

31) Mass = 0.50137 kg
    Displacement = 0.41578 mm

32) Mass = 0.53138 kg
    Displacement = 0.39058 mm

33) Mass = 0.49168 kg
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45) Mass = 0.35272 kg  
Displacement = 1.33420 mm

46) Mass = 0.37765 kg  
Displacement = 0.84169 mm

47) Mass = 0.38042 kg  
Displacement = 0.83512 mm

48) Mass = 0.39381 kg  
Displacement = 0.74909 mm

49) Mass = 0.38319 kg  
Displacement = 0.78136 mm

50) Mass = 0.39427 kg  
Displacement = 0.70691 mm

51) Mass = 0.47552 kg  
Displacement = 0.47187 mm

52) Mass = 0.49445 kg  
Displacement = 0.44306 mm

53) Mass = 0.33702 kg  
Displacement = 1.59613 mm

54) Mass = 0.38596 kg  
Displacement = 0.77029 mm

55) Mass = 0.32871 kg  
Displacement = 2.46341 mm
56) Mass = 0.36380 kg  
Displacement = 0.88687 mm

57) Mass = 0.38919 kg  
Displacement = 0.75064 mm

58) Mass = 0.35503 kg  
Displacement = 1.33332 mm

59) Mass = 0.55539 kg  
Displacement = 0.38359 mm

60) Mass = 0.50645 kg  
Displacement = 0.40637 mm

61) Mass = 0.33887 kg  
Displacement = 1.46811 mm

62) Mass = 0.52215 kg  
Displacement = 0.39820 mm

63) Mass = 0.53046 kg  
Displacement = 0.39159 mm

64) Mass = 0.56047 kg  
Displacement = 0.38284 mm

65) Mass = 0.33933 kg  
Displacement = 1.34973 mm

66) Mass = 0.33425 kg  
Displacement = 1.80314 mm
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

67) Mass = 0.53923 kg  
Displacement = 0.38471 mm

68) Mass = 0.52030 kg  
Displacement = 0.39866 mm

69) Mass = 0.41551 kg  
Displacement = 0.65864 mm

70) Mass = 0.49768 kg  
Displacement = 0.42286 mm

71) Mass = 0.52307 kg  
Displacement = 0.39568 mm

72) Mass = 0.38365 kg  
Displacement = 0.77108 mm
APPENDIX H – Experiment 3 Pareto Set

Cell Types Used = Solid (Steel), Solid2 (Aluminium) & Void, Pareto Set Size = 79
1) Displacement = 0.39260 mm
Stress range = 118.85662 MPa

2) Displacement = 0.39583 mm
Stress range = 118.36486 MPa

3) Displacement = 0.40454 mm
Stress range = 105.33596 MPa

4) Displacement = 0.37969 mm
Stress range = 122.44120 MPa

5) Displacement = 0.38388 mm
Stress range = 121.18442 MPa

6) Displacement = 0.42832 mm
Stress range = 104.40083 MPa

7) Displacement = 0.40576 mm
Stress range = 105.28652 MPa

8) Displacement = 0.37993 mm
Stress range = 121.87043 MPa

9) Displacement = 0.37910 mm
Stress range = 122.46498 MPa

10) Displacement = 0.37877 mm
Stress range = 124.38992 MPa

11) Displacement = 0.44000 mm
Stress range = 103.11065 MPa
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<th>Experiment</th>
<th>Displacement</th>
<th>Stress range</th>
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<tbody>
<tr>
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<td>13)</td>
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<td>14)</td>
<td>0.46732 mm</td>
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<td>15)</td>
<td>0.38409 mm</td>
<td>119.98306 MPa</td>
</tr>
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<td>16)</td>
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</tr>
<tr>
<td>17)</td>
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<td>Stress range</td>
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<td>125.09290 MPa</td>
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<td>0.38189 mm</td>
<td>121.54916 MPa</td>
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<td>32)</td>
<td>0.47798 mm</td>
<td>100.59616 MPa</td>
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<td>Displacement</td>
<td>Stress range</td>
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<tr>
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<td>42)</td>
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<td>102.36780 MPa</td>
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<td>43)</td>
<td>0.43461 mm</td>
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<td>44)</td>
<td>0.37799 mm</td>
<td>124.54469 MPa</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

<p>| | |</p>
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55) Displacement = 0.5096 mm
Stress range = 99.85507 MPa

54) Displacement = 0.47424 mm
Stress range = 100.72869 MPa

53) Displacement = 0.46330 mm
Stress range = 118.95027 MPa

52) Displacement = 0.39268 mm
Stress range = 101.23132 MPa

51) Displacement = 0.48248 mm
Stress range = 100.45438 MPa

50) Displacement = 0.45792 mm
Stress range = 118.51865 MPa

49) Displacement = 0.39108 mm
Stress range = 101.98046 MPa

48) Displacement = 0.51286 mm
Stress range = 99.16953 MPa

47) Displacement = 0.50096 mm
Stress range = 99.85507 MPa

46) Displacement = 0.47424 mm
Stress range = 100.72869 MPa

45) Displacement = 0.47424 mm
Stress range = 100.72869 MPa

Appendix H - Experiment 3 Pareto Set - Darren Watts - September 2008
67) Displacement = 0.38096 mm
   Stress range = 121.72861 MPa

68) Displacement = 0.47327 mm
   Stress range = 100.96085 MPa

69) Displacement = 0.39672 mm
   Stress range = 117.95955 MPa

70) Displacement = 0.40942 mm
   Stress range = 104.59292 MPa

71) Displacement = 0.50286 mm
   Stress range = 99.24049 MPa

72) Displacement = 0.40608 mm
   Stress range = 105.11146 MPa

73) Displacement = 0.38393 mm
   Stress range = 120.96642 MPa

74) Displacement = 0.48393 mm
   Stress range = 100.21192 MPa

75) Displacement = 0.51844 mm
   Stress range = 98.39651 MPa

76) Displacement = 0.42119 mm
   Stress range = 104.42588 MPa

77) Displacement = 0.46364 mm
   Stress range = 101.21257 MPa
Displacement = 0.48502 mm
Stress range = 100.09071 MPa

Displacement = 0.38508 mm
Stress range = 119.70974 MPa
APPENDIX I – Experiment 4 Pareto Set

Cell Types Used = Solid (Steel), Solid2 (Aluminium) & Void, Pareto Set Size = 30
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<td>4</td>
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<td>99.16783</td>
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<td>6</td>
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<td>7</td>
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<td>10</td>
<td>0.30147</td>
<td>231.67453</td>
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<td>11</td>
<td>0.42612</td>
<td>97.59939</td>
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<tr>
<td>Mass</td>
<td>Stress range</td>
<td></td>
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<tr>
<td>--------</td>
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<td></td>
</tr>
<tr>
<td>0.30424 kg</td>
<td>231.66912 MPa</td>
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<tr>
<td>0.22576 kg</td>
<td>10000000 MPa</td>
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<tr>
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<td>420.94428 MPa</td>
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<td>0.38088 kg</td>
<td>146.33209 MPa</td>
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<td>157.58705 MPa</td>
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<td>0.40258 kg</td>
<td>107.25505 MPa</td>
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<td>0.36749 kg</td>
<td>155.13940 MPa</td>
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<tr>
<td>Experiment</td>
<td>Mass</td>
<td>Stress range</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
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</tr>
<tr>
<td>23)</td>
<td>0.38873 kg</td>
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<td>30)</td>
<td>0.29870 kg</td>
<td>231.73461 MPa</td>
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</table>
APPENDIX J – TET Mesh Time Trials

Experimental Methodology

These trials involved the collection of mesh and solve times, at different mesh resolutions, of four beam geometries of increasing complexity, as shown in Figure J.1.

The mesh fidelity was altered in each case by altering the TET mesh maximum element size scale factor parameter, as illustrated by Figure J.2.
Mesh and FE analysis solve times were calculated for various local and remote computational arrangements. These included two local arrangements, one on a laptop PC and one on a desktop PC, in addition to two remote arrangements. Both of the remote configurations used the aforementioned laptop PC to run AML locally, sending meshing jobs remotely to Patran and analysis jobs remotely to Nastran via the TSCRM and TSXM applications, as detailed in Appendix D. The first of the two remote configurations was conducted at Loughborough University, with the remote jobs sent to the same desktop PC used in the local arrangement. The second remote configuration was conducted at Cambridge University, with remote jobs sent to a single node of a supercomputer installation. The general specifications of the network speeds and computational equipment used are given in Table J.1.

<table>
<thead>
<tr>
<th>Table J.1 – Computer specifications &amp; network speeds.</th>
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<tbody>
<tr>
<td>Laptop PC</td>
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<tr>
<td>Number of CPUs</td>
</tr>
<tr>
<td>CPU Speed/Type</td>
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<tr>
<td>RAM</td>
</tr>
<tr>
<td>Operating System</td>
</tr>
<tr>
<td>Network speed at Loughborough = 100 MB/sec</td>
</tr>
<tr>
<td>Network speed at Cambridge = 1 GB/sec</td>
</tr>
</tbody>
</table>

Results & Discussion

Unfortunately, despite extensive efforts, the second remote configuration using the supercomputer node could not be established successfully and therefore could not be used in these trials. This was due to difficulties arising from running multiple operating system platforms on the laptop PC and supercomputer node. AML running on the laptop PC with a 32bit Windows operating system, could not communicate via TSCRM and TSXM to the supercomputer node running a 64bit Linux operating system. Currently, Linux 64bit platforms are unsupported by TechnoSoft, so until this changes it will not be possible for the supercomputer node to be used for this research. The TET mesh time trial results of the remaining three successful computer configurations are displayed in Figure J.3.
Figure J.3 shows clearly how the combined meshing and analysis times for the first beam considered (solid and void) escalates exponentially with respect to TET mesh fidelity effects on numbers of nodes and elements (right) for each of the four beam geometries considered.
fidelity, as does the number of nodes and elements created in the meshed beam representation. Figure J.3 also shows that the overall times required to mesh the second and third beams was slightly higher than the first, but the time required for the fourth beam was noticeably higher in all instances. This extra time required was due to the complexity and intricacy of the various cell structures considered in this case. Additionally, the second, third and fourth plots exhibit uncommon behaviour to the first, in that anomalies occur in their curves. These anomalies occur when the AML system is unable to create a solid TET mesh for the particular input parameters (specific geometry and element size scale factor combination), resulting in a surface mesh being created instead, which is of no interest to this research. Surface meshes do not contain any solid elements (3D), meaning that the anomalies can be readily identified in the plots to the right of Figure J.3 (highlighted pink), and hence ignored.

