Vortex excited structural oscillations of a circular cylinder in flowing water

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Corrigenda

page 3 - line 2:
This definition needs some qualification; the flow forces disappear for phenomena such as galloping or stall hysteresis but obviously remain extant for vortex excitation and buffeting.

page 10 - line 4:
$15\%$ and $2.4 \, \text{d}$ are unrealistically high levels of turbulence intensity and scale only insofar as they are considerably larger than those used by other researchers who have usually attempted to minimise them.

page 24 - caption on fig. 2.3: for "mean drag coefficient $C_d$" read "apparent Strouhal numbers $S$".

page 53 - equations (5.1), (5.2):
The Cauchy number is obtained by simplifying the parametric group defining the bending stiffness of the cantilever cylinder and the dynamic pressure head of the fluid, i.e. $\frac{E t}{\rho V^2 d^4}$.

By considering the case of thin walled cylinders, for which $I \propto d^3 t$ the Cauchy number $\frac{\rho V^2 d^4}{E t}$ is formed. The mass ratio group $\frac{m_e}{\rho d^2}$ derives from the density parameter $\frac{m_s}{\rho d^2}$. Since $m_e = m_s + \rho d^2 x$ constant then $\frac{m_e}{\rho d^2} = \frac{m_s}{\rho d^2} + \text{constant}$ and thus can be substituted for the density parameter provided $\rho$ is the same for both model and full scale.
The use of the combined Stability Parameter $k_s$ is justified by considering the in-phase component of fluid force as inertial and invariant with fluid velocity (i.e. regarding the added mass as constant and 'frozen' to the cylinder). The results of Chapter 8 confirm these assumptions.

page 101 - (iv): for "structural damping" read "structural logarithmic decrement".

page 115 - line 14:
The addition of sufficient mass to the free end suppresses excitation by the twofold effect of increasing $k_s$ and decreasing $n$ thereby reducing the velocity at which excitation occurs and also reducing the energy input into the system. This is discussed in full in section 9.5.

page 217 - Caption on fig. 11.1: for "forces" read "forces/unit length".

page 233 - bottom line: for "no response" read "little response".

page 241 - line 6: for $V_{\text{rm}} = 10$ read $V_{\text{rm}} = 8.5$

for $V_{\text{rm}} = 7.2$ read $V_{\text{rm}} = 7.5$
"Vortex Excited Structural Oscillations of a Circular Cylinder in Flowing Water"

by

Roger King


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M.Ae.S.I.

Head of Department of Transport Technology

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SUMMARY

This thesis assembles the experimental and analytical results of a programme of research into the nature of flow-induced oscillations in water. A total of thirteen different surface piercing circular cylinders were tested over a wide range of experimental variables and, based on observations of amplitude response, wake frequencies, vortex shedding configurations and free surface phenomena, various mechanisms of excitation and stability criteria are proposed.

The practical situations in which flow-induced oscillations have been encountered are described in the Introduction, and a comprehensive selection of laboratory studies are discussed and compared in the Literature Review.

The experiments for this thesis were conducted in a flume designed for this purpose; the overall design philosophy and the commissioning stages are recorded in detail.

A presentation, discussion and analysis of results from the fundamental study is followed by the results from two hydro-elastic models of the full scale marine piles used in a series of field tests.

The fundamental study showed that oscillations could be excited in the flow and cross-flow directions, whereas in air flow, oscillations are almost invariably restricted to the cross-flow direction. The first three normal modes in the flow direction (in-line) and the first two normal modes of oscillation in the cross-flow direction were studied. Oscillations in-line were contained within two distinct but adjacent instability regions. The first of these, \(1.25 < V_r < 2.5\), was characterised by symmetric vortex shedding, and the second, \(2.7 < V_r < 3.7\), by the more usual alternate vortex shedding. In the cross-flow direction, excitation was again caused by alternate vortex shedding and occurred within the range \(4.5 < V_r < 9\).

Close quantitative agreement between the models' and the full-scale results were observed thus confirming the validity of the
modelling criteria developed from a combination of dimensional analysis and the results from the fundamental study.

A transfer matrix analysis of the oscillating cylinder/fluid system was employed to evaluate the fluctuating force coefficients in the flow and cross-flow directions. Graphs of non-dimensional amplitude versus fluctuating force coefficients were compiled, to make the results generally more applicable.

Finally, wake interference effects between two cylinders mounted close to each other in-line were studied, when the cylinders were free to oscillate independently and when coupled by various structural members. A range of cylinder spacings was covered by the tests and the results from the fundamental and second normal modes in-line and the fundamental mode cross-flow showed that the coupling and spacing between the cylinders had consistent but different effects for the two directions.

This thesis was written entirely by the author; the experimental results were recorded solely by the author and were analysed by him in consultation with his supervisors Professor David Johns and Mr. Michael Prosser.

It is always controversial to claim originality for anything and theses are no exception. The programme of research for this thesis was devised in the belief that no similar studies were currently being pursued. Many of the results discussed herein are original in the context that apparently they have not been published elsewhere and also because they are the author's interpretation of highly complex phenomena. The discussions and chapter summaries indicate these areas of originality.
ACKNOWLEDGEMENTS

I am very grateful to the Council of B.H.R.A. Fluid Engineering and its Director (Mr. G.F.W. Adler, and also his predecessor Mr. L.E. Prosser) for financing this project and for providing the experimental facility. I must also thank my departmental head, Mr. George Young, Assistant Director of B.H.R.A., for initially sponsoring, and subsequently supporting my research programme.

To Professor David Johns of Loughborough University of Technology must go the majority of the credit for ensuring that the technical content of this thesis (hopefully) is of the correct standard. His encouragement and thorough, positive criticism are greatly appreciated.

The work for this thesis was completed in part-time study and necessarily committed my wife, Deirdre, to long periods of loneliness - without her understanding and endless encouragement, this form of study would not have been possible.

I have benefitted greatly from discussions with colleagues at B.H.R.A., particularly Michael Prosser, my internal supervisor, David Crow, with whom I shared an office for the 4½ years I was engaged on this work, Don Miller and Steve Irving - I thank them all very sincerely. Another colleague, Bryan Bruce, was invaluable in assisting with many of the experiments and in personally translating references (51), (54) from the original French.

My thanks are extended to Colin Winfield and John Stanton of the B.H.R.A. instrumentation department who installed and maintained the instrumentation, and also to the staff of the B.H.R.A. workshops who built the test flume and the various models tested in it.
Peter Bull took the photographs for this thesis, often under very difficult conditions and I would like to record my thanks for his thoroughness and co-operation.

Finally, my thanks to Mrs. Sue Simper for so conscientiously typing the script, and to Mrs. Sheila Hockenhull who traced the majority of the drawings in this thesis.
NOTATION

A  Cross-sectional area of a cylinder.
A  Upstream cylinder in the coupled cylinder tests (Chapter 12).
B  Downstream cylinder in the coupled cylinder tests (Chapter 12).
b  Thickness of boundary layer.
C  Generalised coefficient of damping.
C_c  Critical coefficient of damping.
C_d  Steady drag coefficient.
C'_{d}  Fluctuating force coefficient in-line.
C'_{L}  Fluctuating force coefficient cross-flow.
C_n  Inertia coefficient in wave motion.
C_s  Coefficient of hysteretic damping.
d  Cylinder diameter.
d'  Depth of depression of water surface.
E  Young's modulus of elasticity.
Ei  Sectional stiffness of cylinder.
f  Second normal mode frequency in water.
f_3  Third normal mode frequency in water.
f_v  Frequency of shedding of vortex pairs.
F  Total force.
F'  Coefficient of Coulomb damping.
Fr  Froude number = V^2/gh
g  Gravitational constant.
G  Coefficient of structural damping.
G/d  Spacing in-line between cylinders tested in Chapter 12.
h  Water depth.
h'  Velocity head recovery at the cylinder front face.
I_t  Intensity of turbulence.
K  Generalised spring stiffness.
\( K_m \) Coefficient of added mass.

\[ \text{Structural Stability Parameter} = 2m_e \delta_s / \rho d^2 \]

\( k_s \) Critical Stability Parameter.

\( L \) Length of cylinder.

\( L_t \) Scale of turbulence.

\( m \) Generalised mass or mass/unit length.

\( m_a \) Added mass/unit length.

\( M_d \) Dynamic Magnifier.

\[ \text{Equivalent mass/unit length} = \int_0^L \! m y^2 \, dx / \int_0^h y^2 \, dx \]

\( m_r \) Mass ratio = \( m_e / \rho d^2 \).

\( m_s \) Structural mass/unit length.

\( M_t \) Mass attached to the cylinder free end.

\( m_w \) Internal water mass/unit length.

\( n \) Fundamental frequency in still water.

\( n_a \) Fundamental frequency in air.

\( N_s \) Size number = \( nd^2 / \nu \)

\( P \) Generalised forcing function/unit length.

\( q \) Damped natural frequency.

\( Q \) Flowrate

\( \text{Re} \) Reynolds number = \( V d / \nu \)

\( S \) Strouhal number = \( f_v d / V \)

\( S^* \) Apparent Strouhal number of an oscillating cylinder.

\( s.n.m. \) Second normal mode frequency.

\( t \) Cylinder wall thickness.

\( T \) Periodic time.

\( T_v \) Periodic time of vortex shedding.

\( U \) Orbital velocity in waves.

\( v \) Oscillatory velocity of cylinder.

\( V \) Velocity of flow.

\( V_h \) Variation of velocity with vertical distance from flume bed.

\( V_r \) Reduced Velocity = \( V / nd \), \( V / f_d \) or \( V / f_3 d \).

\( V_{rc} \) Critical Reduced Velocity.

\( V_{rm} \) Reduced Velocity corresponding to maximum amplitudes of oscillation.

\( V_R \) Resultant velocity vector.
$v_o$  Approach velocity of flow.
$v_1$  Velocity of flow at the cylinder.
$w$  Frequency.
$w_n$  Natural frequency.
$W$  Width of flume.
$y$  Generalised deflection of cylinder.
$y_m$  Maximum deflection of cylinder.
$y_r$  Maximum amplitude of forced oscillations.
$y_t$  Deflection of cylinder free end.
$\delta$  Generalised logarithmic decrement
$\delta_h$  Logarithmic decrement due to a depth of water $h$.
$\delta_s$  Structural logarithmic decrement.
$\delta_t$  Total logarithmic decrement $= \delta_h + \delta_s$.
$\rho$  Mass density of fluid.
$\sigma$  Poisson's ratio.
$v$  Kinematic viscosity of fluid.
$\phi$  Phase angle (Chapter 3),
          or $\int_0^h (y/y_m)^2 \, dz/L$ (Chapter 8).

Coordinate axes:  $x$, $y$, $z$.

Other symbols are defined where they appear in the text.
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1. Introduction

The past twenty years have seen a rapid growth in the construction of fixed or mobile offshore structures. A structure which is located in the open sea is subjected to periodic forces from wind, waves, and steady currents. Further complications can be produced by vibrations transmitted through the structure from machinery mounted on it. As searches for more oil and gas fields are made in increasingly deep water, the industry has acquired a knowledge of the considerable environmental hazards encountered by such operations. Of two hundred mobile drilling rigs constructed in the years 1950-1970, over ten percent collapsed and a further twenty percent suffered severe fatigue failure of structural members. It is a matter of conjecture how many of the losses and failures were due to impulsive wave loadings (wave slam) or wind forces and how many were caused by periodic vortex shedding from flow in the steady water currents (or from quasi-steady flow within the waves). However, there are cases where the causes of failure or of severe oscillations are fully recorded if not completely understood. Wiegel et al (1), investigating wave forces on an experimental pile in sea conditions, reported complete structural fatigue failure after a relatively short period of testing, attributing the source of the periodic forces to vortex shedding produced by the quasi-steady flow within the waves passing the pile.

In the absence of wave effects, periodic vortex shedding was responsible for the large amplitude oscillations of marine piles used in the construction of an oil jetty in the Humber estuary and in subsequent tests on site, one of the 18" diameter experimental piles actually failed after fracturing at bed level and at water level. At certain speeds some submarine periscopes oscillate with amplitudes sufficiently large to prevent their continued use; consequently there are certain speed ranges through which submarines must travel with their periscopes retracted.

Ironically, it is not necessarily the most severe sea con-
ditions which can cause the greatest damage and an example of this was the collapse (in 1961) of one of the "Texas Towers" located on the Atlantic Ocean continental shelf. Imperfect encastre joints in the lower braced members reduced the overall natural frequency from 37 cycles per minute to 19 cycles per minute; collapse ensued in waves whose height was only one third of the maximum that the structure had been designed to withstand. Thus, a disastrous consequence of resonant excitation was produced when the frequency of vortex shedding within the smaller waves and steady currents coincided with the reduced natural frequency of the structure.

This thesis is concerned with the excitation of structural oscillations by vortex shedding in steady water currents. By isolating vortex excitation from all other exciting mechanisms, it is possible to compare and contrast oscillations in water with the well documented vortex excitation of structures in air. It must be appreciated that vortex excitation in water can cause structures to oscillate in both the flow and cross-flow directions whilst structures in air almost invariably oscillate only in the cross-flow direction (there have been exceptional cases and these will be referred to later).

Chimneys have oscillated or failed in the sway and breathing modes and dangerous oscillations of missiles (2) (particularly the Saturn V rocket) on their launching pads have been experienced in moderate, steady winds. Suspended pipelines (3) have exhibited large amplitude oscillations of up to three diameters in low speed winds and roof support members are known to have collapsed under the action of vortex-shedding forces (4).

Water intakes to hydroelectric plants are protected from ingesting debris to the turbines by the use of trash racks. Some designs of trash racks consist of a series of closely spaced metal bars, flow through which can promote objectionable vibrations. Occasional fatigue failures have been recorded, leading to complete plant shutdown or damage to the turbine impellers. Certain arrangements of banks of boiler tubes are particularly prone to interference effects from the flow around and between them, resulting in a history
of severe vibrations or fatigue failures.

Vortex excitation is just one form of mechanism belonging to the general grouping of Flow Induced Oscillations; these are oscillations resulting from complex interactions between the flow of fluid and the motion of the geometric shape about which the fluid flows. When the flow ceases, so also does the excitation mechanism, conversely, when the body is held stationary (so that body motion ceases) the flow forces do not disappear, although they may become less well organised in frequency spectrum and lengthwise correlation. Flow Induced Oscillations have been recorded on many geometric shapes; however in this thesis the structures investigated are slender, circular cylinders.

The following chapter is devoted to a brief history of vortex shedding research followed by a more intensive review of relevant experimental work completed with rigid and flexible (or flexibly mounted) cylinders.
2. Literature review.

2.1 A brief history of vortex shedding research.

The effects of vortex shedding have been recognised for centuries, the Aeolian harp and the sighing of wind through trees being typical examples. In the fifteenth century, Leonardo da Vinci sketched surface flow patterns of water flowing about a body of square section (a wooden fence post?) showing symmetrically arranged vortices in the wake. In 1868 Helmholtz (5) ascribed the drag resistance of a cylinder to separation from and vortex formation at the rear of the cylinder. However, the first scientific study of the phenomenon appears to have been undertaken by Strouhal in 1873 (6). Using a rotating wooden frame to which stretched wires were attached, he showed that in general, the frequency with which vortices were shed from the wires was independent of the tension and elasticity of the wires. He formulated the non-dimensional relationship of shedding frequency \( f_\nu \) relative air velocity \( V \) and wire diameter \( d \) in what has since become known as the Strouhal number \( S \). Within certain limits the Strouhal number for such circular cross-sections was seen to be a constant, \( S = 0.185 \). Strouhal noted that when the Aeolian tones approached one of the natural frequencies of the wires, there was an immediate increase in the intensity of the emitted sound and the pitch of the observed note deviated from the value defined by \( S = 0.185 \). In a study of temperature effects, Strouhal noted, without discussion, a lowering of pitch for a given increase of temperature.

In 1879, Rayleigh (7), using a very crude apparatus consisting of a violin string in the forced draught of an open fire, tackled the problem with characteristic thoroughness and was actually the first person to demonstrate that oscillations induced by air-flow occurred in a plane normal to the direction of flow. Later, in 1896, Rayleigh (8) produced a theoretical study of Strouhal's work, making a correction for changes in viscosity in the form of a truncated McLaurin's expansion. He showed that the correction could be used to explain the variation of frequency with temperature change.
observed by Strouhal in 1878 and mentioned earlier. The first modern pictures showing vortices in the 'staggered' or alternate arrangement in a wake, were (according to Roshko (9)) produced by Ahlborn in 1902 (10). Mallock, in 1907 (11) recorded sketches of the symmetrical and alternate vortex streets observed in experiments with a lamina and circular cylinder in water. In 1908 Benard (12) used a shadow-graph technique to photograph the water surface patterns produced by vortices as they separated from a cylinder to form a vortex street. He showed vortices detaching from alternate sides of the body and his results, together with those of Ahlborn and Mallock, were used to substantiate Rayleigh's observations of cross-flow motion. Karman in 1911 (13) published his celebrated theory of vortex wake stability, in which he showed that only one ratio of longitudinal to lateral spacings of vortices would give a stable wake (all other spacings being classed as unstable). Furthermore he calculated the drag resistance experienced by a circular cylinder from which vortices were shed. In a photographic study, Karman & Rubach in 1912 (14) confirmed Karman's calculations of vortex street configurations.

Kruger and Lauth (1914, (15)) repeated Rayleigh's theoretical study of the Strouhal vortex shedding relationship to explain results from tests with rigid and elastic cylinders. They demonstrated that the frequency of vortex shedding from the vibrating elastic cylinder was governed by the cylinder frequency. In a further investigation they studied the dimensions and velocity of the vortex street formed behind a rigid cylinder, deriving formulae to describe both quantities.

In 1915 Rayleigh (16) returned to the problem of vortex shedding by using a pendulum dipping into a revolving tank of water. (It is interesting to note that this was his first experience of tests in water and arose from a visit to the hot springs at Bath. There he noted that, when his fingers were drawn through the water, they were 'thrown into transverse vibration and struck one another'.) His tests showed excitation to occur in the transverse direction over a range of velocities and he concluded that the Strouhal number giving maximum
excitation was $S = 0.14$. Additionally when the tests were later conducted in heated water ($60^\circ$C) he again recorded $S = 0.14$ for maximum excitation. In repeating his tests of 1879 (the vibrating string in air flow) he showed that $S = 0.16$ for maximum amplitude oscillations.

In 1921, Relf (17) studied the musical notes generated by the motion of wires through air. The wires were tensioned between two circular plates mounted on the shaft of an electric motor, resembling Strouhal's original rotating frame equipment. Concurrently with the air tests, Relf investigated the frequency of vortex shedding from rigid circular section cylinders in a water channel and by a comparison of the two sets of results at equal Reynolds numbers, he confirmed the lack of dependence of vortex shedding frequency on material properties of the wires. He was careful to avoid exciting resonance in the wires by suitably adjusting the motor speed. Fage & Warsap (1927, (18)) investigated the roles of turbulence and surface roughness on the drag of a circular cylinder. They concluded that an increase of turbulence or surface roughness was equivalent to reducing the viscosity (and thereby increasing the Reynolds number).

The majority of research completed by successive workers has been directed at gaining deeper understanding of specific aspects of the overall phenomena, and the rapid sophistication of instrumentation has enabled certain progress to be made in recording properties which in previous years it would have been impossible to detect or measure. However, it must be admitted that few fundamental advances have been made in the general problem and in particular there is still no completely satisfactory theoretical explanation of the vortex shedding mechanism.

Additional research work is extensive and detailed, for convenience it may be categorised into tests on rigid, stationary cylinders, and tests on elastic cylinders free to respond to the influences of the flow forces. The following two sections of this literature review are devoted to an examination of these two categories.
2.2 Stationary cylinder data.

2.2.1 Flow patterns.

In general, the way in which fluid flows about a circular cylinder is determined primarily by the Reynolds number \( \text{Re} = \frac{Vd}{\nu} \). For sufficiently low \( \text{Re} \approx 3 \) the flow resembles that predicted by theoretical mathematics but as the velocity is increased, separation and vortex formation occur. At \( \text{Re} \approx 10 \) a pair of fixed vortices are formed behind the cylinder (these are the so-called Foppl vortices) and these remain extant up to \( \text{Re} \leq 90 \). Between \( \text{Re} \approx 10 \) and \( \text{Re} \approx 90 \) the vortices become elongated in the flow direction until at the upper limit one of the fixed vortices breaks away from the cylinder causing wake pressure asymmetry and thus initiating the alternate vortex shedding process. (However, both Hanna (19) and Kovasny (20) state that at \( \text{Re} \approx 56 \), vortices are not shed from the cylinder but appear some distance downstream as instabilities within the laminar wake.) The vortices shed from the cylinder at \( \text{Re} \approx 90 \) are laminar and viscous and the vortex wake so formed is preserved for many diameters downstream.

From \( \text{Re} \approx 90 \) to \( \text{Re} \approx 300 \) the flow behaviour is said to typify the 'pure Karman range' (21) and the Strouhal number \( S = \frac{fV}{d/V} \) is dependent on flow velocity. Roshko (9) assembled the experimental results from many wind tunnel tests on cylinders of various diameters, showing the Strouhal number variation with Reynolds number through a similar form of equation to that originally postulated by Rayleigh (8). In the \( \text{Re} \) range 50 to 150, \( S = 0.212 \) (4 - \( 21.2/\text{Re} \)) and for \( 150 < \text{Re} < 300 \) considerable scatter was recorded. At approximately \( \text{Re} = 300 \) the 'Subcritical' regime is entered and this is seen to extend until approximately \( \text{Re} = 2 \times 10^6 \). Within this range, Roshko defines the Strouhal number by \( S = 0.212 \) (1 - \( 12.7/\text{Re} \)) and asserts that the vortex shedding has dominant periodicity.

As the flow approaches the cylinder it is accelerated over the front face of the cylinder and the static pressure decreases along the streamlines. The boundary layer is laminar and is subjected to
the decelerating viscous forces within the laminar layer and also the externally exerted pressure field from the essentially inviscid flow outside the boundary layer. At $90^\circ$ from the front stagnation point, the flow encounters a strong adverse pressure gradient, flow reversal thickens the boundary layer, reducing to zero the shear stress at the cylinder surface and laminar separation occurs (22). Transition to turbulence develops in the near wake and the vortices are rapidly dissipated by viscous action, transforming the energy into heat. At $Re \approx 1500$, Gerrard (23) reports that transition waves have been observed and Mair & Maufl (24) propose the possibility of these transition waves and the vortex shedding having similar frequencies leading to intermittent symmetric shedding. Ballengee & Chen (25) have experimentally verified the forward progression of separation points on the cylinder surface with increasing $Re$, showing reductions in the separation angle (from the forward stagnation point) to $70^\circ$ or $80^\circ$ at high $Re$.

The 'Critical' region is quoted by Wootton (26) to lie between $Re \approx 0.2 \times 10^6$ and $0.6 \times 10^6$. Here, the laminar boundary layer senses the adverse pressure gradient before the $90^\circ$ point, separates, becomes turbulent, reattaches and finally separates as a turbulent boundary layer. The strong transverse velocity mixing within the turbulent boundary layer enables it to remain attached for considerable progress into the adverse pressure gradient and the final separation produces a narrow turbulent wake. Periodicity within the wake either ceases or is greatly reduced in strength. A dramatic reduction in steady drag coefficient $C_d$ is experienced in this range and changes of $C_d$ from 1.2 to 0.43 have been recorded (Wootton (26)). The transcritical region extends from $0.6 \times 10^6$ upwards; transition to turbulence occurs on the front face probably at angles of approximately $30^\circ$ from the front stagnation point, separation is turbulent and the wake formed has only weak periodicity.

Fig. 2.1 summarises the flow patterns for the various Reynolds number ranges and fig. 2.2 shows the equivalent behaviour
of steady drag coefficient $C_d$ and Strouhal number $S$.

However, the regimes defined above are only approximate and changes in surface roughness, free stream turbulence, length/diameter ratio and temperature will affect the limits of each regime. As examples of the variations which may be introduced, Fage & Warsap (18) in 1929 demonstrated that the effects of increasing the surface roughness of the cylinder caused the abrupt reduction of drag coefficient to occur at progressively lower Reynolds numbers. They also concluded that the introduction of turbulence into the air stream stabilised the cylinder boundary layer and delayed entry to the Critical regime. Achenbach (27) showed that large surface roughness may cause immediate transition to turbulence within the boundary layer and the spectacular decrease in $C_d$ may be reduced to a decrease more like 1.2 to 0.7 at Reynolds numbers as low as $Re = 6 \times 10^4$ (a confirmation of Fage & Warsap's results). In the same series of tests, Achenbach measured the separation angle for a range of surface roughness, showing that this angle may be changed from $75^0$ to $100^0$ for sufficiently high roughness at $Re = 10^5$.

2.2.2 Measurements of unsteady forces and vortex frequencies.

McGregor (28) measured the r.m.s. pressure distribution around a smooth circular cylinder in a wind tunnel, showing that the fluctuating drag force $C_d'$ oscillated at twice the fundamental vortex shedding frequency associated with the fluctuating lift force $C_L'$. Gerrard (29) also made similar measurements in a wind tunnel and a comparison of his results with those of McGregor for similar ranges of $Re (Re = 10^4$ to $10^5$) indicates that in both tests $C_L'$ is an order of magnitude larger than $C_d'$. However, $C_L'$ varied with Reynolds number, being 0.01 at $Re = 4 \times 10^3$ and reaching a maximum of 0.8 at $Re = 7 \times 10^4$. Wootton (30) has also confirmed these results.

Using the same wind tunnel as McGregor, Surry (31) studied the influence of upstream turbulence on the pressure distribution around a cylinder, recording a large increase in $C_d'$ for increasing turbulence intensity and scale; the fluctuating lift force $C_L'$ was
apparently unaltered by the increase in turbulence and remained constant at approximately $C'_{L} = 0.55$. Surry's results showed that it is possible to reduce the ratio $C'_{L}/C'_{d}$ to 2 by raising the turbulence intensity and scale to the unrealistic levels of 15% and 2.4d respectively. However, the graphs of fluctuating pressure distribution shown in Surry's report suggest there is a forward progression of the resultant pressure which would lead to an amplification of $C'_{L}$ and a diminution of $C'_{d}$. These observations are not substantiated by Surry's direct measurements of $C'_{L}$ and $C'_{d}$; the reasons for this are not clear.

Bishop and Hassan (32) measured fluctuating forces on a cylinder mounted horizontally in a water channel operating in the range $3600 < Re < 11000$. From their results they found $C'_{L} \approx 0.6$, $C'_{L}/C'_{d} \approx 10$ and also confirmed previous observations (28, 29, 30) of $C'_{d}$ fluctuating at twice the frequency of $f'_{L}$.

Ruedy (33) applied stationary cylinder theory to the small amplitude cross-flow oscillations of a cable in a steady wind and deduced $C'_{L} = 0.93$, suggesting that for this case, the assumptions are reasonable (c.f. Gerrards results (29)).

Woodruff & Kozak (34) quote the work of other researchers in defining $C'_{L} = 1.71$ for $Re < 2 \times 10^{5}$ and $C'_{L} = 0.65$ for $Re > 2 \times 10^{5}$, applicable to smooth stationary cylinders. Jones (2) tested a cylinder in a high velocity wind tunnel in the Reynolds number range $4 \times 10^{5} < Re < 1.8 \times 10^{7}$, measuring fluctuating lift coefficients, $C'_{L}$, of between 0.06 and 0.16. Explanations of the wide variation of recorded results in the vicinity of the Critical region may be attributed in part to the uncertain lengthwise correlation of cylinders, and this was demonstrated by Bruun & Davies (35) who obtained fluctuating pressure measurements on a cylinder at Subcritical and Critical Re. At Subcritical Re the frequency spectrum of the pressure fluctuations exhibited the expected peak at the vortex shedding frequency but as the Critical Re was approached the low-frequency component of the spectrum increased significantly, with a rapid drop in the power at
the shedding frequency. These effects were accompanied by a marked change in the lengthwise correlation of the pressure fluctuations. As an example of this at \( \text{Re} = 2.4 \times 10^5 \) the correlation coefficient for a separation of two diameters is 0.7 and at \( \text{Re} = 4.8 \times 10^5 \) the correlation coefficient falls to 0.14. (A correlation coefficient of unity indicates the two quantities are identically correlated and a correlation coefficient of zero implies that the two quantities are only randomly related.)

The collected experimental results of several authors are presented in Table 2.1 where it is seen that the majority of experiments were devised to evaluate \( C'_L \). In many cases \( C'_d \) was not recorded by the authors and it is assumed that this may be interpreted as an indication of the importance attributed by them to the two coefficients.

2.3 Oscillating cylinder data.

2.3.1 Introduction

A considerable number of experiments have been conducted to investigate the complex interactions between flowing fluid (usually either air or water) and oscillating cylinders. The experiments (and full scale reports) reviewed in this section may be categorised in the following way.

1. Rigid sections of cylinders mechanically oscillated in flowing air or water.

2. Cylinders free to oscillate in flowing air or water.
   2a. Elastically mounted rigid sections
   2b. Elastic cylinders

3. Elastic cylinders towed through stationary water.

The general behaviour of the systems outlined above is extremely complicated and it is considered instructive to summarise here the main conclusions from experimental and full scale observations:

a. Cylinders mechanically oscillated (type 1) at frequencies near the Strouhal shedding frequency can control the rate of vortex
shedding such that only the forced frequency is detected in the wake. Under these locked-in conditions there is a dramatic increase in the in-phase flow forces on the cylinder.

b. Under certain conditions, cylinders 2a, 2b may be induced to oscillate under the action of vortex shedding forces. The oscillations are usually excited in the cross-flow direction although oscillations in the in-line direction have also been reported. The fluctuating cross-flow forces are an order of magnitude larger than the fluctuating in-line forces.

c. Response of cylinders 2a, 2b is a function of:
   c.1 damping in the structure or support system (usually defined in terms of the logarithmic decrement $\delta$).
   c.2 ratio of vortex-shedding frequency to cylinder natural frequency, represented by the reduced velocity $V_r$.
   c.3 ratio of equivalent mass of cylinder to the displaced mass of fluid ($= m_r$).
   c.4 slenderness ratio, i.e. length/diameter.
   c.5 mode shape of cylinder.
   c.6 cylinder surface roughness.
   c.7 free stream turbulence.

d. Towed cylinders (type 3) exhibit similar behaviour to cylinders free to oscillate in flowing water, i.e. the two systems are apparently identical at equal relative velocities.

2.3.2 Review of data.

Under certain conditions, cylinders free to respond to periodic vortex shedding forces may be forced to oscillate at the vortex frequency $f_v$ if $f_v$ is remote from the cylinder natural frequency $n$. If the fluid/cylinder system constituted a linear mechanical system, resonance in the cross-flow direction would theoretically be excited when $f_v = n$. Adopting the aerodynamic practice of non-dimensionalising the velocity by the product $n$ we form the Reduced Velocity $V_r$ ($= V/nd$) and by considering flow about a cylinder in the Subcritical Re range, the vortex and structural frequencies should coincide where
$V_r = 5$. However, the cylinder is known to oscillate at its natural frequency for values of $V_r$ less than 5 and in some cases oscillations continue until $V_r = 9$. Attempts have been made to classify the oscillations as self-controlled, self-excited or just 'fluidelastic'; whatever terminology is adopted, the essential fact is that the cylinder does not respond to Strouhal frequency forcing in a mechanical sense. There is a complicated feedback from the cylinder motion to the vortex shedding mechanism as experimental work reviewed here will emphasise.

During tests with a long slender cylinder in flowing water Thoma (36) noted the excitation of fundamental cross-flow oscillations at $V_r = 20$ at which velocity the appropriate stationary cylinder Strouhal frequency should have been four times higher than the frequency of excited oscillations $f_e$. Penzein (37) made similar recordings with an elastically mounted rigid cylinder in a wind tunnel - the oscillations became so severe that the test had to be abandoned to prevent permanent damage to the supporting beams. A tentative explanation may be based on subharmonic excitation and this opinion has since been substantiated by the water channel tests of Leinhard & Liu (38). Meier-Windhorst (39) tested an elastically mounted rigid cylinder in a water channel and showed the excited frequency $f_e$ to vary from the natural frequency of the cylinder by an amount depending upon the mass ratio $m_r$ (see c. 3); largest deviations were produced by smallest values of $m_r$. Maximum amplitudes in the cross-flow direction coincided with $V_r = 6.1$. Scruton (40) and Vickery & Watkins (41) combined the mass ratio $m_r$ with the logarithmic decrement to form the combined Stability Parameter $k_s$ (in water, the added mass of water must be included in the formulation of $m_r$ as it may substantially lower the cylinder natural frequency from the 'in-air' value, (see Chapter 4).

Scruton observed an approximately inverse square law relationship between $k_s$ and the non-dimensional cross-flow amplitudes of oscillation $y/d$, i.e. $y/d \propto (1/k_s)^{1/2}$, showing that if $k_s$ is sufficiently large, oscillations could be reduced to insignificant
levels. Mode shapes were seen to be of great importance in determining the stability of a cylinder in air flow; a $k_s$ of 17 was sufficient to virtually eliminate cross-flow oscillations of a slender elastic cylinder but the equivalent value of $k_s$ for a spring mounted rigid cylinder was approximately 65 (40).

Unemara et al (42) used a wind-tunnel to test an elastically mounted rigid cylinder for which $m_r$ was held constant and $\delta$ varied. A decrease in $\delta$ increased the amplitudes of oscillations and extended the range of $V_r$ over which the natural frequency oscillations were excited.

Wootton (26) tested a range of cylinders in a wind tunnel, experimentally demonstrating the functional relationship between oscillatory amplitudes and the three variables cylinder roughness, slenderness ratio and combined Stability Parameter $k_s$. The values of $V_r$ at which maximum amplitudes were excited depended upon $k_s$ and varied within the range $5 < V_r < 6.5$.

Marris (43) investigated the cross flow oscillations of a Pitot tube in flowing water, proposing that the vibrational velocity ($\nu$) must be comparable with the flow velocity ($V$) before excitation would occur. For a fixed value of $k_s$ he defined the excitation range as $\pm 35\%$ of the Reduced Velocity at which $f_v = n$, i.e. $3.25 < V_r < 6.7$. Den Hartog (44) quotes the excitation range as $\pm 20\%$ of the Reduced Velocity at which $f_v = n$, i.e. $4 < V_r < 6$, although Wootton (26) has shown that the excitation range depends upon cylinder roughness, geometry and Stability Parameter $k_s$.

The recording of amplitude response for increasing and decreasing velocities led Marris to believe the excitation of oscillations was a perfectly reversible process because there were only minor differences between the two sets of results.

Several authors have referred to the oscillating cylinder's ability to control the rate of vortex shedding; in particular, if resonance between the cylinder oscillations and vortex shedding is assumed to be identified with maximum amplitudes of oscillation, the recordings
of resonance at $V_r = 6$ indicate a 'resonant' Strouhal number $S^*$ of 0.16. Further departures of the resonant Strouhal number $S^*$ from the stationary cylinder value $S$ were observed by Glass (45) who recorded an average value of $S^* = 0.12$ in water channel tests using a spring mounted rigid cylinder. The amplitudes and frequencies excited by the flow were shown to depend upon the mass ratio $m_r$ in a manner similar to that shown by the results of Meier-Windhorst (39).

Bishop & Hassan (46) mechanically oscillated a rigid section of a cylinder in flowing water, recording fluctuating and steady forces in the cross-flow and in-line directions. For forcing frequencies $f_f$ remote from the vortex shedding frequency $f_v$ appropriate to the velocity of flow, the fluctuating forces $C'_L$ (cross-flow) and $C'_d$ (in-line) were periodic with frequencies $f_v$ and $2f_v$ respectively. If the forcing frequency was adjusted until it approached the Strouhal frequency $f_v$, the latter was obliterated and only the forcing frequency $f_f$ was detectable in the wake. Their recording of significant hysteresis between amplitude response in increasing and decreasing velocities is at variance with the work of Marris (43) who maintained that the flow excitation of the cylinder was reversible. Bishop & Hassan (46) deduced the fluctuating cross-flow forces $C'_L$ to be an order of magnitude greater than the fluctuating in-line forces $C'_d$. This result has been experimentally verified by several other authors although there is considerable disparity between absolute values of $C'_L$ and $C'_d$. (It is interesting to note that Bishop & Hassan (32) also measured the fluctuating cross-flow and in-line forces during tests on a stationary cylinder, establishing $6 < C'_L/C'_d < 10$).

Fullscale observations have shown that synchronisation (or lock-in) between vortex shedding frequency and cylinder frequency is not restricted to the Subcritical Reynolds number range applying to the tests discussed so far. Sustained oscillations of large chimney stacks at frequencies close to the natural frequencies of the stacks and corresponding to $S^* \approx 0.2$ have been recorded by Den Hartog (44).
Dickey & Woodruff (47) and Pagon (48) at Reynolds numbers up to $7 \times 10^6$. Smoke eddies forming immediately downstream of the chimney served to illustrate the periodic nature of the vortex shedding in Pagon's observations (48). It is perhaps pertinent to recall that in stationary cylinder tests, the Reynolds number range $0.2 \times 10^6$ to $2 \times 10^6$ is characterised by weak, wide-band random vortex shedding followed by a region of narrow band random vortex shedding in which $S \approx 0.3$. The effects of chimney oscillations are apparently to extract energy from the otherwise random shedding at the appropriate frequency of the chimney, and also to improve the lengthwise correlation of the shedding (see (35)).

Fig 2.3 is compiled from the results of wind tunnel tests on a cylinder free to oscillate in the cross-flow direction and the resonant Strouhal number $S^*$ is calculated from the velocity at which maximum dynamic amplitudes resulted, as explained previously. It is seen that $S^*$ has a fairly constant value of $\approx 0.16$ for $Re < 2 \times 10^5$ above which $S^*$ increases to another plateau of $\approx 0.2$ extending from $Re \approx 4 \times 10^5$ to the Reynolds number appropriate to the maximum velocity of the tests ($Re \approx 4 \times 10^6$). A comparison with the corresponding graph from stationary cylinder tests (fig. 2.2) demonstrates the way in which fluidelastic interactions between the oscillating cylinder and the flowing fluid can modify the Strouhal number/Reynolds number relationship. It is interesting to note that the wind tunnel tests would predict $S^* \approx 0.2$ for the $Re$ of the full scale observations of (44), (47), (48). The cylinder surface roughness, length/diameter ratio and Stability Parameters of the references are unlikely to be equal to those used in the wind tunnel tests, and it is proposed that instationary cylinders are perhaps less sensitive than stationary cylinders to variations in environmental conditions, or that the variables of the different tests have combined in such a way that the agreement in $S^*$ is coincidental.

Jones (2) used an hydraulic shaker to mechanically oscillate a section of a large (3 feet diameter) cylinder in a high speed
wind tunnel in which Reynolds numbers of up to $1.8 \times 10^7$ were achieved. For forcing frequencies ($f_f$) far removed from the Strocual frequency $f_V$, there was no significant cross-flow force $C_L'$ due to motion, but as $f_f$ approached $f_V$, $C_L'$ increased with amplitude, building up to several times the stationary cylinder value. Jones' tests were commissioned to investigate the wind induced sustained oscillations of the full scale Saturn V rocket on its launching platform and he includes in his report a graph of amplitude of rocket oscillations versus windspeed for varying degrees of damping.

Having mentioned lock-in and synchronisation between cylinder motion and vortex frequency, Richardson (49) asserts 'a body which is elastically free to be set in resonant vibration by a vortex street of which it then takes control, is similar to the acoustic action of a column of air in an organ pipe; the sound once initiated, takes control of the edge tone at the mouth and imposes on it the natural pipe-tone frequencies'. His description forms a very useful analogy - if a little over-simplified.

In many tests, the experimental arrangements were designed to restrict motion to the cross-flow direction, largely because of the belief that the in-line forces were insignificantly small and therefore of no practical interest. However, in wind tunnels, Walshe (50) and Leon (51) have observed relatively low amplitude oscillations in the in-line direction and Walshe showed that a $k_s$ of 2.2 was sufficient to suppress oscillations in the in-line direction. An indication of the relative power in the cross-flow and in-line directions is obtained from a comparison of the $k_s$ values necessary to prevent oscillations in the two directions (c.f. $k_s \approx 6.5$ for cross-flow (40)). Auger (52) recorded a rare case of full scale in-line oscillations caused by air flow. The cylindrical aluminium members of a space frame building at the 1968 Montreal World Fair were designed to withstand excitation in the cross-flow direction and were relatively flexible in-line; Stockbridge dampers were attached on site, resulting in complete suppression of the excited
oscillations.

In-line oscillations were responsible for the fatigue failure of a full scale test pile at Immingham in the Humber estuary. The test rig was used to investigate the in-line oscillations encountered during construction of an oil terminal at Immingham where the incoming and falling tides have well-defined directions and where exceptionally high velocities (up to 8ft/second) were experienced. King (53), Leon (54) and Vickery & Watkins (41) used small cylinders (<2" diameter) to demonstrate similar oscillations in flowing water showing that in-line oscillations were independent of cross-flow motion. Higher normal modes of oscillation in-line were briefly investigated by King (53) and Leon (54). Using a cantilevered cylinder in a fixed depth of water King (53) showed that in-line oscillations in the fundamental mode could be eliminated by attaching sufficiently large masses to the free end of the cantilever. No attempt was made to relate the masses to a Stability Parameter \( k_s \) and the division between stability and instability was decided in a rather unsatisfactory way. Later analysis of King's results suggests that a \( k_s \) of between 1.0 and 2.0 was sufficient to suppress excitation, although Vickery & Watkins (41) presented graphs of small amplitude in-line oscillations of a cylinder whose \( k_s \) in water was 5.0 (cross-flow motion was evident in the oscillations and this may have influenced the recordings). In air flow Walshe (50) affirmed that \( k_s = 2.2 \) effectively eliminated excitation of in-line oscillations.

The marine piles used in the construction of the Immingham oil terminal started to oscillate at \( V_r \approx 1.2 \) and this behaviour was qualitatively reproduced by King (53) using an approximately hydro-elastic model pile. The model Reynolds numbers were of the order 2 x 10^4 and for the full scale piles in question, Reynolds numbers of up to 1.6 x 10^6 were obtained. King states that "because the model and full scale piles exhibited similar threshold values of \( V_r \) when operating at unscaled Reynolds numbers, it may be concluded that this lack of correspondence of Re was not significant". A fuller
explanation may be inferred from the $S^*/\text{Re}$ relationship of fig. 2.3 if it is assumed that an instationary cylinder is capable of controlling the rate of vortex shedding irrespective of whether motion is predominantly in the cross-flow or in-line directions. Under these assumed conditions, it is not unreasonable for a relatively small model, operating at greatly reduced Reynolds numbers, to reproduce the behaviour of a full scale pile since the $S^*/\text{Re}$ relationships for both situations are approximately similar. Additionally, it is probable that cylinders oscillating in-line will not exhibit the drastic reductions in steady drag coefficient $C_d$ in the region of $\text{Re} = 2 \times 10^5$, implying that small scale models may be used to reproduce accurately the steady and unsteady stresses of full sized piles.

The tests of Walshe (50) and Leon (51) in air, resulted in threshold values of reduced velocities in the range $1.8 < V_r < 2$ for in-line oscillations in the fundamental mode; second normal mode oscillations in-line were observed in the water tests of King (53) and Leon (54) who both recorded the onset of instability in the range $0.8 < V_r < 1.1$. Based on his analysis of vortex-excited breathing oscillations of shell structures in air, Johns (55) postulates that $V_r = 1.2$ is consistent with four cycles of motion for every vortex-pair shed.

Table 2.2 is a collection of experimental results from various sources and contains lists of the Reduced Velocities at which instabilities first occurred ($V_{rc}$) and at which maximum amplitudes were recorded, ($V_{rm}$). Generally, in-line oscillations are promoted more easily in water than in air and in those cases where comparison are possible, instability was excited at lower $V_{rc}$ in water. This naturally leads to speculating that instability is either a function of mass ratio $m_r$ or that Benard (12) possessed astonishing insight on such problems when he insisted "the dynamics and disposition of vortices in water do not conform to those observed in air".

2.4 Discussion.

The literature review has outlined the scope and complexity of the general field of flow-induced oscillations of cylinders. There i
a real need for reliable data relating to flexible cylinders oscillating under the action of vortex shedding in flowing water, particularly when the oscillations are excited in the in-line direction. As a result of this review, the present programme of research was instituted as a thorough investigation of vortex excited oscillations in the in-line and cross-flow directions of slender, flexible cylinders in water.

The influences of mode shape, added mass of water and damping will be established by testing over a wide range of cylinders, cylinder clamping arrangements and water levels. The criteria governing the onset of oscillations of a cylinder in variable depths of water will be compared and contrasted with existing data recorded in wind tunnels and water channels (with very few exceptions, previous research in water channels has been restricted to tests in which the cylinders were completely immersed).

An explanation of the mechanisms of excitation of in-line oscillations at $V_r = 1.2$ is to be sought, based on experimental observations. Tests in the fundamental mode in the cross-flow and in-line directions will be followed by an investigation of in-line second normal mode excitation to establish the stability criteria for each mode.

Two fully hydroelastic models of the full scale piles used in the Immingham field tests will be designed, fabricated and tested to confirm the ability of small scale models operating at unscaled Re to represent the behaviour of large piles in water. Qualitative and quantitative comparisons will be made as the full scale data becomes available.

Simple configurations of coupled and uncoupled cylinders will be tested to observe and describe the wake interactions between two cylinders with various spacings in-line. This is a further complication and previous work on this particular aspect will be reviewed in the relevant chapter.


<table>
<thead>
<tr>
<th>Source</th>
<th>r.m.s. fluctuating force coefficient $C_{L}'$</th>
<th>ratio $C_{L}'/C_{D}$</th>
<th>Reynolds number range</th>
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<tbody>
<tr>
<td>Jones (2)</td>
<td>0.08</td>
<td>10</td>
<td>$0.4 \times 10^6$ to $1.9 \times 10^7$</td>
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<tr>
<td>McGregor (28)</td>
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<td>10</td>
<td>$4.3 \times 10^4$ to $1.3 \times 10^5$</td>
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<td>Gerrard (29)</td>
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<td>2.5 to 10</td>
<td>$4.3 \times 10^4$ to $1.3 \times 10^5$</td>
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<td>$4.4 \times 10^4$</td>
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<td>Bishop &amp; Hassan (32)</td>
<td>0.65</td>
<td>10</td>
<td>$3.6 \times 10^{-3}$ to $1.1 \times 10^4$</td>
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<td>Kozak (34)</td>
<td>1.71</td>
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<td>$0.2 \times 10^6$</td>
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<td>$V_r$ of maximum amplitude $V_{rm}$</td>
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<td>Leon (54)</td>
<td></td>
<td>6.6</td>
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<td>Frank (56)</td>
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<td>Angrilli (57)</td>
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<td>5.3</td>
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<td>Glass &amp; Sayre (58)</td>
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Fig. 2.1 Flow patterns around a stationary circular cylinder for various Reynolds numbers.
Fig. 2.2  Mean drag coefficient $C_d$ versus Reynolds number for a stationary cylinder.

Fig. 2.3  Mean drag coefficient $C_d$ versus Reynolds number for a cylinder free to oscillate.

3.1 Introduction.

Flow-induced oscillations are non-linear and complex; manipulation of the governing differential equations seldom results in compact solutions. Much can be learned from the behaviour of forced, damped linear mechanical systems and these form a useful reference for later experimental work. Analogies can be drawn between linear and non-linear oscillations, and a good understanding of simple linear systems can frequently explain, in qualitative terms, the mechanisms involved in non-linear oscillations.

This Chapter deals with the comparatively simple theory of free and forced, viscous damped linear systems, and introduces the concepts of hysteretic and Coulomb damping.

3.2 Free oscillations with viscous damping.

The governing differential equation of motion for this case is obtained by resolving forces in the vertical direction on fig. 3.1,

\[ \text{i.e. } m \ddot{y} + C \dot{y} + Ky = 0 \]  

and by assuming a solution of the form \( y = Ae^{rt} \) the following quadratic equation in \( r \) is obtained:

\[ (mr^2 + Cr - K) e^{rt} = 0 \]  \( \ldots (3.2) \)

Solutions of (3.2) are:

\[ r_{1,2} = \frac{-C}{2m} \pm \sqrt{\left( \frac{C}{2m} \right)^2 - \frac{K}{m}} \]  \( \ldots (3.3) \)

The physical behaviour of the system depends upon the relative magnitudes of the elements in (3.3) and there are three possibilities:

3.2.1 If \( \left\{ \frac{C}{2m} \right\}^2 > \frac{K}{m} \), both values of \( r \) are real and negative and the solution represents the sum of two decreasing exponential curves. If the mass is displaced and released it will gradually
creep back to the initial equilibrium position and no oscillatory motion occurs;

3.2.2 if \( \left\{ \frac{C}{2m} \right\}^2 = \frac{K}{m} \), the system is critically damped, \( r_1 = r_2 \) and the resulting motion is ill-defined and aperiodic. For this borderline case, the damping coefficient is written \( C_c \) (critical coefficient) where \( C_c = 2m \sqrt{K/m} \). Now the undamped natural frequency \( w_n = \sqrt{K/m} \) and \( C_c \) may be rearranged to read:

\[
C_c = 2m w_n \tag{3.4}
\]

3.2.3 if \( \left\{ \frac{C}{2m} \right\}^2 < \frac{K}{m} \), the roots are complex and the solution represents a sine wave of decreasing amplitude, referred to as the transient

\[
y = e^{-Ct/2m} \left\{ C_1 \cos qt + C_2 \sin qt \right\} \tag{3.5}
\]

where \( q \) is the damped natural frequency

\[
q = \sqrt{\frac{K/m - C^2/4m^2}{}} \tag{3.6}
\]

Comparison of two consecutive maxima of the curve represented by (3.5) gives the rate with which the transient amplitude decays. This is a constant factor of \( e^{\pi C/mq} \) and its natural logarithm is termed the logarithmic decrement, \( \delta \), widely used to describe the damping properties of potentially oscillatory systems. From the above:

\[
\delta = \frac{\pi C}{mq} \tag{3.7}
\]

and from (3.6) and (3.7):

\[
\delta = 2\pi \frac{C}{C_c} \sqrt{1 - \left( \frac{C}{C_c} \right)^2} \tag{3.8}
\]
For lightly damped systems, typical of those with which this thesis is concerned, (3.8) may be modified to read

\[ \delta = 2\pi \frac{C}{C_c} \] \hspace{1cm} \ldots (3.9)

or \[ \delta = Cq \pi /K \] \hspace{1cm} \ldots (3.9a)

The limiting value of \( \delta \) is found by putting \( C = C_c \) in (3.9) to obtain

\[ \delta_c = 2\pi \] \hspace{1cm} \ldots (3.9b)

(A measure of the percentage critical damping may be calculated by dividing the measured logarithmic decrement by \( 2\pi \).)

By non-dimensionalising the damped natural frequency \( q \) it will be noted that the effect of increasing damping is to reduce the frequency \( q \)

\[ \frac{q}{w_n} = \sqrt{1 - \left(\frac{C}{C_c}\right)^2} \] \hspace{1cm} \ldots (3.10)

3.3 Forced oscillations with viscous damping

Returning to figure 3.1 and considering the vertical balance of forces when a periodic forcing function \( P \) is applied to the mass, the following equation is derived:

\[ m\ddot{y} + C\dot{y} + Ky = P\sin wt \] \hspace{1cm} \ldots (3.11)

(\( w \) is the frequency of the periodic force \( P \).)

The solution of (3.11) is:

\[ y = \frac{P/K}{\sqrt{\left(1 - \frac{w^2}{w_n^2}\right)^2 + \left(2\frac{C}{C_c}\frac{w}{w_n}\right)^2}} \sin(wt-\phi) \] \hspace{1cm} \ldots (3.12)

\( P/K \) is the static deflection of \( K \) due to \( P \) and the ratio \( y/(P/K) \) is the Dynamic Magnifier \( M_d' \) and \( \phi \) is the phase shift between the input force \( P \) and the resulting displacement \( y \).
Fig. 3.2(a) shows $M_d$ plotted against dimensionless frequency $w/w_n$ and fig. 3.2(b) is the corresponding phase angle graph. For very slow forced oscillations, i.e. $w/w_n \approx 0$, the forcing approximates to the static case, there is no phase lag ($\dot{\phi} = 0^\circ$) and the Dynamic Magnifier is obviously unity. At frequencies of $w/w_n$ much higher than unity, the input energy is converted into inertia energy, $\dot{\phi} \approx 180^\circ$ and $M_d$ will be less than the static case. Between these extremes of frequencies, maximum amplitudes are found for lightly damped systems in the region of $w/w_n \approx 1$ and $\dot{\phi} \approx 90^\circ$. It is notable that the effects of increasing the damping are to reduce the maximum amplitudes and also the values of $w/w_n$ and $\dot{\phi}$ where they occur.

### 3.3.1 Maximum amplitudes of forced viscous damped oscillations.

Resonance of a mechanical system may be defined in two ways:

(i) Phase resonance, for which $\dot{\phi} = 90^\circ$ and $w = w_n$;

(ii) Amplitude resonance, when $w = q$ and $\dot{\phi} \approx 90^\circ$.

(For very lightly damped systems (i) = (ii).)

Considering (ii) yields the maximum amplitude $y_r$ of forced oscillations:

$$\text{work done by external force} = P \\pi y_r$$

$$\text{energy dissipated by damping} = C \pi w y_r^2$$

By equating (3.13) and (3.14)

$$y_r = \frac{P}{Cw}$$

or, in terms of the Dynamic Magnifier at resonance we have

$$M_d \approx \pi/\delta$$

### 3.4 Hysteretic (or structural) damping.

Hysteretic damping is usually represented by a coefficient $G$ proportional to stiffness but independent of the frequency with which it is in phase,
\[ i.e. \quad C_S = \frac{GK_i}{w} \]  \hspace{1cm} (3.17)

(here, \( i \) is used to denote 90° phase advance.)

Substituting \( C_S \) in (3.1) gives:

\[ m\ddot{y} + \frac{GK_i}{w} \cdot wy + Ky = 0 \]  \hspace{1cm} (3.18)

or

\[ m\ddot{y} + K(1 + iG)y = 0 \]  \hspace{1cm} (3.19)

The solution of (3.19) is a decaying exponential with logarithmic decrement \( \delta_S \) given by:

\[ \delta_S = \frac{2 \pi G}{1 + \sqrt{1 + G^2}} \]  \hspace{1cm} (3.20)

For most engineering materials, \( G \ll 1 \) and (3.20) becomes:

\[ \delta_S \approx \pi G \]  \hspace{1cm} (3.21)

Thus the amplitude decays uniformly and the transient motion resembles that of the viscously damped case. Veubeke (69) shows that the above representation of hysteretic damping is not universally satisfactory because it actually depends upon the past performance of \( y \) and the solutions of (3.19) yield natural frequencies slightly higher than the undamped values, (c.f. viscous damping). However, within a limited frequency band the representation is adequate (69).

For forced motion, the response may be calculated from:

\[ M_d = \frac{1}{\sqrt{1 - \left(\frac{w}{w_n}\right)^2}} + G^2 \]  \hspace{1cm} (3.21a)
3.5 Total logarithmic decrement in the presence of hysteretic and viscous damping.

Consider a system containing both hysteretic and viscous damping. The governing differential equation is:

\[ m\ddot{y} + C\dot{y} + K(1 + iG)y = 0 \quad \ldots \quad (3.22) \]

and the solution of (3.22) leads to

\[ \delta_t = \frac{\pi C}{mq} + \frac{2\pi G}{1 + \sqrt{1 + G^2}} \quad \ldots \quad (3.23) \]

i.e.

\[ \delta_t = \delta + \delta_s \quad \ldots \quad (3.24) \]

Thus, the total logarithmic decrement \( \delta_t \) is the sum of the constituent logarithmic decrements \( \delta \) due to viscous damping and \( \delta_s \) due to structural damping.

3.6 Coulomb damping.

Coulomb damping arises in dry friction effects when a body oscillates in contact with a stationary surface. Fig. 3.3 shows a typical situation in which Coulomb friction is present. The block, of mass \( m \), is held in tension by the two springs of total stiffness \( K \); the coefficient of dry friction between the block and the surface on which it slides, is \( F' \). By resolving forces horizontally the following equations are derived:

\[ m\ddot{y} + Ky - F' mg = 0 \quad \ldots \quad (3.25) \]

i.e.

\[ m\ddot{y} + K(y - F'mg/K) = 0 \quad \ldots \quad (3.26) \]

The solution of this differential equation is a periodic motion with frequency \( \omega_d = \sqrt{K/m} \), i.e. constant damping does not alter the frequency from the undamped value (see 3.2.2). However, the motion is not truly simple harmonic and the shape of the displacement/time transient changes each half cycle. The Coulomb logarithmic decrement
\[ \delta_c = \frac{4F'mg}{Ky} \]  

(3.27)

It is interesting to consider the implication of putting \( y = 0 \) in (3.27); theoretically the logarithmic decrement would be infinite. This case is, of course, never reached because the oscillations cease at the point where the friction force balances the spring force, i.e. the limiting value of \( y \) is \( y = F'mg/K \), and the limiting value of logarithmic decrement is thus \( \delta_c = 4 \).

(For forced oscillations, the amplitude response is determined from particular solutions of (3.26) with the right hand side equal to the periodic disturbing force \( P \). Details of such calculations are given in Den Hartog (44)).

3.7 Theory of dynamic response.

3.7.1 Introduction.

When a cylinder is placed in flowing water it is subjected to fluctuating forces arising from the periodic shedding of vortices. If the cylinder responds to these forces by oscillating with very small amplitudes and if the forces are perfectly correlated along the length of the cylinder, an estimate of the fluctuating force coefficients may be gained from the stationary cylinder data. Surry (31) has published graphs of r.m.s. pressure distribution around the circumference of a stationary cylinder and a typical result is shown in fig. 3.4. The r.m.s. radial pressure distribution can be resolved into the cross-flow and in-line components, giving the magnitudes of the fluctuating forces in the two directions with their corresponding frequencies of \( f_v \) and \( 2f_v \) respectively. However, even small amplitudes of motion are known to influence the vortex shedding frequencies and this added complication is circumvented by considering only those results for which the vortex shedding frequencies are equal to the cylinder natural frequencies. Furthermore, in practical situations it is
reasonable to assume that cylinder motion will influence the magnitudes of the input forces; one way of assessing the coefficients of these forces is through the establishment of an energy balance within the cylinder/fluid system and inferring the values of equivalent force coefficients from substitution of experimental results into the energy balance. Comparison with experimentally determined stationary cylinder force coefficients would indicate the interactions arising between the cylinder motion and the flowing fluid.

3.7.2 Response in the cross-flow direction.

In this and the following section, the fluid input forces and the cylinder response are analysed as mechanical systems in which resonance occurs when the forcing frequency \( (w) \) is equal to the cylinder natural frequency \( (w_n) \).

At resonance, the stiffness and inertia terms are equal and opposite; and the energy input is balanced by the energy dissipated in damping. Adopting the approach of Vickery & Watkins (41) of considering only the structural damping for the cross-flow direction, an energy balance may be constructed as shown below. (Vickery & Watkins established an energy balance for cylinders oscillating in the cross-flow direction in air flow.)

The fluid forcing function extends over the finite depth of water \( h \) which in general will not be equal to \( L \).

(i) energy input from water \( = \pi \int_0^h C_L' \rho \frac{V^2}{2} \cdot d.y(dx) \)

(where \( C_L' \) is the coefficient of fluctuating force in the cross-flow direction and \( \frac{1}{2} \rho V^2 \) is the corresponding dynamic pressure head of the water.)

(ii) energy dissipated in hysteretic damping \( = \)

\[ 4 \delta_s n^2 \pi^2 \int_0^L m_y^2(dx) \]
Equating (i) and (ii) yields

\[ \frac{y_h}{d} = \left( \frac{C_L^t}{L_e} \right)_{\text{num}} \frac{V_r^2}{k_s} \cdot \frac{1}{4\pi} \int_0^h \frac{(y/y_h)(dx)}{\int_0^h (y/y_h)^2 (dx)} \ldots (3.28) \]

\[ m_e = \frac{\int_0^L m y^2 (dx)}{\int_0^h y^2 (dx)} \quad \text{equivalent mass per unit length} \ldots (3.29) \]

\[ \left( \frac{C_L^t}{L_e} \right)_{\text{num}} = \frac{\int_0^h C_L y (dx)}{\int_0^h y (dx)} \quad \text{equivalent force coefficient per unit length} \ldots (3.30) \]

\[ V_r = \text{Reduced Velocity} = \frac{V}{nd} \ldots (3.31) \]

\[ k_s = \text{Stability Parameter} = \frac{2m e^S_s}{\rho d^2} \ldots (3.32) \]

In water the added mass of water (and the water inside the cylinder, if any) must be included in the generalised mass term \( m_e \) of equation (3.29) as it may be comparable with or greater than the structural mass per unit length, \( m_s \).

Equation (3.29) defines an equivalent cylinder of length \( h \), with the mass of the original cylinder/fluid system uniformly distributed over this length \( h \). Three advantages accrue from this definition of an equivalent cylinder:

(a) Direct comparison of the experimental results in water with the results of wind-tunnel tests (where the cylinders naturally are completely immersed, i.e. \( h = L \)). In all probability, the water forces on the cylinder will be amplitude dependent, certainly they exist only over the wetted length \( h \), and the amplitudes influencing the cylinder behaviour will be those.
of the submerged length alone. The portion of cylinder protruding from the water surface obviously affects the mechanics of the system but contributes nothing to the hydroelastic interactions.

(b) The length/diameter ratio of the equivalent cylinder is unchanged from the original configuration.

(c) The equivalent and original cylinders have equal kinetic energy.

3.7.3 Response in the in-line direction.

When the cylinder is excited to oscillate in the in-line direction, its alternate motion induces large variations of local relative velocity. Fig. 3.5 briefly summarises the forces on a cylinder oscillating in-line and from this figure the governing force equations may be established:

\[ F + \Delta F = \frac{1}{2} \rho (C_d + C'_d)(V - w_n y)^2 d.(dx) \quad \ldots \quad (3.33) \]

\[ F - \Delta F = \frac{1}{2} \rho (C_d - C'_d)(V + w_n y)^2 d.(dx) \quad \ldots \quad (3.34) \]

where \( F \) is the mean force in-line
\( \Delta F \) is the fluctuating component of \( F \)
\( C'_d \) is the coefficient of fluctuating force in-line.

From (3.33) and (3.34) \( \Delta F \) is extracted:

\[ \Delta F = \frac{1}{2} \rho V^2 d C'_d (dx) - \rho V d w_n y C_d (dx) \quad \ldots \quad (3.35) \]

The first term of (3.35) is the forcing function and the second, being 90° phase removed is the fluid damping force.

Establishing an energy balance to include structural and fluid damping gives:

(i) energy input from stream = \( \pi \int_0^h \left( C'_d \rho \frac{V^2}{2} d. y(dx) \right) \)
(ii) energy dissipated in damping

\[ = 2 \delta_s \int_0^L \frac{m}{2} (2\pi \gamma n)^2 (dx) + \int_0^h \rho V d 2\pi \gamma n C_d (dx) \]

Equating (i) and (ii) as before leads to:

\[ \frac{y_h}{d} = \frac{V_r^2}{k_s} \cdot \frac{1}{2\pi} \left\{ (C'_d)^e - \frac{2nC_d}{V} \right\} \int_0^h \frac{(y/y_h) \, (dx)}{(y/y_h)^2} \quad (3.36) \]

where

\[ m_e = \frac{\int_0^L \gamma^2 (dx)}{\int_0^h y^2 (dx)} = \text{equivalent mass per unit length} \quad \ldots (3.29) \]

\[ (C'_d)^e = \frac{\int_0^h C'_d y (dx)}{\int_0^h y (dx)} = \text{equivalent force coefficient per unit length} \quad \ldots (3.37) \]

\[ V_r = \frac{V}{nd} \quad \ldots (3.31) \]

\[ k_s = \frac{2m_e \delta_s}{\rho d^2} \quad \ldots (3.32) \]

3.8 Discussion.

By considering the cylinder and fluid as a linear, elastic system, equations (3.28) and (3.36) have been developed to relate the amplitudes of oscillations to the various experimental parameters. In Chapter 2, the ability of a small cylinder (53) to represent the behaviour of a full scale marine pile was explained tentatively by reference to the $S^* / Re$ graph of a cylinder free to oscillate. It was inferred that if the cylinder oscillated in line, the $C_d$ would probably remain constant for Reynolds numbers above $10^3$. The significance of this observation is amplified by equation (3.36), in which the steady
drag coefficient $C_d$ is seen to be one of the variables governing the amplitudes of oscillations in the in-line direction. Thus, if $C_d$ is approximately constant with $Re$ then the fluid forces on the model and the full scale structure will be represented correctly within the limits of the deviations of $C_d$ from the assumed constant value. (In the Imminham full-scale tests, three test piles of different diameters exhibited identical behaviour throughout the $Re$ range $10^3 < Re < 1.5 \times 10^6$, indicating that the assumption of constant $C_d$ was justified.) Substitution of experimentally determined results into the equations (3.28) and (3.36) would yield the equivalent fluctuating force coefficients $(C'_L)_e$ and $(C'_d)_e$ for the cross-flow and the in-line directions. Comparison of these with the stationary cylinder data would demonstrate the hydroelastic interactions between the oscillating cylinder and the flowing water.

Mode shapes are seen to be of paramount importance since the equations involve integrations of the first and second powers of the deflected cylinders. Calculating the mode shapes of partly immersed cylinders using transcendental equations was considered a daunting prospect and the problem was thus transferred to a more convenient solution by computer methods. The cylinder/fluid systems were represented by the transfer matrix equivalents of equations (3.28) and (3.36), correct mode shapes were generated automatically within the program and the integrations completed by a nested subroutine.

Coefficients $(C'_L)_e$ and $(C'_d)_e$ found by calculation and experiment are compared with stationary cylinder data in Chapter 11, and the transfer matrix methods are described fully in Appendix 1.
Fig. 3.1 The elements of a simple oscillatory system.

Fig. 3.2(a) Dynamic magnifier versus forcing frequency for a forced, damped system.
Fig. 3.2(b)  Phase plot versus forcing frequency for a forced, damped system.

Fig. 3.3  Coulomb damping.
Fig. 3.4  R.m.s. pressure distribution around a stationary cylinder.

\[ F - \Delta F = \frac{1}{2} \rho (C_d - C_d') (V + v)^2 d.l \]
\[ F + \Delta F = \frac{1}{2} \rho (C_d + C_d') (V - v)^2 d.l \]

Fig. 3.5  The forces on a cylinder oscillating in-line.
4. Theory of added mass and damping in water.

4.1 Added mass.

4.1.1 Definitions.

When a body oscillates freely in still water, its resulting natural frequency is lower than the frequency of the body oscillating in air. The reduction in frequency is consistent with an increase in the apparent mass of the body, consequently this effect is described as the 'added mass of water'.

Using classical mathematics incorporating the concept of an ideal fluid, Stokes (70, 1850) and Lamb (71, 1911) showed that the added mass of an infinitely long solid cylinder is independent of amplitude and frequency of oscillations and equivalent to the displaced mass of water. Stokes (72, 1901) also showed that the added mass in a real fluid will be increased by an amount depending upon the magnitude of the frequency parameter \( \lambda \), a non dimensional group combining the effects of frequency, viscosity and cylinder diameter.

For the purposes of frequency calculations, the added mass may be represented by a uniform concentric 'collar' of fluid which maintains contact with the cylinder for all cycles of motion. However, Darwin (73, 1952) demonstrated the analogy between added mass and 'drift mass', indicating the limitations of the 'collar' representation. The drift is the distance fluid particles are permanently shifted in the direction of motion during the passage of a body and the drift mass is the quantity of fluid contained within the volume delineated by the integral of drift with respect to distance from the body's line of direction.

The added mass factor \( K_m \) is the coefficient of displaced mass and for an ideal circular cylinder and fluid the theoretical \( K_m \) is unity although this is not true for all geometric shapes.

4.1.2 Previous research.

The first recorded tests designed to investigate added mass are believed to be those of Dubuat (74) in 1786. Using pendulum bobs of lead, glass and wood he determined an experimental value of \( K_m = 0.50 \).
showing added mass to be independent of the density of the oscillating bobs, apparently being a function of geometry alone. Bessel (75, 1826) independently conducted similar tests, finding $0.6 < K_m < 0.9$. Dubuat and Sabine (76, 1829) surmised that the effects of viscosity would increase $K_m$ although no corroborative evidence was sought by them. Poisson (77, 1832) and Green (78, 1836) mathematically proved $K_m = 0.5$ for spherical bodies and this value has been confirmed recently by accurate experiments. In view of this, Dubuat's results ($K_m = 0.58$) were remarkably close to the theoretical value, although the relatively large variations within Bessel's results ($K_m = 0.6$ to 0.9) suggest that other extraneous effects may have influenced the values recorded.

Froude (79, 1861) experimentally investigated the added mass of ships hulls, to determine the frequency of heaving oscillations. Kirchoff (80, 1870), Bassett (81, 1888), Taylor (82, 1928) and Tollmein (83, 1938) carried out theoretical calculations of $K_m$ for odd-shaped bodies and uniform bodies in curving flow.

Luneau (84, 1949) calculated and experimentally determined the added mass of circular discs, finding experimental $K_m$'s of 3 to 9 times the theoretical value.

Stelson & Mavis (85, 1955) tested completely immersed cylinders oscillating in a plane normal to their length. Three-dimensional end effects were investigated by varying the cylinder length ($L$) to diameter ($d$) ratio from 1.2 to 9, and within this range, values of $0.6 < K_m < 0.95$ were recorded. Extrapolation of their results showed $K_m = 1.0$ for the ideal case of $L/d = \infty$. Stelson & Mavis also investigated the added mass of other shapes, finding $K_m = 0.61$ for spheres (c.f. 0.5 by theory) and $K_m = 0.67$ for cubes.

Morison et al (86, 1950) investigated gravity wave forces on cylinders, observing $K_m$ to be strongly dependent upon the length, period and height of the waves. Iverson & Balent (87, 1951) reached a similar conclusion in studies of added mass in periodic flow, stating $K_m$ to be governed by the nature of the motion. However, there are certain basic differences between nominally stationary cylinders in periodic flow and oscillating cylinders in stationary water.
In waves, the cylinder is exposed to three principal forces:

\[ F = F_{\text{drag}} + F_{\text{pressure}} + F_{\text{inertia}} \]  \hspace{1cm} (4.1)

(i) drag force is considered in the usual way and is referred to the local velocity \( U \):

\[ F_{\text{drag}} = \frac{1}{2} C_d |U| U \cdot d \cdot l. \]  \hspace{1cm} (4.2)

(ii) the pressure force arises from accelerations (and thus pressure gradients) within the fluid, such that the pressure on the upstream face of the cylinder is greater than the pressure on the downstream face. Integrating the pressures around the cylinder gives the resultant pressure force

\[ F_{\text{press}} = \int (p \, ds) = \text{mass} \times \text{acceleration of fluid} \]

\[ F_{\text{press}} = \rho A \frac{\partial U}{\partial t}. \]  \hspace{1cm} (4.3)

(iii) the inertia force is produced by the presence of the cylinder, causing the flow to accelerate in a manner analogous to the oscillating cylinder in stationary water

\[ F_{\text{inertia}} = \rho A l \frac{\partial U}{\partial t}. \]  \hspace{1cm} (4.4)

i.e.

\[ F = C_d \frac{1}{2} \rho U |U| U \cdot l \cdot 1 + C_m \rho A l \frac{\partial U}{\partial t}. \]  \hspace{1cm} (4.5)

(where \( C_m = 1 + K_m \))

Equation (4.5) can be manipulated easily for a horizontally mounted cylinder but for a vertical cylinder (as used by Wiegel (1)) the lengthwise variation of wave orbital velocities \( U \) must be measured or assumed and integrated to give the total force \( F \).

McNown (88, 1957) tested a range of rigid lenticular cylinders, thickness \( b \), chord \( c \), completely immersed in periodic flow. \( K_m \) varied with the ratio \( b/c \), being unity for the circular cylindrical case of \( b/c = 1 \).
Keim (89, 1957) experimented with immersed sections of rigid cylinders uniformly accelerated through stationary water. Three-dimensional effects were evident for $L/d < 30$, above which value the cylinders behaved as though infinitely long and an average of $K_m = 0.95$ was recorded.

The influence of boundary conditions on submerged bodies was experimentally investigated by Macagno (90, 1958); solid boundaries increased $K_m$ due to the image effect and for sufficiently low Froude numbers, free surfaces reduced $K_m$, probably through pressure relief associated with the inverse image effect.

Keulegan & Carpenter (91, 1958) tested immersed cylinders in waves, showing $K_m$ to be approximately 1 for low steepness, deep water waves. Jen (92, 1968) also found small amplitude, regular waves gave the most consistent results, and recorded a mean value of $K_m = 1.04$. This is considered logical because it can be shown (93) that inertia forces predominate in low steepness, deep water waves; the Morison equation (4.5) then reduces to Lamb's solution with $C_m = 2$ and $K_m = 1$.

The response of submerged cylinders to impulsive input forces (e.g. to simulate earthquakes) has been experimentally studied by Clough (94, 1960) using a shaking table to which impulsive or sustained periodic forces could be applied. From tests on completely immersed, horizontal nominally rigid cylinders impulsively loaded and in free vibration he found $K_m = 1.0$ for both of the tests. When a flexible vertical cylinder ($L/d = 8$) was tested in the fundamental and second normal modes of oscillation by sustained shaking, a value of $K_m = 0.58$ was obtained for both modes. Clough attributes this lowering of $K_m$ to flexibility effects. Naval architects concerned with calculations of ship hull vibrations use reduction factors of up to 30% to account for the variation of $K_m$ with hull flexibility. However, it is considered that the features of ship vibrations in the presence of a free surface are basically dissimilar to those of the immersed vertical cantilever, which experiences free surface pressure relief over relatively short lengths immediately adjacent to the free surface.
The free end of the vertical cantilever used by Clough extended very close to the water surface and generated surface waves when subjected to sustained excitation. The effects of three dimensional flow about the free end are difficult to assess but it is unlikely that such localised effects could account for the extremely low value of $K_m = 0.58$ for $L/d = 8$. Stelson & Mavis (85) using completely immersed cylinders, found a minimum value of $K_m = 0.6$ for $L/d = 1.2$; their cylinders were rigid and horizontal, necessarily having three dimensional flow effects at both ends of the cylinder. It seems unlikely that three dimensional flow effects of the free surface on the one end of the flexible cylinder used in Clough's tests could explain the observed lowering of $K_m$. The real reason is considered to be connected with peculiarities of the test equipment or data reduction; the flexible cylinders were fabricated from a long flat spring to which short hollow lengths (one diameter long) of circular section formers were attached; the gaps between the formers were covered by rubber membranes.

Clough mentions the difficulty of accounting for the water inside the supposed sealed cylinder but he does not state whether the water was introduced intentionally or accumulated there during tests. This is thought to be a source of potential inaccuracies and may have significantly influenced the results.

4.1.3 Discussion.

The theoretical added mass of a freely oscillating cylinder in stationary fluid is equivalent to the displaced mass of fluid. There is considerable experimental confirmation of this for completely immersed cylinders but the effects of surface piercing structures are not known. The literature survey has shown that three dimensional end effects can influence the value of added mass for length/diameter ratios of up to 30. Although the greatest deviations of $K_m$ occur for $L/d \leq 10$, solid boundaries and free surfaces near the oscillating cylinder can also alter the added mass from the ideal case. Determining the added mass of nominally stationary cylinders in periodic flow is more difficult than
for oscillating cylinders in stationary water; however, well ordered results may be obtained using low steepness, deep water waves.

Section 8.1 details the tests arranged to investigate the following points:
(i) determine the added mass of surface piercing cylinders oscillating freely in various fundamental and second normal modes;
(ii) examine the added mass of cylinders in flowing water, particularly under conditions of vortex excitation. If the excited resonant frequencies should coincide with the still water frequency then the inference would be that added mass is apparently unaffected by streaming flow.

4.2 Theory of hydrodynamic damping.

4.2.1 Definitions

In the preceding section, the concept of added mass was introduced to account for the observed reduction in frequency from the in-air value when a cylinder oscillated freely in water.

Stokes (72, 1901) analysed the fluid resistance forces imposed on a freely oscillating pendulum wire and, assuming small amplitude oscillations and unseparated flow, the following equation was derived to describe the forces:

\[ F = \lambda_1 \frac{d^2y}{dt^2} + \lambda_2 \frac{dy}{dt} \quad \ldots (4.6) \]

\( \lambda_1, \lambda_2 \) are coefficients of acceleration and velocity.

The first term increases the inertia and hence reduces the natural frequency (this is the added mass effect) and the second term is a positive damping effect producing a diminution in amplitude.

Under the assumed conditions, the damping will be viscous and by analogy with the simple theory of Section 3, the logarithmic decrement due to this type of damping will be a constant.

4.2.2 Previous research.

Apart from the experimental and theoretical work of Stokes,
there have been relatively few attempts to determine the magnitude of fluid damping. The total damping will consist of the structural and fluid damping contributions; if equation (4.6) is valid, the fluid damping term will be viscous and the total logarithmic decrement ($\delta_t$) will comprise the constant structural logarithmic decrement ($\delta_s$) and the constant viscous damping log. decrement ($\delta_h$),

$$\delta_t = \delta_s + \delta_h = \text{a constant}$$ \hspace{1cm} (4.7)

Martin (95, 1908) experimentally verified Stokes' equation, using a pendulum oscillating in air. Stuart & Woodgate (96, 1955) tested an oscillating pendulum in air and observed viscous and amplitude-dependent logarithmic decrements, the latter implying separated local flow.

In a full scale test on a marine pile, Wootton et al (97, 1969) concluded that the damping in still water was independent of amplitude effects and the logarithmic decrement was a constant for initial amplitudes of at least one quarter of a diameter.