Figure J.3 indicates the meshing and analysis steps were performed quickest using the local desktop PC arrangement (blue) for all four beam geometries, as expected due to the higher machine specifications. The second fastest arrangement was the local laptop PC (blue) with the remote desktop arrangement (green) being the slowest in all cases, despite using the more powerful desktop PC for executing the remote jobs. The time differences between the locally and remotely executed Patran and Nastran jobs is clearly visible in Figure J.4, which displays the CPU Usage History from Windows Task Manager for a single case.
Figure 4.4a illustrates how the locally executed Patran and Nastran jobs appear as a single almost seamless operation, highlighted by the single red ring. Figure 4.4b demonstrates the remotely executed Patran and Nastran jobs take considerably longer, with the overall process being broken down into six separate steps as follows:

Step 1 – Patran is launched and run remotely to create the automated TET mesh for the current beam topology.

Step 2 – A time delay whilst waiting for the remote Patran job to finish being created and placed in the necessary system directory.

Step 3 – Data transfer across the network of the completed mesh neutral file (*.out).

Step 4 – Nastran is launched and run remotely to solve the FE equations for the current mesh file.

Step 5 – A second time delay, this time whilst waiting for the Nastran job to finish creating the results file (*.f06) and placing it in the necessary system directory.

Step 6 – A second data transfer stage, this time the FEA results file (*.f06) is transferred back across the network from the remote PC to the local instance of AML.

Obviously steps 2 and 5 (highlighted by the yellow rings) cause delays whilst the status of the pending Patran and Nastran jobs are checked, which is usually every three seconds. Additionally, the transferral across the network of the two files created by the remotely executed jobs (steps 3 and 6 highlighted in blue), significantly slow the overall process. The speed of these data transfer steps is also subject to network activity at the time of transfer which will vary over time.

Conclusions

This investigation of TET mesh fidelity effects on computational times required for the meshing and analysis of executable jobs has shown that the fastest result was achieved using the local desktop PC computational configuration. In addition, these trials showed that for more complex beam geometries, it is not always possible to create a solid TET mesh for a given mesh size. The main finding of these trials was that the time required to mesh and solve the sample beam geometries considered, rose exponentially with respect to higher fidelity mesh sizes. Based on these results, a coarse TET mesh is recommended for the advanced experiments to reduce computation.
APPENDIX K – Experiment 9 Pareto Set

Cell Types Used = Solid & Void, Pareto Set Size = 93
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<td>38.04297 MPa</td>
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<td>37.93227 MPa</td>
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46) Mass = 0.05540 kg
Displacement = 1.76252 mm
Stress range = 220.96957 MPa

47) Mass = 0.06094 kg
Displacement = 0.38589 mm
Stress range = 92.68632 MPa

48) Mass = 0.12465 kg
Displacement = 0.06646 mm
Stress range = 42.52934 MPa

49) Mass = 0.07756 kg
Displacement = 0.15476 mm
Stress range = 47.72876 MPa

50) Mass = 0.08310 kg
Displacement = 0.11592 mm
Stress range = 44.82668 MPa

51) Mass = 0.06094 kg
Displacement = 0.31518 mm
Stress range = 105.60609 MPa

52) Mass = 0.11080 kg
Displacement = 0.08660 mm
Stress range = 37.74558 MPa

53) Mass = 0.08033 kg
Displacement = 0.15434 mm
Stress range = 44.23720 MPa

54) Mass = 0.11911 kg
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A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

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APPENDIX L – Experiment 12 Pareto Set

Cell Types Used = Solid, D50 & Void, Pareto Set Size = 120
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Mass = 0.07063 kg
Displacement = 0.23655 mm
Stress range = 219.40982 MPa

Mass = 0.11495 kg
Displacement = 0.07514 mm
Stress range = 69.98816 MPa

Mass = 0.11495 kg
Displacement = 0.07757 mm
Stress range = 39.71755 MPa

Mass = 0.08587 kg
Displacement = 0.17605 mm
Stress range = 123.04573 MPa

Mass = 0.07340 kg
Displacement = 0.21637 mm
Stress range = 186.46293 MPa

Mass = 0.09279 kg
Displacement = 0.11473 mm
Stress range = 95.43710 MPa

Mass = 0.05263 kg
Displacement = 0.69537 mm
Stress range = 569.09998 MPa

Mass = 0.12465 kg
Displacement = 0.06757 mm
Stress range = 57.72957 MPa

Mass = 0.05955 kg
Displacement = 0.84074 mm
Stress range = 300.33244 MPa

Mass = 0.12188 kg
Displacement = 0.07283 mm
Stress range = 39.51741 MPa
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Appendix L – Experiment 12 Pareto Set

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## A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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<tr>
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Appendix L - Experiment 12 Pareto Set - L10 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

100) Mass = 0.13711 kg, Displacement = 0.06279 mm, Stress range = 55.52524 MPa

101) Mass = 0.06509 kg, Displacement = 0.50767 mm, Stress range = 277.11838 MPa

102) Mass = 0.05678 kg, Displacement = 0.86798 mm, Stress range = 305.18072 MPa

103) Mass = 0.02078 kg, Displacement = 10000000 mm, Stress range = 10000000 MPa

104) Mass = 0.04570 kg, Displacement = 1.76847 mm, Stress range = 595.65252 MPa

105) Mass = 0.04432 kg, Displacement = 2.07914 mm, Stress range = 659.07084 MPa

106) Mass = 0.09695 kg, Displacement = 0.11516 mm, Stress range = 58.56744 MPa

107) Mass = 0.13296 kg, Displacement = 0.06335 mm, Stress range = 56.50001 MPa

108) Mass = 0.05124 kg, Displacement = 0.79577 mm, Stress range = 405.33527 MPa

109) Mass = 0.10803 kg, Displacement = 0.08279 mm, Stress range = 51.70311 MPa

110) Mass = 0.11218 kg, Displacement = 0.07872 mm, Stress range = 69.89536 MPa

111) Mass = 0.05401 kg
Displacement = 0.72762 mm
Stress range = 425.05978 MPa

112) Mass = 0.12049 kg
Displacement = 0.06953 mm
Stress range = 40.81216 MPa

113) Mass = 0.11634 kg
Displacement = 0.07396 mm
Stress range = 53.34364 MPa

114) Mass = 0.08033 kg
Displacement = 0.17864 mm
Stress range = 202.63122 MPa

115) Mass = 0.11495 kg
Displacement = 0.07311 mm
Stress range = 77.63969 MPa

116) Mass = 0.06232 kg
Displacement = 0.55485 mm
Stress range = 276.6329 MPa

117) Mass = 0.10110 kg
Displacement = 0.09244 mm
Stress range = 57.99252 MPa

118) Mass = 0.13573 kg
Displacement = 0.06251 mm
Stress range = 55.93356 MPa

119) Mass = 0.12049 kg
Displacement = 0.06922 mm
Stress range = 59.30301 MPa

120) Mass = 0.11218 kg
Displacement = 0.07741 mm
Stress range = 70.28607 MPa
APPENDIX M – Experiment 13 Pareto Set

Cell Types Used = D75 & Void, Pareto Set Size = 42
1) Mass = 0.06025 kg  
Displacement = 0.78520 mm  
Stress range = 455.82060 MPa

2) Mass = 0.06648 kg  
Displacement = 0.38243 mm  
Stress range = 201.75478 MPa

3) Mass = 0.07479 kg  
Displacement = 0.19027 mm  
Stress range = 160.63505 MPa

4) Mass = 0.05194 kg  
Displacement = 1.27356 mm  
Stress range = 496.14403 MPa

5) Mass = 0.06440 kg  
Displacement = 0.38747 mm  
Stress range = 208.14423 MPa

6) Mass = 0.07271 kg  
Displacement = 0.21592 mm  
Stress range = 160.66136 MPa

7) Mass = 0.01246 kg  
Displacement = 10000000 mm  
Stress range = 10000000 MPa

8) Mass = 0.07895 kg  
Displacement = 0.16762 mm  
Stress range = 158.05300 MPa

9) Mass = 0.07895 kg  
Displacement = 0.16283 mm  
Stress range = 158.16341 MPa

10) Mass = 0.06856 kg  
Displacement = 0.34322 mm  
Stress range = 188.54586 MPa

11) Mass = 0.07895 kg  
Displacement = 0.18043 mm  
Stress range = 157.81103 MPa
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

Appendix M – Experiment 13 Pareto Set

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Loughborough University

Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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<th>Stress Range (MPa)</th>
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APPENDIX N – Experiment 14 Pareto Set

Cell Types Used = D75, D50, D25 & Void, Pareto Set Size = 242
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Appendix N – Experiment 14 Pareto Set - N2 - Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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<td>16)</td>
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<tr>
<td>17)</td>
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<td>0.24209 mm</td>
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<td>18)</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