Bramley (98, 1970) tested a rigid section of a circular cylinder and concluded that for small amplitudes only, the damping was viscous, thereby verifying Stokes' analysis. Bramley showed that the size number $N_s = \frac{nd^2}{v}$ is a more meaningful parameter than Reynolds number when considering oscillating cylinders in stationary fluid, also showing that Stokes' theory is most applicable to situations in which the amplitudes are small and the size numbers are large. Stokes' tests were conducted at $N_s \approx 50$, and those of Stuart & Woodgate at $N_s \approx 269$. However, Bramley's tests at $N_s \approx 10^5$ and the full scale tests of Wootton et al at $N_s \approx 5 \times 10^5$ have demonstrated the applicability of the theory at relatively large amplitudes.

In flowing water, the damping will be influenced by local streaming effects. If the cylinder oscillates in response to vortex excitation, separation will obviously be present and the ideal damping will no longer be applicable.
4.2.3 Discussion.

For small amplitudes of oscillation and non-separated local flow the damping is viscous and the amplitude decay constant and independent of amplitude. There is little evidence of the damping associated with very large amplitude oscillations, but it is thought that these could introduce non-viscous effects.

Section 8.2 describes tests to determine the nature of hydrodynamic damping on cylindrical cantilevers and a pendulum freely oscillating in various depths of still water. Stokes' equation was integrated over the immersed lengths of the cylinders to yield an equivalent hydrodynamic logarithmic decrement, and a range of cylinder materials was used to achieve variation of hysteretic logarithmic decrements. The tests investigated the influence of frequency, mode shape and mass on the resulting total logarithmic decrements.

5.1 Design of experimental equipment.

The experiments were planned as a thorough investigation of flow induced oscillations of cylinders in water. Using existing knowledge of the subject, the cylinders were initially designed to oscillate under conditions which could be readily achieved, although in these tests the cylinders themselves are not intended to reproduce any full scale structure.

In the later series of tests, accurately modelled piles were used to simulate the behaviour of the full scale Immingham test piles.

The design of the experiments and cylinders progressed along the following lines:

a. Reynolds numbers of the tests will be restricted to the Subcritical range
   i.e. $2,000 < \text{Re} < 200,000$

b. The cylinders will be tested in an open channel (flume) and previous model testing experience has shown that for accurate flow measurements with the miniature propeller meter (described in Chapter 7) the water velocity $V$ should not exceed 3ft/second or fall below 0.5 ft/second.
   i.e. $0.5\text{ft/sec} < V < 3\text{ft/sec}$.

c. Water depths to be sufficient to permit adequate immersion of the cylinder and to provide nominally two dimensional flow. The literature review of the 'added mass' section suggested that minor three dimensional effects are detectable up to $h/d = 30$ and that drastic departures from the ideal, infinitely long cylinder conditions can result when $h/d \leq 10$. A design value of $h/d \approx 20$ was established as the criterion, thus reducing three dimensional effects to an acceptable level and also restricting the minimum depth to a value comparable with full-scale minima.
   i.e. $h/d \approx 20$
d. From the three conditions above, the minimum depth of water and minimum cylinder diameter are defined:

\[
\frac{Vd}{V} > 2000, \quad V > 0.5 \text{ ft/sec}
\]

\[d \approx 0.6''\]

For convenience, make \(d = 1''\). \[\therefore \quad \text{from c} : \]

\[h \approx 20''\]

e. Relative depths of water and cylinder lengths in practical engineering locations vary from 0.6 < \(h/L\) < 0.9 and adopting this criterion for the flume tests leads to:

\[L \approx 33'' \quad \text{(if the minimum water level is approximately 20'')}\]

f. The maximum anticipated flowrate available for the flume is \(Q = 15\) cusecs (see Chapter 6). By assuming that the maximum water depth \(h\) could be 2.5ft., and that the maximum velocity \(V\) could be 3ft/sec., the width of the flume \(W\) is thus defined:

\[
W = \frac{Q}{Vh}
\]

\[W = 2\text{ft.}\]

From the simple continuity of flow equations, there will be an increase in velocity past the cylinder due to the reduction in flow area caused by the presence of the cylinder in the flume.

Let the approach velocity be \(v_o\) and the velocity at the cylinder be \(v_1:\)

\[v_o \frac{W}{h} = v_1 (W-d) H\]

\[\text{i.e.} \quad \frac{v_1}{v_o} \approx (1 + \frac{d}{W})\]

Inserting the values of \(d\) and \(W\) from above, it is seen that the free stream velocity is increased by nearly 5% due to the partial blockage of the flume. This fact was borne in mind when siting the velocity measuring device during tests (Chapter 7).
g. Two cylinders of different flexibilities will be required, the stiffer one for tests in the fundamental mode in-line and the more flexible one for tests in the fundamental mode cross flow and in the second normal mode in-line. The design natural frequencies of the cylinders can be determined from the water velocity and cylinder diameter through the known formula for the onset of oscillations in the in-line and cross-flow directions,

\[
\begin{align*}
\text{in-line} \\
V/nd & \geq 1.2 \\
V/fd & \geq 1.1 \\
\text{cross-flow} \\
V/nd & \geq 5
\end{align*}
\]

From these, the frequencies are established as:

- \( n > 5\text{Hz} \) in-line
- \( n > 1.25\text{Hz} \) cross-flow
- \( f > 6\text{Hz} \) in-line

Hollow cylinders were selected because the range of natural frequencies can be extended by filling the cylinders with substances of various densities, thus adjusting the equivalent structural mass/unit length. However, manufacturing tolerances on commercially-produced cylinders may influence the test results, therefore it was decided to obtain cylinders of relatively large wall thickness, thereby reducing sectional non-uniformity due to the dimensional tolerances. Excessively long lengths of cylinders were obtained and from these the dimensionally most uniform lengths were selected for use in the flume tests.

The stiffer cylinder was made from aluminium alloy 41" long and 0.036" wall thickness (cylinder II. 1 in Table 5.1) and the more flexible cylinder was a 36" length of 0.083" wall thickness P.V.C. (cylinder II. 2 in Table 5.1). The cylinders were selected from calculations of natural frequencies assuming complete immersion and added mass coefficients of unity. In the event, the aluminium alloy cylinder was filled with water to reduce its natural frequency and thus reduce the critical velocity at which oscillations were initiated.
5.2 Test configurations.

Having determined the sizes of the flume and test cylinders for the basic study, additional test cylinders were required to yield information on more specific aspects as the study progressed. A total of thirteen different cylinders were tested and each was designed to exploit a particular feature of the phenomena. The cylinders may be arranged under three broad headings:

(i) single, isolated flexible cylinders: used in Parts I, II, III, V and VI of the experimental programme;
(ii) single, isolated sectional cylinder: used in Part IV.
(iii) wake interference between two cylinders in-line: used in Part VII.

The test configurations and cylinder details are listed in Table 5.1 and the test programme was conducted as follows:

Part I. An investigation of added mass and damping.
   (a) added mass of cylinders in still water using two different cylinders;
   (b) added mass of cylinders in flowing water also using two different cylinders;
   (c) damping in still water, using four different cylinders and a pendulum.

Part II Fundamental mode oscillations.
   (a) in-line;
   (b) cross-flow.

Part III Second normal mode oscillations in-line.

Part IV A photographic study of vortex-excitation.

Part V Tests on small diameter cylinders.
   (a) in-line;
   (b) cross-flow.

Part VI Hydroelastic model tests.
   (a) 1/30 scale model of 30" diameter pile;
   (b) 1/27 scale model of 18" diameter pile.
Part VII Wake interference effects; cylinders coupled and uncoupled.

(a) in-line oscillations, fundamental modes;
(b) cross-flow oscillations, fundamental modes;
(c) in-line oscillations, higher normal modes.

Variations of frequency were achieved by adjusting water levels, attaching masses or springs to the cylinder free end, or by filling the hollow cylinders with various substances.

In the single cylinder tests of Parts I, II, III, oscillations in the fundamental and second normal modes in-line and in the fundamental mode cross-flow were examined. Vortex shedding frequencies were measured in the wakes of stationary and oscillating cylinders, using two thermister probes. Results of these tests are given in Chapter 9. The photographic study of Part IV was arranged to give greater understanding of the excitation mechanisms responsible for the instabilities recorded in the single cylinder tests. For the photographic tests a solid perspex rod (1'' diameter) was suspended from a variable stiffness support. Potassium permanganate dye and an underwater light were used to illustrate the flow patterns about the cylinder and photographs were taken with a camera, mounted directly over the flume. The behaviour of cylinders at low Reynolds numbers were investigated with small diameter (\( \frac{1}{4}'' \) diameter) cylinders in Part V and in Chapter 9 the results are correlated with the previously recorded results of Parts I, II, III. In these tests of Part V, the second and third normal modes were excited in the in-line direction and the fundamental and second normal modes excited in the cross-flow direction.

Hydroelastic models of two of the piles used in the Immingham full scale tests were designed from the criteria of 5.3. In Chapter 10 their experimental results are qualitatively and quantitatively compared with the full scale results.

In Part VII an investigation was made of the behaviour of two cylinders mounted in separate bases in the flume. The in-line spacing between the two cylinders was varied to determine wake
interference effects when the cylinders were free to oscillate independently and when coupled together with a rigid or flexible member. The results of these tests and a brief discussion of previous work are given in Chapter 12.

5.3 Hydroelastic modelling.

5.3.1 Introduction.

The programme of basic research is intended to serve as a guide to the functional relationship between cylinder instability and the controlled variables of the test facility. However, of more importance to practical engineering situations is the interpretation of this basic data in terms of full scale requirements. To make comparisons between two or more sets of data the governing parameters must be established by dimensional or direct analysis and the data expressed in terms of non-dimensional groups, to remove dependence on size or some other prime variable distinguishing the original data. A true model of a structure is one which yields non-dimensional results, accurately representing the behaviour of the full scale structure. In the following sections, hydro-elastic modelling criteria are presented and discussed; in Chapter 10 these criteria are used to design hydroelastic models of two of the Immingham full scale test piles.

5.3.2 Dimensional analysis.

Dimensional analysis, applied to the particular problem of a flexible cylinder in flowing water, results in the following equation:

\[ \frac{y}{d} = \psi \left\{ \frac{V_d}{v}, \frac{V^2}{gh}, \frac{E_t}{\rho V^2 d}, \delta, \frac{m_e}{\rho d^2}, V_h, I_t, L_t \right\} \]

i.e. \[ \frac{y}{d} = \psi \left\{ \text{Re, Fr, C_n, } \delta, \text{ m_e, V_h, I_t, L_t} \right\} \]  \( \ldots (5.1) \)

For similarity between two cylinders, all the non-dimen-
sional groups of (5.2) should be similar. However before progressing with the definitions of model/full-scale relationships, there are certain aspects of (5.2) which warrant further explanation.

For reasons which will become apparent, it is not possible to satisfy the Reynolds number and Froude number criteria simultaneously if water is used as the fluid medium. It has been shown that cylinders in sustained cross-flow oscillations under the action of vortex shedding exert control over the frequency of vortex shedding throughout the Reynolds number range $10^3 < \text{Re} < 4 \times 10^6$. The ability of a small cylinder to reproduce the in-line oscillations of a full scale pile (53) implies that Reynolds number similarity is of secondary importance when modelling flexible oscillating cylinders. By contrast, Froude number ($Fr$) similarity is of paramount importance in free surface models, ensuring correct reproduction of the magnitude of the fluid dynamic forces and maintaining a realistic balance between these and the free surface gravity effects.

Geometrical reproduction is essential to the generation of similar pressure distributions about the model and full scale cylinders (e.g. one would not use an elliptical model cylinder to represent a circular full scale cylinder).

The cylinder and flowing water constitute a non-linear (hydro-elastic) system in which the motion of the cylinder influences the forcing function originating in the shedding of vortices. Generally, for non-linear oscillations to be excited, some initial finite displacement is necessary (King (99), Zdravkovich (100)) and it is thought that this initial displacement effectively correlates the unsteady forces on the structure. For sustained oscillations to be initiated, the cylinder must respond to the input flow forces in the appropriate manner; this implies that the dynamic and static stiffness of the cylinder and the dynamic forces should also satisfy the Cauchy number criterion.

In the context of these tests, Reduced Velocity $V_r$ is an important parameter. From the Strouhal number relationship $S = f V / d$, we can extract $V / d$ and show that, because $S$ is approx-
imately constant within the Subcritical range, $V/nd$ can be interpreted as a ratio of vortex frequency to structural frequency $f_v/Sn$.

Poisson's ratio is a material property and shows only marginal variations between most common materials, thus it is automatically scaled.

Scruton & Flint (101), Vickery & Watkins (41) and Walshe (102) have demonstrated that under conditions of sustained oscillations the mass ratio $m_r$ and the damping parameter $\delta_s$ may be combined in the Structural Stability Parameter $k_s = \frac{2m_e \delta_s}{\rho d^2}$. Structural damping varies with material and method of construction, e.g. a large offshore drilling rig may have an overall logarithmic decrement of $\delta_s = 0.3$ in the fundamental bending mode; for a lightweight welded steel chimney the equivalent logarithmic decrement could be as low as $\delta_s = 0.03$, and lower values have been recorded in exceptional cases. Vickery & Watkins (41) also found that correct modelling of $m_r$ and was not necessary, provided similarity of the product $2m_r \delta_s (=k_s)$ is maintained.

Correct representation of the velocity profile $V_h$ is probably more important when the cylinder is stationary than when it is oscillating. A stationary cylinder, subjected to a pronounced velocity profile may shed vortices at different frequencies along its length, separated by pockets of incoherent shedding, an effect that can be likened to the shedding of vortices from a tapered cylinder in steady flow (Gaster (103)). The velocity profile can also influence the critical velocity at which oscillations are excited; once instability has been initiated, the shedding becomes synchronised in phase and frequency although the maximum amplitudes of oscillation may be affected.

Turbulence scale $L_t$ and intensity $I_t$ are statistical properties of the flow. The scale is a measure of the correlation between fluctuations at two points in the flow, and the intensity statistically describes the magnitudes of velocity fluctuations referred to the three
coordinate axes. In the Immingham full scale tests, turbulence properties were not measured and thus there is no data on which to base the modelling of turbulence. However, Walshe (102) asserts that the incorrect modelling of turbulence generally has only slight influence on the amplitudes of oscillations and the critical velocity at which oscillations are initiated.

5.3.3 Application of Scaling Laws.

In the following comparisons, suffixes m and p refer to the model and full scale structures respectively and s is the linear scale factor \( s > 1 \).

(a) Geometry.

\[
\begin{align*}
\frac{d_m}{d_p} = \frac{h_m}{h_p} = s
\end{align*}
\]

(b) Reynolds number.

For similarity of \( \text{Re} \),

\[
\begin{align*}
\left\{ \frac{Vd}{\nu} \right\}_m = \left\{ \frac{Vd}{\nu} \right\}_p
\end{align*}
\]

but, because the same fluid is used in the full scale and model:

\[
\begin{align*}
\nu_m &= \nu_p; \text{ also } \frac{d_m}{d_p} = 1/s
\end{align*}
\]

\[
\begin{align*}
\therefore \quad V_m &= V_p/s \quad \ldots (5.3)
\end{align*}
\]

(c) Froude number.

\[
\begin{align*}
\left\{ \frac{V^2}{gh} \right\}_m = \left\{ \frac{V^2}{gh} \right\}_p
\end{align*}
\]

but \( g_m = g_p \) and \( h_m / h_p = 1/s \)

\[
\begin{align*}
\therefore \quad V_m^2 &= V_p^2/s \quad \ldots (5.4)
\end{align*}
\]

\[
\begin{align*}
\therefore \quad V_m &= V_p/\sqrt{s} \quad \ldots (5.5)
\end{align*}
\]
Note that Re and Fr cannot both be satisfied if the same fluid is used in full scale and model. (Equations 5.4 and 5.5)

(d) Cauchy number.

\[
\left( \frac{E_{m}}{\rho V^{2}d} \right)_{m} = \left( \frac{E_{p}}{\rho V^{2}d} \right)_{p}
\]

\[
\therefore \frac{E_{m}^{t}}{E_{p}^{t}} = \frac{V_{m}^{2}d_{m}}{V_{p}^{2}d_{p}}
\]

\[
E_{m}^{t} = E_{p}^{t} s^{2}
\]

...(5.6)

(e) Reduced Velocity

\[
\frac{(V/nd)_{m}}{(V/nd)_{p}} = \frac{d_{m}}{d_{p}} = \frac{1}{s}
\]

and \[
\frac{V_{m}}{V_{p}} = \frac{1}{\sqrt{s}}
\]

\[
\therefore \quad n_{m} = n_{p} \sqrt{s}
\]

...(5.7)

(f) Combined Stability Parameter.

\[
\left( \frac{2m \varepsilon}{\rho d^{2}} \right)_{m} = \left( \frac{2m \varepsilon}{\rho d^{2}} \right)_{p}
\]

\[
\text{but} \quad \frac{d_{m}}{d_{p}} = \frac{1}{s}
\]

\[
\therefore \quad (m \varepsilon)_{m} = \frac{(m \varepsilon)_{p}}{s^{2}}
\]

...(5.3)
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<tr>
<th>Cylinder</th>
<th>Cylinder material</th>
<th>Outer diameter d inches</th>
<th>Wall thickness t inches</th>
<th>Length L inches</th>
<th>Section stiffness EI lbf*ft²</th>
<th>Equivalent stiffness K lbf*ft</th>
<th>Mass/ unit length mg/ft</th>
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<td>-</td>
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<td>-</td>
<td>15.0 to 120.0</td>
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<td>94.5</td>
<td>7.107</td>
<td>2.060</td>
</tr>
</tbody>
</table>

× I.C.I. Grey Darvic
† Acrylonitrile Butadiene Styrene
* Hollow
▲ Filled with water
▼ Filled with lead shot
• Filled with sand, lead shot and water
<table>
<thead>
<tr>
<th>Cylinder</th>
<th>Structural logarithmic decrement</th>
<th>Frequency range in water $n$, $f$, $f_3$ Hz</th>
<th>Reduced velocity range $V_r$</th>
<th>Maximum Reynolds No $Re$</th>
<th>Baseplate material</th>
<th>Method of fixing to baseplate</th>
<th>Comments</th>
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<td>Araldite adhesive</td>
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<td>1.2 to 4.0</td>
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<td>Hot air welding</td>
<td></td>
</tr>
<tr>
<td>VII.2</td>
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<td>2.0 to 2.5</td>
<td>3.3 to 10.0</td>
<td></td>
<td>Darvic &amp; steel</td>
<td>Hot air welding</td>
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6. Designing and commissioning the test flume.

6.1 Introduction.

The writer designed the flume, drew and detailed the working drawings, supervised construction and subsequently commissioned the facility.

6.2 Overall design philosophy.

The flume was designed to satisfy the experimental requirements of Chapter 5 (i.e. 2ft wide, 3ft deep and a maximum velocity of 3ft/sec.), and to provide a useful laboratory test facility at the completion of the thesis programme.

Basically the final design centred on two considerations:

(i) water tanks in the laboratory are usually constructed from standard wooden tank panels, the sizes of which are 4ft x 4ft, 8ft x 4ft or 12ft x 4ft.

(ii) existing available equipment consisted of:

a 10 H.P. d.c. electric motor;

two variable pitch axial flow impellers and their shafts;

a converging/diverging section of metal housing for the impellers.

There are obvious economical advantages to be gained from utilising existing equipment and it is good engineering practice to use standard items where possible, even when this results in modifications in the design.

From (i) the overall size of the facility was determined as 24ft long x 4ft wide (see 6.3) and from (ii) the existing housing for the impellers decided the sizes of the metal ducting through which the flow circulated. To economise on floor space, the flume was mounted on a metal frame over the ducting. Fig. 6.1 is a general arrangement of the facility and also shows the features of the metal frame and ducting with the wooden flume removed for clarity.

The electric motor drove the two axial flow impellers via vee belts and reduction pulleys; the water was impelled through the
horizontal ducting, round a 180° bend and into the flume, returning to the impellers through two orifices in the floor of the flume, each connected to the metal housing by a 'lobster back' bend. For further economy of space the bends at the pump inlet and flume inlet were made sharper than general practice would suggest as the optimum; remedial measures were needed later to correct for this departure from general practice.

6.3 Flume design.

From experience it was known that adequate lengths of flume must be provided upstream and downstream of the test area. The upstream length assists the establishment of a uniform, fully developed velocity profile and the downstream length must be sufficient to allow a vortex street wake to form. It was decided that four flume widths upstream and two downstream were required, thus making the total lengths 12ft. However, a flume with a width of 2ft, depth of 3ft and length of 12ft has only limited general applications for the fairly large flow rate of 15 cusecs; a more universal facility was achieved by locating the flume in a larger tank, and using false walls to delineate the limits of the flume. The larger tank, built from standard panels, was 4ft deep x 4ft wide and its overall length 24ft, to give adequate lengths for contracting the flow at inlet to the flume and diffusing it again at outlet. Fig. 6.2 shows the arrangement consisted of the basic wooden tank with two false walls (also of wood) at 2ft separation, running parallel with the longer sides of the tank to form the test flume. A complex contraction reduced the 4ft inlet width of the tank to the 2ft inlet width of the flume and a short diffuser was incorporated in the outlet end of the flume. The symmetrical contraction of area ratio 2:1 was designed with the assistance of reference (104) and was essential to the establishment of a uniform velocity distribution in the test area. The diffusing section was installed as a head recovery device to reduce the overall circuit losses. The included angle of the diffuser was made 10° to prevent separation and unstable flow conditions at entry to the pumps. The false walls
were sealed to the floor of the tank only along the straight flume length and diffusing section. In the contracting section, a small gap between the bottom of the false walls and the tank floor equalised the hydrostatic pressures across the false walls. The elimination of unbalanced hydrostatic forces reduced the structural strength required of the walls. The advantages of this were that the walls were cheaper to build because less material was necessary, and that at the end of the thesis programme the walls could be removed from the tank without causing major structural modifications to the facility. The test area of the flume extended for about 4ft and within this area the floor was reinforced considerably with timber and steel joists, to accommodate the test cylinders which were fixed to the floor of the test area. Perspex windows were incorporated in one tank wall and in the corresponding flume wall to permit visual observation of the cylinders under test. The flume was plumbed into one of the laboratory sumps, and filled by a small auxiliary pump and filtering unit. A bypass loop in the network enabled the auxiliary pump to continuously filter the water contained in the flume, thus reducing the risk of algae growth and the resulting discolouration of the water. The flume was emptied by gravity flow back to the sump or by pumping out with the auxiliary pump, by-passing the filter and discharging to the sump.

6.4 Pump drive and axial thrust on bearings.

The impellers were 15" diameter with 7 aerofoil blades of variable pitch. Previously, they had formed part of another experimental rig and fortunately, characteristics were obtained during their operation (Sutton (105)). Axial flow pumps were essentially low head, comparatively high flow devices and figs. 6.3 and 6.4 show the impeller details and performance at the optimum blade angle of 20°, for two speeds, 400 rpm and 500 rpm. The estimated flume circuit losses are shown superimposed - these were difficult to assess and the two lines embrace the maximum and minimum anticipated operational losses under realistic conditions. Although theoretically possible, in practice the impeller speed could not be increased above 500 rpm because the blades
stalled and performance declined rapidly (105). Thus, the anticipated maximum flowrate lay between 13 to 16 cusecs if the head loss in the flume could be confined to 1.5 ft. (An energy balance of the flume showed that the energy from the electric motor was converted into 'water horse power' and frictional losses. A 10 H.P. motor and pump unit delivering say 14 cusecs at 1.5ft head was operating with an efficiency of approximately 25% and represented an incredibly inefficient device.) The maximum speed of the motor was 1000 rpm and a gearing of 2:1 was therefore necessary to limit the maximum speed of the impellers to 500 rpm. The vee pulley drive arrangement is shown in fig 6.1; the 6" diameter motor pulley has four vee grooves and each 12" diameter impeller pulley has two. Selection of the driving belts was based on the relative angles of contact on the three pulleys, the maximum torque and maximum rotational speed as outlined in the manufacturer's handbook. A variable transformer (variac) and rectifier were electrically connected to the motor in order to give infinite adjustment within the operational speed range of the motor.

The 'motor ends' of the impeller shafts were mounted on tapered roller bearings in plummer blocks, acting as thrust-and journal bearings; the 'impeller ends' of the shafts were supported in water-lubricated plain journal bearings. Axial thrusts on the plummer blocks were estimated from the product of maximum head developed across the impellers and their mean area. This showed that thrusts of up to 600 lbf per plummer block could be developed at certain conditions, and the metal frame on which the blocks were mounted was designed accordingly.

6.5 Whirl and vibration of the impeller shaft.

Resonance at the fundamental frequency of shaft bending vibrations could be excited if this frequency corresponded to the impeller blade frequency. The excitation mechanism was created in the non-uniform peripheral distribution of velocity (if, for example, the flow on the inside of the lobster back bend was separated, or at any rate slower than that on the outside of the bend). Each blade then
would be subjected to a fluctuating bending moment which constituted an exciting force at the blade frequency. The stiffness of the shaft and the low rotational speed ensured that neither whirl nor bending vibrations of the shaft were excited.

6.6 Impeller shaft seals.

The impeller shafts passing through the 'lobster back' bends presented problems of sealing the shaft housings against water leakage under stationary and dynamic conditions. The solution is shown in fig. 6.5; the gland was fabricated from readily available water piping and piping reducers. The annular body of the gland was stuffed with a carbon and silicon based material, and the stuffing nut tightened to compress the sealing compound thus forming a watertight, self-lubricating seal.

6.7 Commissioning the flume.

6.7.1 Surging, velocity distribution, turbulence and hydraulic gradient.

The initial tests were arranged to determine the maximum flowrate available for various water levels. During these tests and particularly at low water levels, large, intermittent air-entraining vortices were seen forming at each pump intake orifice; under these conditions, the flow showed a distinct tendency to surge. A splitter plate was built between the two orifices and the whole area covered by a sloping hood to form a convergent intake unit. It was hoped that vortex formation would be destroyed by the combined actions of separating the two orifices, reducing the free surface area, and contracting and accelerating the flow. Perforated screens were fitted over the intake of the hood and the final arrangement completely suppressed the surging effects. Attention was then focused upon the velocity distribution in the approach to the test area - the presence of the inlet bend immediately upstream of the channel contraction produced a distorted velocity profile, probably worsened by separation and the secondary flow generated within the bend. Flow resistance screens were installed to even out the spatial variations in velocity,
reduce turbulence and prevent separation from the inner radius of
the inlet bend. The screens were placed immediately upstream of
the contraction to reduce the overall head loss through the circuit
(the head loss through the screens was proportional to the square of
velocity and placing the screens in the contracted section would increase
the screen head loss by a factor of four). The screens, consisting
of a perforated metal plate with an open area of 0.23 and a one inch
thick layer of 'hairlok' matting, were mounted on a separate wooden
framework which could be removed from the flume to permit periodic
cleaning. This was considered important to the maintenance of
similar test conditions; otherwise the screens would become blocked
with water-borne debris and the resistance characteristics would vary
with time. The operation of screens is a study in itself and the function
of 'hairlok' matting is not completely understood. With the open area
ratio used here, the flow through the perforated plate would be con-
verted into discrete jets and interactions between these could, in
fact, worsen the velocity distribution if this plate was used in isolation.
The presence of the 'hairlok' probably suppressed the interactions
between, and assisted coalescence of, the jets emerging from the
perforated plate. Subsequent measurements of velocity showed a
satisfactorily uniform distribution across the test section and typical
traverses are shown in fig. 6.6.

However, the modifications to the inlet necessarily increased
the system losses and the flow rate through the final test arrangement
was measured as 13 cusecs - coinciding with the more pessimistic of
the two estimated performance curves of fig. 6.4. The increased
resistance of the circuit doubtless enhanced the smooth running of the
pump and this damped out the major velocity fluctuations and probably
helped to improve the velocity profile. The velocity variations with
depth and distance from the centre-line of the flume are shown in
fig. 6.6. The measurements were recorded at the entrance to the
test area, approximately 8 ft. (4 flume widths) from the contraction,
and at mean velocities of 1.0, 2.0 and 2.7 ft/second. The velocity
distributions apparently were independent of the mean velocity setting.

The variation of velocity across the flume was symmetrical about the flume centre-line and the boundary layer on each wall was between two and three inches thick for the velocity range of these tests. It was noted that the wall boundary layers were formed at the end of the contraction and increased in thickness with distance along the flume, similar to the formation of a boundary layer on a flat plate. At the site where the velocity measurements were made, the Reynolds numbers for the velocity settings, based on the distance from the end of the contraction, were $6.4 \times 10^6 < \text{Re} < 17.3 \times 10^6$. The boundary layer thickness, $b$, theoretically varied as $\text{Re}^{-0.2}$ and for these Reynolds numbers $1.3'' < b < 1.6''$; thus the theoretical boundary layer thickness was essentially constant and of the same order of magnitude as the experimentally inferred values.

A fairly pronounced velocity variation with vertical distance was noted near the flume floor in fig. 6.6. At the floor, the velocity was zero and the local velocity increased rapidly to within 4% of $V_{\text{max}}$ at a distance of one quarter the water depth measured from the floor (i.e. $h/4$). From $h/2$ to $h$ the velocity varied by only 1% from $V_{\text{max}}$, and all subsequent velocity measurements, during the experimental programme, were made between these two levels. To improve further the velocity distribution would have required the use of graded screens, resulting in additional losses in head and flowrate. It was considered imperative to maintain the maximum flowrate at not less than 13 cusecs, and, since the velocity profiles were satisfactory and apparently independent of absolute water depths, no further remedial action was taken. The absence of dependence upon absolute water depths implied that each test cylinder would be subjected to equal amounts of velocity variation with relative depth.

The turbulence intensity in the direction of flow was inferred from measurements made with the miniature propeller meter. The low frequency velocity fluctuations were passed through an r.m.s.
meter and their r.m.s. value compared with the mean velocity. In all cases the turbulence intensity was between 1% and 2%.

During subsequent tests, the water level in the test area of the flume was observed to decrease as the velocity was increased, due to the establishment of an hydraulic gradient within the circuit. The small auxiliary pump, used for filling and emptying the flume, was used to overcome the hydraulic gradient difficulties by maintaining a constant water level in the test area. During increasing velocity tests, water was pumped into the flume, and conversely, water was extracted during decreasing velocity tests.
Fig. 6.1  General arrangement of flume metalwork and impeller drive.
Fig. 6.2 Details of experimental flume.
Blade details

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<td>0.204&quot;</td>
<td>6%</td>
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</tr>
<tr>
<td>Boot</td>
<td>3.62&quot;</td>
<td>0.412&quot;</td>
<td>6.5%</td>
<td>3.75&quot;</td>
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Fig. 6.3 Impeller details.
Fig. 6.4 Estimated impeller performance (105) with assessed losses superposed.

Fig. 6.5 Detail of impeller shaft seal.
Fig. 6.6 Velocity traverses in the test area.
7. Instrumentation and experimental technique.

7.1 Instrumentation.

The instrumentation used in the tests was selected for its reliability, accuracy and simplicity. The basic equipment, principles of operation and calibration (where appropriate) are described below.

7.1.1 Strain gauges.

Foil resistance strain gauges were cemented to each test cylinder (see fig. 7.1) at convenient distances from the base and along carefully chosen longitudinal axes (Section 7.2.1) aligned with the cross-flow and in-line directions. The gauges were connected to form one half of a Wheatstone bridge (fig. 7.2) to eliminate drift from localised temperature fluctuations and the gauges themselves were selected for their comparative lack of temperature sensitivity. The reference voltage was maintained by a stabilised power supply and the electrical output from the bridge circuit was passed through a buffer amplifier unit and thence to a galvanometer in an Ultra Violet Recorder (U.V.R.) from which an analogue trace was obtained on light-sensitive paper (fig. 7.3). A timer periodically triggered a light in the U.V.R. thus marking a time base on the paper. The width of the U.V. paper was marked in centimetres and millimetres, so that one trace yielded the amplitude and frequency of motion. The width of the trace line was approximately 0.5 mm and the accuracy with which the results could be interpreted was a function of the ratio of this width to the oscillatory amplitudes recorded by the time history of the trace line. For consistency throughout the tests, the limiting oscillatory amplitudes interpreted from the U.V. traces were \( \pm 0.25 \) cm. This was equivalent to \( \pm 0.02 \) diameters on all the cylinders tested in the course of the research programme.

Each strain gauge was protected from the water environment by two layers of 'Gagekote 4', a resinous, non-hardening compound that was painted on in a fluid form. This protection did not greatly increase the local diameter as seen in fig. 7.4, but afforded adequate
insulation and had no measurable effect on the local stiffness.

To calibrate the strain gauges, the cylinder base was fixed to the reinforced floor of the flume test area. A horizontal force was applied to the cylinder free end, using a system of weights, a low friction pulley and a light string (fig. 7.5). For each applied force, the galvanometer deflection in centimetres was measured from the U.V. trace and after correction for pulley friction effects, the results were used to plot graphs of bending moments versus galvanometer deflections for each strain gauge (fig. 7.6). This method of calibration was employed for the in-line and cross-flow directions in sequence, and yielded a measure of the cross coupling between the two directions. Fig. 7.6 shows that less than 10% of the signal in one direction was detected by the gauges in the orthogonal direction.

The calibration of the strain gauges in terms of bending moments meant that during subsequent tests in flowing water, the signal registered by a galvanometer represented a known bending moment for all mode shapes. In view of the anticipated large variation of forces in the two normal directions, the galvanometers were protected by a bank of potentiometers connected between the buffer amplifier and the U.V.R.

7.1.2 Correction for friction in the pulley.

The low friction pulley used in the calibration of the strain gauges was manufactured from a 3" diameter light alloy casting with a $\frac{3}{4}$" diameter ball bearing race insert mounted on a $\frac{1}{4}$" diameter spindle. To calculate the frictional constant, the pulley was loaded as shown in fig. 7.7: the loads $W_1$ and $W_2$ were initially equalised and weights added to $W_2$ until static balance was no longer possible. The weights were again equalised and the test repeated by increasing $W_1$ to detect hysteresis or directional effects - none was found. The frictional constant ($\mu$) was calculated as shown below:

$$\mu = \frac{W}{1 + W_1 / \Delta W}$$  \hspace{1cm} (7.1)
Substitution of the experimental variables into (7.1) led to:

\[
\mu = 0.008 \quad \ldots (7.2)
\]

This value is comparable with declared frictional constants for ball races.

In the strain gauge calibration tests, the friction in the pulley would reduce the effective loading to which the cylinder was subjected and the results were corrected for this pulley effect in the following way:

Let the tension in the horizontal string be \( T_1 \) and the tension in the vertical string be \( T_2 \). Then

the applied torque \( = R(T_1 - T_2) \) and

the friction torque \( = \mu r (T_1^2 + T_2^2)^{\frac{1}{2}} \)

(because \( T_1 \) and \( T_2 \) are orthogonal vectors).

For static balance:

\[
R(T_1 - T_2) = \mu r (T_1^2 + T_2^2)^{\frac{1}{2}} \quad \ldots (7.3)
\]

and

\[
\frac{T_2}{T_1} = \frac{1}{1 - \sqrt{\frac{2\mu r}{R}}} \quad \ldots (7.4)
\]

Thus the force at the cylinder was reduced consistently by 3.6%.
The experimental results were factorised to compensate for this, and the corrected results used to plot the graph of fig. 7.6.

7.1.3 Ultra Violet Recorder U.V.R.

The basic operation of the U.V.R. was described in 7.1.1 and this section is concerned with the more detailed calibration of the timing device. The timing unit of the U.V.R. consisted of a mains voltage-controlled free running oscillator, an extremely accurate device when adjusted correctly. The accuracy of the U.V.R. timer was verified prior to embarking on the experimental programme by calibration with a high resolution digital counter. Each calibration
test lasted for twenty four hours, after which time the U.V.R. timing unit was adjusted and the test re-run until the difference between the timer and the counter was better than 0.1%.

7.1.4 Miniature propeller meter.

The velocity in the flume was measured with a miniature propeller meter, a digital instrument consisting of a small six-bladed propeller mounted in jewelled bearings at the end of a long coaxial stem (fig. 7.8). When the propeller rotated due to local relative velocities, the passage of the blades produced a variation in resistance across the coaxial pick-up. An analogue readout of velocity was inferred from a meter calibrated in terms of impulses from the propeller. The resolution of the propeller meter was examined in a preliminary exercise, which showed that each velocity setting could be read from the meter to within 0.5% of the true mean reading. The oscillations of the meter readings were caused partly by the turbulence of the flow, estimated in Chapter 6 as 1% to 2%, and partly by the response of the instrument.

The declared calibration was carefully checked by two methods, the 'whirling arm' test and comparison with an orifice plate flowmeter.

The 'whirling arm' facility consisted of a free surface annular tank of water with a horizontal arm mounted coaxially over it. The arm was driven by an electric motor, and speed control achieved with a transistor-thyrister controlled unit. The propeller stem was attached to the free end of the arm and immersed vertically in the water. A range of arm rotational speeds were covered by the tests and for each test the propeller meter reading was checked with the velocity determined from the product of arm radius and angular velocity. The declared calibration of the propeller meter was observed to deviate by less than 1% from the values determined by the 'whirling arm'. A comparison between the propeller meter and an orifice plate meter resulted in differences of less than 1% between the two sets of readings. These tests were conducted in a narrow
flume supplied by a centrifugal pump with the orifice plate mounted in the pipeline between the pump and the flume.

In the subsequent test programme, the declared calibration of the propeller meter was used to determine flow velocities.

7.1.5 Thermister probes.

Two thermister probes were used to measure the vortex frequencies in the wake of the cylinders (see fig. 7.9). In air flow, hot wires are often used for this purpose but water generally is too contaminated for their use. The thermisters were essentially hot wires in ceramic coatings and were much more robust than the normal hot wire probes. A variation of local velocity (e.g. the passage of a vortex) caused a corresponding variation in the heat transferred from the thermister and hence a change in electrical resistance.

The two thermisters were connected to form two limbs of a Wheatstone bridge, the output of which was used to energise a galvanometer in the U.V.R.

No calibration was necessary with the probes because they were used only as qualitative, digital (i.e. frequency) devices.

Initially the Wheatstone bridge was supplied from a d.c. voltage but this was changed later to an alternating supply to prevent anodic erosion.

7.1.6 Manometer tube.

Water levels in the flume were measured using a manometer tube, flush-mounted in the false wall of the flume and connected to a stilling well on the outer wall of the tank. The stilling well, as its name implies, was designed to damp out minor fluctuations in water level and give the mean static head; water levels in the stilling well were measured to an accuracy of ± 0.02" with a hook gauge and vernier micrometer.

7.2 Experimental technique.

7.2.1 Manufacturing the cantilevers.

Essentially, the cantilevers were circular cylinders
mounted in baseplates which in turn were fastened with long bolts to specially reinforced beams in the floor of the flume test area. A total of thirteen different cylinders were tested in various sections of the experimental programme.

Excessive lengths of cylinders were obtained, and from these the dimensionally most consistent portions were selected to form the cantilevers. Prior to mounting in their bases, the cylinders were carefully weighed and all dimensions measured. The cylinders were then fixed to their bases by adhesives, solder or interference fitting, the method of fixity being determined primarily by the materials of the two components, as detailed in Table 5.1.

Occasionally, when the cantilever free end was deflected and released, the transient oscillatory motion would not be confined exclusively to the plane of the initial displacement and appeared to execute a 'figure of eight' type of orbit. This effect was considered to arise from either non-linearity in the base stiffness (baseplate, adhesive or floor of the flume) or dimensional variations produced during manufacture resulting in the establishment of orthogonal principal planes into which the strain energy was gradually diverted. By attaching the cylinder bases to massive steel joists and repeating the tests, the first explanation was excluded and the slight dimensional variations were identified as the real cause of the observed behaviour.

The orthogonal principal planes of each cylinder were then located by rotating the plane of initial displacement until transient oscillations were confined to one plane only; strain gauges were cemented onto the cylinder surface along corresponding longitudinal axes.

7.2.2 Calibrating the cylinders.

The stiffness of the cantilevers was determined by applying a horizontal force to the free end, measuring the corresponding deflection with a vernier micrometer and obtaining, from the graph of force versus deflection, an equivalent stiffness. The results were corrected for friction in the pulley in a similar way to the results of
7.1.1. A.U.V. trace of transient motion in air was recorded for each cylinder, and from this, the natural frequency and structural logarithmic decrement were obtained. The experimentally determined natural frequencies were compared with the equivalent calculated values, based on the measured variables of each cylinder; maximum differences of less than 2% were observed. The calibration of the strain gauges in terms of bending moments was described in section 7.1.1.

7.2.3 Testing sequence.

A standard testing method was adopted for all tests and this is summarised below.

(i) Establish the correct water level and allow conditions to settle.

(ii) Displace and release the free end of the cylinder; record transient motion on the U.V.R. for in-line and cross-flow directions in still water.

(iii) Gradually increase the flow velocity from zero to maximum, recording fluctuating and steady components of strain in both directions. Visually observe the behaviour of the cylinder under test.

(iv) Reverse step (iii) and repeat step (ii) noting drift in galvanometers (if any).

Certain detailed modifications to the programme were made for some tests but in general the main series of tests was conducted using steps (i) to (iv) as detailed. In the 'added mass' tests of Part I(a) and in the still-water damping tests of Part I(c) only steps (i) and (ii) were necessary.
Fig. 7.1 A strain gauge cemented to one of the test cylinders.

Strain gauge circuit

Fig. 7.2 Electrical circuit of strain gauge measurement system.
Fig. 7.3 The instrumentation used in the experimental programme.

Fig. 7.4 A strain gauge encapsulated in waterproof adhesive.
Fig. 7.5 Calibration of the strain gauges Cylinder II.2.

![Diagram of calibration setup]

Fig. 7.6 Bending moments versus galvanometer deflection to determine cross-coupling. Cylinder II.2.

![Graphs showing bending moments versus galvanometer deflection]
pulley coefficient of friction = \( \mu \)
\[ \Delta W = W_2 - W_1 \]
\[ \mu = \frac{R}{\ell} \left( \frac{1}{1 + \frac{W_1}{\Delta W}} \right) \]

Fig. 7.7  Corrections for the pulley used in the strain gauge calibration tests.
Fig. 7.9 One of the thermistor probes.
8. Experimental results: Part I of the experimental programme, Added mass and damping.

8.1 Added mass.

8.1.1 Experimental results and discussion.

The tests were designed to investigate the added mass of cylinders oscillating in the fundamental and second normal modes in various depths of water. Cylinders I1, I2 were approximately three feet long and were tested in depths of water ranging from 11 inches to complete immersion ($h = L$). The experimental conditions were intended to illustrate the departure of real systems from the ideal situation analysed by Lamb (71) and Stokes (70) who considered cylinders of infinite length and sufficiently far removed from the effects of solid boundaries and free surfaces. Chapter 4 showed that the added mass of an ideal cylinder, calculated using potential flow theory, is represented by a uniformly distributed loading equal to the displaced mass of fluid.

In the initial tests, natural frequencies of I1, I2 were determined from the transient amplitude decay traces of the cylinders in still water. The transients of the fundamental mode were obtained by recording cylinder motion on the U.V.R. when the cylinder free end was released from a deflected position. To excite the second normal modes of I1, the cylinder was struck impulsively at a point corresponding to the antinode for that mode (between one half and three quarters of the length from the clamped end of the cantilever). Transient results were recorded for the fundamental and second normal modes for a range of water levels. Springs and concentrated masses were attached to the cantilevers free ends to vary the natural frequencies and mode shapes.

In a preliminary exercise (53), a cylinder in flowing water was observed to oscillate in line under the action of vortex shedding at frequencies close to its still-water natural frequency. This implied that streaming flow did not alter the added mass from the still-water value, and where possible, in the present tests, the
observation was investigated further by carefully noting the frequencies excited initially in the in-line direction.

The cylinders I1, I2 were tested in sequence in the water circulation channel of a large civil engineering model at the British Hydromechanics Research Association at Cranfield. Flowing water tests were limited to medium depths of water, thus excluding an examination of very high and very low water levels. Each cylinder was mounted in the channel as a vertical cantilever and natural frequencies obtained from the U. V. recordings of transient motion in various depths of still water. A period of ten minutes was allowed to elapse between consecutive tests in a fixed depth of water, to re-establish still water conditions, although there was no evidence of extreme sensitivity to local flow disturbances.

Tables 8.1, 8.2 record experimental and calculated fundamental natural frequencies for the free-ended cylinders I1, I2. The frequencies in air were recorded to confirm the accuracy of the transfer matrix method of calculation using easily defined structural details as input data. It is seen that the agreement is very satisfactory. The still-water natural frequencies of I1, I2 showed only minor variations from the 'in-air' values at water levels less than one half the cylinder length. In low water levels, the presence of three dimensional free surface effects cannot be excluded entirely from consideration, although reference to the calculated values shows that the frequency variations may be explained by comparison of the relative water and structural loadings.

By considering the cylinder and masses as a two degrees of freedom system and by assuming the second normal mode frequency to be relatively high compared with the fundamental (see Appendix 2), the following equation may be established for defining the fundamental frequency of a cylinder in water:

$$n = \frac{1}{2\pi} \sqrt{\frac{3EI}{M_tL^3 + 0.243(m_sL^4 + m_a h^4)}}$$  \(\ldots (6.1)\)
Examination of (8.1) shows that a concentrated mass at the free end is equivalent to a distributed loading of approximately four times the concentrated mass. Also shown is the relationship between frequency, water depth and cylinder length. For example, consider the case of a free ended cantilever in water \( (i.e. \: M_t = 0 \) in (8.1))

\[
\frac{n}{2\pi} = \sqrt[4]{\frac{3EI/L^4}{0.243 \left( m_s/m_a \right)^4 (h/L)^4}} \quad \ldots (8.2)
\]

The added mass term, \( m_a \), is associated with a number, less than or equal to unity, raised to the fourth power. Thus, for small values of \( h/L \) the resultant may be negligible although \( m_a \) may be much larger than \( m \). An example of this is given by reference to the tests on the free ended fibreglass cylinder \( I_2 \), the natural frequency of which differed by only 4% from the in-air value when the water level was \( h/L = 0.3 \). The reason for this is easily seen by rearranging (8.2) to obtain

\[
\left( \frac{n_a}{n} \right)^2 = 1 + \frac{m_a}{m_s} \cdot \left( \frac{h}{L} \right)^4 \quad \ldots (8.3)
\]

and

\[
\left( \frac{n_a}{n} \right) = 1.04 \quad \therefore \frac{m_a}{m_s} = 9
\]

Thus although the added mass/unit length is nine times larger than the structural mass/unit length, the water depth must be comparable with the cylinder length before significant reductions in natural frequency result. In the present case, a water depth of \( h/L = 0.75 \) would be sufficient to halve the in-air natural frequency. By contrast, the frequency of the lighter, fibreglass cylinder was reduced from approximately 20 Hz in air to 6.37 Hz when fully immersed. This emphasised the implications of siting lightweight piling in large depths of water, and the necessity for including the added mass in the generalised mass terms when calculating natural frequencies.

In flowing water, the excited frequencies of sustained
oscillations were very close to the still-water frequencies, demonstrating that streaming flow has little effect upon the added mass, at least for the frequencies and amplitudes initially excited by vortex shedding. The flowing water tests were arranged to qualify the assumed added mass function and, once excitation had been initiated, that test was terminated. The subsequent variation of excited frequency with increasing velocity is recorded and discussed in Chapter 9.

Tables 8.3, 8.4, 8.5 compare experimental and calculated fundamental frequencies of cylinders 11, 12 with concentrated springs and masses attached to the free ends. These appendages resulted in large variations of natural frequencies and mode shapes; however, there is close agreement between experimental and calculated results (figs. 8.1, 8.2, 8.3) and this is interpreted as confirmation of the assumed ideal added mass function representing the added mass of a cylinder in real fluid. The simple equation (8.1) cannot be modified to compute frequencies when concentrated springs are attached to the free end, and the calculated values of frequency shown in Table 8.4 result from transfer matrix analysis. In common with the previous tests, the experimentally recorded and calculated still-water frequencies were in close agreement, and the flow excited frequencies showed only minor departures from these still-water frequencies. The tests in the fundamental mode have emphasised that variations in mode shape and frequencies do not lead to variations of added mass in still-water or streaming flow, for the range of variables covered by the tests. These results are in general agreement with the results from previous research with completely immersed cylinders for which $L/d > 10$, thus confirming the lack of three-dimensional free surface effects in still-water. The minimum depth of water in which flow-excited oscillations were recorded was $h/L = 20$, and there was no evidence of departure from the still-water frequencies at this $L/d$. Iverson & Balent (87), Stelson & Mavis (85) and McNown (88) deduced that a value of $K_m = 1.0$ applied for fully immersed oscillating rigid
cylinders for $L/d > 10$. Keim (89) recorded $K_m = 0.95$ for all $L/d > 15$ and it is interesting to note that in his tests the cylinders were uniformly accelerated through the water (consideration of Darwin's (73) drift mass explanation shows that ideally $K_m = 1.0$ for Keim's test arrangement).

In the present tests, the free end displacements of the cylinders without springs were approximately one diameter and an examination of the U.V. transient traces showed that for each trace the frequency was invariant with time. In the fundamental mode the amplitudes of oscillation are determined by the distance along the cylinder from the fixed base. The recording of uniform frequency for time along the U.V. trace demonstrates that within the amplitude range up to one diameter the net added mass of the cantilever is not influenced by local amplitude effects. The combination of very high frequencies and large amplitudes could lead to the establishment of a bow wave at the front face of the cylinder, and a ventilated and depressed free surface at the rear face. These effects could lead to departures from a uniformly distributed added mass function, with consequent variations in the natural frequency (this is discussed in detail in Section 9.3.6). In Tables 8.6, 8.7 the results of tests in the second normal mode for 11 with and without concentrated tip masses are shown. The experimental still-water frequencies are very close to the calculated values and compare almost exactly with the flow excited frequencies. The obvious indications are that the added mass function for the second normal modes is identical to the added mass function for the fundamental modes, i.e. $K_m = 1.0$. However, the maximum amplitudes of these tests were restricted to $\pm 0.2d$ and they are, therefore, less comprehensive than the tests in the fundamental modes.

The results of this section are at variance with the work of Clough (91) who also tested a cantilever oscillating in the second normal mode, recording $K_m = 0.58$. The reason for such a large difference between the two sets of results was attributed (Chapter 4)
to peculiarities in Clough's test arrangement. Full scale site tests on surface piercing piles have confirmed $K_m = 1.0$ for both the fundamental and second normal modes of oscillation.

In summary the present tests have established the following points:

(i) the added mass of a cantilevered cylinder oscillating in the fundamental or second normal modes may be described by the ideal potential flow function considered as a uniformly distributed loading;

(ii) within the range of variables used in the tests the added mass was not influenced by frequency, amplitude, mode shape, free surface effects or streaming flow;

(iii) maximum differences of 1% were observed between calculated and experimental natural frequencies and this is about the assessed accuracy of frequency measurement.

8.2 Hydrodynamic damping.

8.2.1 Experimental results and discussion.

This section examines the hydrodynamic damping of a cylinder oscillating freely in the fundamental mode in still water. A preliminary test using a PVC cylinder revealed that rate of decay of amplitude was uniform with time, indicating that the overall damping demonstrated the same characteristics as viscous damping (see Chapter 4). A brief visual study of the flowfield in the immediate vicinity of the oscillating cylinder showed the water 'slipped round' the cylinder without separation. Many fluid engineering problems exhibit separated flow with quadratic damping (i.e. proportional to square of velocity), resulting in amplitude-dependent effects, and the unseparated flowfield observed in the visual study explained the origins of the recorded viscous damping. On this evidence, it seemed reasonable to apply Stokes' (72) element damping theory to the problem and the mode shape of the immersed length of the oscillating cylinder.
was integrated to yield the energy degraded by viscous damping. This led to the definition of an equivalent coefficient of damping from which a theoretical logarithmic decrement was calculated (see Appendix 3). The initial series of tests were arranged to test one of the cylinders comprehensively in order to confirm the general form of the expression for the theoretical logarithmic decrement. Following this, a pendulum and three other cylinders were tested for comparison with the theoretical calculations to establish the operating range over which the analysis was applicable. Reynolds number has only limited significance when applied to an oscillating cylinder and a more comprehensive form of comparison is provided through the Size number $N_s$, the full meaning of which will be explained later in this section.

The cylinders were tested in sequence; each was mounted in the flume as a vertical cantilever and transient amplitude decay traces obtained by the standard procedure of displacing and releasing the cantilever free end and recording the resulting motion on the UVR. Cylinder 11 was used to confirm the general form of the expression for the theoretical damping coefficient and the results of these tests are shown in Table 8.8 and in the graph of fig. 8.4. It is seen that the three curves demonstrate similar trends; for water levels less than 0.5L the total logarithmic decrements are very close to the structural values $\delta_s$ and for water levels greater than 0.5L the damping increases very sharply. The calculated hydrodynamic damping ($\delta_h$) is additional to the structural damping ($\delta_s$) and Chapter 3.3 shows the total logarithmic decrement ($\delta_t$) of a cylinder oscillating freely in a depth h (h < L) of still water is determined by the sum of the structural and hydrodynamic contributions, i.e. $\delta_t = \delta_s + \delta_h$ \hspace{1cm} (3.8)

where $\delta_h$ is defined by equation(A3.8) and derived fully in Appendix 3.

For the hollow cylinder the logarithmic decrement recorded in the maximum water level was a factor of four greater than $\delta_s$. The agreement between experiment and theory is consistently close.
throughout the range of variables tested, showing that within this range, the damping can be described by an equivalent viscous effect. The logarithmic decrement in air ($\delta_s$) varied with the loading attached to the free end as noted in Table 8.8. To vary the mass/unit length, the cylinder was tested when hollow and also when filled with water. The logarithmic decrement of the water-filled cylinder showed a pronounced variation with amplitude and frequency. Typically for the cylinder oscillating freely in air, the damping was approximately twice that recorded when the cylinder was hollow. The increase of damping was attributed to the sloshing of free surface waves within the cylinder and a calculation of the sloshing frequency based on the cylinder internal dimensions resulted in a frequency of 5.7 Hz compared with the cantilever frequency of 4.5 Hz. Interference between the two frequencies was considered inevitable and a small cork of negligible mass was used to seal the open end of the cylinder. When the tests were repeated with the sealed cylinder, the results were free from amplitude effects. Johns [106] quotes an interesting case history of sloshing being used to suppress the wind-induced oscillations of horizontal hollow structural members of an oil jetty. The members were partly filled with water to detune their natural frequencies by increasing their mass, and to augment the level of damping by the sloshing of free surface waves within the members. This expedient is reported to have been completely successful.

Table 8.9 records the results of tests on the PVC cylinder II when helical springs of various stiffnesses were attached to the free end. The structural logarithmic decrement is extremely sensitive to the stiffnesses of the attached springs and to the frequency ratio between the cylinder with and without the springs. For a spring of stiffness five times greater than the stiffness of the free ended cylinder the apparent structural logarithmic decrement was halved. To a first approximation, $\delta_s$ varied linearly with frequency ratio. The tests were conducted in air and in maximum depth of water only, for four different spring rates and therefore give a limited insight into the complex mechanisms of structural and hydrodynamic damping. An analysis of the results in water showed that the hydrodynamic
logarithmic decrement could be related approximately as follows, where subscript s denotes operation with tip springs:

\[
\frac{\delta_h}{\delta_s} = \frac{\phi_s}{\phi} \cdot \frac{n_s^2}{(n_0)_s^2}
\]  \hspace{1cm} \ldots (8.4)

and as previously noted, the apparent structural logarithmic decrement can be described by:

\[
\frac{\delta_s}{\delta_s} \propto \frac{n_a}{(n_0)_s}
\]  \hspace{1cm} \ldots (8.5)

Thus, attaching springs to the free end effectively lowers the logarithmic decrements in air and in water. This is reasonable and the definition of logarithmic decrement in the presence of viscous damping (see Chapter 3.1.1) would predict the recorded behaviour of \(\delta_h\), although the analysis is not applicable to \(\delta_s\). Great care was exercised throughout the test programme to ensure that cylinder motion was restricted to one plane only. Motion in more than one plane would doubtless increase the damping observed in any one of the planes and the analysis would be invalid. The cylinder tip was displaced with a light string whilst observing the strain in the cylinder's two principal planes, as displayed on the UVR. The direction of pull was approximately aligned with one of the planes and shifted slightly until zero strain appeared in the plane perpendicular to the string; the cylinder was then released and the transient recorded. However, not all amplitudes of recorded motion could be used in the subsequent analysis due to modal variation. When released, the cylinder mode shape changed from the deflection curve imposed by the application of a point force at the free end to one consistent with a freely oscillating cylinder having a distributed loading. The analysis involved integrating the square of the mode shape and errors in the assumed mode shape are amplified by the squaring process. As an example
of the errors likely to arise, two typical mode shapes are compared in Table 8.10 where differences of 20% are recorded between amplitudes defined by the two modes. On each transient trace the first three cycles were discarded during subsequent analysis, a practice employed by Stokes (72) during tests to confirm his theoretical predictions. The initial deflection was regulated to give an amplitude at the free end of one diameter for the first cycle included in the analysis (except for the cantilever with springs fitted to the free end - the springs limited the maximum amplitude to 0.2d). It was felt that these discarded cycles may contain amplitude-dependent damping effects. For this reason the rigid pendulum I3 was tested; the statically deflected and freely oscillating mode shapes were identical and all recorded amplitudes were included for analysis. The pendulum (details of which are shown in fig. 8.5) also had the advantage of maintaining a uniform, easily predicted mode shape whilst permitting variations of natural frequency. However, the tension springs and strain gauge link restricted the maximum amplitude of motion to 0.2d. The results of the pendulum tests are shown in Table 8.11 and figure 8.6; close agreement is evident, simultaneously confirming the lack of amplitude-dependent damping effects and emphasising the validity of the assumed theoretical treatment. Total logarithmic decrements (δₜ) increased with increases in water depth according to equation (A3.17) in Appendix 3. The apparent rapid increase of δₜ with h/L (fig. 8.6) would suggest inconsistency with the graphs of tests using the cantilever II; the vertical axis of the graph for the pendulum tests is drawn to a larger scale, whilst the horizontal scale is maintained as for the previous graphs and this has the effect of exaggerating the variations of δₜ with h/L. In absolute terms the hydrodynamic logarithmic decrement δₜ of the pendulum is considerably lower than the δₜ recorded for the hollow PVC cylinder II in Table 8.8. At first sight this is a surprising result as the frequencies in the two tests are virtually identical, the pendulum has twice the projected area of the PVC cylinder and intu-
tively one may expect the larger cylinder to experience a more rapid amplitude decay. But, by returning to the definition of logarithmic decrement due to viscous damping (eqn. 3.9a) and using this to compare values of $\delta_h$ for 11 and 13 at equal frequencies, and for the one case of $h = L$, equation (8.6) is obtained:

$$\frac{(\delta_h)_{I3}}{(\delta_h)_{I1}} = \frac{C_{I3}}{C_{I1}} \cdot \frac{K_{I1}}{K_{I3}} \quad \ldots (8.6)$$

The ratio of damping coefficients is calculated from:

$$\frac{C_{I3}}{C_{I1}} = \frac{d_{I3}}{d_{I1}} \cdot \frac{L_{I3}}{L_{I1}} \cdot \frac{1}{\phi} \quad \ldots (8.7)$$

i.e. $\frac{C_{I3}}{C_{I1}} = 2.3$

and the stiffness of the two cylinders is:

$$K_{I1} = \frac{3EI}{L^3} = 7.5 \text{ lbf/ft}$$

$$K_{I3} = 34.7 \text{ lbf/ft}$$

$$\frac{K_{I1}}{K_{I3}} = 1/4.6, \quad \phi = 0.249$$

The calculated ratio of logarithmic decrements is therefore

$$\frac{(\delta_h)_{I3}}{(\delta_h)_{I1}} = 0.5 \quad \text{c.f.} \quad \frac{0.128}{0.260} = 0.49 \text{ measured.}$$

Similarly, when the equation (3.9a) is applied to the PVC cylinder filled with water and also to the pendulum:

$$\frac{(\delta_h)_{I3}}{(\delta_h)_{I1}} = \frac{C_{I3}}{C_{I1}} \cdot \frac{K_{I1}}{K_{I3}} \cdot \frac{w_{I3}}{w_{I1}} \quad \ldots (8.8)$$
from which the calculated values of logarithmic decrements are

\[
\frac{(\delta_h)_{13}}{(\delta_h)_{11}} = 0.69 \text{ c.f. } \frac{0.128}{0.187} = 0.69 \text{ measured.}
\]

This type of analysis can also be used to compare the two sets of results from the pendulum tests with the two spring stiffnesses 34.7 lbf/ft and 60.4 lbf/ft.

\[
i.e. \quad \frac{(\delta_h)_{34.7}}{(\delta_h)_{60.4}} = \frac{C_{34.7}}{C_{60.4}} \cdot \frac{K_{60.4}}{K_{34.7}} = \left( \frac{n_{34.7}}{n_{60.4}} \right)^{3/2}
\]

\[
= \frac{60.4}{34.7} \left( \frac{n_{34.7}}{n_{60.4}} \right)^{3/2}
\]

\[
= 1.15 \text{ calculated}
\]

\[
c.f. \quad \frac{0.128}{0.108} = 1.18 \text{ measured}
\]

These cross-checks again emphasise the truly viscous nature of the damping recorded in the tests.

Fig. 8.7 records the results of tests on three fibreglass cylinders. The tests were conducted in a separate facility and amplitudes were measured by a capacitance device, the two plates of which were formed by a fixed probe and a light brass shim attached to the cylinder. As the cylinder oscillated, the separation (and hence the capacitance) between the plates varied; the recorded values of capacitance, being inversely related to separation, were converted to cylinder displacements through a calibration graph prior to use in the calculations of logarithmic decrement. Restrictions within the test rig limited the maximum water level to 0.75L. It was thought that the additional step of converting recorded values of capacitance
into cylinder displacements may increase the errors associated with each recorded value of $\delta_t$, and the limited range of water depths meant that the results were recorded over the region most sensitive to errors. In view of this the agreement seen in Fig. 8.7 is considered reasonable and the graphs of $\delta_t$ versus water depth all demonstrate overall shapes similar to the graphs from the PVC cylinder 11 and the pendulum 13, with $\delta_t$ increasing rapidly for water depths greater than 0.5L.

The recorded results of Tables 8.9, 8.11, 8.12, for which $y/d \approx 0.25$ are in general agreement with the quoted work of previous researchers testing in air or using completely immersed cylinders. This comparison demonstrates freedom from free surface influences and confirms the application of Stokes element damping theory to the present test arrangements, within the limitations of the experimental variables. The test results are quoted in terms of size number, $N_s$, and amplitude of oscillations $y/d$, and it is convenient now to consider the relevance of these two parameters. Prandtl (107) showed the boundary layer thickness $b$ of an oscillating cylinder to be of the order $\sqrt{v/n}$ and by substituting this value into the equation for $N_s$, the relationship between $b$ and $d$ is obtained:

$$N_s = \frac{nd^2}{v}$$
and $$b^2 = \frac{v}{n} \quad \text{(Prandtl)}$$

$$\therefore \quad N_s = \frac{d^2}{b^2}$$

... (6.10)

Boundary layer theory is most applicable to situations in which $d \gg b$, i.e. from (6.10) when the size number $N_s$ is large. Boundary layer theory was not formally defined until 1904 (Prandtl) and it is remarkable that Stokes (in 1851) used in his analysis one of the most important factors of boundary layer theory - that there is no
relative motion at the interface between the fluid and the surface of
the body over which it flows.

Bramley (98) demonstrated that the amplitude ratio $y/d$
may be regarded as the ratio of the rate of convection through a
length $d$ to the rate of diffusion through the boundary layer $b$.
Bramley concluded that the ratio $y/d$ must be small to avoid estab-
lishing large convective terms which cause non-linearities in the
Navier-Stokes equations of motion.

Stuart & Woodgate (96), testing a rigid pendulum in air
at size number $N_s \approx 40$ showed that damping increased linearly with
$y/d$ for $y/d \geq 0.4$, although independent of amplitude effects for $y/d$
$\leq 0.3$. At $y/d = 1$, the damping was approximately three times
higher than at $y/d = 0.3$. In Bramley's (98) tests with a rigid section
of a horizontal circular cylinder in water, non-viscous damping was
detected for various values of $y/d$ depending on $N_s$. The most con-
sistent results were achieved at the lowest size number, $N_s \approx 3600$,
and approximately uniform logarithmic decrements were recorded
up to $y/d = 0.2$. At the highest size number tested, $N_s \approx 180,000$,
amplitude-dependent damping was observed for all amplitudes of
motion; in those tests, a rotational mode of oscillations was excited
and it is not known to what extent that influenced the results. Viscous
damping effects have been recorded on tests with full scale piles in
water (97) at size numbers up to $N_s \approx 50,000$ and amplitudes of up to
$0.12d$; in the present tests (Table 8.8) initial free end displacements
of up to one diameter confirmed viscous damping at size numbers
$N_s \leq 3,500$. The experimental evidence does not establish clearly
defined relationships between the recorded damping and the two prime
variables $N_s$ and $y/d$. There is general agreement that damping is
viscous for $y/d \leq 0.2$ (in accordance with Bramley's (98) analysis of
$y/d$) but the range of $N_s$ over which this applied was from $40$ to $50,000$
and the original tests of Stokes were made at $N_s \approx 50$. From equation
(8.10) it was deduced that large $N_s$ were most amenable to boundary
layer considerations; the recordings of viscous damping by Stokes
and Stuart & Woodgate at small $N_s$ were unexplainable until the recently published review paper by Williams & Hussey (108). These authors refer to Stokes' original paper, pointing out that in his derivation of the expression for the fluid force on the cylinder, Stokes neglected the non-linear term in the fluid equation of motion, (the Navier-Stokes equations). Neglecting non-linear terms is considered valid by Williams & Hussey if either of two conditions are satisfied:

\[ (1) \quad y \ll \frac{d}{2} \quad \text{and} \quad \left( \frac{2 \nu}{n} \right)^{\frac{1}{2}} \gg \frac{d}{2} \]

or

\[ (2) \quad \frac{d}{2} \cong \left( \frac{2 \nu}{n} \right)^{\frac{1}{2}} \quad \text{and} \quad \text{Re} \ll 1 \]

The first condition leads to the conditions already proposed, i.e. $y/d \ll 1$, $d/b \gg 1$ (i.e. $N_s$ large), and the second states that $N_s \approx 1$ and $\text{Re} \ll 1$. The Reynolds number $\text{Re}$ is calculated from the maximum amplitude, frequency and kinematic viscosity of the cylinder/fluid system. However, $\text{Re}$ may be written as shown below:

\[
\text{Re} = \frac{2 \pi n y}{d} \cdot \frac{d^2}{v}
\]

i.e. $\text{Re} = 2 \pi n \frac{y}{d} \cdot N_s \quad \ldots (8.11)$

and for $\text{Re} \ll 1$, and $N_s \approx 1$ (from (2)) substitution in equation (8.11) gives:

\[
\frac{2 \pi n y}{d} \cdot 1 \ll 1
\]

i.e. $y/d \ll 1$ approximately.

Thus it appears that there are two ranges of size numbers (high and low) over which Stokes' theoretical analysis is applicable - but for each, a prerequisite is that $y/d \ll 1$. It is not clear how the non-
linear effects of larger \( y/d \) would influence the results in the undefined area separating the two \( N_S \) regions. Speculatively, the non-linear effects of larger \( y/d \) are functions of \( N_S \). Williams & Hussey (108) describe their own tests on torsion pendula in various fluids; the vertical pendula rods were surface piercing and the results showed that Stokes' analysis applied for the largest amplitude tested \( (y/d < 1.2) \) at very low size numbers \( (N_S \approx 5) \) and Reynolds numbers \( (Re \approx 0.6) \). Free surface effects were not detected, even for the higher Reynolds number of \( Re \approx 36 \). It is seen that the physical behaviour of a cylinder oscillating in a fluid is not described uniquely by satisfaction of the conditions (1) or (2), and it appears that the actual limitations of the applicability of Stokes' theory are less rigorous than specified by Williams & Hussey (108). It must be admitted that explanations of behaviour at the larger \( y/d \) values are little more than speculative. However, in this connection it is appropriate to examine the implications of oscillatory velocity as a product of frequency and amplitude, and the way in which this can lead to separation and a predominance of pressure forces over the viscous damping forces. An oscillatory motion, starting from rest in a real fluid, is initially irrotational and free from separation. As oscillatory velocity increases, edge vortices will form and coalesce into a wake; the degree of completeness to which a wake develops will depend upon the duration of flow in one direction. In steady flow past a stationary cylinder the time taken to form a vortex pair at the maximum velocity of the oscillatory motion is given by 

\[
T_v = \frac{d}{(S \times \omega)}
\]  

(8.12)

McNown & Keulegan (109) state that the ratio \( T/T_v \) determines the nature of the flow field adjacent to the cylinder; they classify the flow as follows:

(i) if \( T/T_v \) is 0.1 or less, separation and vortex formation are relatively unimportant.
(ii) if $T/T_v$ is greater than 10, the motion is quasi-steady and the pressure drag may be calculated from the drag coefficient derived for steady flow.

McNown and Keulegan also state that vorticity from the preceding cycle retards the formation of a wake and suppresses separation.

The present tests were conducted in varying depths of water but by examining the results of the tests for which $h = L$, the ratio of $T/T_v$ varied from zero at the fixed end of the cylinder II to a maximum of 1.26 at the free end. The average value of $T/T_v$ along the length would be much less than 1.26, thus defining an unseparated boundary layer. In this case, the energy dissipation would be due to viscous shear forces and it is therefore gratifying that viscous damping alone was recorded in the tests.

In summary, the experimental tests of this section and their analysis have established the following:

(i) Hydrodynamic damping of the natural oscillations of a cantilever and a pendulum in still water is purely viscous and may be described by Stokes' (72) element damping theory. This has been verified over the size number range $619 < N_s < 34,020$ using free end deflections of up to one diameter.

(ii) Viscous damping increases rapidly with water depth for $h/L > 0.5$.

(iii) Transient motion contains no free-surface amplitude-dependent damping effects.

(iv) Structural damping is influenced by cantilever free end loading.

(v) It is not clear how the variables $y/d$ and $N_s$ control the damping when departures are made from the theoretical limitations quoted in the text.
### Table 8.1 Comparison of fundamental frequencies P.V.C. cylinder II: free ended.

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Observed (Hz)</th>
<th>Calculated (Hz)</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.840</td>
<td>6.849</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>6.780</td>
<td>6.783</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>6.670</td>
<td>6.722</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>6.164</td>
<td>6.246</td>
<td>6.180</td>
</tr>
<tr>
<td>25</td>
<td>5.670</td>
<td>5.668</td>
<td>5.660</td>
</tr>
<tr>
<td>28</td>
<td>5.140</td>
<td>5.142</td>
<td>5.140</td>
</tr>
<tr>
<td>30</td>
<td>4.780</td>
<td>4.795</td>
<td>4.670</td>
</tr>
<tr>
<td>33</td>
<td>4.280</td>
<td>4.276</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>3.756</td>
<td>3.794</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8.2 Comparison of fundamental frequencies fibreglass cylinder II; free ended.

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Observed (Hz)</th>
<th>Calculated (Hz)</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>20.000</td>
<td>20.001</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>19.550</td>
<td>19.715</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>15.330</td>
<td>15.333</td>
<td>15.320</td>
</tr>
<tr>
<td>24</td>
<td>12.960</td>
<td>12.858</td>
<td>12.880</td>
</tr>
<tr>
<td>30</td>
<td>9.110</td>
<td>9.212</td>
<td>9.110</td>
</tr>
<tr>
<td>35</td>
<td>6.980</td>
<td>7.055</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>6.460</td>
<td>6.327</td>
<td></td>
</tr>
</tbody>
</table>

### Table 8.3 Comparison of fundamental frequencies P.V.C. cylinder II; with masses attached to the free end.

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Observed (Hz)</th>
<th>Calculated (Hz)</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.750</td>
<td>3.738</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3.718</td>
<td>3.719</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>3.402</td>
<td>3.446</td>
<td>3.410</td>
</tr>
<tr>
<td>30</td>
<td>3.285</td>
<td>3.293</td>
<td>3.310</td>
</tr>
<tr>
<td>36</td>
<td>2.900</td>
<td>2.908</td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>2.205</td>
<td>2.224</td>
<td></td>
</tr>
</tbody>
</table>

\[
\text{M}_{t}/m_{g}L = 0.577
\]

\[
\text{M}_{t}/m_{g}L = 1.160
\]

\[
\text{M}_{t}/m_{g}L = 2.510
\]
Table 8.4 Comparison of fundamental frequencies
P.V.C. cylinder II; with springs attached to free end -
\[ \frac{M_t}{m_5 L} = 0.577 \]

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Frequencies</th>
<th>Stiffness Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed (Hz)</td>
<td>Calculated (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>4.180</td>
<td>4.182</td>
</tr>
<tr>
<td>0</td>
<td>5.920</td>
<td>5.929</td>
</tr>
<tr>
<td>0</td>
<td>6.480</td>
<td>6.483</td>
</tr>
<tr>
<td>36</td>
<td>3.280</td>
<td>3.235</td>
</tr>
<tr>
<td>36</td>
<td>4.430</td>
<td>4.432</td>
</tr>
<tr>
<td>36</td>
<td>5.000</td>
<td>4.960</td>
</tr>
</tbody>
</table>

Table 8.5 Comparison of fundamental frequencies
Fibreglass cylinder I2 with masses attached to free end.

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Frequencies</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed (Hz)</td>
<td>Calculated (Hz)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \frac{M_t}{m_5 L} = 2.507 ]</td>
</tr>
<tr>
<td>0</td>
<td>5.960</td>
<td>5.040</td>
</tr>
<tr>
<td>20</td>
<td>5.830</td>
<td>5.821</td>
</tr>
<tr>
<td>24</td>
<td>5.660</td>
<td>5.648</td>
</tr>
<tr>
<td>35</td>
<td>4.680</td>
<td>4.692</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[ \frac{M_t}{m_5 L} = 4.757 ]</td>
</tr>
<tr>
<td>0</td>
<td>4.460</td>
<td>4.409</td>
</tr>
<tr>
<td>20</td>
<td>4.330</td>
<td>4.356</td>
</tr>
<tr>
<td>24</td>
<td>4.290</td>
<td>4.285</td>
</tr>
<tr>
<td>35</td>
<td>3.850</td>
<td>3.822</td>
</tr>
</tbody>
</table>

Table 8.6 Comparison of Second Normal mode frequencies
P.V.C. cylinder II; free ended.