34) Mass = 0.06232 kg  
Displacement = 0.23734 mm  
Stress range = 163.28204 MPa

35) Mass = 0.06094 kg  
Displacement = 0.24012 mm  
Stress range = 262.32296 MPa

36) Mass = 0.06579 kg  
Displacement = 0.21575 mm  
Stress range = 178.80780 MPa

37) Mass = 0.07341 kg  
Displacement = 0.22892 mm  
Stress range = 159.64649 MPa

38) Mass = 0.05955 kg  
Displacement = 0.26257 mm  
Stress range = 162.2617 MPa

39) Mass = 0.05955 kg  
Displacement = 0.25853 mm  
Stress range = 162.90618 MPa

40) Mass = 0.05332 kg  
Displacement = 0.31076 mm  
Stress range = 276.96654 MPa

41) Mass = 0.04363 kg  
Displacement = 0.45527 mm  
Stress range = 277.63040 MPa

42) Mass = 0.06025 kg  
Displacement = 0.27896 mm  
Stress range = 160.31654 MPa

43) Mass = 0.04432 kg  
Displacement = 0.43929 mm  
Stress range = 220.25897 MPa

44) Mass = 0.04778 kg  
Displacement = 0.37421 mm  
Stress range = 277.15300 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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Appendix N – Experiment 14 Pareto Set - N6 - Darren Watts – September 2008
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### A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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<td>76</td>
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<td>77</td>
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<td>0.54263 mm</td>
<td>253.88866 MPa</td>
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## Appendix N: Experiment 14 Pareto Set

**Loughborough University**

### Mass Displacement Stress range

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<tr>
<td>79)</td>
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<td>0.35549 mm</td>
<td>192.44157 MPa</td>
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<tr>
<td>80)</td>
<td>0.05055 kg</td>
<td>0.33604 mm</td>
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<td>82)</td>
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<td>1.77793 mm</td>
<td>748.5981 MPa</td>
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<td>83)</td>
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<td>84)</td>
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<td>86)</td>
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<tr>
<td>87)</td>
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<tr>
<td>88)</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

<table>
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<tr>
<td>91)</td>
<td>0.03462 kg</td>
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<td>92)</td>
<td>0.03185 kg</td>
<td>2.01639 mm</td>
<td>927.90776 MPa</td>
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<td>93)</td>
<td>0.03324 kg</td>
<td>1.25958 mm</td>
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<tr>
<td>94)</td>
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<td>0.40734 mm</td>
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<td>95)</td>
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<td>0.97999 mm</td>
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<td>96)</td>
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<td>194.75648 MPa</td>
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<tr>
<td>97)</td>
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<td>98)</td>
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<td>0.26385 mm</td>
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<td>99)</td>
<td>0.03947 kg</td>
<td>0.55404 mm</td>
<td>262.32574 MPa</td>
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Appendix N - Experiment 14 Pareto Set - N10 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

100) Mass = 0.07202 kg
Displacement = 0.19949 mm
Stress range = 162.73713 MPa

101) Mass = 0.05471 kg
Displacement = 0.29561 mm
Stress range = 165.28627 MPa

102) Mass = 0.04917 kg
Displacement = 0.35703 mm
Stress range = 178.86990 MPa

103) Mass = 0.05124 kg
Displacement = 0.33076 mm
Stress range = 162.08728 MPa

104) Mass = 0.06232 kg
Displacement = 0.22760 mm
Stress range = 189.39823 MPa

105) Mass = 0.03809 kg
Displacement = 0.69386 mm
Stress range = 354.69077 MPa

106) Mass = 0.05955 kg
Displacement = 0.25947 mm
Stress range = 162.65692 MPa

107) Mass = 0.03739 kg
Displacement = 0.70660 mm
Stress range = 358.23498 MPa

108) Mass = 0.07202 kg
Displacement = 0.23247 mm
Stress range = 159.85810 MPa

109) Mass = 0.07548 kg
Displacement = 0.21001 mm
Stress range = 160.14708 MPa

110) Mass = 0.07617 kg
Displacement = 0.18533 mm
Stress range = 163.46484 MPa

Appendix N – Experiment 14 Pareto Set

Darren Watts – September 2008
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<tr>
<th>No.</th>
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<th>Displacement</th>
<th>Stress range</th>
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<td>113</td>
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<td>0.54233 mm</td>
<td>254.84009 MPa</td>
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<tr>
<td>114</td>
<td>0.06856 kg</td>
<td>0.21087 mm</td>
<td>163.75174 MPa</td>
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<tr>
<td>115</td>
<td>0.04086 kg</td>
<td>0.52725 mm</td>
<td>258.65198 MPa</td>
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<tr>
<td>116</td>
<td>0.04224 kg</td>
<td>0.51062 mm</td>
<td>233.29400 MPa</td>
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<tr>
<td>117</td>
<td>0.05748 kg</td>
<td>0.25612 mm</td>
<td>163.05106 MPa</td>
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<tr>
<td>118</td>
<td>0.06925 kg</td>
<td>0.20318 mm</td>
<td>277.67032 MPa</td>
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<tr>
<td>119</td>
<td>0.07341 kg</td>
<td>0.19310 mm</td>
<td>276.04065 MPa</td>
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<tr>
<td>120</td>
<td>0.07063 kg</td>
<td>0.20216 mm</td>
<td>162.76528 MPa</td>
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<tr>
<td>121</td>
<td>0.05609 kg</td>
<td>0.26889 mm</td>
<td>163.09477 MPa</td>
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</table>
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

122) Mass = 0.06025 kg
Displacement = 0.24224 mm
Stress range = 163.25412 MPa

123) Mass = 0.05055 kg
Displacement = 0.33965 mm
Stress range = 167.79085 MPa

124) Mass = 0.05678 kg
Displacement = 0.26742 mm
Stress range = 182.44413 MPa

125) Mass = 0.03047 kg
Displacement = 2.69904 mm
Stress range = 1189.29154 MPa

126) Mass = 0.05471 kg
Displacement = 0.36147 mm
Stress range = 162.89568 MPa

127) Mass = 0.04917 kg
Displacement = 0.36147 mm
Stress range = 162.89568 MPa

128) Mass = 0.05609 kg
Displacement = 0.28615 mm
Stress range = 162.28771 MPa

129) Mass = 0.04293 kg
Displacement = 0.48907 mm
Stress range = 239.43687 MPa

130) Mass = 0.03393 kg
Displacement = 1.28404 mm
Stress range = 593.04057 MPa

131) Mass = 0.0457 kg
Displacement = 0.40056 mm
Stress range = 187.40258 MPa

132) Mass = 0.04917 kg
Displacement = 0.35694 mm
Stress range = 179.44310 MPa
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<thead>
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<th>Displacement (mm)</th>
<th>Stress range (MPa)</th>
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<td>0.91599</td>
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<td>136)</td>
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<tr>
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<td>1.58801</td>
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<td>140)</td>
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<td>141)</td>
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<td>143)</td>
<td>0.05401</td>
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<td>183.24911</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

144) Mass = 0.05817 kg  
Displacement = 0.26945 mm  
Stress range = 161.95737 MPa

145) Mass = 0.07617 kg  
Displacement = 0.21058 mm  
Stress range = 160.05632 MPa

146) Mass = 0.06786 kg  
Displacement = 0.20822 mm  
Stress range = 193.98854 MPa

147) Mass = 0.07687 kg  
Displacement = 0.20241 mm  
Stress range = 160.18240 MPa

148) Mass = 0.06717 kg  
Displacement = 0.22013 mm  
Stress range = 162.82325 MPa

149) Mass = 0.04986 kg  
Displacement = 0.34358 mm  
Stress range = 299.70083 MPa

150) Mass = 0.03878 kg  
Displacement = 0.70749 mm  
Stress range = 317.22800 MPa

151) Mass = 0.07271 kg  
Displacement = 0.21636 mm  
Stress range = 159.98808 MPa

152) Mass = 0.05817 kg  
Displacement = 0.26568 mm  
Stress range = 162.72927 MPa

153) Mass = 0.06302 kg  
Displacement = 0.23098 mm  
Stress range = 162.43148 MPa

154) Mass = 0.05540 kg  
Displacement = 0.28582 mm  
Stress range = 162.37411 MPa
155) Mass = 0.03255 kg
Displacement = 1.54935 mm
Stress range = 919.16470 MPa

156) Mass = 0.05471 kg
Displacement = 0.29678 mm
Stress range = 161.77686 MPa

157) Mass = 0.03116 kg
Displacement = 2.54202 mm
Stress range = 1174.62509 MPa

158) Mass = 0.07479 kg
Displacement = 0.19176 mm
Stress range = 193.81500 MPa

159) Mass = 0.06440 kg
Displacement = 0.22171 mm
Stress range = 186.75221 MPa

160) Mass = 0.07964 kg
Displacement = 0.18502 mm
Stress range = 183.49479 MPa

161) Mass = 0.06371 kg
Displacement = 0.22617 mm
Stress range = 162.79053 MPa

162) Mass = 0.07964 kg
Displacement = 0.18149 mm
Stress range = 183.64691 MPa

163) Mass = 0.05955 kg
Displacement = 0.24924 mm
Stress range = 160.36970 MPa

164) Mass = 0.06371 kg
Displacement = 0.22553 mm
Stress range = 177.45428 MPa

165) Mass = 0.06579 kg
Displacement = 0.25733 mm
Stress range = 160.3697 MPa
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<th>Stress range</th>
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<tr>
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<td>178</td>
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<td>179</td>
<td>0.06509 kg</td>
<td>0.22221 mm</td>
<td>163.46108 MPa</td>
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<td>180</td>
<td>0.06302 kg</td>
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<td>163.22946 MPa</td>
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<td>181</td>
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<td>0.23473 mm</td>
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<td>182</td>
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<td>183</td>
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<td>184</td>
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<td>0.23643 mm</td>
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<tr>
<td>187</td>
<td>0.02077 kg</td>
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<td>10000000 MPa</td>
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</table>
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