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Frequencies</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed (Hz)</td>
<td>Calculated (Hz)</td>
</tr>
<tr>
<td>0</td>
<td>42.900</td>
<td>42.900</td>
</tr>
<tr>
<td>12</td>
<td>37.800</td>
<td>38.020</td>
</tr>
<tr>
<td>15</td>
<td>33.150</td>
<td>33.139</td>
</tr>
<tr>
<td>18</td>
<td>28.820</td>
<td>29.160</td>
</tr>
<tr>
<td>21</td>
<td>27.250</td>
<td>26.777</td>
</tr>
<tr>
<td>24</td>
<td>25.540</td>
<td>25.470</td>
</tr>
<tr>
<td>25</td>
<td>25.100</td>
<td>25.150</td>
</tr>
<tr>
<td>30</td>
<td>25.000</td>
<td>24.480</td>
</tr>
<tr>
<td>33</td>
<td>24.450</td>
<td>22.998</td>
</tr>
<tr>
<td>36</td>
<td>22.830</td>
<td></td>
</tr>
</tbody>
</table>
Table 8.7 Comparison of Second Normal Mode frequencies
P.V.C. cylinder II; with end masses

<table>
<thead>
<tr>
<th>Water Depth (inches)</th>
<th>Observed (Hz)</th>
<th>Calculated (Hz)</th>
<th>Flow excited (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mt/m_L = 0.882</td>
<td></td>
</tr>
<tr>
<td></td>
<td>31.600</td>
<td>31.600</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>29.500</td>
<td>29.480</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>25.950</td>
<td>26.042</td>
<td>26.000</td>
</tr>
<tr>
<td>18</td>
<td>22.970</td>
<td>23.030</td>
<td>22.940</td>
</tr>
<tr>
<td>24</td>
<td>18.530</td>
<td>18.400</td>
<td>18.460</td>
</tr>
<tr>
<td>27</td>
<td>17.390</td>
<td>17.375</td>
<td>17.410</td>
</tr>
<tr>
<td>30</td>
<td>16.250</td>
<td>16.382</td>
<td>16.230</td>
</tr>
<tr>
<td>33</td>
<td>16.000</td>
<td>16.117</td>
<td>16.160</td>
</tr>
<tr>
<td>36</td>
<td>17.080</td>
<td>17.093</td>
<td></td>
</tr>
</tbody>
</table>

Mt/m_L = 0.577

Table 8.8 Results of Main Tests, using Cylinder II.

\[ N_s = 1.080 - 3.500 \]

<table>
<thead>
<tr>
<th>h/L</th>
<th>Frequency Hz</th>
<th>( \phi )</th>
<th>( \delta_h )</th>
<th>( \delta_L )</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>calculated</td>
<td>experiment</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.537</td>
<td>6.306</td>
<td>0.0188</td>
<td>0.023</td>
<td>0.107</td>
<td>0.104</td>
</tr>
<tr>
<td>0.634</td>
<td>5.835</td>
<td>0.0398</td>
<td>0.045</td>
<td>0.129</td>
<td>0.130</td>
</tr>
<tr>
<td>0.732</td>
<td>5.256</td>
<td>0.0732</td>
<td>0.079</td>
<td>0.163</td>
<td>0.160</td>
</tr>
<tr>
<td>0.756</td>
<td>5.104</td>
<td>0.0839</td>
<td>0.090</td>
<td>0.173</td>
<td>0.180</td>
</tr>
<tr>
<td>0.805</td>
<td>4.797</td>
<td>0.1078</td>
<td>0.115</td>
<td>0.199</td>
<td>0.200</td>
</tr>
<tr>
<td>0.878</td>
<td>4.347</td>
<td>0.1516</td>
<td>0.158</td>
<td>0.242</td>
<td>0.254</td>
</tr>
<tr>
<td>1.000</td>
<td>3.660</td>
<td>0.2499</td>
<td>0.256</td>
<td>0.344</td>
<td>0.340</td>
</tr>
<tr>
<td>0.537</td>
<td>3.887</td>
<td>0.0179</td>
<td>0.015</td>
<td>0.095</td>
<td>0.064</td>
</tr>
<tr>
<td>0.634</td>
<td>3.770</td>
<td>0.0380</td>
<td>0.030</td>
<td>0.110</td>
<td>0.113</td>
</tr>
<tr>
<td>0.732</td>
<td>3.600</td>
<td>0.0706</td>
<td>0.054</td>
<td>0.134</td>
<td>0.140</td>
</tr>
<tr>
<td>0.878</td>
<td>3.263</td>
<td>0.1497</td>
<td>0.122</td>
<td>0.192</td>
<td>0.193</td>
</tr>
<tr>
<td>0.927</td>
<td>3.136</td>
<td>0.10563</td>
<td>0.142</td>
<td>0.222</td>
<td>0.227</td>
</tr>
<tr>
<td>1.000</td>
<td>2.939</td>
<td>0.2499</td>
<td>0.187</td>
<td>0.267</td>
<td>0.268</td>
</tr>
<tr>
<td>0.537</td>
<td>2.917</td>
<td>0.0153</td>
<td>0.009</td>
<td>0.081</td>
<td>0.074</td>
</tr>
<tr>
<td>0.732</td>
<td>2.796</td>
<td>0.0633</td>
<td>0.037</td>
<td>0.109</td>
<td>0.114</td>
</tr>
<tr>
<td>1.000</td>
<td>2.448</td>
<td>0.2417</td>
<td>0.137</td>
<td>0.209</td>
<td>0.212</td>
</tr>
<tr>
<td>1.000</td>
<td>2.183</td>
<td>0.2405</td>
<td>0.116</td>
<td>0.184</td>
<td>0.182</td>
</tr>
<tr>
<td>1.000</td>
<td>1.843</td>
<td>0.2390</td>
<td>0.089</td>
<td>0.149</td>
<td>0.152</td>
</tr>
</tbody>
</table>

Tip mass = 0
\( \delta_s = 0.0836 \)
cylinder hollow

Tip mass = 0
\( \delta_s = 0.0800 \)
cylinder filled with water

Tip mass = 0.147
\( \delta_s = 0.0721 \)

Tip mass = 0.777
\( \delta_s = 0.0682 \)

Tip mass = 1.227
\( \delta_s = 0.0601 \)
**Table 8.9** Experimental results for the cylinder II with springs fitted to the free end.

\[ N_b = 1,800 - 3,500 \]

<table>
<thead>
<tr>
<th>Frequency Hz.</th>
<th>Frequency ratio</th>
<th>((\delta_L)_b)</th>
<th>(\delta_S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.940</td>
<td>1.000</td>
<td>0.268</td>
<td>0.0800</td>
</tr>
<tr>
<td>3.780</td>
<td>1.285</td>
<td>0.226</td>
<td>0.0784</td>
</tr>
<tr>
<td>4.840</td>
<td>1.648</td>
<td>0.157</td>
<td>0.0643</td>
</tr>
<tr>
<td>5.826</td>
<td>1.982</td>
<td>0.134</td>
<td>0.0485</td>
</tr>
<tr>
<td>6.890</td>
<td>2.345</td>
<td>0.116</td>
<td>0.0400</td>
</tr>
</tbody>
</table>

**Table 8.10** Comparison of mode shapes of cantilevers with uniformly distributed and concentrated loads.

<table>
<thead>
<tr>
<th>(x/L)</th>
<th>Deflection at (x/L)</th>
<th>Difference %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>distributed load</td>
<td>concentrated load</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0087</td>
<td>0.0070</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0304</td>
<td>0.0260</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0594</td>
<td>0.0540</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0917</td>
<td>0.0879</td>
</tr>
<tr>
<td>0.9</td>
<td>0.1083</td>
<td>0.1063</td>
</tr>
<tr>
<td>1.0</td>
<td>0.1250</td>
<td>0.1250</td>
</tr>
<tr>
<td>$h/L$</td>
<td>Frequency Hz.</td>
<td>$\delta_h$</td>
</tr>
<tr>
<td>-------</td>
<td>---------------</td>
<td>-----------</td>
</tr>
<tr>
<td>0</td>
<td>5.732</td>
<td>0.0063</td>
</tr>
<tr>
<td>0.321</td>
<td>5.586</td>
<td>0.0379</td>
</tr>
<tr>
<td>0.632</td>
<td>4.846</td>
<td>0.0739</td>
</tr>
<tr>
<td>0.808</td>
<td>4.231</td>
<td>0.0948</td>
</tr>
<tr>
<td>0.878</td>
<td>4.000</td>
<td>0.1282</td>
</tr>
<tr>
<td>0.972</td>
<td>3.643</td>
<td></td>
</tr>
</tbody>
</table>

$K_e = 34.7 \text{ lbf/ft.}$

<table>
<thead>
<tr>
<th>$h/L$</th>
<th>Frequency Hz.</th>
<th>$\delta_h$</th>
<th>$\delta_t$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7.581</td>
<td>0.0012</td>
<td>0.0081</td>
<td></td>
</tr>
<tr>
<td>0.233</td>
<td>7.512</td>
<td>0.0213</td>
<td>0.0103</td>
<td>0.0128</td>
</tr>
<tr>
<td>0.530</td>
<td>6.780</td>
<td>0.0389</td>
<td>0.0294</td>
<td>0.0247</td>
</tr>
<tr>
<td>0.670</td>
<td>6.212</td>
<td>0.0571</td>
<td>0.0470</td>
<td>0.0528</td>
</tr>
<tr>
<td>0.770</td>
<td>5.709</td>
<td>0.1081</td>
<td>0.0652</td>
<td>0.0647</td>
</tr>
<tr>
<td>0.967</td>
<td>4.825</td>
<td></td>
<td>0.1152</td>
<td>0.1160</td>
</tr>
</tbody>
</table>

$K_e = 60.4 \text{ lbf/ft.}$
Table 8.12 Results for the fiberglass cylinders I4, I5, I6.

$N_g = 619 - 21,600$

<table>
<thead>
<tr>
<th>$h/L$</th>
<th>frequency Hz</th>
<th>$\delta_h$ calculated</th>
<th>$\delta_h$ experiment</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6.33</td>
<td>0.0965</td>
<td>0.1000</td>
<td>Cylinder I4. 0.505&quot; dia.</td>
</tr>
<tr>
<td>0.417</td>
<td>6.10</td>
<td>0.1046</td>
<td>0.1071</td>
<td>EI = 6.841 Lbf.ft.²</td>
</tr>
<tr>
<td>0.488</td>
<td>5.80</td>
<td>0.1210</td>
<td>0.1240</td>
<td>$X_o = 0.37&quot;$</td>
</tr>
<tr>
<td>0.585</td>
<td>5.24</td>
<td>0.1330</td>
<td>0.1352</td>
<td></td>
</tr>
<tr>
<td>0.634</td>
<td>4.93</td>
<td>0.1491</td>
<td>0.1480</td>
<td></td>
</tr>
<tr>
<td>0.683</td>
<td>4.61</td>
<td>0.1662</td>
<td>0.1624</td>
<td></td>
</tr>
<tr>
<td>0.730</td>
<td>4.25</td>
<td>0.1750</td>
<td>0.1673</td>
<td></td>
</tr>
<tr>
<td>0.750</td>
<td>4.10</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>11.10</td>
<td>0.0667</td>
<td>0.0643</td>
<td>Cylinder I6. 1.000&quot; dia.</td>
</tr>
<tr>
<td>0.417</td>
<td>9.80</td>
<td>0.0740</td>
<td>0.0732</td>
<td>EI = 204.5 Lbf.ft.²</td>
</tr>
<tr>
<td>0.488</td>
<td>9.20</td>
<td>0.0952</td>
<td>0.0974</td>
<td>$X_o = 0.5&quot;$</td>
</tr>
<tr>
<td>0.585</td>
<td>7.96</td>
<td>0.1080</td>
<td>0.1131</td>
<td></td>
</tr>
<tr>
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<tr>
<td>0</td>
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<td>0.1130</td>
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<td>Cylinder I5. 1.812&quot; dia.</td>
</tr>
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<td>0.417</td>
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Fig. 8.1  Transient fundamental mode frequencies versus water depth. Fibreglass cylinder 1.2

Fig. 8.2  Transient fundamental mode frequencies versus water depth. P.V.C. cylinder 1.1
Fig. 8.3 Transient second normal mode frequencies versus water depth. P.V.C. cylinder I.1.
Fig. 8.4 Calculated and experimental total logarithmic decrements versus water depth. P.V.C. cylinder 1.1
Fig. 8.5 Details of the pendulum hinge.
Fig. 8.6 Calculated and experimental total logarithmic decrements versus water depth. Pendulum, Cylinder 13.
Fig. 8.7 Calculated and experimental logarithmic decrements. Fibreglass cylinders 14, 15, 16.

9.1 General introduction.

The following sections 9.2 to 9.5 describe, in their correct chronological sequence, the tests conducted to complete the experimental programme of basic research using single cylinders. The results are discussed in detail in the text and where appropriate, the results and conclusions from each test are summarised at the end of each section; for continuity, a brief synopsis of the immediately successive tests and the information that these tests were designed to reveal, is also included in the discussions. The results of this section have been published by King, Prosser & Johns in the Journal of Sound & Vibration (110).

9.2 Flowing water tests using the single aluminium alloy cylinder II.1.

9.2.1 Introduction.

In the first part of these tests, the aluminium alloy cylinder II.1 was used to investigate the Reduced Velocity range $1.25 < V_r < 3$. The cylinder was fixed to the floor of the flume as a free-ended vertical cantilever, and the amplitude response of the sustained oscillations examined in a fixed depth of water as the flow velocity was increased from zero to maximum and then reduced to zero. The cylinder, instrumented with eight strain gauges (four along each of the two orthogonal axes) was installed such that the orthogonal axes were aligned with the cross-flow and in-line directions. Simultaneous recordings of strain in these two directions were possible, although subsequent analysis was facilitated by recording results in each direction individually.

A range of water depths were used in the tests, to determine the relationship between water depth, velocity of flow and cylinder response. The water depths were expressed as non-dimensional ratios of water depth/cylinder length ($= h/L$) and for $h/L = 0$ obviously there were no exciting forces and the cylinder was infinitely stable.
As $h/L$ was increased, and assuming the other necessary conditions were satisfied, a value of $h/L$ was determined at which sustained oscillations were initiated within the speed range of the flume. The results were analysed to define a Stability Parameter, $k_s^*$ for each water level, based on the theoretical approach of Chapter 3, using calculations incorporating the transfer matrix method, (Appendix 1). Hysteresis effects were investigated by comparing the results obtained in increasing flow velocity with those results recorded when the flow velocity was reduced.

In the second part of the tests, a similar test procedure was adopted when, for a fixed depth of water, concentrated masses were attached to the free end of the cantilever to modify the mode shapes and reduce the natural frequencies, thus extending the Reduced Velocity range to $1.25 < V_r < 8$. The addition of sufficient mass to the free end of a cantilever is known to increase $k_s$ and suppress excitation of the fundamental mode frequencies in the in-line direction (53). The minimum Stability Parameter for which the cantilever remains stationary is denoted $k_{sc}$ (the Critical $k_s$). The present series of tests was arranged to permit correlation of the values of $k_{sc}$ resulting from varying the water depth (first part of tests) and altering the mass of the cylinder in a fixed depth of water (second part of tests).

**9.2.2 Tests on the free ended cantilever.**

With the cylinder installed in the flume as a free-ended vertical cantilever, water was pumped into the flume until the required level was reached. When stable test conditions became established, and prior to each flowing water test, the still-water logarithmic decrement ($\delta_l$) was obtained in the standard way, (Chapter 8); the still-water natural frequency $n$, measured from the U. V. trace used for determining $\delta_l$, was compared with the natural frequency in air ($n_a$) and the added mass of water deduced as in Chapter 8. In all cases, the still-water natural frequency could be calculated by incorporating the ideal added mass function.
described in Chapters 4 & 8 (see Table 9.1). The total logarithmic
decrement, \( \delta_t \), comprising the viscous hydrodynamic damping and
the hysteretic damping of the cylinder material (see Chapter 3) was
used for further comparison with the theoretical calculations of
Chapters 4 & 8, as shown in figure 9.1. The recordings of \( \delta_t \) and
\( n \) were examined carefully for amplitude-dependent effects because
it was thought that prolonged testing under 'resonant' conditions
could result in the base fixity becoming less than perfect. Chapter
8 showed that \( \delta_t \) was a constant for each water level, frequency,
cylinder material and cylinder geometry; recordings of amplitude-
dependent \( \delta_t \) and \( n \) would indicate the existence of non-linearity in
the base fixity. Both the cross-flow and in-line directions were
investigated in this way, before each flowing water test. Non-linear
damping and frequency effects were not recorded and it was confi-
dently assumed that the base fixity remained fully encastre throughout
the experimental programme. As the flow velocity was increased
from zero in small steps, the steady and fluctuating deflections of
the U.V.R. galvanometers, determined by the electrical output from
the strain gauges, were recorded on the U.V.R. At Reduced Vel-
cities below \( V_r = 1.2 \) only steady deflections were registered in
the in-line direction whilst in the crossflow direction, small ampli-
tude forced oscillations were recorded, at the Strouhal vortex shedding
frequency \( f_v \) given by \( f_v = SV/d \) (similar recordings were made in the
earlier tests of reference (53) ). This was not surprising because
the cylinder was lightly damped and cross-flow forces exceeded the
very small in-line forces by an order of magnitude (28), (29), (30),
(31), thus explaining the absence of corresponding forced motion
in-line.

For high water levels, and with \( V_r \sim 1.25 = V_{rc} \), the
cylinder started to oscillate in-line at its still water natural frequency;
the establishment of instability was achieved in a relatively short
time and motion increased to a limit cycle within 20 to 40 cycles
from the oscillations first being detected (see fig. 9.2). Once
reached, the limit amplitude remained uniform in both magnitude and frequency, until the velocity setting was changed. Rather surprisingly, the small amplitude forced oscillations persisted in the cross-flow direction, although they were confined to bursts of variable duration; the bursts were longer, and the amplitudes larger and more uniform when \( n/f_v = 4, 3.5, 3, 2.5 \).

As \( V_r \) was increased, the amplitude of in-line oscillations also increased, being a constant for each velocity, and reaching a maximum at \( 2 < V_r < 2.3 \). Further increases of \( V_r \) were accompanied by beating effects in the recorded amplitudes (fig. 9.3) and the mean value abruptly reduced to zero at, or below, \( V_r \geq 2.5 \).

The maximum velocity through the flume limited the maximum \( V_r \) to \( V_r \leq 2.8 \) and exploration at higher values of Reduced Velocity was not possible under the test arrangement for this series of tests. However, the reappearance of in-line oscillations at the still-water natural frequency of the cylinder at \( V_r \approx 2.7 \) suggested the existence of a second instability region, and this was confirmed in the later tests on the cylinder with masses attached to the free end.

Reducing the velocity demonstrated hysteresis effects of a type frequently encountered in non-linear oscillations (111). The \( V_r \) range over which zero amplitudes were recorded, extended down to approximately \( V_r = 2.1 \) at which velocity the amplitude increased abruptly to the maximum recorded in the increasing velocity test at the same water level. Additional decreases in \( V_r \) demonstrated only minor differences between the two sets of results, as shown in fig. 9.4.

The overall results, recorded in the increasing velocity tests are shown in fig. 9.5(a) and it will be seen that below a water depth of \( h/L \geq 0.49 \), (\( h/d \geq 20 \)), no significant excitation was recorded. This implied a Critical Stability Parameter (\( k_{sc} \)) of approximately 1.0 for in-line motion compared with Scrutton's (40) value of \( k_{sc} = 17 \) for suppressing cross-flow oscillations.

Fig. 9.5(a) shows that \( V_{rc} \) was approximately constant.
although there were slight increases in $V_{rc}$ for the higher values of $k_s$. The values of Reduced Velocities at which maximum amplitudes occurred were fairly constant for $k_s \leq 0.34$ but decreased for $k_s \geq 0.6$. The variation of $k_s$ was achieved by altering the water depths and it is possible that this observed $V_r$ effect was due to the influence of flow three dimensionality at low water levels appropriate to these higher $k_s$ values. A Stability Parameter of 0.6 is equivalent to a water depth/diameter (h/d) ratio of 23 and it will be remembered that Bénard (12), using a water channel, maintained that three dimensional effects were detectable up to h/d $\leq 23$. The results of the next section (9.2.3) and those of the tests in the cross-flow direction (9.3) also suggest that three dimensional effects could be partly responsible for similar $V_r$ behaviour recorded in those two sections. The results of fig. 9.5 shown in terms of base bending moments (the strain gauges were calibrated to yield these directly), were generally consistent with the wind tunnel test results discussed in Chapter 2 in that amplitudes of bending moments varied inversely as the magnitudes of mass and damping. However, in the figure 9.5(a) it is seen that the maximum oscillatory bending moment for run 8 ($k_s = 0.34$) exceeded that of run 6 ($k_s = 0.16$) and those of runs 10 and 14, although comparison of Stability Parameters would suggest the converse. This apparent anomaly is explained by considering the amplitudes at the water surface as defined by $k_s$ (Chapter 3) and illustrated in the analysis of the later tests on the cylinder with masses attached to the free end. Calculated and recorded bending moment distributions along the cylinder length exhibited almost identical agreement (less than 1% variation, see Table 9.2) and the recorded bending moments were converted into cylinder amplitudes through similar calculations. The accuracy of the calculated amplitudes corresponding to the recorded base bending moments was confirmed in supplementary tests with a pointer, mounted on the free-end of the cylinder and moving over a calibrated scale.
At $V_{rc}$, the excited frequency was equal to the still water frequency, though variations of up to 5% between these two frequencies were observed at higher velocities and amplitudes (fig. 9.3(b)). The reasons for this are thought to be twofold:

(i) three dimensional flow effects due to the free surface;
(ii) interactions between the exciting forces and the cylinder motion.

At the higher velocities, a measurable hydraulic gradient was established through the flume circuit, thus reducing by up to one inch the water level recorded at the test section. The static water level in the flume test section was maintained at the required value by a small auxiliary pump. Velocity head recovery at the cylinder was evidenced by the creation of a bow wave at the front face of the cylinder and a depression in the water surface immediately downstream (see figure 9.6). The water surface was obviously three dimensional and extremely agitated; bubbles of air were sucked down through the downstream water surface to be released from the cylinder at depths of up to three diameters below the upstream water surface. These three-dimensional effects are comparable with those experienced on stacks where the flow over and around the top of the stacks causes a thickening of the wake and a downwash of the smoke plumes on the leeward side of the stacks. The existence of three-dimensionality indicates a departure from the concept of uniform distribution of hydraulic forces, implying corresponding modifications to the added mass and true immersed length. This is discussed in detail in section 9.3.6. However, other researchers have noted similar variations of excited frequencies with velocity and amplitude, in air flow where there were no free surface effects, and in the present tests, the recorded variations of frequency were undoubtedly governed by a complex combination of (i) and (ii) above.

Bursts of small amplitude forced oscillations in the cross-flow direction, at the Strouhal frequency for a stationary cylinder were recorded throughout the experimental range. This intermittent cross-flow forcing at frequencies which varied linearly with velocity
served to obscure the complete understanding of the mechanisms responsible for energising and sustaining in-line oscillations at a comparatively uniform frequency throughout the same velocity range, particularly since it is shown by the photographic study of Section 9.4 that the instability region $1.25 < \frac{V}{r} < 2.5$ is characterised by symmetric vortex shedding (see fig. 9.39). This also prompted the study reported in 9.2.4 in which the cross-flow motions were inhibited.

The equivalent steady drag coefficients $C_d$ of the oscillating cylinder were derived from the graph of steady base bending moments versus square of velocity. Thus if the steady fluid loading is

$$\frac{1}{2} \rho V^2 dC_d$$

per unit length, the base bending moment due to this loading is $M_b$,

$$M_b = \frac{1}{2} \rho V^2 \frac{h^2}{2} dC_d$$  \hspace{1cm} (9.1)

or

$$C_d = \frac{M_b}{V^2 K_b}$$  \hspace{1cm} (9.2)

where

$$K_b = \frac{4}{\rho d h^2}$$  \hspace{1cm} (9.3)

Drag coefficients derived in this way varied in the range $1.26 < C_d < 1.31$ and a typical result is shown in fig. 9.7; in view of the observed three-dimensionality of the water surface, the values of $C_d$ were considered satisfactory and essentially constant. Values of stationary cylinder $C_d$, quoted in the literature (Chapter 2) for similar Reynolds numbers, varied from 1.0 to 1.4 depending on cylinder surface roughness, and length to diameter ratio. From the present tests it is seen that cylinder oscillations did not influence the steady drag coefficient appreciably and the cylinder oscillated about the mean position determined by the steady drag forces.

9.2.3 Tests on cylinder II.1 with masses attached to the free end.

In the tests with the free ended cylinder the maximum base
bending moments were recorded for a water depth of \( h/L \approx 0.73 \) and this was used as the fixed water depth of the present tests. Masses were attached to the free end and the experimental procedure of 9.2.2 repeated.

For the smallest additional mass tested, the amplitude response with increasing velocity was identical with the results of the free ended tests for \( 1.25 < V_r < 2.5 \). The maximum in-line amplitude was recorded at \( V_r = 2.2 \), followed by an abrupt reduction in amplitude at \( V_r \approx 2.3 \); oscillations ceased at \( V_r \approx 2.5 \). Further increase in velocity caused the cylinder to start oscillating once more at \( 2.6 < V_r < 2.75 \); the amplitude of oscillations increased with increasing velocity reaching a maximum at \( V_r \approx 3.2 \). This maximum was followed by a fairly abrupt reduction in amplitude to zero at \( V_r \approx 3.8 \). A total of five masses were used in the tests, results of which are given in fig. 9.8(a). The most striking feature of the graph is the confirmation of the two instability regions of in-line motion suggested in the free ended tests (9.2.2). At about this stage of the experimental programme, the results of the Immingham full-scale tests became available (112) and fig. 9.9 compares the results of this section, with the results from the Immingham tests. Satisfactory qualitative agreement is seen although there is some discrepancy between the Reduced Velocities appropriate to the maxima of the two instability regions. However, the mode shapes for the two sets of results were markedly different and exact agreement could not be expected. Hydroelastic modelling of two of the piles resulted in close quantitative agreement, (Chapter 10). Increasing the tip mass had the effect of increasing \( k_s \) and it is seen that base bending moments and tip masses are inversely related. At similar values of \( k_s \), substantial variations were noted between the base bending moments recorded on the free ended cylinder (Section 9.2.2) and the base bending moments recorded when masses were attached to the cylinder's free end. However, the formulation of \( k_s \) was derived (Chapter 3) for a cylinder whose length was equal to the water depth, thereby
defining the cylinder deflection at the water surface (i.e. the maximum amplitude) as the correlating factor, and not the base bending moments (this is emphasised later in the cross-flow tests). When the 'equivalent cylinder' criteria were applied to the two sets of results, close agreement was obtained, as detailed in table 9.3. This interpretation of $k_s$ also explained the apparently anomalous behaviour of base bending moments with $k_s$ recorded in the free-ended cylinder tests (Section 9.2.2) where, it will be remembered, the values of $k_s$ were obtained by varying the water depth $h/L$. For the present tests, fig. 9.8(a) shows that $V_{rc}$ increased with increasing $k_s$ in the first instability region but remained reasonably constant in the second, at $V_{rc} \simeq 2.7$. The Reduced Velocity at which maximum amplitudes were recorded ($V_{rm}$) was approximately constant for both regions for all $k_s$, being $V_{rm} \simeq 2.1$ in the first and $V_{rm} \simeq 3.2$ in the second. These tests were conducted in a uniform water depth $h/L = 0.73$ ($h/d = 30$), and reference back to the graph of the free ended cylinder tests (fig. 9.5(a)) shows that for $h/L \geq 0.63$ ($h/d \geq 26$), maximum amplitudes were also recorded at approximately constant $V_{rm} \simeq 2.1$. By comparing the first instability region of both sets of results in this way it was deduced that water depth/diameter effects contributed to the variation of $V_{rm}$ in the lower water levels. However the interpretation of tests in variable water depths was complicated further by the way in which $k_s$ varied with water depth. These details are discussed in the presentation of the cross-flow results in Section 9.3.2 and the bow wave effect is studied in Section 9.3.6.

The response curves of the free ended cylinder tests (Section 9.2.2) showed that $k_s$ must exceed approximately 1.0 to prevent excitation of sustained in-line oscillations. The results of the present series of tests demonstrated not only the existence of a second instability region but also that a larger $k_s$ was required to suppress excitation at the correspondingly higher $V_r$ of this region. In common with the free ended cylinder tests, it is seen that $k_{sc} \simeq 1.0$ for the first instability region and that $k_{sc} \simeq 1.2$ for the second instability.
region. The recordings of similar $k_{sc}$ for the first instability region of the two series of tests with the Stability Parameters derived in two different ways, suggested that although free-surface effects may alter the $V_r$ at which maximum amplitudes occurred, apparently they did not alter the stability boundary. This was considered logical, because free surface influences are velocity dependent and at the lower (critical) velocities their effects would be less than at $V_{rm}$. However, comparison of maximum amplitudes of the immersed lengths showed a well ordered variation with $k_s$ for both series of tests (Table 9.3) implying freedom from unaccountable three dimensional free surface effects.

King & Prosser (113) quoted $0.6 < k_{sc} < 1.1$, but when their tests were conducted only the first instability region was investigated. The present investigation qualifies their experimental results by establishing $k_{sc} \simeq 1.0$ for the first instability region.

In the preceeding series of tests with the free ended cylinder, the frequency of excited in-line oscillations was observed to increase gradually with velocity (fig. 9.5(b)). The results were consistent and repeatable, and a similar effect was noted in the present series of tests, as shown in fig. 9.8(b). Maximum deviations from still water frequencies were observed in the greater depths of water, indicating that this effect probably was associated with variations in effective depths of immersion (free surface phenomena were almost certainly complicated by the cylinder motion; maximum amplitudes of the immersed length occurred at the free surface). Section 9.3.6 discusses free surface effects in detail, showing that they can decrease the true immersed length of the cylinder, consequently reducing the effective added mass and thereby increasing the frequency. Equation (8.1) demonstrates the parametric relationships between the cylinder frequency $n$ and the added mass and water depth for a given cylinder; it is seen that the added mass term is greatest in the higher water levels. A typical calculation for the cylinder showed that a 3% increase in frequency was equivalent to a decrease of 15% in effective immersed
length. For this calculation, it was assumed that the added mass per unit length remained uniform over the effective immersed length found from the calculation. This was a rather extravagant assumption as it excluded consideration of interaction between cylinder motion and the exciting mechanism. However, Section 9.3.6 indicates that the calculations incorporating this assumption resulted in correct orders of magnitude.

On the full scale test rig (112) the frequency of excited oscillations was consistently lower than the still water frequency and this represents a reversal of the present observations. Wootton (112) explained the reduction in frequency in terms of the increased effective volume of the pile oscillating in-line; the same explanation should also apply to the present tests in the absence of scale effects, and this lack of agreement between the two sets of results is not explainable currently. It is possible that the points of fixity of the full scale piles varied with amplitude (the points of fixity were approximately 4 metres below the bed level) and it is logical to assume that variations in fixity would result in increases in the free lengths of the piles and thus in reductions of natural frequencies. A very simple calculation of the effects of variation of fixity level showed that a 6% decrease of frequency was equivalent to a 3% increase in pile length and resulted in a point of fixity approximately 4.9 metres below bed level. This was considered not unreasonable.

The ventilated bow waves of the fullscale and present results would not be related identically (see 9.3.6) but their effects in both cases should result in increases in frequency. An alternative explanation for the observed differences is that scaling effects were present and the results represented differences in size.

Cross-flow forced motion was detected throughout the tests, although the two instability regions were characterised by two different types of forcing. In the first instability region, and similar to the results from the free-ended cylinder tests, bursts of small amplitude, forced oscillation were recorded in the cross-flow direction at
frequencies $f_v$ which increased linearly with increasing $V_r$ and defined by the stationary cylinder Strouhal number. The bursts were longer and the amplitudes more uniform when $n$ and $f_v$ were in simple proportions like $n/f_v = 4, \ 3.5, \ 3, \ 2.5$. In the second instability region, the frequency of forced motion was observed to be constant at one half the natural frequency of in-line oscillations. These observations were clarified by the measurements of vortex frequencies in the wake of the oscillating cylinder (Section 9.2.5) and also by photographing the wake (Section 9.4). It is shown that the two instability regions were identified by essentially dissimilar types of vortex shedding; the first region was characterised by an apparent symmetric vortex shedding and the second by alternate vortex shedding.

Adopting the aerodynamic practice of assessing the apparent Strouhal number $S^*$ of oscillating cylinders by noting the $V_r$ at which the maximum amplitudes occurred, it may be shown that in the second instability region of in-line motion, $S^* \approx 0.16$, a value which compares very closely with the results of wind tunnel tests of cross-flow motion. In the first instability region, the symmetric vortex shedding invalidated a true comparison with the apparent Strouhal numbers defined in terms of alternate shedding.

9.2.4 The effects of restricting motion to the in-line direction: aluminium alloy cylinder II.1, free-ended.

The influences of the obviously powerful cross-flow forces on the response in-line were inferred from tests in which motion was restricted to the in-line direction.

Two methods were employed to prevent motion in the cross-flow direction:

(i) in-line slides;

(ii) transverse tensioned cables.

The slides consisted of two lengths of rectangular section P.T.F.E. with the contact edges feathered to give minimum contact area (fig. 9.10). However, this method was abandoned because it was impossible to reduce the Coulomb friction to an acceptable level;
initially setting up the slides and cylinder with low contact pressure resulted in the 'pseudo-Coulomb' logarithmic decrement shown in fig. 9.11. During testing, the alternate cross-flow forces caused corresponding variations in the contact pressure, and the recorded results bore little resemblance to the results of 9.2.2.

As an alternative, long pretensioned stainless steel cables ('Laystrate', model aeroplane multistrand control cable) were selected. The cables, each of which was approximately 3ft long and tensioned to approximately 10 lbf, were located, normal to the flow direction, on diametrically opposing points on the cylinder surface. Variations in tension due to the maximum amplitudes of cylinder oscillation (± 0.2") were less than 0.5%, thus excluding non-linear spring effects.

Generally, the effects of eliminating cross-flow motion were a slight reduction in the in-line $V_{rc}$ and an increase of up to 10% in the maximum amplitudes recorded (fig. 9.12). These findings suggested that although cross-flow motion might increase the length-wise correlation of cross-flow forces, its effect in the in-line direction was an apparent reduction of correlation. This may also be explained by the cross-flow motion causing the circumferential r.m.s. pressure distribution due to vortex shedding to have its resultant acting less in the in-line direction and more in the cross-flow direction.

9.2.5 Wake observations and measurements of frequencies in the wake of cylinder II.1 oscillating in the fundamental mode in-line.

The tests thus far have centred on the recordings of cylinder structural response in the in-line direction for a given Reduced Velocity range. More information was required concerning the mechanisms responsible for promoting the oscillations, and particularly in clarifying the differences between the two instability regions. The recordings of two distinctly different types of cross-flow forced motion in the two instability regions indicated that completely dissimilar mechanisms were responsible for the observed behaviour. In most
wind-tunnel experiments, the terms 'lock-in' or 'wake-capture' inevitably arise in attempts to describe the recorded phenomena. The two terms are generally synonymous and refer to the region of instability in the cross-flow mode where the cylinder motion apparently controls the frequency of vortex shedding. Bruun & Davies (35) found evidence of vortex shedding occurring in discrete cells along the cylinder length, separated by regions of aperiodic shedding. One theory to which some researchers subscribe is that at the onset of instability the cylinder motion causes the cells to become synchronised in phase and period, with a consequent rapid increase in the effective applied force and the resulting deflection.

A critical $V_{rc}$ of 5 for the initiation of cross-flow oscillations suggests that lock-in occurred when $n = f'_{v'}$ and in view of the 2:1 frequency relationship between $F_d$ and $F_L$, it would seem logical to propose $V_{rc} = 2.5$ for in-line excitation. The results of the present tests have shown that $V_{rc} = 1.25$ was possible; Surry's (31) work applied here, could attribute the low $V_{rc}$ to the effects of a second harmonic of the fundamental vortex shedding frequency. Wootton et al (97) suggested that $V_{rc} = 1.25$ was consistent with the cylinder completing two oscillations for every vortex shed and thus satisfying the 4:1 ratio between $n$ and $f'_{v'}$. Johns (114) also demonstrated that similar factors can apply not only to the lateral (bending) oscillations of a shell structure but also to the ovalling or breathing oscillations of the shell cross-section.

Because the in-line oscillations of the cylinder II.1 were thought to result from vortex excitation it seemed logical to examine the predominant vortex frequencies in the wake behind the cylinder oscillating in both instability regions. For this purpose, two thermometer probes (described in Chapter 7) were used as frequency counting devices. The probes were each 10" long and their electrical connections were not water-proofed, thus restricting the depth below the water surface at which measurements could be made. Several preliminary experiments were run in order to determine the optimum
location of the probes downstream of the cylinder. The final configuration was similar to that used in the original tests of King (53) and also similar to the arrangement in the Immingham full scale tests (112) suggesting close agreement between the configurations of vortex streets formed behind oscillating cylinders at substantially different Re. In the present tests, the two probes were spaced equally from the vortex street centre line, as shown in fig. 9.13 and the full scale arrangement is shown for comparison.

The graph of fig. 9.14 shows the predominant wake frequency $f_v$ plotted against $V_r$ with the base bending moments superposed. In this test, motion was restricted to the in-line direction alone. At velocities below the first critical $V_{rc}$, the vortex frequencies were close to the frequencies determined by the Strouhal number relationship for a stationary cylinder, i.e. $f_v = 0.2 V/d$ and the straight line on the graph shows the frequencies determined by this equation for the velocity range of the flume. Throughout the first instability region, the predominant wake frequency continued to vary approximately as $f_v = 0.2 V/d$ whilst the cylinder oscillated at its constant still-water natural frequency $n$. There was some evidence of simple ratios between $n$ and $f_v$ and indications that the predominant vortex frequency changed in small steps to maintain simple proportions such as $n/f_v = 4:1, 3:1, 2:1$. At the null point of $V_r \approx 2.5$, the cylinder ceased to oscillate, remaining motionless until the second instability region was entered at $V_{rc} \approx 2.7$. Within the null range $2.5 < V_r < 2.7$ the vortex shedding frequency $f_v$ continued to increase linearly with velocity; with progress into the second instability region, the vortex shedding frequency $f_v$ and the cylinder still-water natural frequency $n$ became synchronised. In the Immingham full-scale tests (112) measurements were made of the power spectra of velocity fluctuations in the wake of the largest pile (30" diameter), demonstrating that the dominant frequency was equal to the stationary cylinder vortex shedding frequency for all $V_r \leq 1.58$, the $V_r$ range of the site. Thus there were striking similarities between the wakes of the full-scale piles.
and the smaller diameter cylinders used in the present tests, i.e.

(a) identical observations of dominant wake frequencies within
the first instability region of in-line oscillations. The
maximum Reynolds number of the full scale tests was $1.6 \times 10^6$
and for the present tests $Re \approx 2 \times 10^4$.

(b) wake frequencies were measured using probes (yaw probe
for the full-scale and thermister probes for the smaller
cylinder tests); the location of the probes relative to the
pile or cylinder was determined experimentally as the point
of maximum response to the wake being measured. It was
noted that the relative location was virtually identical in
both cases, indicating a common geometrical disposition of
the wakes behind the two bodies.

Because the in-line component of vortex shedding oscillates
at twice the vortex frequency $(29), (32)$, synchronisation arose when
$f_v = n/2$ and thus only $n/2$ was recorded in the wake. At the end of
the second instability region, cylinder motion ceased at $V_r \approx 3.8$
and $f_v$ reverted to the stationary cylinder value determined by the
straight line on the graph.

The recordings of variable $f_v$ in the first instability region
partly explained the mechanisms responsible for the intermittent
cross-flow forced oscillations. However, the energy source for
the in-line oscillations was identified only during the photographic
study (Chapter 9.4) where it was shown that symmetric vortex shedding
occurred throughout this instability region. In the second instability
region, the assumed mechanism was consistent with established
reports of synchronisation between cylinder motion and vortex shedding
frequency. Synchronisation would also explain the small amplitude
cross-flow oscillations at the constant frequency of $n/2$. The photo-
graphic study proved later that this was due to alternate vortex shedding
at the synchronised frequency.

In order to determine the generality of these results and to
gain an improved understanding of the excitation mechanisms, similar
tests were arranged for the P.V.C. cylinders II.2, II.3, III.1 as described in the next section (9.3). The results for the P.V.C. cylinders confirmed the findings of this section (9.2.4) and also demonstrated their application to the second normal mode of in-line oscillations. Cross-flow oscillations are recorded in Section 9.3.2.
9.3 Results from the P.V.C. cylinders II.2, II.3, III.1.

9.3.1 Tests in the in-line direction: Cylinder II.2, fundamental mode.

The P.V.C. cylinder II.2 was tested briefly in the first instability region of in-line motion, to confirm the results obtained from the tests with the aluminium alloy cylinder II.1 in Chapter 9.2.3. For the tests, II.2 was hollow, free ended and fixed to the reinforced floor of the test area of the flume. Before each test, the stillwater natural frequency and logarithmic decrement were obtained, and compared with the theoretical values. The results are shown in fig. 9.15 and close agreement is seen.

In the first of the flow-excited tests, for which \( h/L = 0.67 \), \( h/d = 24 \), and \( k_s = 0.32 \), the P.V.C. cylinder exhibited behaviour similar to that recorded with the aluminium alloy cylinder II.1. Fig. 9.16 records the response of II.2 with the response of II.1 superposed. Both tests were conducted at similar values of \( k_s \), \( V_r \), and water levels although the cylinder material properties differed quite markedly. The two sets of results showed identical trends both qualitatively and quantitatively, thereby establishing \( k_s \) and \( V_r \) as the factors through which correlations of amplitude response should be made. Vickery & Watkins (41) previously demonstrated similar effects when correlating results from tests in wind tunnels with those from tests in water tunnels, showing that correct reproduction of \( m_e \) and \( \delta \) (independently) was unnecessary provided equality of \( k_s \) as a group was maintained.

When the P.V.C. cylinder II.2 was tested at \( h/L = 0.86 \), \( h/d = 32 \), with \( k_s = 0.16 \), the graph of fig. 9.17 was obtained. This shows the first instability region maximum amplitude at \( V_r = 2.2 \), followed by a slight decrease in, and then a linear increase of, amplitude with increasing velocity. The frequency in the in-line direction also increased linearly, being consistently twice the Strouhal stationary cylinder vortex frequency (the maximum in-line forced frequency was 8 Hz, compared with the still-water natural frequency
of approximately 5 Hz).

A similar effect was noted with the aluminium alloy cylinder II.1, also at a very low $k_s$ of 0.12; however, when motion in the cross-flow direction was eliminated using cables, the in-line forcing was no longer observed. The two sets of results (with and without cross-flow motion) are presented in fig. 9.18 and it was concluded that in-line forced oscillations were induced only when $k_s$ was very low and when the cylinder was free to oscillate in the cross-flow direction simultaneously.

From recordings of the cylinder's steady deflections over the range of Reduced Velocity used in the tests, the equivalent steady drag coefficient was calculated as $C_d = 1.21$, as shown in fig. 9.19. It was noted that this was in very close agreement with the $C_d$ for the aluminium alloy cylinder II.1.

The tests of this first section have established or confirmed three points, viz.:

i) correlation between sets of data should be made through the Stability Parameter, $k_s$, and the Reduced Velocity, $V_r$.

ii) for very low $k_s$, forced oscillations may be induced in-line, if motion in the cross-flow direction is also permitted.

iii) the Critical Reduced Velocity for in-line oscillations is $V_{rc} = 1.2$ for $k_s \leq 0.3$.

9.3.2 Tests in the cross-flow direction: cylinders II.2, II.3, III.1, fundamental mode.

The majority of tests completed by other researchers have been confined almost exclusively to examinations of oscillations in the cross-flow direction. Usually air has been the fluid medium and in the few cases of tests in flowing water, the cylinders were completely immersed and therefore devoid of free surface effects.

The P.V.C. cylinders II.2, II.3, III.1 were tested over a wide range of Stability Parameters, to establish the behaviour of the
cylinders with variations of $k_s$ and in the presence of a free surface, thus forming a basis of comparison with the aerodynamic tests, and less numerous hydrodynamic tests from other sources. Variations of $k_s$ were achieved by adjusting the water depth, filling the cylinders with either water or lead shot, and also by attaching masses to the free end. Fig. 9.20 shows a representative selection of test results, plotted in the form of base bending moments versus Reduced Velocity $V_r$ and each curve is identified by the run number and corresponding $k_s$. It is seen that bending moments and $k_s$ are approximately inversely related, a feature noted in the previous tests in-line and also in wind tunnel tests (41). All the curves of fig. 9.20 have approximately similar shapes, being symmetrical about the Reduced Velocity of the maximum amplitudes ($V_{\text{rm}}$). The low $k_s$ values ($k_s = 0.21, 0.32$) were achieved by testing in a water depth of $h/L = 0.86$ ($h/d = 32$); the lower $k_s$ refers to the hollow cylinder II.2 and the higher of the two to the water-filled cylinder II.3. All the other tests were conducted with the lead filled cylinder III.1.

Runs 32, 33 demonstrated the instability that could be excited at low $V_r$ and sufficiently low $k_s$; unfortunately, there was insufficient flowrate to explore fully the instability ranges of the low $k_s$ cylinders. The extents of the instability regions were reduced by the higher values of $k_s$ and it is seen that for $k_s \geq 1.0$ the maximum amplitudes were recorded at fairly constant $V_{\text{rm}} \leq 7.5$. The results of run 36 ($k_s = 7.6$) showed a departure from the general trend, with the maximum amplitude recorded at $V_{\text{rm}} = 5.9$. It was significant that the water level of this run ($h/d = 24$) was considerably lower than for the other tests ($h/d = 32$). Similar exceptions from generality were recorded in the in-line tests with the cylinder II.1 and it seems probable that the explanation for this lies in the free surface effects of the lower water depth/diameter ratios. This is discussed in detail in Section 9.3.6 where it is shown that these effects can influence the true immersed length, the added mass and the vortex shedding correlation length.
Wootton (26), testing a spring mounted rigid circular cylinder (L/d = 10) in a wind tunnel recorded two peaks in the amplitude versus $V_r$ response curves for very low $k_s$. As $k_s$ was increased, the second peak, originally larger than the first, decreased in magnitude and finally disappeared, leaving only the first peak (which was reduced in amplitude by the increased $k_s$). Wootton attributed the two peaks to vortex shedding from the upper and lower parts of the cylinder; the first peak at $V_{rm} \approx 5.2$ represented vortex shedding from the main body of the cylinder, and the second, at $V_{rm} \approx 6$ represented the influence of the free end on the vortex shedding from the upper part of the cylinder. The free end effects were greatest when $k_s$ was low and thus, when the amplitudes were correspondingly large. Throughout the $V_r$ and $k_s$ ranges tested (26), $V_{rm}$ remained approximately constant, although $V_{rc}$ showed minor increases with increasing $k_s$. Wootton further shows that decreasing the L/d ratio for fixed $k_s$, resulted in a downward shift in $V_{rm}$ and a corresponding reduction of the amplitudes recorded at $V_{rm}$. For L/d = 11.5, $V_{rm} \approx 6.8$ (second peak $\gg$ first) and for L/d = 9, $V_{rm} \approx 6$ (first peak $\gg$ second); L/d values of 8, 8.5 were not excited for the Stability Parameter of the tests. The tests also demonstrated that when the length/diameter ratio was greater than a certain value and the Stability Parameter sufficiently low, the free end effects governed the vortex shedding lengthwise correlation. The correlation starts at the free end (26) and it would seem logical to record maximum amplitudes of well correlated vortex excitation when L/d was large and $k_s$ was small. (Note that the length/diameter ratios of (26) must be doubled to convert to the h/d ratio of these tests in water since the ground board of (26) was a plane of reflection for the flow, (see (112)). This would give an approximate comparison only, because the water surface and free-end effects would not be truly comparable.)

In the present study, similar effects were observed in the in-line and crossflow tests and by analogy it was probable that the variations in $V_{rm}$ for water depths below a certain value were caused
by reductions in water depth/diameter ratios rather than increases in $k_s$. Maximum amplitudes recorded in fig. 9.20 corresponded to $5.9 < V_{rm} < 7.5$ and were in agreement with the results from water channel tests (39), (45) and wind tunnel tests (26). The one cross-flow test with the aluminium alloy cylinder II.1 is shown in fig. 9.22 where it is seen that $V_{rm} \approx 6.7$ for $k_s \approx 1.4$ and $h/d = 30$. From the preceding tests in-line and cross-flow it was tentatively proposed that length/diameter ratio effects significantly affected the $V_{rm}$ for $h/d < 0.63$; thus a departure of $V_{rm}$ from the average 7.5 to 6.7 for the cylinder II.1 cannot be explained entirely on the basis of variations in $h/d$. It is possible that surface roughness may have influenced the results - the aluminium cylinder II.1 was of slightly rougher surface finish than the P.V.C. cylinders II.2 or II.3 and Wootton (30) has demonstrated the sensitivity of cross-flow oscillations to variations in surface roughness. However, comparison of the results from the fundamental modes in-line gave exact agreement of $V_{rm}$ for II.1 and II.2, indicating that surface roughness did not influence results in the in-line direction.

Assuming the vortex shedding natural frequency from the oscillating cylinder to be equal to the cylinder natural frequency at the velocity of maximum recorded amplitudes, the apparent Strouhal numbers $S^*$ appropriate to these $V_{rm}$ are $0.12 < S^* < 0.17$.

The greatest deviations of $S^*$ from the stationary cylinder value of $S = 0.2$ have been recorded fairly consistently in water channels (16), (39), (45), (54) although Marris (43) and Angrilli (57) observed $S^* = 0.2$ also in water tests. The present results from II.3 showed that larger values of $S^*$ were associated with higher degrees of damping or larger $k_s$. Angrilli used an experimental arrangement in which uncertain amounts of non-linear damping could have arisen and this may have influenced his results. Marris tested a cantilevered Pitot tube, but did not disclose the damping characteristics of the tube; there appears to be no obvious explanation for his recordings of $S^* = 0.2$ other than cylinder roughness or length/diameter ratio (Wootton).
(26), using a wind tunnel, demonstrated that $S^*$ was determined by these two variables plus $k_s$, recording $0.15 < S^* < 0.2$. Glass (43) established the variation of $S^*$ with mass ratio $m_r$, defining $S^* = 0.12$ for $1 < m_r < 3$; however, the damping and cylinder diameter were maintained constant and the variations of $m_r$ can be interpreted as changes in $k_s$. Mass ratios in wind-tunnel tests can be an order of magnitude greater than the equivalent water channel tests - clearly this is one of the more important fundamental differences between tests in the two media, and may explain the comparatively low occurrence of in-line oscillations excited in air flow.

The complete experimental results for the series of tests with cylinder II.3 were replotted in fig. 9.21 in the form of maximum amplitudes versus $k_s$; the collected results of other research sources are shown superposed. The cylinder and fluid forcing functions were obviously inter-related, probably through non-linear amplitude effects, and in order to maintain the generality of the comparison of results and to remain consistent with the interpretation of $k_s$ for the in-line tests, the maximum amplitudes recorded on the graph (fig. 9.21) were those maxima which the fluid and cylinder experienced mutually. In the case of the partly immersed cylinder, the maximum amplitude occurred at the water surface and this was the tip deflection of the equivalent completely immersed cylinder defined by $k_s$ in Chapter 3.

A common 'best fit' curve through the data complied with the least squares curve through the combined data of Scruton (40) and Vickery & Watkins (41) as analysed by Blevins (115). This emphasized the validity of using the 'hydrodynamic' $k_s$ as a factor for correlating tests in water with those in air. Additionally it implies that Bénard's (12) contention concerning the dissimilarity between the disposition of vortex streets in air and in water, although possibly applicable to the wakes behind stationary cylinders, apparently ceased to apply when the cylinders were free to oscillate.

In tests to determine the Critical Stability Parameter, a value of $k_{sc}$ was deduced, using cylinder II.3 and varying the water
level to achieve the variation of $k_s$. This result is in good agreement with the collected data of figure 9.21, which showed that for $k_{sc} > 17$ sustained oscillations would not be excited in air or water. (However, Scruton (40) has shown that a spring mounted rigid cylinder required a considerably higher value of $k_{sc}$ to suppress excitation, implying that mode shapes and Stability Parameters are not necessarily defined uniquely.) Comparison of oscillatory amplitudes at similar values of $k_s$ is valid only when similar modes are considered, thus, it is meaningless to compare amplitudes in-line with amplitudes cross-flow at equal $k_s$. As an illustration of this, fig. 9.22 shows the results of tests in-line and cross-flow with cylinder II.1; at $k_s = 1.42$ the in-line oscillations were negligibly small whereas the amplitude of cross-flow oscillations were over $\pm 1d$.

It is interesting to note that as $k_s$ tended to zero, the cross-flow amplitudes did not approach infinity, demonstrating that whereas cylinder amplitudes influence the fluid forcing functions, the range of hydroelastic interactions was limited to operation within finite departures of less than $\pm 2d$ (approximately) from the stationary cylinder flow configuration. Presumably, the reported structural failures of stacks were consequences of the $\pm 2d$ maximum deflection inducing stresses which exceeded the yield stress of the stack material. This suggests that if the $k_s$ and $V_r$ criteria are satisfied, short stacks oscillating in the fundamental mode are most likely to fail. Indeed, Zorilla (116) reported that even in 1971 there was a widespread practice, in the American petroleum industry, of estimating the stability of stacks by the sole criterion of slenderness ratio $L/d$. Generally, he stated, for $L/d > 20$ oscillations were considered likely and for $L/d < 20$ the stacks were regarded as stable. As examples of shortcomings of this practice, Zorilla cited cases where a stack of $L/d = 16.7$ suffered 'very violent' oscillations, another of $L/d = 18.8$ actually failed and a further case of a stack of $L/d = 14$ oscillated with an amplitude of $\pm 9''$.

In summary, the tests of this section have established the
limits of cross-flow oscillations, showing that when $r > 0.5$, $k_s < 17$, sustained oscillations occurred. Also, the relationships between excited amplitudes and Stability Parameters have been examined. It appeared that the maximum amplitudes of limit cycle oscillations in the cross-flow direction were restricted to approximately $\pm 2d$ under the steady flow conditions of the tests.

The wake observation tests of the following section were arranged to examine the lock-in phenomenon of cross-flow oscillations for correlation with the similar recordings from wind tunnel tests.

9.3.3 Wake measurements behind cylinder II. 3 oscillating in the fundamental cross-flow mode.

Measurements of frequencies were made in the wake of the cylinder oscillating in the cross-flow direction, using the thermister probes described in Chapter 7, and arranged relative to the cylinder as shown in fig. 9.13.

Fig. 9.23 is a graph of predominant wake frequency $f_v$ versus $V_r$. As the velocity was increased from zero, the vortex frequencies followed the straight line defining the stationary cylinder vortex frequencies, until the Critical $V_r$ for cross-flow motion was reached at point A, where lock-in occurred. Throughout the lock-in range, $f_v = n$, until at point B, the amplitude of oscillations decreased abruptly; the motion of the cylinder in the region of B was extremely complex.

Fig. 9.24 shows the response of the cylinder recorded from the four strain gauge locations, and it is seen that the base and middle sections were responding at different frequencies, signifying a loss of lengthwise correlation. This feature was not observed with the aluminium alloy cylinder II. 1 probably because it was almost an order of magnitude stiffer than the P. V. C. cylinder II. 3. The cause of the sudden loss of correlation is not evident from the recordings but it may be inferred that the lock-in region was one in which lengthwise correlation was a significant factor and that the end of the lock-in range is accompanied by drastic reductions in lengthwise correlation.
There is no completely satisfactory explanation for the initiation of oscillations, and the termination of excitation appears to be equally inexplicable.

The following section deals with excitation of the second normal mode in-line oscillations to determine and compare the criteria for this mode with those of the fundamental modes in-line and cross-flow.

9.3.4 Tests in the in-line direction. Cylinders II.3 & III.1: second normal mode (s.n.m.).

The problems encountered during construction of an oil terminal at Immingham (Chapter 2), arose from isolated piles oscillating in the fundamental mode in-line, and member piles of completed mooring dolphins oscillating in the second normal mode in-line. A brief examination of the second normal mode (s.n.m.) of a free-ended cantilever was made by King (53) in conjunction with the contractors responsible for the piled structures at Immingham. The principal findings of this brief examination were: (a) a demonstration of the s.n.m. in-line oscillations on a comparatively small cylinder at Reynolds numbers at least one order of magnitude lower than on the full scale, (b) agreement between the $V_{rc}$ for the 'model' and full scale, i.e. $V_{rc} \approx 1.1$ in both cases, (c) in some tests, masses and helical coil springs were attached to the free end of the cylinder, to illustrate, very approximately, the effects of variations in deck mass and stiffness imposed by adjacent braced members.

Very little experimental work has been devoted to normal modes of oscillation higher than the fundamental - indeed, the only other reference of which the writer is aware is the work of Leon (54). Leon demonstrated the excitation of higher normal modes up to the fourth harmonic. His recording of $V_{rc} = 0.8$ for the second normal mode in-line was considerably lower than that recorded in (53). However, Leon's test rig consisted of a cylinder mounted in a pipe expansion tee joint and projecting into the flow such that the free end oscillated
with a small clearance between it and the pipewall. It is not known by how much Leon's results were influenced by the peculiarities of his test rig.

The later full scale site tests at Immingham (112) concentrated on investigations of flow induced oscillations of marine piles under various degrees of end fixity (clamped-pinned to clamped-clamped). In this section of the thesis, the second normal mode was examined in detail to determine whether the criteria of this mode were comparable with the criteria of the fundamental mode. Wake observations and measurements are discussed and recorded in the following section, (9.3.5).

The second normal mode frequency \( f \) of the free ended cantilever in water was approximately 4 to 6 times higher than the fundamental frequency \( n \) in similar depths of water; in order to reduce \( f \) and thus extend the \( V_r \) range (based on the s.n.m. frequency \( f \)) available in the flume, masses were attached to the free end of the cylinder. For sufficiently large masses, a virtual node was created at the free end, and the fundamental mode shapes of the Immingham tests were approached (see fig. 9.25). This is explained by reference to the transcendental equation for a uniform ideal beam with a mass \( M_t \) attached to the free end (117)

\[
\frac{1 + \cosh z \cos z}{\cosh z \sin z - \sinh z \cos z} = \frac{M_t}{m_s} \cdot \frac{1}{L}
\]

where

\[
z = aL
\]

and

\[
a = \frac{w^2 m_s}{EI}
\]

Put \( \frac{M_t}{m_s} = \infty \) in (9.4) and obtain

\[
\cosh z \sin z - \sinh z \cos z = 0
\]

This is the equation for a clamped, pinned cylinder.
Once the virtual node was achieved, the addition of extra masses obviously had no further influence on the natural frequency or mode shape. Even in the approximate clamped-pinned mode, it was possible to test only a very limited range of $V_r$. As an initial exercise, the cylinder II.3 was tested in various water depths to determine the variation of $V_{rc}$ with $h/L$ and also to deduce the Critical Stability Parameter $k_{sc}$. The calculated Stability Parameters for the fundamental and second normal modes are shown in fig. 9.26 where it is seen that at equal water depths, the $k_s$ for the s.n.m. was considerably lower than the $k_s$ corresponding to the fundamental mode, except of course for the one case of the free ended cylinder completely immersed where the $k_s$ values were equal. Calculations of $k_s$ for the s.n.m. were based on the transfer matrix method of Appendix 1 and incorporated the experimentally measured logarithmic decrement of the fundamental mode. Estimates of damping for the two modes confirmed they were of similar orders of magnitude (fig. 9.27); however, the fundamental mode was considerably more amenable to measurement, and greater accuracy was thought to accrue from the convention adopted. The Immingham full scale tests (97), (112) have also shown a close comparison between logarithmic decrements for the fundamental and second normal modes, although each mode result contained unspecified amounts of hydrodynamic damping. More recently, Jeary & Winney (118), testing a full sized multi-flue chimney, calculated the damping of the second normal mode to be approximately twice that of the fundamental. The relationship between damping and the various normal modes of oscillation was undoubtedly influenced greatly by methods of fabrication, foundation support and other factors detailed by Scruton & Flint (101).

Figure 9.27 summarises the results of the stability tests in the second normal mode in-line and the fundamental mode tests of the cross-flow and in-line directions. It is seen that a limiting value of $k_{sc} \leq 1.1$ applied to the (assumed) first instability region of the second normal mode in-line and that instability was recorded in the range
0.42 < h/L < 1 compared with the corresponding instability range of 0.93 < h/L < 1 for the fundamental mode in-line. Table 9.4 records the experimental conditions and results from the s.n.m. stability tests; the variation of $V_{rc}$ with $k_s$ was similar to that measured in the fundamental mode in-line. $V_{rc}$ was seen to vary in the range $1.2 < V_{rc} < 1.8$ - the higher values of $V_{rc}$ corresponding to the larger values of $k_s$.

Filling the cylinder with lead shot reduced the second normal mode frequency $f$ from approximately 16 Hz to approximately 11 Hz and this greatly extended the $V_r$ range (based on $f$) available in the flume. The characteristics of this cylinder are shown in Table 5.1 as cylinder III.1. Fig. 9.29 records the results of one test with cylinder III.1, and comparison with the results from the series of tests examining the fundamental mode in-line demonstrated close agreement between the two sets of results. Second normal mode oscillations in-line were first excited at $V_{rc} \approx 1.3$, the maximum amplitude coincided with $V_{rm} \approx 2.2$ and the abrupt decline to zero amplitude coincided with $V_r \approx 2.5$. Thus there were very definite similarities between the nature of the instabilities recorded in-line in the fundamental and second normal modes. Comparison with the Immingham full-scale tests in the pinned end mode (to which the cylinder III.1 approximated) showed that the shapes of the instability curves gave qualitative agreement although there was less agreement in the values of $V_r$ defining the two instability regions. This aspect is discussed in more detail in the hydroelastic model tests of Chapter 10.

The major results of this section (9.3.4) may be summarised by the following:

(i) from the tests so far, the results have shown that vortex-excited instabilities were excited in the sequence -

- in-line, fundamental mode $1.25 < V_{nd} < 2.5$
- & $2.5 < V_{nd} < 3.8$
- cross-flow fundamental mode $4.5 < V_{nd} < 8$
in-line, second normal mode \[ 1.25 < \frac{V}{fd} < 8 \]  
(note \( 4n \leq f \leq 6n \))

(ii) Second normal mode oscillations were excited in-line in low depths of water for which the fundamental modes in-line and cross-flow were stable;

(iii) the Critical Stability Parameter for in-line oscillations in the second normal mode was \( k_{sc} \approx 1.1 \) compared with \( k_{sc} \approx 1.2 \) for in-line oscillations and \( k_{sc} \approx 17 \) for cross-flow oscillations in the fundamental modes.

The following section 9.3.5 is devoted to measurements of vortex shedding frequencies in the wake of cylinder III.1 oscillating in the second normal mode in-line. It is shown that the vortex shedding of this mode was identical with the fundamental mode in-line. Chapter 9.3.6 studies the free surface effects at the Froude numbers of the tests and Chapter 9.4 is the photographic study in which the symmetric and alternate vortex shedding instability regions are illustrated.

9.3.5 Wake measurements; cylinders II.3 & III.1 oscillating in the second normal mode in-line. 

This section describes a series of tests in which measurements were made in the wake of the P.V.C. cylinders II.3 and III.1 oscillating in the second normal mode in-line, in order to detect and define similarities between these observations and those of section 9.2.5. The wake frequencies were measured using the two thermister probes described in Chapter 7 and arranged as shown in fig. 9.13. The results of this exercise are shown in fig. 9.30 demonstrating that there were striking similarities between this instability region of the second normal mode and the first instability region of the fundamental mode in-line. Throughout the Reduced Velocity range of the tests the dominant wake frequency showed only minor deviations from the stationary cylinder Strouhal vortex shedding frequency. However, there was further evidence of the steplike changes of \( f \) noticed in
the tests of section 9.2.5 using cylinder II.1. Table 9.5 assembles the results from which fig. 9.30 was compiled, showing that the ratios $f/f_v$ apparently varied in such a way that simple proportions were maintained. Instability was first recorded at $V/f_d = 1.50$ and at this point $f/f_v = 4:1$, corresponding to $S = V/f_v d = 0.166$ (fig. 9.31(a)). This result represented the largest deviation from the stationary cylinder value of $S = 0.20$; the cylinder was also oscillating fairly uniformly with small amplitudes in the cross-flow direction at the stationary-cylinder natural frequency corrupted by the second normal mode frequency from the in-line direction and this may have influenced the result. Other recordings at $V_{rc}$ (e.g. fig. 9.32) have shown that at the onset of oscillations in-line, $f/f_v = 4:1$, although $V_{rc}$ may vary from 1.2 to 1.9.

Fig. 9.31(b) contains the results at $V_r = 1.82$ where the ratio $f/f_v$ was reduced to 3:1. The amplitudes of in-line motion were uniform and the excited frequency was equal to the stillwater natural frequency. The implications of this are discussed more fully in Chapter 9.5.

In Section 9.2.2 it was stated that the end of the first instability region was accompanied by a pronounced beating effect. Fig. 9.29 shows that the maximum amplitude for run 51 occurred at $V_r = 2.2$ and it will be observed from the vortex frequency measurements at $V_r = 2.2$ for run 51 (fig. 9.31(c)) that two vortex shedding frequencies were recorded. Both frequencies formed relatively simple proportions of $f/f_v$ as shown and further increase of velocity confirmed the existence of more than one vortex frequency as the cylinder amplitude decreased to zero when only the stationary cylinder Strouhal vortex shedding frequency was recorded in the wake. Thus there were impressive similarities between the first instability region of the fundamental mode in-line oscillations and the first instability region of the second normal mode in-line oscillations. The flow field immediately downstream of a cylinder oscillating in-line under the action of vortex shedding was studied photographically and recorded.
in Section 9.4. It is shown that the first instability region in-line was identified by symmetric vortex shedding and the second by alternate vortex shedding. Only the fundamental mode was studied, and the recordings of similar wake characteristics in the vortex shedding frequency measurements of sections 9.2.5 and 9.3.5 indicated that symmetric vortex shedding was responsible for the excitation of sustained oscillations in-line in the first instability region of both the fundamental and second normal modes.

9.3.6 Bow wave and free surface effects.

This section describes a series of tests which were undertaken to examine visually the surface wave effects on stationary and oscillating cylinders in flowing water, in order to illustrate the variations in effective immersed length with increasing flow velocity. Two cylinders (1" and $\frac{1}{2}$" diameter) were tested independently to determine the way in which the cylinder size contributed to the observed flow effects.

Previous work on this subject has usually been related to the flow about hydrofoil struts (119, 120, 121) and in particular to the conditions necessary for the inception of ventilation in the cavity formed immediately downstream of a strut (the creation of ventilated cavities is an unpredictable occurrence and their creation can cause dramatic variations in handling characteristics of the craft. Stability is generally achieved by designing for full ventilation under all conditions and sacrificing some lift at the foil).

Reference (119) contained data most pertinent to the present series of tests although the cylinders tested ($\frac{1}{4}$" to 1" diameter) in that reference were stationary cantilevers piercing the water surface from above, and immersed to a depth of up to 14". The results from the larger cylinder tests were inevitably influenced by three dimensional end effects particularly in the lower water depths. However, at flows up to 2 ft/sec the two sets of results showed good agreement and (119) also provided a guide to the behaviour of the free surface
waves at extremely high velocities of up to 9 ft/sec.

In the present tests, and for velocities of less than 1 ft/sec. a three-dimensional hump appeared in the water surface in contact with the upstream face of the cylinder and a water surface depression at the rear of the cylinder, as shown in fig. 9.33. The height of the hump could be calculated from the assumption that one velocity head would be recovered at the cylinder stagnation point, and, not surprisingly the heights of the humps were independent of the cylinder's diameter. It was noted that the downstream water surface depression was consistently greater than the height of the hump on the upstream face, and that the depression at the rear of the $\frac{1}{2}$" cylinder appeared to be slightly smaller than for the 1" diameter cylinder. Reference (119) gives an approximate formula for the maximum depth of depression ($d'$) in the unventilated condition, i.e.

$$d' = 0.057 d^{0.22} v^{1.55} \text{ (ft)} \quad \ldots (9.6)$$

and this equation predicts smaller cavities for the smaller diameter cylinder, as observed in the test results above.

Table 9.6 compares the calculated and experimentally derived values of hump height ($h'$) and cavity depth ($d'$) where

$$h' = \frac{v^2}{2g} \text{ (ft)} \quad \ldots (9.7)$$

Good agreement was observed throughout the velocity range, although the cavities occasionally became ventilated; the depth of the ventilated cavities always exceeded the unventilated depths recorded in Table 9.6.

As the velocity was increased (1.0 ft/sec < $v$ < 1.5 ft/sec) the hump developed into a wave with a falling crest, having a waveform similar to the bow wave formed about surface boats. The cavity downstream of the cylinder resembled a glassy, curved membrane with small bubbles occasionally passing through the cavity walls and into the low pressure region beneath. Further increases in velocity above
V = 1.5 ft/sec caused the falling crest to extend around the cylinder and the glassy cavity became unstable, fluctuating between the ventilated and unventilated states.

Air entrainment through the cavity walls was intermittent; large bubbles were drawn down the cylinder surface, coalescing to form a deep, ventilated cavity. Once established, the ventilated cavity was washed away by the flow, presumably because the air demand exceeded the air supply. Fig. 9.34(a) illustrates the manner in which the ventilated cavity was formed from discrete bubbles drawn through the cavity walls; some of the bubbles were washed away before the cavity was fully ventilated. Fig. 9.34(b) shows the glassy, unventilated cavity, and a large bubble being drawn down the cylinder surface, at the start of the ventilating process.

The depths to which the bubbles were drawn down were determined by the ratio of the dynamic pressure forces generated by the flow, to the bubble buoyancy forces. This introduced a complication into the use of free-surface Froude scale hydroelastic models of full sized structures in water. This is because a free surface model must be operated at Froude velocities if the main flow paths are to be reproduced, and since these model velocities are reduced by the square root of the scale factor, the corresponding dynamic forces are very much smaller. In a model, air entrainment is lower, the air bubbles are larger and the bubbles separate out much faster. (122). These considerations show that the 'entrainment depth' measured in the model would underestimate the equivalent depth in the full scale conditions. Furthermore, when the cavity is ventilated, surface tension effects cannot be ignored. There is a division of opinion concerning the correct use of Froude models to reproduce air entrainment conditions (particularly with regard to air entraining 'stationary' vortices over pump intakes etc); however, it is generally agreed that the Froude criteria, although realistic for unventilated flows, ceases to be truly representative for ventilated conditions.

When the cylinder was free to oscillate in response to vortex
excitation the ventilated cavity periodically formed and collapsed in sympathy with the in-line oscillations when the flowrates and oscillatory velocities of motion were sufficiently high.

During cross-flow oscillations, the cavity was unstable and swung across the flow direction, lagging the cylinder motion by up to approximately one eighth of a cycle, although this phase lag was velocity- and amplitude-dependent.

The size of the bow wave hump appeared to be dictated by the velocity head equivalent to the resultant vector of free stream velocity and cylinder oscillatory velocity, for both the in-line and crossflow oscillations. Fig. 9.35 shows the cylinder oscillating in the crossflow direction; the ventilated cavity at the rear of the cylinder extended back into the plane of the figure and lagged the cylinder motion as mentioned previously. Both the downstream cavity and the entrained air would reduce the hydrodynamic loading on the downstream face of the cylinder and similarly, the bow wave on the front face would increase the loading locally. The combination of these effects would result in a departure from the ideal added mass function assumed in the calculations of natural frequencies.

The excited frequencies of oscillation in-line were consistently lower than the stillwater natural frequencies and the causes tentatively were attributed to the variations in added mass, water depth/cylinder diameter effects and interactions between the oscillating cylinder and the vortex shedding mechanisms. Table 9.7 assembles the calculated variations in water depth resulting from assuming the ideal added mass function distributed over a calculated 'effective' water depth for the increased frequencies of excitation. It is seen that the variations in water depth resulting from these calculations could not be explained by direct reference to the calculated or measured cavity depths, to $k_s$ or to the water depth/diameter ratio. It was not possible to identify the individual contributions of these parameters, and the observed variations of excited frequencies were ascribed to complex interactions between the cylinder’s oscillatory
motion in-line, the exciting mechanism and (possibly) the periodically ventilated cavity. In view of these observations, it seems almost certain that the increased frequencies of excitation recorded in the full-scale tests were due to variations in fixity level rather than flow/cylinder motion interactions.

In the cross-flow direction the behaviour was equally difficult to analyse and the 'effective' water depths failed to form well ordered variations with any one of the parameters.

In summary, the tests of this section have shown that the water surface cavities forming downstream of a stationary or oscillating cylinder were stable at relative velocities below 1 ft/sec. and were unstable and either ventilated or unventilated at velocities above 1.5 ft/second. In the discussion of the results it was argued that the ventilated cavities and depths of air entrainment would underestimate, and be unrepresentative of corresponding effects in full scale situations.
9.4 Photographic study of the near wake of cylinder IV.1 oscillating in-line. Part IV of the experimental programme.

In this section, the near wake of a cylinder oscillating in-line was studied to examine the mechanisms of excitation in the first and second instability regions.

The small capillary waves, and later the large bow wave generated by the flow velocities necessary to sustain the oscillations of the cantilevered cylinders tested so far made it impossible to look through the water surface at the vortex wake formed behind the cylinders. The cylinders could be viewed through the perspex side windows but these were not in the right plane for photographing the wake geometry. As an alternative, the vertical sectional cylinder IV.1 was used in these tests; its low natural frequency and the correspondingly low water velocities appropriate to the two instability regions enabled the vortex wake to be observed through the water surface. A sketch of the test arrangement is shown in fig. 9.36 and it is seen that the cylinder was a cylinder of 1" diameter solid perspex attached to a length of 1/4" diameter solid brass rod. The perspex cylinder was comparatively rigid and its frequencies of mass oscillations were varied by adjusting the effective clamping point on the brass bar to alter the support stiffness, or by changing the depth of immersion and hence the added mass of the cylinder; these two methods of varying the frequency could be completed independently. For the photographic study the immersed length was held constant at 18" and the frequency adjusted until the two instability regions could be traversed in the absence of capillary or bow waves. It was noted that excitation in the first instability region in-line occurred only when the immersed length was greater than 5d, although oscillations in the cross-flow mode could be recorded for a depth of immersion of only one diameter.

Potassium permanganate (KMnO₄) was injected through a hypodermic tube immediately upstream of the cylinder and the resulting flow patterns illuminated by an underwater light. Photographs were
taken with the overhead camera as the velocity was increased gradually from zero, to permit both instability regions in-line to be recorded in detail.

Fig. 9.37 shows the wake formed downstream of the stationary cylinder IV.1 at Re = 6,600. Flow separation was laminar and the transition to turbulence apparently occurred at approximately one diameter from the cylinder, followed by the vortex formation region at between two and three diameters from the cylinder. The shear layers in the photograph resemble the vertical flow of smoke from a lighted cigarette held in still air; initially the smoke drifts upwards as a laminar filament, becoming turbulent at a certain distance from the cigarette.

The recorded vortex formation region of fig. 9.37 agreed well with previous observations by Bloor (123); Schiller & Linke (124) showed that the length of the laminar shear layers decreased from 1.4 to 0.7 diameters after separation when the Reynolds number was increased from 3,500 to 8,500 and it thus appears that in fig. 9.37 the transition to turbulence immediately preceded the formation of the turbulent vortices. With reference to the decreasing length of the laminar shear layers with increasing Re, it will be remembered that the Critical region was defined by the Re at which the laminar length was reduced to zero and turbulent separation ensued (21).

A careful study of the Critical Reduced Velocity area of the first instability region illustrated that for all \( V_r < V_{rc} \) the flow field resembled that of fig. 9.37 but when \( V_r \approx V_{rc} \) the vortex formation region fluctuated between zero and one to two diameters. At \( V_r = V_{rc} \) several alternate vortices were shed from the cylinder which then oscillated with rapidly increasing amplitudes in-line; the limit amplitudes of motion were characterised by the symmetrical shedding of fig. 9.38. These vortices were shed symmetrically; one pair of vortices were shed for each complete cycle of motion as the cylinder reversed direction from downstream to upstream at the point of maximum oscillatory amplitude. The appearance of
the symmetrically shed vortices resembled the 'starting vortices' shed from a body accelerated (from rest or from one velocity to another) in a stationary fluid, as investigated by Pierce (125).

In this respect it was understandable that the symmetric vortices were shed from the cylinder at the point of zero oscillatory velocity since that was also the point of maximum oscillatory acceleration.

Downstream from the cylinder, the symmetric wake entered a 'transition range' from which it emerged as a turbulent, alternate street. Within this 'transition range' some vortices were drawn forward to coalesce with vortices shed from the previous cycle, or backwards to the succeeding vortex pair.

The vortices were not always of equal strength and the weaker vortex was drawn across the wake centre-line to coalesce with the stronger vortex. Fig. 9.39 shows this effect quite clearly and also shows the alternate vortex street formed from previously shed vortices. At several diameters from the cylinder the vortex street configuration and frequency complied with Karman's (13) criteria for vortex shedding from a stationary cylinder. This means that of the two vortices shed per cycle of cylinder in-line motion, variable numbers of vortices must be 'lost' in the 'transition range' in order to create the requisite numbers of vortices appropriate to the Strouhal vortex street at the corresponding velocities. Thus, a $V_r$ of 1.25 implies that one vortex in four must be preserved; at $V_r = 1.65$, one vortex in three must be preserved, and at $V_r = 2.5$, one in two. Between these values of $V_r$, the numbers of vortices 'lost' are not related to whole numbers (i.e. $V_r = 1.9$ implies a loss of one in 2.63 vortices) and the apparent steplike changes in dominant wake frequencies observed in sections 9.2 & 9.3, may be explained by the wake 'losing' preferred arrangements of vortices. (See also the descriptive sketches of the 'transition range' in reference (126).) Fig. 9.40 shows the progression from the laminar symmetric pair, through the 'transition range' and finally to the first large, turbulent vortex of the alternate street. It is seen (fig. 9.40) that
the overall transformation from the symmetric to the alternate street was completed within approximately six diameters from the cylinder at $V_r = 1.6$ (this is in exact agreement with the results of (126) at the equivalent $V_r$).

The cylinder amplitudes of oscillation increased to a maximum at $V_{rm} \approx 2.1$ followed by the beating motion and abrupt decrease to zero amplitude noted in section 9.2.2. This region of beating and reduction to zero amplitude was observed to contain mixed shedding of symmetric and alternate vortices; neither type of shedding predominated and there were short periods of either or both. In the 'null' range of $V_r$, following the abrupt reduction to zero amplitude, the shedding was regular, resembling the separation from, and subsequent vortex formation downstream of, the stationary cylinder (fig. 9.37). Progress into the second instability region of in-line motion at $V_r \approx 2.7$ resulted in a repetition of the onset of oscillations in the first instability region; the alternate vortices were formed and shed at the cylinder which then oscillated in-line at its still water natural frequency. However, in this instability region, the vortex shedding was exclusively alternate and locked-in to the cylinder frequency (figure 9.41). The vortices were shed from alternate sides of the cylinder at each reversal of motion into the flow direction, defining the locked-in condition of this instability region as $f_V = n/2$.

Thus, it is seen that for the symmetric and alternate vortex shedding instability regions, the vortices were shed when the cylinder oscillatory velocity was zero and its oscillatory acceleration a maximum.

Downstream from the cylinder, the geometry of the alternate vortex street was similar to the stable Karman (13) vortex street configuration shown in fig. 2.1(d). The motion of the cylinder oscillating in-line would influence the relative velocity of the vortex street and its in-line spacing, similar to the way in which cross-flow oscillations are known to alter the cross-flow spacing of the vortex street, (127). There was no coalescence of the vortices and it is considered
reasonable to propose that the wake velocity and geometry combined to satisfy Karman's criteria.

The $k_s$ of the cylinder IV. 1 used in these photographic studies was extremely low, and violent cross-flow forced oscillations prevented the complete exploration of this second instability region of in-line motion. A brief study of cross-flow oscillations at the natural frequency of the cylinder oscillating exclusively in the cross-flow direction demonstrated that the vortices were shed alternately (see fig. 9.42) at each cross-flow reversal in the direction of motion, i.e. $f_v = n$. The observation was consistent with recordings from other sources.

The principal findings of this section have been the identification of two completely different types of vortex shedding, viz, symmetric shedding in the first instability region and alternate shedding in the second instability region of in-line oscillations. In the full-scale tests (112) both symmetric and alternate vortex shedding were detected although the authors of (112) were unable to associate them with the two instability regions. The present work and the results of (126) were believed to be amongst the first recordings of sustained oscillations caused by symmetric vortex shedding. However, Laird (136) quotes Mandini (128) who noted "a range of velocities in which cylinders oscillated in the flow direction, shedding symmetrical vortices"; all efforts to trace this quoted work have failed.

It will be remembered that in the first instability regions of the present series of tests and also in the full scale tests (112), probes mounted downstream of the cylinder detected dominant wake frequencies governed by the stationary cylinder Strouhal frequency. Visual observation of this region showed regular, symmetric shedding at the (constant) still water natural frequency of the cylinder. Additionally, the test cylinders in the present tests occasionally exhibited cross-flow forcing at the stationary cylinder Strouhal shedding frequency, as defined by the instantaneous velocity and the cylinder diameter.
Consideration of the symmetric shedding process showed that if the two vortices of the symmetric pair were identical, there would be no cross-flow component of the fluctuating forces arising from the shedding. If one vortex was consistently larger than the other, the cross-flow force component of this imbalance would then have a constant frequency of $n$. Consequently, it would appear that the recordings of dominant wake frequencies equal to the stationary cylinder Strouhal shedding frequencies cannot be explained uniquely in terms of symmetric vortex shedding.