188) Mass = 0.03878 kg
Displacement = 0.62578 mm
Stress range = 322.05122 MPa

189) Mass = 0.08102 kg
Displacement = 0.18387 mm
Stress range = 164.05799 MPa

190) Mass = 0.03047 kg
Displacement = 2.68998 mm
Stress range = 1240.58188 MPa

191) Mass = 0.06856 kg
Displacement = 0.20683 mm
Stress range = 187.28491 MPa

192) Mass = 0.03739 kg
Displacement = 0.75986 mm
Stress range = 332.45962 MPa

193) Mass = 0.06509 kg
Displacement = 0.22426 mm
Stress range = 163.14458 MPa

194) Mass = 0.07618 kg
Displacement = 0.20965 mm
Stress range = 159.92901 MPa

195) Mass = 0.03878 kg
Displacement = 0.69838 mm
Stress range = 319.10894 MPa

196) Mass = 0.06509 kg
Displacement = 0.21659 mm
Stress range = 189.45022 MPa

197) Mass = 0.03462 kg
Displacement = 1.26621 mm
Stress range = 610.70997 MPa

198) Mass = 0.06025 kg
Displacement = 0.25526 mm
Stress range = 162.84033 MPa
199) Mass = 0.07410 kg
   Displacement = 0.21753 mm
   Stress range = 159.86648 MPa

200) Mass = 0.07618 kg
   Displacement = 0.20806 mm
   Stress range = 160.06096 MPa

201) Mass = 0.03393 kg
   Displacement = 1.27235 mm
   Stress range = 595.68329 MPa

202) Mass = 0.06025 kg
   Displacement = 0.25682 mm
   Stress range = 162.5309 MPa

203) Mass = 0.03462 kg
   Displacement = 1.02030 mm
   Stress range = 810.52997 MPa

204) Mass = 0.04224 kg
   Displacement = 0.50384 mm
   Stress range = 276.80918 MPa

205) Mass = 0.06509 kg
   Displacement = 0.21749 mm
   Stress range = 183.13306 MPa

206) Mass = 0.06025 kg
   Displacement = 0.24916 mm
   Stress range = 162.37721 MPa

207) Mass = 0.04155 kg
   Displacement = 0.53216 mm
   Stress range = 243.81896 MPa

208) Mass = 0.05540 kg
   Displacement = 0.28058 mm
   Stress range = 162.51789 MPa

209) Mass = 0.06302 kg
   Displacement = 0.22433 mm
   Stress range = 192.18781 MPa

Appendix N - Experiment 14 Pareto Set - N20 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

Appendix N - Experiment 14 Pareto Set - N21 - Darren Watts - September 2008
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing’s Design Freedom

221) Mass = 0.04293 kg
        Displacement = 0.50046 mm
        Stress range = 272.05405 MPa

222) Mass = 0.06648 kg
        Displacement = 0.20975 mm
        Stress range = 184.44665 MPa

223) Mass = 0.05817 kg
        Displacement = 0.30695 mm
        Stress range = 159.87799 MPa

224) Mass = 0.05055 kg
        Displacement = 0.34271 mm
        Stress range = 163.59756 MPa

225) Mass = 0.05332 kg
        Displacement = 0.30538 mm
        Stress range = 162.13876 MPa

226) Mass = 0.06232 kg
        Displacement = 0.23318 mm
        Stress range = 163.43233 MPa

227) Mass = 0.05817 kg
        Displacement = 0.25575 mm
        Stress range = 163.36793 MPa

228) Mass = 0.05955 kg
        Displacement = 0.26164 mm
        Stress range = 162.63762 MPa

229) Mass = 0.04709 kg
        Displacement = 0.38071 mm
        Stress range = 305.31662 MPa

230) Mass = 0.06648 kg
        Displacement = 0.21276 mm
        Stress range = 177.78281 MPa

231) Mass = 0.07271 kg
        Displacement = 0.23123 mm
        Stress range = 159.57663 MPa

<table>
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<tr>
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<th>Mass</th>
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<th>Stress range</th>
</tr>
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<td>0.23288 mm</td>
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<td>0.05194 kg</td>
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<td>162.58995 MPa</td>
</tr>
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<td>234)</td>
<td>0.06579 kg</td>
<td>0.27870 mm</td>
<td>158.95477 MPa</td>
</tr>
<tr>
<td>235)</td>
<td>0.05955 kg</td>
<td>0.29226 mm</td>
<td>159.93718 MPa</td>
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<td>236)</td>
<td>0.04847 kg</td>
<td>0.35843 mm</td>
<td>275.55566 MPa</td>
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<td>237)</td>
<td>0.06925 kg</td>
<td>0.21650 mm</td>
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<td>238)</td>
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<td>239)</td>
<td>0.06509 kg</td>
<td>0.21608 mm</td>
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<tr>
<td>240)</td>
<td>0.06371 kg</td>
<td>0.22590 mm</td>
<td>163.75855 MPa</td>
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<td>241)</td>
<td>0.04986 kg</td>
<td>0.34783 mm</td>
<td>265.15570 MPa</td>
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<tr>
<td>242)</td>
<td>0.04917 kg</td>
<td>0.35504 mm</td>
<td>303.05312 MPa</td>
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APPENDIX O – Experiment 15 Pareto Set

Cell Types Used = B75 & Void, Pareto Set = 75
<table>
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<td>155.27158 MPa</td>
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<td>0.37223 mm</td>
<td>241.24197 MPa</td>
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<td>3</td>
<td>0.09349 kg</td>
<td>0.13885 mm</td>
<td>83.05502 MPa</td>
</tr>
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<td>4</td>
<td>0.08102 kg</td>
<td>0.14461 mm</td>
<td>108.61877 MPa</td>
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<td>5</td>
<td>0.08518 kg</td>
<td>0.15775 mm</td>
<td>86.62905 MPa</td>
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<td>6</td>
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<td>0.12722 mm</td>
<td>86.27339 MPa</td>
</tr>
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# A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

<table>
<thead>
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34) Mass = 0.08518 kg
    Displacement = 0.13339 mm
    Stress range = 91.77766 MPa

35) Mass = 0.08726 kg
    Displacement = 0.13319 mm
    Stress range = 90.26346 MPa

36) Mass = 0.07895 kg
    Displacement = 0.15035 mm
    Stress range = 95.91221 MPa

37) Mass = 0.07479 kg
    Displacement = 0.17224 mm
    Stress range = 118.92173 MPa

38) Mass = 0.06233 kg
    Displacement = 0.50271 mm
    Stress range = 288.86887 MPa

39) Mass = 0.05609 kg
    Displacement = 0.91653 mm
    Stress range = 481.20713 MPa

40) Mass = 0.05609 kg
    Displacement = 0.96028 mm
    Stress range = 406.23642 MPa

41) Mass = 0.08518 kg
    Displacement = 0.15628 mm
    Stress range = 86.99834 MPa

42) Mass = 0.08310 kg
    Displacement = 0.14052 mm
    Stress range = 87.73579 MPa

43) Mass = 0.08726 kg
    Displacement = 0.13459 mm
    Stress range = 89.55648 MPa

44) Mass = 0.06856 kg
    Displacement = 0.33520 mm
    Stress range = 211.72720 MPa
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56) Mass = 0.09556 kg
Displacement = 0.11851 mm
Stress range = 90.15326 MPa

57) Mass = 0.02701 kg
Displacement = 17.15687 mm
Stress range = 2024.65351 MPa

58) Mass = 0.09556 kg
Displacement = 0.11940 mm
Stress range = 86.98670 MPa

59) Mass = 0.08726 kg
Displacement = 0.12847 mm
Stress range = 91.15339 MPa

60) Mass = 0.07895 kg
Displacement = 0.14802 mm
Stress range = 100.11112 MPa

61) Mass = 0.07687 kg
Displacement = 0.15725 mm
Stress range = 97.32548 MPa

62) Mass = 0.10180 kg
Displacement = 0.10880 mm
Stress range = 89.33939 MPa

63) Mass = 0.08933 kg
Displacement = 0.12265 mm
Stress range = 90.27805 MPa

64) Mass = 0.08102 kg
Displacement = 0.14809 mm
Stress range = 92.40589 MPa

65) Mass = 0.05194 kg
Displacement = 1.38966 mm
Stress range = 394.08095 MPa

66) Mass = 0.09556 kg
Displacement = 0.11816 mm
Stress range = 90.98326 MPa
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<td>75)</td>
<td>0.08102 kg</td>
<td>0.14732 mm</td>
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APPENDIX P – Experiment 16 Pareto Set

Cell Types Used = B75, B50, B25 & Void, Pareto Set = 234
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

1) Mass = 0.06163 kg  
   Displacement = 0.28282 mm  
   Stress range = 176.39265 MPa

2) Mass = 0.04640 kg  
   Displacement = 0.46783 mm  
   Stress range = 251.98887 MPa

3) Mass = 0.06302 kg  
   Displacement = 0.27735 mm  
   Stress range = 171.56646 MPa

4) Mass = 0.04432 kg  
   Displacement = 0.52520 mm  
   Stress range = 309.37355 MPa

5) Mass = 0.05124 kg  
   Displacement = 0.38659 mm  
   Stress range = 228.93267 MPa

6) Mass = 0.05471 kg  
   Displacement = 0.33747 mm  
   Stress range = 210.09861 MPa

7) Mass = 0.04570 kg  
   Displacement = 0.47839 mm  
   Stress range = 252.52924 MPa

8) Mass = 0.04293 kg  
   Displacement = 0.58098 mm  
   Stress range = 270.55144 MPa

9) Mass = 0.05609 kg  
   Displacement = 0.32783 mm  
   Stress range = 198.72267 MPa

10) Mass = 0.03324 kg  
    Displacement = 20.27185 mm  
    Stress range = 6187.593 MPa

11) Mass = 0.04501 kg  
    Displacement = 0.51501 mm  
    Stress range = 230.08279 MPa
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Appendix P – Experiment 16 Pareto Set

Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

23) Mass = 0.06717 kg
   Displacement = 0.25259 mm
   Stress range = 140.11819 MPa

24) Mass = 0.06232 kg
   Displacement = 0.28253 mm
   Stress range = 175.10448 MPa

25) Mass = 0.04501 kg
   Displacement = 0.48704 mm
   Stress range = 263.29575 MPa

26) Mass = 0.05678 kg
   Displacement = 0.32519 mm
   Stress range = 172.25099 MPa

27) Mass = 0.06232 kg
   Displacement = 0.28801 mm
   Stress range = 162.70055 MPa

28) Mass = 0.06163 kg
   Displacement = 0.28265 mm
   Stress range = 195.37807 MPa

29) Mass = 0.04986 kg
   Displacement = 0.40557 mm
   Stress range = 197.82479 MPa

30) Mass = 0.04293 kg
    Displacement = 0.56817 mm
    Stress range = 301.71624 MPa

31) Mass = 0.06371 kg
    Displacement = 0.27789 mm
    Stress range = 157.59839 MPa

32) Mass = 0.04709 kg
    Displacement = 0.45394 mm
    Stress range = 230.61566 MPa

33) Mass = 0.04501 kg
    Displacement = 0.47992 mm
    Stress range = 267.15581 MPa
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<td>43</td>
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<td>44</td>
<td>0.06094 kg</td>
<td>0.28577 mm</td>
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45) Mass = 0.06994 kg
Displacement = 0.21949 mm
Stress range = 171.13704 MPa

46) Mass = 0.03878 kg
Displacement = 0.89938 mm
Stress range = 496.05802 MPa

47) Mass = 0.03324 kg
Displacement = 20.10685 mm
Stress range = 6730.65088 MPa

48) Mass = 0.07825 kg
Displacement = 0.20674 mm
Stress range = 148.59106 MPa

49) Mass = 0.05817 kg
Displacement = 0.30459 mm
Stress range = 178.32979 MPa

50) Mass = 0.07687 kg
Displacement = 0.20142 mm
Stress range = 174.21251 MPa

51) Mass = 0.07271 kg
Displacement = 0.22010 mm
Stress range = 151.28326 MPa

52) Mass = 0.06786 kg
Displacement = 0.23462 mm
Stress range = 158.05736 MPa

53) Mass = 0.08310 kg
Displacement = 0.18933 mm
Stress range = 165.90758 MPa

54) Mass = 0.03116 kg
Displacement = 85.23432 mm
Stress range = 10975.9199 MPa

55) Mass = 0.07410 kg
Displacement = 0.21883 mm
Stress range = 152.04241 MPa
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Appendix P - Experiment 16 Pareto Set

- P8 -

Darren Watts – September 2008
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<tr>
<th>Mass</th>
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<tr>
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<tr>
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<td>0.45932 mm</td>
<td>221.87199 MPa</td>
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<td>0.05817 kg</td>
<td>0.30592 mm</td>
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<tr>
<td>0.06856 kg</td>
<td>0.23658 mm</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

89) Mass = 0.04709 kg  
Displacement = 0.45181 mm  
Stress range = 272.67714 MPa

90) Mass = 0.06579 kg  
Displacement = 0.25385 mm  
Stress range = 166.61131 MPa

91) Mass = 0.06302 kg  
Displacement = 0.26936 mm  
Stress range = 180.77325 MPa

92) Mass = 0.04570 kg  
Displacement = 0.48369 mm  
Stress range = 228.39368 MPa

93) Mass = 0.06786 kg  
Displacement = 0.24931 mm  
Stress range = 140.24399 MPa

94) Mass = 0.03393 kg  
Displacement = 1.72232 mm  
Stress range = 978.88140 MPa

95) Mass = 0.06232 kg  
Displacement = 0.28551 mm  
Stress range = 166.47720 MPa

96) Mass = 0.06994 kg  
Displacement = 0.22826 mm  
Stress range = 149.03579 MPa

97) Mass = 0.04778 kg  
Displacement = 0.44036 mm  
Stress range = 269.00511 MPa

98) Mass = 0.06648 kg  
Displacement = 0.24804 mm  
Stress range = 160.10497 MPa

99) Mass = 0.06717 kg  
Displacement = 0.24447 mm  
Stress range = 147.72441 MPa
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<td>0.26133 mm</td>
<td>159.12398 MPa</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

111) Mass = 0.05886 kg
Displacement = 0.30264 mm
Stress range = 198.75274 MPa

112) Mass = 0.06786 kg
Displacement = 0.24444 mm
Stress range = 149.02155 MPa

113) Mass = 0.07964 kg
Displacement = 0.22404 mm
Stress range = 127.95122 MPa

114) Mass = 0.05886 kg
Displacement = 0.32000 mm
Stress range = 168.90306 MPa

115) Mass = 0.04778 kg
Displacement = 0.44743 mm
Stress range = 249.91003 MPa

116) Mass = 0.04847 kg
Displacement = 0.44258 mm
Stress range = 200.20947 MPa

117) Mass = 0.03878 kg
Displacement = 0.86131 mm
Stress range = 678.43491 MPa

118) Mass = 0.07271 kg
Displacement = 0.23209 mm
Stress range = 137.61279 MPa

119) Mass = 0.06440 kg
Displacement = 0.25679 mm
Stress range = 161.13256 MPa

120) Mass = 0.06509 kg
Displacement = 0.25440 mm
Stress range = 142.69110 MPa

121) Mass = 0.04224 kg
Displacement = 0.58237 mm
Stress range = 267.51603 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

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<td>124</td>
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<td>125</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

144) Mass = 0.05332 kg
Displacement = 0.36734 mm
Stress range = 185.90305 MPa

145) Mass = 0.06717 kg
Displacement = 0.23993 mm
Stress range = 159.59766 MPa

146) Mass = 0.05609 kg
Displacement = 0.31927 mm
Stress range = 200.41573 MPa

147) Mass = 0.08033 kg
Displacement = 0.23993 mm
Stress range = 140.62141 MPa

148) Mass = 0.06440 kg
Displacement = 0.25823 mm
Stress range = 164.09475 MPa

149) Mass = 0.04778 kg
Displacement = 0.44725 mm
Stress range = 226.39492 MPa

150) Mass = 0.04570 kg
Displacement = 0.47333 mm
Stress range = 263.35003 MPa

151) Mass = 0.04086 kg
Displacement = 0.60166 mm
Stress range = 269.43084 MPa

152) Mass = 0.06371 kg
Displacement = 0.26886 mm
Stress range = 192.82852 MPa

153) Mass = 0.04778 kg
Displacement = 0.44601 mm
Stress range = 265.05279 MPa

154) Mass = 0.06371 kg
Displacement = 0.26886 mm
Stress range = 192.82852 MPa
155) Mass = 0.03601 kg
    Displacement = 1.16600 mm
    Stress range = 706.67891 MPa

156) Mass = 0.04570 kg
    Displacement = 0.46151 mm
    Stress range = 259.03529 MPa

157) Mass = 0.06925 kg
    Displacement = 0.23620 mm
    Stress range = 145.43194 MPa

158) Mass = 0.07894 kg
    Displacement = 0.20025 mm
    Stress range = 150.04044 MPa

159) Mass = 0.04432 kg
    Displacement = 0.49261 mm
    Stress range = 268.95883 MPa

160) Mass = 0.06440 kg
    Displacement = 0.26506 mm
    Stress range = 170.77033 MPa

161) Mass = 0.05194 kg
    Displacement = 0.37744 mm
    Stress range = 202.75317 MPa

162) Mass = 0.06232 kg
    Displacement = 0.28109 mm
    Stress range = 156.64027 MPa

163) Mass = 0.07687 kg
    Displacement = 0.20619 mm
    Stress range = 164.03358 MPa

164) Mass = 0.05124 kg
    Displacement = 0.38812 mm
    Stress range = 226.94069 MPa

165) Mass = 0.06717 kg
    Displacement = 0.23949 mm
    Stress range = 156.03811 MPa
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<td>138.37752 MPa</td>
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<td>168</td>
<td>0.06302</td>
<td>0.28081</td>
<td>162.16321 MPa</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

Appendix P - Experiment 16 Pareto Set  

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<td>0.27264 mm</td>
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<td>0.04640 kg</td>
<td>0.46072 mm</td>
<td>304.71769 MPa</td>
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<tr>
<td>0.06232 kg</td>
<td>0.27800 mm</td>
<td>175.09702 MPa</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

188) Mass = 0.04363 kg
Displacement = 0.54063 mm
Stress range = 243.27365 MPa

189) Mass = 0.04847 kg
Displacement = 0.43067 mm
Stress range = 225.55167 MPa

190) Mass = 0.07133 kg
Displacement = 0.22466 mm
Stress range = 149.02792 MPa

191) Mass = 0.06786 kg
Displacement = 0.24575 mm
Stress range = 142.06492 MPa

192) Mass = 0.06856 kg
Displacement = 0.23641 mm
Stress range = 150.61851 MPa

193) Mass = 0.04570 kg
Displacement = 0.46738 mm
Stress range = 254.28266 MPa

194) Mass = 0.04640 kg
Displacement = 0.46355 mm
Stress range = 227.93066 MPa

195) Mass = 0.06925 kg
Displacement = 0.23653 mm
Stress range = 139.47458 MPa

196) Mass = 0.05748 kg
Displacement = 0.31010 mm
Stress range = 181.54614 MPa

197) Mass = 0.06371 kg
Displacement = 0.27159 mm
Stress range = 165.64211 MPa

198) Mass = 0.01870 kg
Displacement = 10000000 mm
Stress range = 10000000 MPa
### Appendix P - Experiment 16 Pareto Set