The probes used in both sets of tests were mounted in similar positions relative to the cylinders and pile (i.e. approximately one diameter downstream and one diameter cross-flow measured from the cylinder surface). It is proposed that the alternate wake pressure field, fluctuating at the stationary cylinder shedding frequencies extended back to the probes and occasionally to the cylinder without adversely influencing the symmetric shedding.

Additionally, the photographic study was made using fairly low water velocities and it was shown that increasing $V_r$ reduced the distance between the cylinder and the 'transition range'. The full-scale tests and the series of tests for this thesis were made using higher water velocities and it was thought that the distance from the cylinder to the formation of an alternate vortex street was also a function of velocity (similar to the stationary cylinder data of refs. (123, 124)). At these higher velocities, the vortices would have been shed symmetrically and the alternate street would form in the vicinity of one diameter from the cylinder and so be sensed by the probes. It was noted that the vortices of the alternate street were considerably larger than the vortices of the symmetric street thus their effects would be detectable at larger distances from the vortex cores.

In the second instability region of in-line motion the vortex shedding was from alternate sides of the cylinder and locked in to the cylinder natural frequency, i.e. $f_v = n/2$. 
Cross-flow motion was also characterised by alternate shedding, locked in to the cylinder natural frequency, i.e. \( f_v = n \).

The fluctuating exciting and damping forces arising in both instability regions of in-line motion were analysed in Chapter 3 and the experimental results of Chapter 9 are treated by this analysis in Chapter 11. The photographs of symmetric shedding were used in Chapter 11 to assess the limits of the fluctuating drag coefficient, and the computed results, incorporating these drag coefficients yielded realistic values for the amplitude response.
9.5 An investigation of Reynolds number effects and the excitation of higher normal modes in the in-line and cross-flow directions. Part V of the experimental programme.

9.5.1 Reynolds number effects, cylinder IV. 1, V. 1, V. 2, V. 3.

References (56) and (101) both cite cases in which the excitation of objectionable oscillations of structures exposed to airflow were suppressed by lowering the natural frequencies. The dynamic pressures generated by the velocities at which the vortex frequencies and structural frequencies were equal were insufficient to initiate excitation, and structural stability resulted. This implied the existence of two possible effects: stability was achieved either because the structural deflections were too small to develop the 'threshold' fluidelastic interactions, or because Reynolds number effects were inhibiting excitation. In the water tests of this thesis, it was observed that lowering the fundamental frequency in-line suppressed excitation of that mode but frequently led to instability in the second normal mode in-line. However, lowering the frequency was usually achieved by increasing the Stability Parameter for the fundamental mode and the lack of excitation was cursorily attributed to increased $k_s$ influences. The tests recorded in the present section were devised to examine further the conditions determining the Critical Reduced Velocity $V_{rc}$ and in particular, the contributions of Reynolds number effects.

The one inch diameter perspex cylinder IV. 1, mounted as shown in fig. 9.36, was tested briefly in the in-line direction to determine the limiting Re for excitation of sustained oscillations. In these tests the depth of immersion of the cylinder was held at a value which permitted excitation at $V_{rc} = 1.25$, thereby indicating a sufficiently low $k_s$ value for the tests. At this fixed depth of water, the natural frequency was gradually reduced until the excitation of sustained oscillations in-line was no longer possible; in this way the frequency and stiffness were varied without affecting the Stability Parameter. As $n$ was decreased below 5 Hz, $V_{rc}$ increased from the original 1.25...
and excitation could not be initiated for $\text{Re} \leq 1200$ (with $n = 0.9$ Hz). This was not conclusive evidence that $V_{rc}$ was Reynolds number-dependent although simple reasoning would show that the dynamic forces and the stiffness were each reduced by similar amounts; thus the observed behaviour was more probably due to features of the flow rather than structural properties.

Following this, the stainless steel cylinders V.1, V.2 and the aluminium alloy cylinder V.3 (see Table 5.1) were tested in their fundamental and second normal modes in the in-line direction. The cylinders were mounted as cantilevers from the floor of the flume and concentrated masses and springs were attached to the cylinders' free ends. The water depth was maintained at $h/L = 0.8$ to ensure very low $k_s$ values. Similar results were recorded in individual tests with the three cylinders and it was concluded that the minimum Reynolds numbers for the excitation of sustained oscillations in the in-line direction (fundamental or second normal mode) was $1250 < \text{Re} < 1400$.

It was interesting to note that the minimum Reynolds number for cross-flow oscillations, found in supplementary tests using a length of stainless steel hypodermic tubing, was approximately $\text{Re} = 110$.

9.5.2 Excitation of higher normal modes. Second and third normal modes in-line; second normal mode cross-flow.

The cylinders V.1, V.2 and V.3 were fixed to the floor of the flume and tested in turn using the test sequence outlined in Chapter 7. The results of the tests are shown in figures 9.43 and 9.44(a), (b); it is seen that the $V_r$ ranges of sustained oscillations in the second and third normal modes in-line and the second normal mode in the cross-flow direction were consistent with those recorded in the respective fundamental modes. The two higher normal modes in-line each exhibited the two instability regions recorded in the fundamental mode of that direction, i.e. $1.25 < V_r < 2.5$. 
2.7 < V_r < 3.7, provided that Re > 1200. The single instability region for the second normal mode in the cross-flow direction was similar to the results from the fundamental mode tests of the P.V.C. cylinder II.2 with very low k_s (see fig. 9.20, run 33). However, it will be noted from fig. 9.44(b) that the first instability region of the third normal mode in-line and the (single) instability region of the second normal mode cross-flow were excited simultaneously and throughout the V_r range of the tests the frequencies in the two directions were related by 3:1. Reference back to the tests in-line with cylinders II.1, III.1 and the photographic study of Section 9.4 showed that the first instability region in-line, although characterised by symmetric vortex shedding was apparently accompanied by step-like changes of f_v. In particular, stretches of cross-flow oscillations at the forced frequencies of f_v (where f_v = 0.2 V/d) were recorded when n and f_v were related by simple ratios such as 4:1, 3:1, 2:1. In the full scale tests (112) the dominant wake frequency was measured up to V_r < 1.6 where a 3:1 relationship was observed between n and f_v. In the present tests the 3:1 ratio between the in-line frequency (50 Hz) and the cross-flow frequency (16.6 Hz) established the sustained oscillations in the cross-flow direction when V/f_3.d = 1.68, f_3/f_v = 3 and thus for the cross-flow mode V_rc = 5. However, once excited, lock-in apparently occurred between f_v and the cross-flow frequency. The mechanism responsible for supporting cross-flow oscillations at a fixed frequency, and in-line oscillations at a frequency consistently three times higher than the former was not easily defined. Although the results from the two directions demonstrated an otherwise apparent lack of interaction of one with the other, it was notable that the cross-flow oscillations ceased at the end of the first instability region in-line; the second instability region contained alternate vortex shedding from the cylinder and at the locked-in frequency of the cylinder oscillating in-line. The cross-flow component of the vortex shedding forces (at one half the in-line frequency) then would not coincide with the natural frequency.
of the cross-flow direction and stability resulted. This suggested that in the first instability region the wake and the vortex shedding process were separate phenomena. It has been shown in previous parts of this chapter that the in-line oscillations of the first instability region were generated by symmetric vortex shedding at the cylinder frequency whilst the dominant wake frequency continued to be governed by the stationary cylinder Strouhal number. The fluctuating pressure field of the wake extended back to the thermister probes (one diameter from the cylinder) and occasionally, when the cylinder frequency and $f_v$ were suitably related, small amplitude cross-flow forced oscillations at $f_v$ were recorded, although these did not alter the response in-line. In the present tests, the cylinder presumably shed symmetric vortices in-line; these were rearranged in an alternate wake downstream, and the frequency of this alternate wake was locked-in to the cross-flow direction motion. The Karman configuration was satisfied when the locked-in wake was re-aligned still further downstream. Lock-in in the cross-flow direction, recorded previously, was seen to be a region in which alternate vortices were formed at the cylinder and shed at the locked-in frequency; clearly in the present tests, alternate and symmetric vortex shedding could not coexist and this was regarded as further evidence of the independence of the in-line shedding and the downstream wake. The mechanics by which the wake apparently influenced only the cross-flow mode, and the way in which symmetric vortices were shed from the cylinder oscillating cross-flow at a completely different frequency, are currently not fully explainable.

A surprising and interesting effect was noted at the higher $V/nd$ values in tests with the aluminium alloy cylinder V. 3, when, for $V/nd > 16$, forced, cross-flow oscillations were recorded consistently at one quarter the dominant wake frequency $f_v$. The amplitudes of these oscillations increased with velocity (or $V_r$) and at $V_r \approx 20$, where $f_v/4 = n$, the motion cross-flow was comparatively violent. This observation could be relevant to the recordings of
Thoma (36) and Penzein (37) who both reported cross-flow oscillations at $V/\nu \approx 20$. In Chapter 2, results of (36) and (37) were explained by reference to the subharmonic excitation of cables in the paper by Lienhard and Liu (38). It was not evident why excitation was restricted to one quarter the vortex shedding frequency, except that it was nearer the structural natural frequency than $f_\nu/2$, the first harmonic of the fundamental vortex shedding frequency, where greater power would be expected. The present recordings, those of (36), (37), (38) and the stationary cylinder tests of (28), (29), (31) all indicated that vortex shedding was not restricted to one single frequency and that harmonics of the fundamental frequency, although of lower power content, were measurable and, under certain conditions, could lead to instabilities.
Table 9.1 Comparison of calculated and experimental values of log decrement and frequency. Cylinder II.1.

<table>
<thead>
<tr>
<th>$h/L$</th>
<th>log decrement $\delta_t$</th>
<th>frequency n (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>calculated</td>
<td>experimental</td>
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<tr>
<td>0.487</td>
<td>0.026</td>
<td>0.026</td>
</tr>
<tr>
<td>0.560</td>
<td>0.029</td>
<td>0.029</td>
</tr>
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<td>0.635</td>
<td>0.033</td>
<td>0.033</td>
</tr>
<tr>
<td>0.685</td>
<td>0.038</td>
<td>0.038</td>
</tr>
<tr>
<td>0.805</td>
<td>0.054</td>
<td>0.054</td>
</tr>
<tr>
<td>0.878</td>
<td>0.071</td>
<td>0.068</td>
</tr>
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Table 9.2 Comparison of calculated and experimentally recorded bending moment distributions along cylinder II.1.

<table>
<thead>
<tr>
<th>$h/L$</th>
<th>Calculated</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>base 0.23L 0.45L 0.67L</td>
<td>base 0.23L 0.45L 0.67L</td>
</tr>
<tr>
<td>0.633</td>
<td>2.30 1.52 0.85 0.35</td>
<td>2.30 1.48 0.79 0.34</td>
</tr>
<tr>
<td>0.633</td>
<td>4.90 3.30 1.82 0.72</td>
<td>4.90 3.25 1.80 0.70</td>
</tr>
<tr>
<td>0.681</td>
<td>2.00 1.34 0.73 0.27</td>
<td>2.00 1.29 0.72 0.28</td>
</tr>
<tr>
<td>0.681</td>
<td>3.00 2.00 1.10 0.41</td>
<td>3.00 1.95 1.10 0.39</td>
</tr>
<tr>
<td>0.805</td>
<td>1.80 1.21 0.66 0.24</td>
<td>1.80 1.18 0.65 0.25</td>
</tr>
<tr>
<td>0.805</td>
<td>4.30 2.90 1.58 0.57</td>
<td>4.30 2.85 1.60 0.59</td>
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</table>
Table 9.3 Comparison of amplitudes of equivalent cylinders and Stability Parameters. In line oscillations.

<table>
<thead>
<tr>
<th>Run</th>
<th>$k_s$</th>
<th>$h/L$</th>
<th>$(y/d)_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.16</td>
<td>0.805</td>
<td>0.148</td>
</tr>
<tr>
<td>8</td>
<td>0.34</td>
<td>0.634</td>
<td>0.128</td>
</tr>
<tr>
<td>10</td>
<td>0.30</td>
<td>0.685</td>
<td>0.132</td>
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<td>0.500</td>
<td>0.027</td>
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<tr>
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<td>1.06</td>
<td>0.464</td>
<td>0.005</td>
</tr>
<tr>
<td>22</td>
<td>0.38</td>
<td>0.732</td>
<td>0.124</td>
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<tr>
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</tr>
<tr>
<td>25</td>
<td>0.52</td>
<td>0.732</td>
<td>0.090</td>
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Table 9.4 Second normal mode stability tests $k_s$ and $V_{rc}$

<table>
<thead>
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<th>$h/L_{max}$</th>
<th>$k_s$</th>
<th>$V_{rc}$</th>
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<tbody>
<tr>
<td>0.890</td>
<td>0.20</td>
<td>1.28</td>
</tr>
<tr>
<td>0.555</td>
<td>0.30</td>
<td>1.35</td>
</tr>
<tr>
<td>0.510</td>
<td>0.40</td>
<td>1.41</td>
</tr>
<tr>
<td>0.486</td>
<td>0.50</td>
<td>1.43</td>
</tr>
<tr>
<td>0.446</td>
<td>0.55</td>
<td>1.85</td>
</tr>
<tr>
<td>0.400</td>
<td>0.75</td>
<td>not excited</td>
</tr>
</tbody>
</table>

Table 9.5 Evidence of the steplike changes in the ratio $n$ $f_v$
Second normal mode in line Cylinder III.1

<table>
<thead>
<tr>
<th>Reduced velocity $V_r$</th>
<th>frequency ratio $n/f_v$</th>
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</thead>
<tbody>
<tr>
<td>1.34</td>
<td>4.0</td>
</tr>
<tr>
<td>1.42</td>
<td>4.0</td>
</tr>
<tr>
<td>1.51</td>
<td>3.75</td>
</tr>
<tr>
<td>1.58</td>
<td>3.5</td>
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<td>1.66</td>
<td>3.25</td>
</tr>
<tr>
<td>1.74</td>
<td>3.0</td>
</tr>
<tr>
<td>1.82</td>
<td>2.75</td>
</tr>
<tr>
<td>1.90</td>
<td>2.75</td>
</tr>
<tr>
<td>1.98</td>
<td>2.5</td>
</tr>
<tr>
<td>2.12</td>
<td>2.25</td>
</tr>
</tbody>
</table>
Table 9.6 Free surface effects of flow past the 1" diameter cylinder IV.1.

<table>
<thead>
<tr>
<th>V (ft/sec)</th>
<th>h' inches</th>
<th>d' inches</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Calculated</td>
<td>experimental</td>
</tr>
<tr>
<td>0.5</td>
<td>0.046</td>
<td>0.135</td>
</tr>
<tr>
<td>1.0</td>
<td>0.186</td>
<td>0.396</td>
</tr>
<tr>
<td>1.5</td>
<td>0.418</td>
<td>0.742</td>
</tr>
<tr>
<td>2.0</td>
<td>0.744</td>
<td>1.159</td>
</tr>
<tr>
<td>2.25</td>
<td>0.942</td>
<td>1.392</td>
</tr>
<tr>
<td>2.50</td>
<td>1.162</td>
<td>1.638</td>
</tr>
</tbody>
</table>

Table 9.7 Apparent variations in water depth during the excitation of in-line oscillations. Cylinder II.1.

<table>
<thead>
<tr>
<th>k_s</th>
<th>Water depth</th>
<th>V (ft/sec)</th>
<th>Δh (inches)</th>
<th>calculated h'</th>
<th>d'</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.34</td>
<td>24</td>
<td>1.5</td>
<td>2.0</td>
<td>0.42</td>
<td>0.74</td>
</tr>
<tr>
<td>0.37</td>
<td>26</td>
<td>2.0</td>
<td>1.5</td>
<td>0.74</td>
<td>1.16</td>
</tr>
<tr>
<td>0.27</td>
<td>28</td>
<td>2.0</td>
<td>2.2</td>
<td>0.74</td>
<td>1.16</td>
</tr>
<tr>
<td>0.38</td>
<td>30</td>
<td>1.5</td>
<td>2.5</td>
<td>0.42</td>
<td>0.74</td>
</tr>
<tr>
<td>0.16</td>
<td>33</td>
<td>2.0</td>
<td>2.45</td>
<td>0.74</td>
<td>1.16</td>
</tr>
<tr>
<td>0.12</td>
<td>36</td>
<td>2.0</td>
<td>2.4</td>
<td>0.74</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Δh/h
Fig. 9.0  Definitive sketch of experimental arrangement
Fig. 9.1  Total logarithmic decrement versus water depth for the cylinder II.1, calculated using the results of Chapters 4 & 8.

Fig. 9.2  The onset of instability in the in-line direction Cylinder II.1.

Fig. 9.3  Beating effects recorded in tests with the aluminium alloy cylinder II.1. The end of the first instability region.
Fig. 9.4 Hysteresis effects. Cylinder II.1 oscillating in-line.
**Fig. 9.5(a)** Bending moments versus Reduced Velocity. Free ended cylinder II.1 oscillating in-line in various depths of water.

**Fig. 9.5(b)** Frequencies excited in the structure expressed as percentage of the water natural frequencies. Cylinder II.
Fig. 9.6  Bow wave effects - note the air sucked down the downstream face of the cylinder.

Fig. 9.7  Determination of the steady drag coefficient $C_d$ for cylinder II.1.

slope = 0.245 (equation 9.2)
$C_d = 1.29$
• increasing velocity
• decreasing velocity
Fig. 9.8(a) The two instability regions recorded in the in-line direction Cylinder II.1 with various masses attached to the free end. Water level = 0.732L.

Fig. 9.8(b) Frequencies excited in the in-line direction expressed as proportions of the stillwater natural frequencies. Cylinder II.1.
Fig. 9.9 The full-scale results (112).

Fig. 9.10 The P.T.F.E. slides to restrict motion to the in-line direction.
Fig. 9.11 Pseudo-Coulomb logarithmic decrement recorded with cylinder II.1 and the slides shown in Fig. 9.10.

Fig. 9.12 Amplitude response of cylinder II.1 with restraining wires limiting motion to the in-line direction.
Fig. 9.13  Arrangement of the 'model' and full-scale probes.

Fig. 9.14  Dominant vortex frequencies in the wake of cylinder II.1 oscillating in-line.
Fig. 9.15  Total logarithmic decrements and natural frequencies. P.V.C. cylinders II.2, II.3.

PVC cylinder II.2, $h/L = 0.67$, free ended $k_s = 0.37$

Al. cylinder II.1, $h/L = 0.73$, $M_t = 0.3$ lbs, $k_s = 0.38$

Fig. 9.16  Comparison of amplitude response of the aluminium alloy cylinder II.1 with the P.V.C. cylinder II.2.
Fig. 9.17 Experimental results recorded with cylinder II.2 for $k_s = 0.16$

Fig. 9.18 Amplitude of cylinders II.1, II.2 with and without restraining wires for very low $k_s$
Fig. 9.19  Determination of the steady drag coefficient $C_d$ for cylinder II.2.
Fig. 9.20  Bending moments versus Reduced Velocity for cross-flow motion. Cylinders II, 2, 3, 4.
The collected results of cross-flow vibration stability parameters, expressed as maximum deflection divided by diameter, for various piles:

- PVC pile
- Al pile

Stability Parameter: $2m_e\delta_s/\rho d^2$

Key:
- ■ ■ ■ King (in water) $h/d = 20-33$ thesis results
- ○ Vickery & Watkins (in water) $L/d = 15$ (ref. (41))
- ▲ Vickery & Watkins (in air) $L/d = 14.2$ (ref. (41))
- + Feng (in air) $L/d = 9$ (ref. (144))
- □ Hartlen (in air) $L/d = 13.8$ (ref. (145))
- ▼ Scruton (in air) $L/d = 27.5$ (ref. (40))
Comparison of in-line and cross-flow oscillations Cylinder II.1.

Fig. 9.23 Dominant vortex frequencies measured in the wake of cylinder III.1 oscillating in the fundamental mode cross-flow.
Fig. 9.24  U.V. recording of straingauge output at the end of the lock-in range.
Fig. 9.25 The mode shapes of cylinder III.1 with various masas attached to the free end.
Fig. 9.26  Stability Parameter versus water depth h/L for the fundamental and second normal modes. Cylinder III.1.

Fig. 9.27  U.V. trace of transient motion. Cylinder III.1 oscillating freely in the second normal mode. h/L = 0.685.
Fig. 9.28  Stability limits for the in-line and cross-flow directions, fundamental and second normal modes.

Fig. 9.29  The first instability region of in-line oscillations Cylinder III. 1 oscillating in the second normal mode.
Fig. 9.30 Dominant wake frequencies measured in the wake of cylinder III. Oscillating in the second normal mode in-line.

Fig. 9.31(a) Strain gauge outputs and dominant wake frequencies Cylinder III. Oscillating in-line
**Fig. 9.31(b) Strain gauge outputs and dominant wake frequencies**

Cylinder III. 1 oscillating in-line

**Fig. 9.31(c) Strain gauge outputs and dominant wake frequencies**

Cylinder III. 1 oscillating in-line
Fig. 9.32 U.V. trace of the onset of instability in the second normal mode, with the corresponding wake measurements superposed.
Fig. 9.33  Free surface phenomena. Water velocity = 10 inches/second.

Fig. 9.34(a)  $V = 1.5$ ft/second. Ventilated cavity forming from coalescence of small bubbles.
Fig. 9.34(b) \( V = 1.5 \text{ ft/second} \). Cavity unventilated - bubble drawn down at the start of the ventilating process.

Fig. 9.35 The cavity formed when the cylinder oscillated in the cross-flow direction.
Fig. 9.36  Diagram of the experimental arrangement for the photographic study.

Fig. 9.37  The vortex street formed behind a stationary cylinder.
Fig. 9.38  Vortex shedding in the first instability region in-line. Note the vortices are shed symmetrically.

Fig. 9.39  Symmetrically disposed vortices of unequal strength. Note the coalescence in the near wake. In-line oscillations.
Fig. 9.40  The formation of an alternate vortex street from the symmetric vortices. In-line oscillations.

Fig. 9.41  Alternate vortex shedding in the second instability region of in-line oscillations.
Fig. 9.42  Alternate vortex shedding in the cross-flow instability region.

Fig. 9.43  Results from the $\frac{1}{4}$" diameter cylinder V. 3 Higher normal modes in-line and cross-flow.
Fig. 3.44(a) Results from the free-ended $\frac{3}{4}$" diameter cylinder V.2. Second normal mode in-line.

Fig. 9.44(b) Results from the $\frac{1}{4}$" diameter cylinder V.2 with masses attached to the free end. Higher normal modes in-line and cross-flow.
10. Tests with hydroelastic models of two of the piles used in the full-scale tests at Immingham.

This section describes briefly the application and limitations of the scaling laws developed in Chapter 5.

The tests so far have confirmed, in general qualitative terms, the ability of small cylinders oscillating in-line to represent the behaviour of larger cylinders operating at considerably higher Reynolds numbers. The series of tests described in this section were designed to establish the validity of using hydroelastic models to provide reliable quantitative information, and for this purpose two model cylinders were tested for comparison with the full scale results.

The two full-scale piles modelled were 30\" and 18\" in diameter and for these, geometric scaling factors of 30 and 27 were selected, to suit the water levels and flowrates available in the test flume. The model of the 30\" diameter cylinder was tested in a variety of configurations and the model 18\" diameter cylinder was tested in the in-line direction only. The features of a true hydroelastic model of the 30\" diameter pile are shown in Table 10.1 (columns 1 and 2). Such a cylinder would be difficult to obtain commercially and it was decided to use materials which were available more readily. P.V.C. was selected as the most appropriate modelling medium and as in most hydroelastic models some compromises were inevitable because of the impossibility of satisfying all the requirements demanded by dimensional analysis. Limited departures were made from correct model values of stiffness and mass, but their ratios, and thus frequencies, were reproduced exactly. The tests of Vickery & Watkins (11) and the results of Chapter 9 showed that precise reproduction of \( m_c \cdot \delta \) was not necessary provided that the Stability Parameter group

\[ k_s = 2m_c \delta / \rho d^2 \]

was modelled correctly. Kolkman (129) stated that experience at the Delft Hydraulics Laboratory showed that departures from correct model values of mass and stiffness should not exceed 200%.

The equivalent mass per unit length \( m_c \) could be written (113)
\[ m_e = (m_a + m_w) + m_s \int_0^L y^2 \, dx / \int_0^h y^2 \, dx \] \quad \ldots \quad (10.1)

and by evaluating the integrals with the aid of the transfer matrix technique described in Appendix 1, the expression for \( m_e \) appropriate to the water depth of the full-scale tests (\( h/L = 0.77 \)) became:

\[ m_e = (m_a + m_w) + 1.2 \, m_s \] \quad \ldots \quad (10.2)

By substituting in (10.2) the various values for correct modelling it was shown that an increase of 150% in \( m_s \) would result in only a 25% increase in \( m_e \). Table 10.1, column 3, records the features of the actual model pile and column 4 expresses the percentage departures from correct modelling.

The natural frequencies in air and in water were achieved by adjusting the structural mass/ unit length using sand, lead shot and water. The stiffness of the capping beam across the top of the pile was more easily scaled and the model beam was built from thin brass sheet with soldered joints. The capping beam was rotatable about the vertical axis of the pile and by this means the fixity in the in-line direction was varied from fully encastre to pinned.

A false floor in the flume was used to simulate the imperfectly encastre bed conditions of the full-scale site and figure 10.1 shows the overall layout and a detail of the capping beam.

The angle of the capping beam relative to the flow direction was set to the required value and the outputs from the four strain gauges two in-line and two cross-flow, were monitored as the flow velocity was increased from zero. Recordings were made for motion across the beam (by rotating the model through the same angle as the beam) or in-line with the flow.

Figures 10.2(a), (b) and 10.3(a), (b) compare model and full-scale (30" diameter pile) results for motion across the capping beam and in the in-line direction. The Critical Reduced Velocity \( V_{rc} \) was approximately 1.3 for both sets of results and good quantitative
agreement was recorded throughout the ranges of $V_r$ common to the full-scale and model tests.

There were inevitable variations in corresponding individual data points but clear similarities were shown between the overall shape of the model and full-scale response curves of Figs. 10.2(a), (b) and 10.3(a). Fig. 10.3(b) exhibited the greatest variation between the two sets of results at the lower values of $V_r$ but showed remarkable quantitative agreement for $V_r > 1.45$. Fig. 10.2(b) illustrates close qualitative comparison between the shape of the two response curves although there was an apparent upward shift of $V_r$ for corresponding values of amplitude in the model results.

The model results showed a general increase in amplitude response with increases in the angle between the flow direction and the capping beam, and maximum amplitudes of ±0.1 diameters were recorded when the capping beam was normal to the flow direction. These results were confirmed by reference to the appropriate full-scale results.

Fig. 10.2 shows the abrupt reduction to zero amplitude at $V_r \Delta 2.3$ and it is interesting to note that when motion restarted from this null region, the model pile was forced to oscillate at frequencies corresponding to the shedding of individual vortices from a stationary cylinder. A similar effect was noted previously in section 9.2 during tests with cylinders of very low $k_s$ and in common with the present tests the amplitudes of this forced motion apparently increased linearly with velocity.

As a supplementary test, a 1/27 scale model of the 18" diameter Immingham pile was tested in the in-line direction. The material of this model pile (Acrylonitrile Butadiene Styrene - A. B. S.) possessed a structural damping and mass density lower than the P.V.C. pile and these properties combined to give the hydroelastic model detailed in Table 10.2. Some corrections for equivalent mass/unit length were necessary and these were achieved by the method employed with the model 30" diameter pile.
Fig. 10.4 records the model and full-scale test results and it is seen that there is impressive agreement between the shapes of the two sets of results; the model quantitatively reproduced the full-scale amplitudes but with an offset in $V_r$. The abrupt reduction to zero amplitude at the end of the first instability region coincided with $V_r = 2.4$ and was close to the $V_r$ values recorded in the thesis tests of the previous chapter.

The model tests for both the 30" and 18" piles were conducted using a fixed depth of water and fixed natural frequencies; they therefore differed from the full-scale tests in which tidal ranges of up to 16ft were encountered in maximum water depths of 80ft. These tidal variations caused changes in natural frequencies and also slightly modified the Stability Parameters. Furthermore, the full-scale piles oscillated with a 'figure of eight' (Lissajous) motion in the first instability region but this feature was not observed in these model tests and this may have contributed to the variations between the model and full-scale values of $V_r$ at corresponding amplitudes of oscillations. No attempt was made to model the surface roughness of the piles and in this respect there were differences between the full-scale 30" diameter and 18" diameter piles. The surface of the smaller pile was covered by a uniform growth of crustacea and silt deposits with a short length of barnacle growth between the high and low water levels. The 30" diameter pile was also covered by the crustacea and silt, but over an appreciable length from the bed level the pile surface growth included large roughness marine life and the barnacle population, near the high water level, extended for approximately twice as far as for the 18" diameter pile. The uncertainty of the full-scale fixity levels was discussed in Section 9.3 where it was argued that the fixity level probably decreased during excitation. The full scale piles were driven close to each other and similar variations of apparent fixity level would be expected for all three piles. It will be noted from figures 10.2, 10.3 that the maximum departures from the fullscale results were recorded when the
capping beam was normal to the flow direction. For this condition the recorded amplitudes were greater than for any other angle, and assuming fixity levels to be amplitude-dependent, the observed departures also could be explained through changes in effective fixity levels. The time-dependent velocity profile of the Immingham site was not reproduced and it is possible that this effect may have influenced the results. In introducing the topic of hydroelastic modelling in Chapter 5, the importance of the velocity profile \( V_h \) was emphasised; distorted profiles could lead to non-uniform vortex shedding, variations in lengthwise correlation and departures from the values of \( V_{rc} \) associated with uniform flow. The full-scale water surface velocity was at times 20% greater than the velocity at \( h/L = 0.65 \), and if correlation was initiated at the water surface by velocities considerably higher than the recorded mean, this would lead to an apparent reduction in \( V_{rc} \) for those tests. Finally, the turbulence scale and intensity were not modelled and it was presumed that the lack of agreement between the model and the full-scale results was due to a complex functional relationship between the surface roughness, the fixity variations, velocity profile and turbulence effects.

The graph of the apparent Strouhal number \( S^* \) of a cylinder free to oscillate in the cross-flow direction (fig. 2.3) showed that \( S^* \) was a function of Reynolds number for a given cylinder, varying in the range 0.16 for Subcritical Re to 0.21 for Supercritical Re. The definition of \( S^* \) would not apply to the symmetric vortex shedding instability region, although there may be similar Reynolds number effects between the full-scale and model cylinders. (The preceeding tests of Chapter 9, and the results of references (39), (45) showed that \( S^* \) was also a function of Stability Parameter \( k_s \); however, the full-scale \( k_s \) was correctly reproduced for these hydroelastic modelling tests to avoid introducing another variable.)

The modelling criteria employed in these tests resulted in very satisfactory qualitative and quantitative comparisons with the full-scale tests. The agreement demonstrated the validity of the
criteria selected (i.e. $V_p$, $C_n$, $Fr$, $k_s$, $V_h$), and the discussion above has suggested areas where more specific information should be sought, as these could lead to the definition of even more comprehensive criteria.

The results of this Chapter form the basis of a paper (130) to be presented by the writer at the 6th International Offshore Technology Conference in Houston, U.S.A. in May 1974.
<table>
<thead>
<tr>
<th>Property to be scaled</th>
<th>1:30 model</th>
<th>Actual value obtained in model</th>
<th>% Departure from true modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L)</td>
<td>43.2&quot; *</td>
<td>43&quot;</td>
<td>0</td>
</tr>
<tr>
<td>Diameter (d)</td>
<td>1.0&quot;</td>
<td>1.055&quot;</td>
<td>5</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.016&quot;</td>
<td>0.080&quot;</td>
<td>400</td>
</tr>
<tr>
<td>Modulus of elasticity (E)</td>
<td>$1 \times 10^6$ lbf/in$^2$</td>
<td>$0.5 \times 10^6$ lbf/in$^2$</td>
<td>100</td>
</tr>
<tr>
<td>Cantilever stiffness</td>
<td>2.66 lbf/ft</td>
<td>6.43 lbf/ft</td>
<td>141</td>
</tr>
<tr>
<td>Effective structural mass/unit length ($m_e$)</td>
<td>0.172 lbf/ft</td>
<td>0.410 lbf/ft</td>
<td>140</td>
</tr>
<tr>
<td>Natural frequency in water (n)</td>
<td>9.8 Hz</td>
<td>10.4 Hz</td>
<td>6</td>
</tr>
<tr>
<td>Log decrements in water (5)</td>
<td>0.087</td>
<td>0.090</td>
<td>3</td>
</tr>
<tr>
<td>Stability parameter ($k_s$)</td>
<td>0.34</td>
<td>0.43</td>
<td>25</td>
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* This includes 5.4" for imperfect encastre conditions on the full scale rig - a false floor was fitted round the base of the model pile to simulate the effects.
<table>
<thead>
<tr>
<th>Property to be scaled</th>
<th>Correct value for 1:27 model</th>
<th>Actual value obtained in model</th>
<th>% Departure from true modelling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L)</td>
<td>48&quot; *</td>
<td>48&quot;</td>
<td>0</td>
</tr>
<tr>
<td>Diameter (d)</td>
<td>0.667&quot;</td>
<td>0.675&quot;</td>
<td>1.2</td>
</tr>
<tr>
<td>Wall thickness</td>
<td>0.014&quot;</td>
<td>0.138&quot;</td>
<td>892</td>
</tr>
<tr>
<td>Modulus of elasticity (E)</td>
<td>1.11 x 10^6 lbf/in^2</td>
<td>0.25 x 10^6 lbf/in^2</td>
<td>77</td>
</tr>
<tr>
<td>Cantilever stiffness</td>
<td>0.55 lbf/ft</td>
<td>0.75 lbf/ft</td>
<td>36</td>
</tr>
<tr>
<td>Effective structural mass/unit length (m_e)</td>
<td>0.0766 lbf/ft</td>
<td>0.104 lbf/ft</td>
<td>36</td>
</tr>
<tr>
<td>Natural frequency in water (n)</td>
<td>6.08</td>
<td>6.75</td>
<td>11</td>
</tr>
<tr>
<td>Log decrements in water (S)</td>
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<td>0.067</td>
<td>8</td>
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<tr>
<td>Stability parameter (k_s)</td>
<td>0.305</td>
<td>0.316</td>
<td>3.6</td>
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</table>

* This includes 5.4" for imperfect encastre conditions on the full scale rig - a false floor was fitted round the base of the model pile to simulate the effects.
Fig. 10.1 The hydroelastic model of the 18" diameter pile and capping beam and experimental arrangement.
Fig. 10.2  Comparison of model and full-scale results. Cross beam motion. 30" diameter pile.
Fig. 10.3  Comparison of model and full-scale results.  In-line motion.  30" diameter pile.
Fig. 10.4 Comparison of model and full-scale results. In-line motion. 18'' diameter pile.
11. Analysis of the fluctuating force coefficients $C_d'$ and $C_L'$.

This Chapter compares the experimental results of Chapter 9 with those obtained from a linear mathematical representation of the cylinder/water system, using the extended transfer matrix technique described in Appendix 1.

The limiting amplitude of oscillatory motion in the in-line direction, found from the computer analysis, was in agreement with the maximum amplitude recorded in the experimental section. The limiting value of $C_d'$, to which this limiting amplitude was appropriate, was deduced from an analysis of the vortex shedding photographs from the photographic study (9.4). Similarly, the computed cross-flow force coefficients $C_L'$, when compared with the collected results of Vickery & Watkins (41) demonstrated very close agreement between cylinders of comparable materials, although there was a suggestion of an additional dependence upon structural damping.

The mathematical representation was a development of the linear theory described in Chapter 3; the cylinder material properties (mass, geometry, elasticity and hysteretic damping), the added mass of water and the hydrodynamic damping (for oscillations in the in-line direction) were converted into matrix form for each element of the cylinder length. The vortex excitation was represented by a periodic forcing function dominated by either $C_d'$ or $C_L'$ and assumed uniformly distributed over the cylinder immersed length. Appendix 1 describes the detailed matrix manipulation and methods of solution; the forces on a representative element of a cylinder are illustrated in figure 11.1.

Initially in the analysis, the eigen frequency of the cylinder was calculated for the water velocity at which the values of $C_d'$ and $C_L'$ were required; at this eigen frequency, calculations were made of the amplitude response corresponding to a range of values of $C_d'$ and $C_L'$. By an iterative process, the $C_d'$ and $C_L'$ values were continuously revised until the calculated and experimental amplitudes of oscillation were in exact agreement. From this the correct values of $C_d'$ and $C_L'$ were deduced, and the following sections describe and
discuss the implications of their magnitudes.


The hydrodynamic forces of excitation and damping exerted on a cylinder oscillating in-line are shown in fig. 3.5. An analysis of the free shear layers in the vortex shedding process illustrated in the photographic study of section 9.4 showed that during the first instability region, identified by symmetric shedding, the fluctuating force coefficient $C_d'$ should not exceed 0.86. The physical reasoning for this was based on the variation in the location of the separation points on the cylinder oscillating in-line as shown in fig. 11.2(a), (b). In fig. 11.2(b), the cylinder was oscillating in the upstream direction and the flow separation resembled that associated with Critical or Supercritical Reynolds numbers for which the steady drag coefficient $C_d \approx 0.4$. Fig. 11.2(a) shows the cylinder oscillating in the downstream direction; the free shear layers were similar to those separating from a very bluff body such as a flat plate, for which $C_d \approx 2.1$. When the cylinder was stationary with respect to the observer, at the ends of each half cycle of motion, the flow patterns were similar to those characterising the stationary cylinder in flowing water situation, where $C_d$ was measured as 1.26 (see Section 9.1). Thus the instantaneous steady drag coefficient was $C_d = 1.26 \pm 0.86$ and the fluctuating force coefficient was obviously $C_d' = 0.86$. Fig. 11.3 shows the results of the analysis of the experimental data for the fundamental and second normal modes in-line. For the analysis of the second normal mode, the hysteretic damping was increased by an amount determined from the logarithmic decrement of that mode. In general, the hysteretic damping constant ($G$) for the first two normal modes were approximately $G$ and $1.2G$ respectively. It is seen that the maximum amplitudes of oscillations and the fluctuating force coefficients $C_d'$ were related linearly, although there was some scatter in the computed values of $C_d'$ for
amplitudes less than the maxima. The greatest departures from the straight line through the maxima were recorded at comparatively low amplitudes where the results were subject to the largest uncertainty. The calculated $C'_d$ values were deduced from the assumption that the excitation forces were distributed uniformly over the cylinder length; this was an approximation as the force coefficients were almost certainly amplitude dependent and this was demonstrated in a supplementary series of flow visualisation studies. The vorticity associated with the larger amplitudes was considerably greater than that of the smaller amplitudes at the same $V_r$ and at different locations on the same oscillating cylinder. Furthermore, the point in the water at which the alternate street was formed, appeared to be a function of amplitude and $V_r$, and it was observed that this was consistent with amplitude-dependent variations of exciting forces at various distances along the cylinder length. Thus, the calculated value of $C'_d = 0.69$ for $y/d = 0.15$ (the maximum amplitude recorded) represented the uniformly distributed equivalent of $C'_d$ values ranging from 0.86 at the maximum amplitudes, to the pseudo-stationary $C'_d$ near the base. Extrapolation of the $y/d$ versus $C'_d$ line towards the origin showed that for the stationary cylinder case of $y/d = 0$, $C'_d = 0.12$. This extrapolation assumed the vortex shedding was perfectly correlated along the cylinder length, although previous research showed this was rarely achieved with stationary cylinders, and values of $0.04 < C'_d < 0.08$ were more usual (in the second instability region, a similar extrapolation yielded $C'_d \approx 0.08$ for $y/d = 0$). However, it must be remembered that the stationary cylinders were subjected to alternate vortex shedding whereas the first instability region, considered here, was characterised by symmetric vortex shedding. The extrapolation therefore was only a qualitative representation of the stationary cylinder conditions although the form of the graph indicated that the extrapolation was justified.