<table>
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<tr>
<th>Mass</th>
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<th>Stress range</th>
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<td>0.45450 mm</td>
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<td>0.27268 mm</td>
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<td>185.83114 MPa</td>
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<td>0.49742 mm</td>
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<td>0.07825 kg</td>
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<td>0.03255 kg</td>
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<td>6855.54590 MPa</td>
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<tr>
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<td>0.44661 mm</td>
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<tr>
<td>0.06094 kg</td>
<td>0.28589 mm</td>
<td>171.39403 MPa</td>
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Appendix P – Experiment 16 Pareto Set

<table>
<thead>
<tr>
<th>Mass</th>
<th>Displacement</th>
<th>Stress range</th>
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<tr>
<td>0.07687 kg</td>
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<td>0.2058845 mm</td>
<td>6492.01416 MPa</td>
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<tr>
<td>0.07617 kg</td>
<td>0.21056 mm</td>
<td>151.19731 MPa</td>
</tr>
<tr>
<td>0.05124 kg</td>
<td>0.41019 mm</td>
<td>191.78929 MPa</td>
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<td>0.54770 mm</td>
<td>288.85365 MPa</td>
</tr>
<tr>
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<td>0.24665 mm</td>
<td>141.35755 MPa</td>
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<tr>
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<td>221.67636 MPa</td>
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<td>0.45391 mm</td>
<td>266.30266 MPa</td>
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<td>0.23583 mm</td>
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<tr>
<td>0.06786 kg</td>
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<td>155.92408 MPa</td>
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<tr>
<td>0.07756 kg</td>
<td>0.20028 mm</td>
<td>170.67106 MPa</td>
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</table>
A Genetic Algorithm Based Topology Optimisation Approach
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Mass = 0.06925 kg
Displacement = 0.22453 mm
Stress range = 183.20559 MPa

Mass = 0.06925 kg
Displacement = 0.22455 mm
Stress range = 173.27817 MPa

Mass = 0.06440 kg
Displacement = 0.26318 mm
Stress range = 175.74861 MPa

Mass = 0.03185 kg
Displacement = 22.78723 mm
Stress range = 6554.96191 MPa

Mass = 0.07894 kg
Displacement = 0.19284 mm
Stress range = 174.78821 MPa

Mass = 0.07617 kg
Displacement = 0.20900 mm
Stress range = 178.33423 MPa

Mass = 0.03947 kg
Displacement = 0.73203 mm
Stress range = 653.33288 MPa

Mass = 0.05124 kg
Displacement = 0.39594 mm
Stress range = 195.96346 MPa

Mass = 0.07617 kg
Displacement = 0.20764 mm
Stress range = 199.11852 MPa

Mass = 0.07617 kg
Displacement = 0.22682 mm
Stress range = 131.48993 MPa

Mass = 0.04709 kg
Displacement = 0.46149 mm
Stress range = 229.89230 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

232) Mass = 0.06648 kg
Displacement = 0.25269 mm
Stress range = 143.77865 MPa

233) Mass = 0.06094 kg
Displacement = 0.28610 mm
Stress range = 190.16225 MPa

234) Mass = 0.05471 kg
Displacement = 0.33183 mm
Stress range = 204.24874 MPa
APPENDIX Q – Experiment 17 Pareto Set

Cell Types Used = D75, B50, D25 & Void, Pareto Set Size = 170

[Diagrams showing the Pareto set for different stress ranges and mass values]
<table>
<thead>
<tr>
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<th>Mass</th>
<th>Displacement</th>
<th>Stress range</th>
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<tbody>
<tr>
<td>1</td>
<td>0.04432 kg</td>
<td>0.55592 mm</td>
<td>326.95599 MPa</td>
</tr>
<tr>
<td>2</td>
<td>0.04709 kg</td>
<td>0.51485 mm</td>
<td>306.24200 MPa</td>
</tr>
<tr>
<td>3</td>
<td>0.04986 kg</td>
<td>0.48857 mm</td>
<td>319.52334 MPa</td>
</tr>
<tr>
<td>4</td>
<td>0.04016 kg</td>
<td>0.66794 mm</td>
<td>369.22703 MPa</td>
</tr>
<tr>
<td>5</td>
<td>0.04986 kg</td>
<td>0.47766 mm</td>
<td>483.19238 MPa</td>
</tr>
<tr>
<td>6</td>
<td>0.05609 kg</td>
<td>0.33929 mm</td>
<td>312.13572 MPa</td>
</tr>
<tr>
<td>7</td>
<td>0.04986 kg</td>
<td>0.50174 mm</td>
<td>304.27069 MPa</td>
</tr>
<tr>
<td>8</td>
<td>0.05332 kg</td>
<td>0.38415 mm</td>
<td>302.64090 MPa</td>
</tr>
<tr>
<td>9</td>
<td>0.04293 kg</td>
<td>0.58607 mm</td>
<td>311.30024 MPa</td>
</tr>
<tr>
<td>10</td>
<td>0.04363 kg</td>
<td>0.55847 mm</td>
<td>352.89144 MPa</td>
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<tr>
<td>11</td>
<td>0.03601 kg</td>
<td>0.79579 mm</td>
<td>388.51659 MPa</td>
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</table>
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

12) Mass = 0.02908 kg
Displacement = 10000000 mm
Stress range = 10000000 MPa

13) Mass = 0.05263 kg
Displacement = 0.42386 mm
Stress range = 288.53928 MPa

14) Mass = 0.04917 kg
Displacement = 0.49959 mm
Stress range = 352.39170 MPa

15) Mass = 0.04570 kg
Displacement = 0.52602 mm
Stress range = 325.07627 MPa

16) Mass = 0.04432 kg
Displacement = 0.55253 mm
Stress range = 348.25879 MPa

17) Mass = 0.04293 kg
Displacement = 0.59304 mm
Stress range = 311.1321 MPa

18) Mass = 0.04986 kg
Displacement = 0.49722 mm
Stress range = 343.19711 MPa

19) Mass = 0.04917 kg
Displacement = 0.49748 mm
Stress range = 345.01967 MPa

20) Mass = 0.04778 kg
Displacement = 0.52517 mm
Stress range = 304.82111 MPa

21) Mass = 0.04293 kg
Displacement = 0.58292 mm
Stress range = 322.00003 MPa

22) Mass = 0.03532 kg
Displacement = 0.81229 mm
Stress range = 386.80181 MPa
23) Mass = 0.05263 kg
Displacement = 0.42376 mm
Stress range = 287.70216 MPa

24) Mass = 0.03462 kg
Displacement = 0.79721 mm
Stress range = 416.34107 MPa

25) Mass = 0.04363 kg
Displacement = 0.59125 mm
Stress range = 284.32951 MPa

26) Mass = 0.04640 kg
Displacement = 0.53943 mm
Stress range = 294.14577 MPa

27) Mass = 0.06302 kg
Displacement = 0.27053 mm
Stress range = 167.73868 MPa

28) Mass = 0.05748 kg
Displacement = 0.32319 mm
Stress range = 303.13668 MPa

29) Mass = 0.04501 kg
Displacement = 0.55247 mm
Stress range = 306.41765 MPa

30) Mass = 0.04016 kg
Displacement = 0.65592 mm
Stress range = 370.26355 MPa

31) Mass = 0.06509 kg
Displacement = 0.25796 mm
Stress range = 154.88799 MPa

32) Mass = 0.04086 kg
Displacement = 0.65555 mm
Stress range = 337.73881 MPa

33) Mass = 0.04501 kg
Displacement = 0.54789 mm
Stress range = 345.69971 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

34) Mass = 0.05540 kg
Displacement = 0.34328 mm
Stress range = 251.29845 MPa

35) Mass = 0.03809 kg
Displacement = 0.73412 mm
Stress range = 388.99891 MPa

36) Mass = 0.05124 kg
Displacement = 0.44522 mm
Stress range = 302.57192 MPa

37) Mass = 0.04986 kg
Displacement = 0.47010 mm
Stress range = 323.46749 MPa

38) Mass = 0.05194 kg
Displacement = 0.40851 mm
Stress range = 316.87082 MPa

39) Mass = 0.06371 kg
Displacement = 0.26433 mm
Stress range = 169.21186 MPa

40) Mass = 0.04086 kg
Displacement = 0.63788 mm
Stress range = 428.85524 MPa

41) Mass = 0.06579 kg
Displacement = 0.25366 mm
Stress range = 149.04775 MPa

42) Mass = 0.05886 kg
Displacement = 0.30604 mm
Stress range = 321.10194 MPa

43) Mass = 0.04570 kg
Displacement = 0.56489 mm
Stress range = 292.9455 MPa

44) Mass = 0.05886 kg
Displacement = 0.30632 mm
Stress range = 319.24189 MPa
45) Mass = 0.03601 kg  
Displacement = 0.75274 mm  
Stress range = 406.67213 MPa

46) Mass = 0.05886 kg  
Displacement = 0.30092 mm  
Stress range = 194.09444 MPa

47) Mass = 0.05678 kg  
Displacement = 0.32321 mm  
Stress range = 308.71086 MPa

48) Mass = 0.05332 kg  
Displacement = 0.36562 mm  
Stress range = 335.24728 MPa

49) Mass = 0.05055 kg  
Displacement = 0.44639 mm  
Stress range = 327.66829 MPa

50) Mass = 0.04570 kg  
Displacement = 0.53005 mm  
Stress range = 351.09991 MPa

51) Mass = 0.04847 kg  
Displacement = 0.51231 mm  
Stress range = 323.63001 MPa

52) Mass = 0.03878 kg  
Displacement = 0.67401 mm  
Stress range = 369.86273 MPa

53) Mass = 0.05609 kg  
Displacement = 0.32698 mm  
Stress range = 318.98096 MPa

54) Mass = 0.05679 kg  
Displacement = 0.38647 mm  
Stress range = 239.71446 MPa

55) Mass = 0.04570 kg  
Displacement = 0.54089 mm  
Stress range = 323.43058 MPa

Appendix Q – Experiment 17 Pareto Set - Q6 - Darren Watts – September 2008
56) | Mass = 0.04086 kg | Displacement = 0.64862 mm | Stress range = 356.82015 MPa
57) | Mass = 0.05263 kg | Displacement = 0.39427 mm | Stress range = 299.31645 MPa
58) | Mass = 0.04570 kg | Displacement = 0.53733 mm | Stress range = 344.60581 MPa
59) | Mass = 0.05678 kg | Displacement = 0.34622 mm | Stress range = 281.16018 MPa
60) | Mass = 0.03462 kg | Displacement = 0.78125 mm | Stress range = 437.44571 MPa
61) | Mass = 0.05332 kg | Displacement = 0.41915 mm | Stress range = 282.73022 MPa
62) | Mass = 0.03739 kg | Displacement = 0.73982 mm | Stress range = 352.49097 MPa
63) | Mass = 0.06025 kg | Displacement = 0.29047 mm | Stress range = 330.45713 MPa
64) | Mass = 0.04293 kg | Displacement = 0.61346 mm | Stress range = 285.37947 MPa
65) | Mass = 0.04917 kg | Displacement = 0.49490 mm | Stress range = 371.89415 MPa
66) | Mass = 0.05332 kg | Displacement = 0.39009 mm | Stress range = 291.00405 MPa
A Genetic Algorithm Based Topology Optimisation Approach
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67) Mass = 0.04501 kg
     Displacement = 0.54327 mm
     Stress range = 343.27494 MPa

68) Mass = 0.05609 kg
     Displacement = 0.33266 mm
     Stress range = 339.28088 MPa

69) Mass = 0.05055 kg
     Displacement = 0.48217 mm
     Stress range = 297.05860 MPa

70) Mass = 0.05609 kg
     Displacement = 0.33752 mm
     Stress range = 319.06668 MPa

71) Mass = 0.05401 kg
     Displacement = 0.41516 mm
     Stress range = 264.84551 MPa

72) Mass = 0.05332 kg
     Displacement = 0.37134 mm
     Stress range = 307.40962 MPa

73) Mass = 0.04432 kg
     Displacement = 0.56587 mm
     Stress range = 297.07229 MPa

74) Mass = 0.03462 kg
     Displacement = 0.85025 mm
     Stress range = 387.30021 MPa

75) Mass = 0.03739 kg
     Displacement = 0.70597 mm
     Stress range = 393.58639 MPa

76) Mass = 0.03532 kg
     Displacement = 0.79514 mm
     Stress range = 398.59875 MPa

77) Mass = 0.05817 kg
     Displacement = 0.31105 mm
     Stress range = 330.90263 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

78) Mass = 0.04778 kg
Displacement = 0.51939 mm
Stress range = 294.10839 MPa

79) Mass = 0.04501 kg
Displacement = 0.53446 mm
Stress range = 361.10231 MPa

80) Mass = 0.03255 kg
Displacement = 1.12402 mm
Stress range = 551.78442 MPa

81) Mass = 0.04709 kg
Displacement = 0.51418 mm
Stress range = 323.06087 MPa

82) Mass = 0.04501 kg
Displacement = 0.54420 mm
Stress range = 367.78158 MPa

83) Mass = 0.05540 kg
Displacement = 0.39902 mm
Stress range = 262.09042 MPa

84) Mass = 0.04155 kg
Displacement = 0.62362 mm
Stress range = 385.71041 MPa

85) Mass = 0.06302 kg
Displacement = 0.27191 mm
Stress range = 157.53289 MPa

86) Mass = 0.06025 kg
Displacement = 0.29251 mm
Stress range = 326.22955 MPa

87) Mass = 0.04501 kg
Displacement = 0.54793 mm
Stress range = 325.22168 MPa

88) Mass = 0.05748 kg
Displacement = 0.31719 mm
Stress range = 318.78454 MPa

Appendix Q – Experiment 17 Pareto Set - Q9 - Darren Watts – September 2008
### Appendix Q – Experiment 17 Pareto Set

#### Mass, Displacement, and Stress Range

<table>
<thead>
<tr>
<th>Model</th>
<th>Mass (kg)</th>
<th>Displacement (mm)</th>
<th>Stress Range (MPa)</th>
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<td>0.65610</td>
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<td>90)</td>
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<td>91)</td>
<td>0.06509</td>
<td>0.25302</td>
<td>162.34324</td>
</tr>
<tr>
<td>92)</td>
<td>0.05955</td>
<td>0.28459</td>
<td>339.02560</td>
</tr>
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<td>93)</td>
<td>0.04709</td>
<td>0.50202</td>
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<td>94)</td>
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<td>96)</td>
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<td>97)</td>
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<td>99)</td>
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<td>0.30912</td>
<td>172.77980</td>
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100) Mass = 0.04363 kg
Displacement = 0.55823 mm
Stress range = 358.73911 MPa

101) Mass = 0.03947 kg
Displacement = 0.71085 mm
Stress range = 365.72526 MPa

102) Mass = 0.04501 kg
Displacement = 0.53820 mm
Stress range = 355.45666 MPa

103) Mass = 0.06232 kg
Displacement = 0.27420 mm
Stress range = 171.52783 MPa

104) Mass = 0.06094 kg
Displacement = 0.30542 mm
Stress range = 167.14482 MPa

105) Mass = 0.03601 kg
Displacement = 0.76420 mm
Stress range = 397.72865 MPa

106) Mass = 0.04640 kg
Displacement = 0.51806 mm
Stress range = 332.85252 MPa

107) Mass = 0.05679 kg
Displacement = 0.32364 mm
Stress range = 299.55354 MPa

108) Mass = 0.05055 kg
Displacement = 0.44530 mm
Stress range = 330.3433 MPa

109) Mass = 0.04155 kg
Displacement = 0.63903 mm
Stress range = 348.16274 MPa

110) Mass = 0.05263 kg
Displacement = 0.42670 mm
Stress range = 251.02151 MPa
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<thead>
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<th>Experiment</th>
<th>Mass</th>
<th>Displacement</th>
<th>Stress range</th>
</tr>
</thead>
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</tr>
<tr>
<td>112)</td>
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<td>0.74072 mm</td>
<td>409.28177 MPa</td>
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<td>113)</td>
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<td>0.58871 mm</td>
<td>284.11029 MPa</td>
</tr>
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<td>114)</td>
<td>0.03670 kg</td>
<td>0.81100 mm</td>
<td>386.50669 MPa</td>
</tr>
<tr>
<td>115)</td>
<td>0.04778 kg</td>
<td>0.52707 mm</td>
<td>279.92357 MPa</td>
</tr>
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<td>116)</td>
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<td>0.41358 mm</td>
<td>293.23541 MPa</td>
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<tr>
<td>117)</td>
<td>0.04709 kg</td>
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<td>118)</td>
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<td>0.03393 kg</td>
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<td>426.29828 MPa</td>
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<tr>
<td>121)</td>
<td>0.05332 kg</td>
<td>0.34585 mm</td>
<td>265.07339 MPa</td>
</tr>
</tbody>
</table>
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

122) Mass = 0.04847 kg, Displacement = 0.50025 mm, Stress range = 318.52931 MPa

123) Mass = 0.05886 kg, Displacement = 0.31688 mm, Stress range = 246.96023 MPa

124) Mass = 0.06163 kg, Displacement = 0.29973 mm, Stress range = 158.60803 MPa

125) Mass = 0.03947 kg, Displacement = 0.68613 mm, Stress range = 346.98531 MPa

126) Mass = 0.05748 kg, Displacement = 0.32933 mm, Stress range = 180.45805 MPa

127) Mass = 0.05263 kg, Displacement = 0.40112 mm, Stress range = 340.40736 MPa

128) Mass = 0.04086 kg, Displacement = 0.66481 mm, Stress range = 335.48387 MPa

129) Mass = 0.05817 kg, Displacement = 0.31790 mm, Stress range = 157.02902 MPa

130) Mass = 0.04640 kg, Displacement = 0.52865 mm, Stress range = 288.91008 MPa

131) Mass = 0.04709 kg, Displacement = 0.53158 mm, Stress range = 288.46897 MPa

132) Mass = 0.04224 kg, Displacement = 0.59394 mm, Stress range = 301.01414 MPa

Appendix Q – Experiment 17 Pareto Set - Q13 - Darren Watts – September 2008
<table>
<thead>
<tr>
<th>Number</th>
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<td>142)</td>
<td>0.04917</td>
<td>0.52090</td>
<td>261.25024</td>
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<td>143)</td>
<td>0.05609</td>
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A Genetic Algorithm Based Topology Optimisation Approach
for Exploiting Rapid Manufacturing's Design Freedom

144) Mass = 0.04847 kg
Displacement = 0.51479 mm
Stress range = 279.67540 MPa

145) Mass = 0.05886 kg
Displacement = 0.29324 mm
Stress range = 322.51379 MPa

146) Mass = 0.03670 kg
Displacement = 0.80238 mm
Stress range = 383.32064 MPa

147) Mass = 0.05817 kg
Displacement = 0.30985 mm
Stress range = 320.12366 MPa

148) Mass = 0.04363 kg
Displacement = 0.58621 mm
Stress range = 301.56659 MPa

150) Mass = 0.06163 kg
Displacement = 0.28411 mm
Stress range = 175.07286 MPa

151) Mass = 0.05194 kg
Displacement = 0.41951 mm
Stress range = 303.80939 MPa

152) Mass = 0.04709 kg
Displacement = 0.53991 mm
Stress range = 285.66890 MPa

153) Mass = 0.03670 kg
Displacement = 0.79900 mm
Stress range = 386.21594 MPa

154) Mass = 0.05955 kg
Displacement = 0.29565 mm
Stress range = 319.93200 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing's Design Freedom

155) Mass = 0.03601 kg
Displacement = 0.79228 mm
Stress range = 389.71865 MPa

156) Mass = 0.06509 kg
Displacement = 0.26039 mm
Stress range = 151.67439 MPa

157) Mass = 0.06025 kg
Displacement = 0.29003 mm
Stress range = 333.09316 MPa

158) Mass = 0.05886 kg
Displacement = 0.30758 mm
Stress range = 240.22758 MPa

159) Mass = 0.03185 kg
Displacement = 1.53191 mm
Stress range = 711.69934 MPa

160) Mass = 0.03878 kg
Displacement = 0.70421 mm
Stress range = 366.10426 MPa

161) Mass = 0.06579 kg
Displacement = 0.25652 mm
Stress range = 157.43219 MPa

162) Mass = 0.06232 kg
Displacement = 0.26933 mm
Stress range = 174.67595 MPa

163) Mass = 0.06440 kg
Displacement = 0.25404 mm
Stress range = 157.80418 MPa

164) Mass = 0.06579 kg
Displacement = 0.25525 mm
Stress range = 145.27236 MPa

165) Mass = 0.06717 kg
Displacement = 0.24693 mm
Stress range = 141.34781 MPa

Appendix Q - Experiment 17 Pareto Set - Q16 - Darren Watts - September 2008
<table>
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<td>167)</td>
<td>0.03324 kg</td>
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<td>431.65974 MPa</td>
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<td>168)</td>
<td>0.05540 kg</td>
<td>0.33338 mm</td>
<td>322.92678 MPa</td>
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<tr>
<td>169)</td>
<td>0.06302 kg</td>
<td>0.26964 mm</td>
<td>173.36677 MPa</td>
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<tr>
<td>170)</td>
<td>0.04363 kg</td>
<td>0.55897 mm</td>
<td>301.67503 MPa</td>
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APPENDIX R - Experiment 18 Pareto Set

Cell Types Used = B75, D50, B25 & Void, Pareto Set Size = 137
<table>
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<tr>
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<th>Stress range</th>
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<tbody>
<tr>
<td>1)</td>
<td>0.05401 kg</td>
<td>0.46951 mm</td>
<td>265.57599 MPa</td>
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<tr>
<td>2)</td>
<td>0.05055 kg</td>
<td>0.48455 mm</td>
<td>329.13981 MPa</td>
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<td>3)</td>
<td>0.06510 kg</td>
<td>0.25628 mm</td>
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<td>4)</td>
<td>0.04640 kg</td>
<td>0.77913 mm</td>
<td>383.53561 MPa</td>
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<td>5)</td>
<td>0.07479 kg</td>
<td>0.18107 mm</td>
<td>129.22767 MPa</td>
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<td>6)</td>
<td>0.05748 kg</td>
<td>0.36368 mm</td>
<td>210.16963 MPa</td>
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<td>7)</td>
<td>0.05748 kg</td>
<td>0.36323 mm</td>
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<td>0.03255 kg</td>
<td>4.53021 mm</td>
<td>1515.73206 MPa</td>
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<td>9)</td>
<td>0.04293 kg</td>
<td>1.12095 mm</td>
<td>530.26398 MPa</td>
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<tr>
<td>10)</td>
<td>0.03809 kg</td>
<td>1.84195 mm</td>
<td>686.32225 MPa</td>
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<td>11)</td>
<td>0.04016 kg</td>
<td>1.24719 mm</td>
<td>729.18753 MPa</td>
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A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

12) Mass = 0.06025 kg  
Displacement = 0.31350 mm  
Stress range = 191.46818 MPa

13) Mass = 0.06925 kg  
Displacement = 0.22019 mm  
Stress range = 147.77450 MPa

14) Mass = 0.04363 kg  
Displacement = 0.78693 mm  
Stress range = 418.09872 MPa

15) Mass = 0.04363 kg  
Displacement = 1.00225 mm  
Stress range = 541.41760 MPa

16) Mass = 0.05401 kg  
Displacement = 0.40338 mm  
Stress range = 314.29390 MPa

17) Mass = 0.03739 kg  
Displacement = 1.72680 mm  
Stress range = 951.58351 MPa

18) Mass = 0.07341 kg  
Displacement = 0.19013 mm  
Stress range = 110.57578 MPa

19) Mass = 0.05194 kg  
Displacement = 0.46652 mm  
Stress range = 265.77428 MPa

20) Mass = 0.05401 kg  
Displacement = 0.39730 mm  
Stress range = 269.63321 MPa

21) Mass = 0.07617 kg  
Displacement = 0.17680 mm  
Stress range = 136.39421 MPa

22) Mass = 0.05124 kg  
Displacement = 0.57915 mm  
Stress range = 266.49339 MPa

Appendix R – Experiment 18 Pareto Set - R3 - Darren Watts – September 2008
<table>
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<td>25)</td>
<td>0.07825 kg</td>
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<td>26)</td>
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<td>0.54337 mm</td>
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IN
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<td>Mass</td>
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<td>Mass</td>
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<td>87)</td>
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<td>88)</td>
<td>0.03878 kg</td>
<td>1.67060 mm</td>
</tr>
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</table>
89) Mass = 0.05540 kg  
Displacement = 0.39918 mm  
Stress range = 220.21084 MPa

90) Mass = 0.03116 kg  
Displacement = 4.83041 mm  
Stress range = 1687.04395 MPa

91) Mass = 0.06925 kg  
Displacement = 0.20715 mm  
Stress range = 157.13250 MPa

92) Mass = 0.06509 kg  
Displacement = 0.22901 mm  
Stress range = 201.01186 MPa

93) Mass = 0.06717 kg  
Displacement = 0.25309 mm  
Stress range = 143.42779 MPa

94) Mass = 0.06579 kg  
Displacement = 0.24717 mm  
Stress range = 172.96969 MPa

95) Mass = 0.06579 kg  
Displacement = 0.24044 mm  
Stress range = 157.36909 MPa

96) Mass = 0.03324 kg  
Displacement = 3.90001 mm  
Stress range = 952.61157 MPa

97) Mass = 0.06440 kg  
Displacement = 0.26914 mm  
Stress range = 158.98450 MPa

98) Mass = 0.06717 kg  
Displacement = 0.22415 mm  
Stress range = 181.03499 MPa

99) Mass = 0.07548 kg  
Displacement = 0.18230 mm  
Stress range = 112.47807 MPa
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

100) Mass = 0.06094 kg
Displacement = 0.29607 mm
Stress range = 181.92669 MPa

101) Mass = 0.05124 kg
Displacement = 0.46496 mm
Stress range = 322.60107 MPa

102) Mass = 0.01662 kg
Displacement = 10000000 mm
Stress range = 10000000 MPa

103) Mass = 0.03739 kg
Displacement = 1.55333 mm
Stress range = 1122.41100 MPa

104) Mass = 0.03670 kg
Displacement = 1.87437 mm
Stress range = 976.51152 MPa

105) Mass = 0.06648 kg
Displacement = 0.23867 mm
Stress range = 143.48480 MPa

106) Mass = 0.05609 kg
Displacement = 0.39606 mm
Stress range = 128.33344 MPa

107) Mass = 0.05767 kg
Displacement = 0.23867 mm
Stress range = 175.72839 MPa

108) Mass = 0.06579 kg
Displacement = 0.24658 mm
Stress range = 134.11410 MPa

109) Mass = 0.07687 kg
Displacement = 0.17867 mm
Stress range = 134.11410 MPa

Appendix R - Experiment 18 Pareto Set - R11 -

Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

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<td>0.28472 mm</td>
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<td>121</td>
<td>0.04709 kg</td>
<td>0.67115 mm</td>
<td>308.67664 MPa</td>
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</table>

Appendix R – Experiment 18 Pareto Set

 Darren Watts – September 2008
A Genetic Algorithm Based Topology Optimisation Approach for Exploiting Rapid Manufacturing’s Design Freedom

122) Mass = 0.07133 kg
Displacement = 0.20003 mm
Stress range = 126.84554 MPa

123) Mass = 0.07064 kg
Displacement = 0.21088 mm
Stress range = 132.52277 MPa

124) Mass = 0.04293 kg
Displacement = 1.15793 mm
Stress range = 579.20982 MPa

125) Mass = 0.03393 kg
Displacement = 2.35888 mm
Stress range = 943.13470 MPa

126) Mass = 0.04363 kg
Displacement = 1.14264 mm
Stress range = 527.78749 MPa

127) Mass = 0.07687 kg
Displacement = 0.17078 mm
Stress range = 124.60167 MPa

128) Mass = 0.05194 kg
Displacement = 0.44602 mm
Stress range = 282.55055 MPa

129) Mass = 0.03947 kg
Displacement = 1.54077 mm
Stress range = 940.54947 MPa

130) Mass = 0.08379 kg
Displacement = 0.16415 mm
Stress range = 99.68275 MPa

131) Mass = 0.03947 kg
Displacement = 1.29749 mm
Stress range = 695.17830 MPa

132) Mass = 0.04709 kg
Displacement = 0.65294 mm
Stress range = 321.73830 MPa

Appendix R - Experiment 18 Pareto Set
- R13 -
Darren Watts – September 2008
133) Mass = 0.07825 kg
Displacement = 0.18573 mm
Stress range = 105.24679 MPa

134) Mass = 0.06925 kg
Displacement = 0.22586 mm
Stress range = 122.32282 MPa

135) Mass = 0.03739 kg
Displacement = 1.79052 mm
Stress range = 730.72308 MPa

136) Mass = 0.03809 kg
Displacement = 1.68580 mm
Stress range = 758.42570 MPa

137) Mass = 0.04917 kg
Displacement = 0.52273 mm
Stress range = 474.68926 MPa