Hardwick & Wootton (126), in their analysis of the vortex excitation process applied to the fundamental mode of in-line oscillations, predicted a limiting amplitude of $y/d \approx 0.11$ corresponding to a maxi-
mum $C'_d = 0.8$. The results of their laboratory tests with an elastically supported rigid cylinder, of $L/d = 5$, agreed with their theoretical reasoning ($y/d \approx 0.1$, c.f. $y/d = 0.11$ by theory) although Wootton (112) previously recorded maximum amplitudes of $y/d = 0.15$ in the Immingham full scale tests and in the present tests, amplitudes of up to $y/d = 0.15$ were noted. The reasons for the low $y/d$ predicted by the theory of (126) were not confirmed, as the $V_{rm}$ recorded by those authors and at which their theory was evaluated, coincided with the present test results of $V_{rm} = 2.1$. Also the basic forms of the forces defined in the rigid body analysis (126) were similar to those employed herein. However, the rigid cylinder used in their laboratory tests possessed equal amplitudes at all sections and it was considered that in the present tests with elastic cantilevers, local departures from the limiting $y/d = 0.1$ (as defined by (126)) could be generated although these would be restricted to relatively short lengths of the cantilevers.

In the present analysis of both the fundamental and second normal mode tests, the hydrodynamic damping was a function of flow velocity, oscillatory amplitude and frequency, and the mathematical representation of exciting forces should be refined by the inclusion of an amplitude-dependent effect. When the exciting forces were represented by a quantity varying linearly with local amplitude, the results did not correlate well with the recorded data. It was concluded that more comprehensive expressions for the fluid and cylinder interactions must be developed. Many of the mathematical or heuristic models of flow-induced oscillations incorporate modified forms of the Van der Pol equation (111) and recent publications by Griffin et al (131), and Skop & Griffin (132) have demonstrated further modifications and their specific application to cylinder oscillations.

11.2 Second instability region in-line. Evaluation of $C'_d$.

The results from tests in the second instability region were analysed by a method similar to that employed in section 11.1. The relative magnitudes of the exciting and damping forces of this insta-
bility region were assumed to be of the same order of magnitude as those for the symmetric shedding region. However, the limitation of \( C'_d \) defined from visual observation of the free shear layers obviously was of no relevance in this alternate shedding region.

The results of the analysis, shown in fig. 11.4 revealed that the maximum amplitudes of the fundamental and second normal modes were linearly related to the fluctuating force coefficient \( C'_d \). By extrapolating the \( y/d \) versus \( C'_d \) line towards the origin, a stationary cylinder value of \( C'_d = 0.08 \) was obtained. This was in agreement with the values recorded during wind tunnel tests with stationary cylinders (28), (29), (30). In the present tests, the second instability region and the stationary cylinder (null) range immediately preceding instability were both identified by alternate vortex shedding and the extrapolation was considered rather more justifiable in these tests than in the previous tests of 11.1. At the maximum recorded amplitude of \( y/d = 0.14 \), \( C'_d = 0.45 \), compared with \( C'_d = 0.65 \) at a similar amplitude in the first instability region. In the first instability region, the oscillatory velocity included in the hydrodynamic damping term, was comparable with the flow velocity, the damping was relatively large, and the fluctuating force coefficient \( C'_d \) correspondingly high. In the second instability region, the flow velocity was higher, although the oscillatory velocity remained approximately constant. The damping term \( (\alpha P V) \) thus increased linearly with flow velocity whilst the excitation force increased as the square of the flow velocity. Hence, for equal amplitudes in the two instability regions, lower force coefficients were necessary in the second of these. Photographs of the vortex shedding in the second instability region showed that the vortices separated from the cylinder at angles of approximately 130° to 160° from the forward stagnation point. The larger angle was observed at the maximum amplitude. An analysis of Sury’s (31) graphs of rms pressure distribution around a stationary circular cylinder, gave \( C'_d = 0.08 \), at a separation of approximately 100°. By comparing the in-line components of the two separation angles recorded
in the photographs, and making the elaborate assumption that in the cases considered, the vortex strengths were uniform, the equivalent values of $C_d'$ were $0.22 \leq C_d' \leq 0.43$. It will be noted that these two values of $C_d'$ were close to those deduced from the matrix analysis of the oscillating cylinder. It was appreciated that these analogies with the stationary cylinder $C_d'$ were approximate and that the results obtained thus were of a qualitative nature only. However, the $C_d'$ recorded in tests with stationary cylinders (31) usually represented the average experimental reading at the instrumented section of the cylinders length; in view of the generally low lengthwise correlation of such cylinders (35), the average $C_d'$ for the whole cylinder would be considerably less than the $C_d'$ from the instrumented section. Thus, in the present tests, a calculated uniformly distributed $C_d'$ equal to the stationary cylinder $C_d'$ from (31) would denote well-correlated vortex shedding without necessarily implying a variation in vortex strength. Furthermore, although the vortex strengths appeared to be proportional to the oscillatory amplitudes of motion, increases in in-line force coefficients could also be caused by increases in separation angle, which in turn probably were functions of amplitude, $V_r$ and oscillatory velocity.

The ratio of oscillatory velocity ($v$) to the flow velocity ($V$) is:

$$\frac{v}{V} = \frac{2\pi}{V_r} \cdot \frac{y}{d} \quad \ldots \quad (11.1)$$

and by substituting $y/d = 0.14$ in (11.1), the following were obtained:

$$\frac{v}{V} = 0.419 \text{ when } V_r = 2.1 \text{ (first instability region } V_{r_{m}} \text{)}$$

$$\frac{v}{V} = 0.274 \text{ when } V_r = 3.2 \text{ (second instability region } V_{r_{m}} \text{)}$$
It was noted that the ratio of force coefficients in the two regions were related approximately as the ratio of $v/V$:

\[
\begin{align*}
C_d' \text{ (first)} &= 0.65 \\
C_d' \text{ (second)} &= 0.45 \\
C_d' \text{ ratio} &= 1.44 \\
v/V \text{ (first)} &= 0.419 \\
v/V \text{ (second)} &= 0.45 \\
v/V \text{ ratio} &= 1.53
\end{align*}
\]

This was considered logical from the discussion of excitation and damping above, and the inclusion of the hysteretic damping term in this simple comparison probably would reduce the disparity (6%) between the quoted ratios of $C_d'$ and those of $v/V$. It was further noted that the $C_d'$ ratio remained sensibly constant throughout the experimental range.

11.3 Cross-flow oscillations. Evaluation of $C_L'$

The analysis developed for the preceding sections was modified for application to the results from tests in the cross-flow direction; in the absence of a reasonable description of fluid damping in that direction, only the hysteretic damping was considered in the dissipative terms of the governing matrix. However, this analysis enabled the results to be compared directly with those of Vickery & Watkins (41) who deduced values of $C_L'$ from the simplified theory of Chapter 3.7, for situations in which $h = L$; they assumed that the fluid damping was negligible compared with the hysteretic damping.

The results of the present analysis, and those of (41) are shown in fig. 11.5. It is seen that for amplitudes of $y/d \leq 0.4$, the figure resembles the graphs of figs. 11.3, 11.4, and the calculated
force coefficients $C_L'$ increased with increasing $y/d$. For $y/d \geq 0.6$ the results demonstrated an inverse linear relationship between $C_L'$ and $y/d$.

The one result from tests with the aluminium cylinder II.1 coincided with, and the results from tests with the stainless steel cylinder V.2 were slightly below, the calculated results of (41). The P.V.C. cylinder results were consistently higher than the results of Vickery \& Watkins, recorded in tests with brass cylinders. The sets of results showed an apparent dependence upon the type of materials and the magnitude of its hysteretic damping although there were no obvious mass ratio effects. Rather surprisingly, the most highly damped cylinder oscillated with the largest amplitudes; however, the aluminium cylinder $V_{rm} \approx 5.0$, for the stainless steel cylinder $V_{rm} \approx 7.5$, and for (41) $V_{rm} \approx 5.5$. Thus, although the P.V.C. cylinder contained the greatest hysteretic damping, the flow velocities and hence the excitation forces were considerably larger than for the other cylinders considered. This observation emphasised the importance of considering both $k_s$ and $V_r$ when comparing data from a variety of sources.

Vickery \& Watkins' (41) explanation of the increase, followed by the decrease, of calculated $C_L'$ with $y/d$ was that the total energy of the vortices in the wake was invariant with $y/d$ and that at the smaller amplitudes, the graph demonstrated a lack of vortex shedding correlation in the presence of low damping. As $y/d$ increased, the lengthwise correlation improved, the hydrodynamic damping increased, and the steady state limiting conditions, it was argued, were defined by $C_L' = 0$ and the (constant) total energy of the wake was dissipated by the hysteretic and hydrodynamic damping. This explanation should be qualified by making the total wake energy velocity dependent, otherwise their analysis implied that complete correlation was achieved only at the maximum amplitude, or that the amplitudes of oscillations were restricted through the agency of another undefined medium. With this velocity dependence included, the explanation then became
quite plausible and the $y/d$ versus $C_L'$ graph (fig. 11.5) was divided into two parts. The lower part, in which the amplitudes were restricted by the poor lengthwise correlation (and the low damping) was followed by the upper part in which the hydrodynamic damping energy increased more rapidly than the energy input from the exciting forces, until the limiting amplitude was established at $C_L'=0$. Fig. 11.5 indicated that complete lengthwise correlation was obtained when $y/d \geq 0.5$ and reference to the graphs of figs. 11.3, 11.4 showed that the maximum recorded amplitudes were $y/d \leq 0.15$, thus implying that for in-line motion, the vortex shedding was never completely correlated. The first instability region of in-line motion shown in fig. 11.3 and examined in section 11.1, necessarily could not be discussed in the same context as the alternate vortex shedding regions of figures 11.4, 11.5 as the mechanisms of excitation were basically dissimilar. However, consideration of those two figures showed that over a corresponding range of amplitudes, the amplitudes and fluctuating force coefficients exhibited similar trends. Possibly, the vortex shedding was not completely correlated in the cross flow direction (and extrapolation of the lower part of the graph indicated that this was so) but the results from the in-line direction (fig. 11.4), where considerably higher fluid damping was experienced, confirmed well correlated vortex shedding as the maximum fluctuating force coefficient in that mode exceeded by an order of magnitude the stationary cylinder $C_d'$. Furthermore, extrapolation of the $C_d'$ versus $y/d$ relationship yielded $C_d'=0.08$ where $y/d = 0$, and this would represent the perfectly correlated stationary cylinder value.

An additional examination of the results from the cross-flow direction showed that the limiting amplitude could be linked to the ratio of oscillatory velocity to mean flow velocity. By applying equation (11.1) to this mode, the following were obtained for the three sets of results (fig. 11.5) at their maximum amplitudes and $V_{rim}$:

$$\frac{v}{V} = \frac{2\pi}{V} \cdot \frac{y}{d}$$

(11.1)
(a) P.V.C. cylinder II.3, III.1.

\[
\left( \frac{v}{V} \right)_{\text{max}} = 1.66 \quad \ldots (11.2)
\]

(b) Stainless steel cylinder V.2

\[
\left( \frac{v}{V} \right)_{\text{max}} = 1.76 \quad \ldots (11.3)
\]

(c) Vickery & Watkins results.

\[
\left( \frac{v}{V} \right)_{\text{max}} = 1.72 \quad \ldots (11.4)
\]

The two velocities \( v \) and \( V \) were orthogonal and their vector resultants \( V_R \), therefore, were greater than the mean flow velocity \( V \). It would appear that the limiting amplitudes of cylinders with very low \( k_s \) were defined by the condition \( v/V \lesssim 1.7 \), or alternatively where the resultant maximum resultant velocity vector \( V_R \) was \( V_R \approx 2V \).

By extrapolating the results from the 'well correlated' part of the \( y/d \) versus \( C_L' \) graph in fig. 11.5, the stationary cylinder \( C_L' \) for the three sets of results was determined as \( C_L' = 0.78 \); this was in agreement with recordings of \( C_L' \) made during wind tunnel tests with stationary cylinders (28), (29), (62).

It should be emphasised that the limiting values of \( y/d \) during the present analysis applied to cylinders having very low Stability Parameters.

In summary, this Chapter illustrated the value of analysing experimental results with the assistance of a linear, mathematical analogue; it was shown that the limiting amplitude for cross-flow oscillations (with \( k_s \rightarrow 0 \)) was approximately 2 diameters, representing the amplitude for which the oscillatory velocity \( v \) was twice the mean flow velocity \( V \). The absolute values of limiting cross flow amplitudes.
were determined by $V_{rm}^*$ which in turn were complex functions of mass ratio $m_r$ and the ratio water depth/cylinder length $h/d$.

In the in-line direction, the limiting amplitude recorded was $y/d \simeq 0.15$ and this was equivalent to $C_d' = 0.69$ in the first instability region and $C_d' = 0.44$ in the second instability region. These represented significant deviations from the stationary cylinder values of $0.04 < C_d' < 0.08$. It was also seen that the values of $C_d'$ recorded in the two instability regions were related inversely with the ratio $v/V$.

The following Chapter details the testing of two cylinders, coupled and uncoupled at various spacings in-line. The tests were arranged as a preliminary examination of the problems involved, and the results from oscillations excited in the cross-flow direction correlated well with wind tunnel test results.
\[ \text{Fluid force function} = \frac{1}{2} \rho V^2 C_d' \frac{d^2}{dz} \]

Structural damping
- \(vGK\)

Fluid damping
- \(fVvC_d \frac{d}{dz}\)

Fig. 11.1  The forces on a cylinder element.

**Fig. 11.2**  Derivation of limiting values of \(C_d'\) from a consideration of the types of vortex shedding in-line.

<p>| | |</p>
<table>
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<tr>
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<tr>
<td>(a) (C_d = 2.1)</td>
<td>Cylinder moving downstream</td>
</tr>
<tr>
<td>(b) (C_d = 0.4)</td>
<td>Cylinder moving upstream</td>
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Fig. 11.3  Calculated values of the fluctuating force coefficient $C_d$ versus non-dimensional amplitude. First instability region in-line.
Fig. 11.4(a) Generation of unsteady forces during alternate vortex shedding. Second instability region in-line.

Fig. 11.4(b) Calculated values of the fluctuating force coefficient $C'_d$ versus non-dimensional amplitude. Second instability region in-line.
Fig. 11.5(a) Generation of unsteady forces during alternate vortex shedding. Cross-flow motion.

Fig. 11.5(b) Calculated values of the fluctuating force coefficient $C'_f$ versus non-dimensional amplitude $L/a_h$. Cross-flow motion.
PILE FILLED

PILE LENGTH = 42.00 INCHES
PILE MASS/UNIT LENGTH = 0.00391 SLUGS/FT
STRUCTURAL LOGARITHMIC DECREMENT = 0.02600
NUMBER OF STRUCTURAL ELEMENTS = 8
DAMPED RESONANT FREQUENCY = 5.775761 HZ.

TIP LOAD = 1.060 LBS
STOKES FREQUENCY PARAMETER = 5527
REYNOLDS NUMBER = 8285
WATER VELOCITY = 13.60 INCHES/SECOND
WATER DEPTH = 30.000 INCHES

SECTION STIFFNESS = 845.000 LBF. FT²
ELEMENT LENGTH = 6.000 INCHES
G FACTOR = 0.00826
FREQUENCY BY FORMULA = 5.737106 HZ.

MASS RATIO = 2.404
SIZE NUMBER = 3518
PRESSURE COEFFICIENT = 0.48 BASED ON DIAMETER
V/ND = 2.355
DEPTH RATIO = 0.714

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<th>BENDING MOMENTS</th>
<th>CYLINDER DEFLECTION</th>
<th>PHASE ANGLE OF DISPLACEMENTS</th>
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STATIC DEFLECTION = 0.0040 INCHES
MAGNIFICATION FACTOR = 39.9156

IN-LINE OSCILLATIONS
## Cross-Flow Oscillations

**Cubic Solution**

**Pile Filled**

- **Pile Length**: 42.00 inches
- **Pile Mass/Unit Length**: 0.00391 slugs/ft
- **Structural Logarithmic decrement**: 0.02600
- **Number of Structural Elements**: 8
- **Damped Resonant Frequency**: 11.018065 Hz.
- **Tip Load**: 0.000 lbs
- **Stokes Frequency Parameter**: 10543
- **Reynolds Number**: 9137
- **Water Velocity**: 15.00 inches/second
- **Water Depth**: 30.000 inches

**Section Stiffness**: 855,000 lbf-ft²

**Element Length**: 6,000 inches

**G Factor**: 0.00828

**Frequency by Formula**: 10.910029 Hz

**Determinate of Governing Matrix**: 0.00006349

**Mass Ratio**: 0.000

**Size Number**: 6712

**Pressure Coefficient**: 1.00 based on diameter

**V/νν**: 1.561

**Depth Ratio**: 0.714

### Cross-Flow Response to Strouhal Forcing

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<th>Bending Moments (lbf ft)</th>
<th>Cylinder Deflection (inches)</th>
<th>Phase Angle of Displacements (degrees)</th>
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### Frequency Ratio and Amplitude

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<th>Determinant</th>
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12.1 Introduction

The stability criteria developed in the tests so far were derived for an isolated cylinder, and therefore not immediately applicable to a complex structure, although it may be argued that at certain spacings, individual cylindrical components may be treated as isolated. One of the prime objectives of the tests described in this section was to determine the interactions which arose when two cylinders, located at various distances from each other in-line were free to oscillate independently (coupled by the fluid) and when they were structurally coupled at the free ends by elastic or rigid members.

12.2 Background and previous work.

It is an acknowledged fact that cylindrical bodies in the wakes of other cylinders do not conform to the pattern of behaviour recorded when the cylinders are isolated. Ad hoc tests on proposed arrangements, using aeroelastic models in wind tunnels have resulted in few generally applicable criteria. That this is an extremely complex problem is borne out by considering the vast numbers of variables involved. Heat exchanger tube banks, groups of tower block buildings, chimney stacks closely spaced, cooling tower arrays, support piles and braced members of offshore structures and members of light space frames are included in the list of situations that are prone to the effects of flow interactions. The interactions referred to take two forms, viz:

(i) vortex shedding: the bodies are excited within a limited resonant frequency band; oscillatory amplitudes increase fairly sharply and reach peak values before declining to zero at the end of the band.
(ii) buffetting: initially, small amplitude forced random motion; oscillatory amplitudes increase linearly with velocity.

Whitbread & Wootton (133) encountered both types of interaction during wind tunnel tests on two octagonal tower block buildings. In those tests, the upstream tower (A) was rigid, and the elastic downstream tower (B) was tested at various distances from A. For a gap to diameter (G/d) ratio of less than 1.25, the response of B was apparently forced oscillations caused by the vortex street of the combined wakes of the two towers and not by the vortex street of A alone. Peak amplitudes were recorded in the range $7 < V_r < 9$. Between $1.25 < G/d < 3$, buffetting oscillations were evident and the amplitudes increased with velocity without reaching a peak. For $G/d > 3$, the rear tower was in that part of the wake from A where definite peaks in the velocity spectrum were recorded; B oscillated with large amplitudes, reaching a peak at uniform values of $V_r = 7.2$. In tests with two elastic towers at a spacing of $G/d = 0.78$, both A and B exhibited enormous increases of peak amplitudes compared with the solitary tower tests (for example, the amplitude of A increased by a factor of 12 and that of B by a factor of 20 for uniform damping parameters). It was noted that the damping necessary to suppress excitation of an isolated tower was not sufficient to prevent oscillations during tests on the two towers. From a comparison of tests in smooth and turbulent flow, the authors of (133) concluded that turbulence had little influence on vortex excitation of the towers - mutual interference was by far the greatest contributor to instability.

During tests on an instrumented horizontal rigid cylinder (4.33" diameter) in the wake of a parallel cylinder in water, Laird et al (134) showed that depending on the spacing, the mean drag force could be reduced to between one half and one fifth of the isolated cylinder value. Unmeasured lift forces were evidently large and in one case the 450 lbs. towing carriage was lifted off the rails. In a later paper, Laird et al (135) investigated the sheltering effects of
groups of rigid vertical cylinders attached to a pendulum frame performing simple harmonic motion in still water. They concluded that the presence of an upstream cylinder and the separation G/d influenced the steady and fluctuating drag forces. For $0 < G/d < 1$ negative steady drag was developed in the absence of fluctuations; when $1 < G/d < 3$, fluctuating lift and drag forces of the order of the steady drag forces were developed. For larger spacings, sheltering effects decreased and the cylinder approached the isolated cylinder situation for $G/d = 10$. In another paper, Laird & Warren (136) experimentally determined the sheltering effects of a symmetrical group of 24 vertical cylinders towed through still water. They showed the isolated cylinder drag coefficient was approached for $G/d \geq 7$.

Interference effects on steady drag forces on two rigid cylinders in-line were experimentally investigated by Nagai & Kurata (137) in tests with an open channel (flume). For very small spacings ($G/d \approx 0.1$) they photographically showed that the flow enclosed both cylinders, causing a 10% reduction in steady drag force on the upstream cylinder and negative drag forces on the downstream cylinder. At larger spacings ($1.2 < G/d < 5$) there was still a slight reduction of drag force on the upstream cylinder and a recovery of positive drag on the downstream cylinder. Sheltering and vortex shedding effects from the upstream cylinder were evident for at least $G/d = 15$. In general, these observations were in agreement with those of Laird & Warren (136) although there is some disparity between their quoted values of the separation ($G/d$) at which the isolated cylinder situation was approximated.

Zdravkovich (100) tested two cylinders in a wind tunnel; various combinations of cylinders were used (rigid, elastic, coupled) for a range of longitudinal spacings $G/d$ and transverse spacings $T/d$. (In a preliminary exercise he measured the static pressure distributions around two rigid stationary cylinders for a range of $G/d$, confirming the findings of Laird et al (135) and Nagai & Kurata (137) in that negative steady drag forces were recorded on the downstream cylinder when $G/d \approx 0.5$). With the upstream cylinder fixed and the downstream cylinder
attached to a pivoted arm, large amplitude, low frequency oscillations were recorded for all \( G/d \leq 2.5 \). The corresponding Strouhal number was 0.01 and both amplitude and frequency were strongly dependent upon velocity. No instability could be excited for \( G/d \geq 2.5 \). Springs were then attached to the pivoted arm thus defining a natural frequency for the arm and cylinder system. Zdravkovich (100) showed that provided \( G/d \leq 2.5 \), sustained crossflow oscillations could be induced when the downstream cylinder was given sufficiently large initial amplitudes to capture the free shear layers from the upstream cylinder. For \( G/d \geq 2.5 \) excitation could not be induced. However, by laboratory standards, the quoted values of structural logarithmic decrement for the springs and pivoted arm were rather high (\( \delta_s \approx 0.24 \)) and as discussed later in this chapter, that effect may have been responsible for the observed behaviour. When both rigid cylinders were attached in-line to the pivoted arm (\( G/d \leq 6 \)) no oscillations were recorded and Zdravkovich concluded that excitation existed only when there was relative motion between the cylinders. This conclusion was at variance with the results of the tests of Laird (128) and the difference in densities of the two fluid media was tentatively proposed as the explanation for the recorded differences, although the tests of this Chapter suggested the results of (100) were in error. Mair & Maull (138) considered the flow about a circular cylinder and demonstrated that if another cylinder was placed downstream, the potential flow theory dictated a resultant force directed away from the centre line of the wake. Experiments showed the converse was true and an explanation of this was based on the entrainment of fluid into the wake, producing the restoring force necessary for sustained instability.

12.3 The test arrangement.

The two P. V. C. cylinders VII. 1, VII. 2 were each mounted in identical base units as shown in figure 12.1. Each base unit was fabricated from two massive pieces of mild steel with a one inch square
section brass bar recessed into the main vertical block from which it projected for ten inches. Each cylinder was welded into a composite P. V. C. ("Darvic", I. C. I. trade name) mounting block, using a hot air welding technique perfected during the construction, testing and subsequent modification of hydroelastic river control gates, (99) (139). A one inch wide slot, one inch deep was cut into the mounting block which slid on the matching square brass bar of the base unit. Bolts passing through the mounting block and into a Darvic plate were used to clamp the cylinder in the required position on the bar. The two base units were bolted to the floor of the flume as shown in figure 12.2; the relative separation between the cylinders was achieved by sliding and clamping one or both of the Darvic mounting blocks. A similar arrangement was incorporated at the cylinders' free ends; a slotted Darvic cap was welded to each cylinder and a bar passed through the slotted caps to link the two cylinders and maintain the correct spacing. The clamping arrangement was a device similar to that at the base end. A false floor was fitted around the two base units to simulate the smooth flume floor to which the bases of the single cylinders were fixed (Chapter 9), thus ensuring that the cylinders were tested in similar boundary layers.

Four strain gauges were cemented to each cylinder, two in-line and two crossflow; when mounted in the flume, these gauges were calibrated by static loading as described in Chapter 7.

The initial tests were concerned with an investigation of the fundamental mode in-line. When tested, the cylinders were hollow and (a) uncoupled, (b) coupled with a slender flexible Darvic member 1/32" thick x 1/2" wide. Spacing ratios in the range 1.25 ≤ G/d ≤ 6 were covered in these tests. It was appreciated that in practical situations, foundation consolidation and other physical reasons restrict the minimum spacing between marine piles to G/d ≈ 5. The tests at the smaller G/d were designed to yield data comparable with the results from the experimental investigations outlined in the brief review of previous work. Three different water levels
were selected for the tests, in order to give three significantly different values of $k_s$.

In the second part of the tests, the cylinders were filled with lead shot to reduce their natural frequencies and permit an examination of fundamental cross-flow and second normal mode in-line oscillations when the cylinders were (a) uncoupled, (b) coupled. Three different members were used to couple the cylinders for the second normal mode tests, viz.: the slender Darvic member from the initial tests, a stiffer Darvic member $\frac{1}{4}''$ thick x $\frac{3}{2}''$ wide and a rigid brass member $\frac{1}{4}''$ thick x $\frac{1}{2}''$ wide. In the crossflow tests only the $\frac{1}{4}''$ thick Darvic member was used. Coupling the cylinders altered the mode shape from that of an isolated cylinder. The single cylinder tests demonstrated the importance of mode shapes in determining the stability of a cylinder and for this reason the computer program detailed in Appendix 1 was applied to the overall coupled 'bent'. In common with all computer programs, a period of time was occupied by proving the analogue by comparing computed and experimental results. The work involved in this proving period is described in Section 12.4 followed by a summary of the experimental programme in Section 12.5 and the presentation and discussion of experimental results from the flowing water tests, in Section 12.6.

12.4 Analysis of computed and experimental mode shapes and frequencies in air and still water.

Before progressing with the analysis of the coupled cylinders (bent) a brief check was made to confirm the absence of interactions between the bases of the two uncoupled cylinders by deflecting and releasing one cylinder and observing the response of the other. No measurable interactions were recorded and thereafter the cylinders were confidently assumed to be perfectly isolated.

The cross-flow and in-line frequencies of each cylinder were obtained in the standard manner and from these the effective mass of each Darvic cap was evaluated. The mass densities of the
three coupling members were determined by weighing and accurate dimensional measurement. These results were used as input data for the bent program which generated the mode shapes and frequencies for a range of $G/d$.

12.4.1 Fundamental mode. Cylinders VII.1

The computed and experimental results for the $\frac{1}{4}$" thick x $\frac{1}{2}$" wide Darvic member are shown in fig. 12.3, which also includes the results from the proving tests in still water. In all cases close agreement was observed, thus confirming the accuracy of the transfer matrix analysis. Initially a comparison of bending moments indicated that the mode shapes were in reasonable agreement but a visual check on the bent for unit base bending moment suggested the program was under-estimating the deflections of the coupling member. In formulating the program, the length of the caps had been omitted; the program was modified to include these effects and employed in calculating the results of fig. 12.3.

Obviously, the cross-flow frequency was a constant for all $G/d$. The very slender $1/32"$ thick x $\frac{1}{2}$" wide Darvic member did not alter the mode shapes or the frequencies in-line from the free-ended values and subsequent analysis of the bent with this member utilised the less complex single cylinder program.

It was realised that the rigidly coupled cylinders could be analysed as an equivalent single cylinder system in which the coupled end carried a mass of one half the rigid coupling and was constrained to move in a horizontal slider (hereafter termed the clamped-sliding mode). The single cylinder program was modified to incorporate this form of end fixity and all additional results were computed with this program.

Typical mode shapes in the fundamental mode are shown in fig. 12.4.

Damping, as evidenced by recordings of logarithmic decrements $\delta_s$ did not vary greatly from the free ended cylinder value
using the 1/32" Darvic and the ¼" brass members; a 20% increase in $\delta_s$ was recorded with the ¼" Darvic member.

12.4.2 Second normal mode. Cylinders VII.2.

Second normal mode natural frequencies appropriate to each coupling member were recorded in air and in still water. Rather surprisingly, and in contrast with the fundamental mode tests, the 1/32" Darvic member produced a considerable reduction from the corresponding free-ended frequencies. The modified bent program predicted these reductions with great accuracy and subsequently was relied upon to supply results for this member. The rigidly coupled cylinders (¼" brass member) were analysed by the single cylinder clamped-sliding program which yielded calculated frequencies in close agreement with experimentally measured values.

The bent program accurately calculated the frequencies and mode shapes produced by coupling the cylinders with the ¼" Darvic member and the results for the air and water proving tests are shown in fig. 12.5. Typical mode shapes produced by each of the three members are shown in fig. 12.6. Recordings of logarithmic decrement ($\delta_s$) for the bent coupled with the ¼" Darvic member were 20% higher than for the fundamental mode with this member.

In calculating the stability parameters for both the fundamental mode and second normal mode only one cylinder and one half the mass of the coupling member were included in the integrations defining $m_e$.

12.5 Experimental programme.

The tests were designed to investigate wake and structural coupling effects between two identical cylinders at various spacings in the in-line direction. The programme was divided into three parts, viz. fundamental mode in-line, fundamental mode cross-flow and second normal mode in-line; for all three parts of the programme, the range of spacings covered by the tests was $1.25 \leq C/d \leq 6$. In
the fundamental mode only the 1/32" and 1/4" thick Darvic members were employed and in the second normal mode those two and the rigid brass deck were tested.

For each of the three water levels each cylinder was tested individually throughout the velocity range and the results of these were used as the bases of reference for the wake interference tests.

The testing sequence for the wake interference tests consisted of setting the cylinders at the correct spacing in air, filling the flume to the required water level and, when conditions were settled, the stillwater transient was recorded and examined for amplitude dependent effects as these would indicate a departure from the initially perfect encastré conditions. The velocity range was traversed in a manner similar to that of the single cylinder tests described in Chapter 9. During these tests the amplitude and frequency response of both cylinders were carefully recorded. Each water level and G/d test was recorded a total of four times; once for no structural coupling and once for each of three coupling members. The water level was changed and the tests repeated for the one value of G/d; when results had been recorded in all three water levels the flume was drained and the spacing altered to another value of G/d.

12.6 Results.

Note: cylinder A is upstream, cylinder B is downstream.

In these tests, six spacings (G/d) were investigated in each of three water depths h/L = 0.603, 0.712, 0.822; results were recorded when the cylinders were uncoupled and when coupled with the 1/32" thick Darvic member. The water levels were selected to give three different values of Stability Parameter, viz. $k_s = 1.0, 0.5, 0.25$. Tests on the isolated cylinder A showed $k_{sc} \approx 1$ compared with $k_{sc} \approx 1.2$ for cylinders II.1 and II.2; there was no obvious reason
for this difference although it had been noted that tests in the neighbour- 
hood of $k_{sc}$ could produce unrepeatable results and the actual 
value of $k_{sc}$ was rather difficult to define.

Figs. 12.7, 12.8, 12.9, summarise the overall results and figs. 12.10, 12.11, 12.12 show typical "as recorded" results for 
the three different water levels. In all cases there was either 
no excitation or oscillations exhibiting characteristic well defined 
peaks usually associated with vortex shedding excitation.

(i) Water level $h/L = 0.603$; $k_s = 1.0$

No response was recorded at either of the uncoupled 
cylinders in the first instability region $1.25 \leq V/nd \leq 2.5$
although both cylinders oscillated in the second instability region 
$2.6 \leq V/nd \leq 4$. For all $G/d$ the amplitudes of $B$ were con- 
siderably larger than those of $A$; the peak amplitude for $A$
($\pm 0.08d$, compared with $\pm 0.11d$ for the isolated cylinder)
coincided with $G/d = 3.75$ and at higher and lower values of $G/d$,
$A$ became virtually stationary. $B$ oscillated with an amplitude of $\pm 0.08d$ at $G/d = 1.25$, rose to a plateau of $\pm 0.14d$ for $1.75 \leq G/d \leq 3.75$ and declined abruptly to zero at $G/d = 5$.

Coupling the cylinders together resulted in minor excitation of 
the first instability region ($\pm 0.03d$) at $G/d = 1.75$. In the 
second instability region the bent oscillated with amplitudes in 
the range $\pm 0.08d$ to $\pm 0.12d$ for $1.25 \leq G/d \leq 3.75$ after which, 
excitation ceased. It is seen that the amplitudes of the coupled 
cylinders were almost exactly equal to the arithmetic mean of 
the amplitudes recorded when the two cylinders were uncoupled 
(the exception was for $G/d = 1.25$ where maximum amplitudes 
were recorded).

(ii) Water level $h/L = 0.712, 0.82$; $k_s = 0.5, 0.25$.

The results from these two water levels were similar in 
many aspects and they will be described under the same heading.

In tests on the uncoupled cylinders, no response was
recorded on B in the first instability region $1.25 \leq V/nd \leq 2.5$ for spacings greater than $G/d \leq 1.75$. The maximum amplitude for B was approximately equal to the isolated cylinder amplitudes and coincided with $G/d = 1.25$. Cylinder A oscillated with amplitudes of approximately $\pm 0.1d$ for $1.75 \leq G/d \leq 6$; minimum amplitudes were recorded at $G/d = 1.25$ (note this was the spacing corresponding to the maximum amplitude of B). In the second instability region, $2.6 \leq V/nd \leq 4$, cylinder A oscillated with amplitudes of approximately $\pm 0.1d$ for $2.75 \leq G/d \leq 6$ followed by a decrease in amplitude for $G/d < 2.75$. In all cases the amplitudes of B were consistently higher than the amplitudes of A, by a factor of two or three. The maximum amplitude for B occurred at $G/d = 1.75$ where amplitudes of $\pm 0.32d$ were recorded.

Coupling the cylinders produced substantial oscillations in the first instability region for $G/d \leq 2.75$ for both water levels; for $G/d > 3.75$, oscillations were recorded only when $h/L = 0.82$. In the second instability region, peak amplitudes were recorded for $G/d = 1.25$ and $3.75$ (this latter spacing corresponded to the $G/d$ for minimum excitation in the first instability region). Amplitudes ranged from the isolated cylinder amplitude to over twice this value. The amplitudes of the bent in the second instability region were equal to the arithmetic mean of the amplitudes recorded when the cylinders were uncoupled as mentioned in (i) above.

12.6.2 Discussion of results for 12.6.1.

It was not clear how some effects arose and the following discussion is intended as a tentative explanation of certain aspects of the overall and extremely complex mechanisms involved. Previous experimental work with single cylinders (Chapter 9) showed that the first instability region was characterised by symmetric vortex shedding which degenerated into an alternate street some distance downstream of the cylinder. It was reasoned that if a second elastic cylinder (I)
was placed in the wake of the oscillating cylinder (A) at a distance sufficiently far downstream for it to have no influence on (A), it would have imposed on it an alternate or mixed street, which could suppress the formation of symmetric shedding at (B). Thus, in the tests above, for \( G/d > 1.75 \) no excitation was recorded on the downstream cylinder B. To test this premise a little further, photographs were taken of the vortex formation between two cylinders separated by \( G/d = 1 \) and \( G/d = 3 \), the results are shown in figs. 12.13, 12.14. In the first case both cylinders were shedding symmetric vortices and both were oscillating in the first instability region. In the second case the upstream cylinder shed symmetric vortices which became staggered before reaching the downstream cylinder, and only the upstream cylinder oscillated. Hatfield & Morkovin (140) recently published their results from wind tunnel tests on a cylinder exposed to an oscillating free stream. Their results demonstrated that the sinusoidally oscillating component of the free stream velocity greatly influenced the formation of vortices from the cylinder, as evidenced by the measurements of r.m.s. radial pressure distribution. The oscillating flow was symmetric with respect to the cylinder whilst the Karman vortex field was alternate or asymmetric; thus, any transfer in one region of the flow tended to be counteracted by an opposite transfer in its mirror region. The analogy with the present tests is one of relative motions; the circulation contained within the alternate street from the upstream cylinder A would enhance the formation of one vortex of the symmetric pair forming at B, but the sense of this circulation would be counter to that of the other symmetric vortex and truly symmetric shedding would be inhibited. This argument obviously applied only when the vortices shed from A were at a frequency compatible with the symmetric shedding frequency from B. It is appreciated that this interpretation relied on rather extravagant assumptions and further research is needed to formulate more rigorous explanations.

The amplitude of A differed from the isolated cylinder
value only when \( G/d \) was less than approximately 2.5, suggesting that B ceased to influence A at spacings greater than \( G/d = 2.5 \). The amplitude of B was consistently larger than that of A indicating that the alternate vortex shedding from the downstream cylinder was reinforced by the alternate shedding from A. However, in the tests, the two cylinders possessed identical natural frequencies and later, when the upstream cylinder was held stationary, the amplitude of B was severely reduced. In that case, the stationary cylinder A shed vortices at a frequency which varied with the flow velocity, so that, in general, the shedding frequency and the frequency of the downstream cylinder B were not coincident. Previously, in Chapter 9, it was shown that in the second instability region in-line, lock-in occurred between the cylinder natural frequency and the frequency of vortex shedding from the cylinder. It would seem that the maximum augmentation of one alternate vortex street by another occurred when the frequencies of both were equal; and that at unequal frequencies attenuation resulted.

Instability was recorded in the first instability region when the cylinders were coupled at \( G/d \geq 3.75 \), i.e. outside the range for which the downstream cylinder was thought to influence A. In this case A was supposed to be driving B since B was stationary in the uncoupled condition (and therefore had no exciting forces at the correct frequency). It was noted that instability was recorded when \( h/L = 0.82 \); if A did in fact drive B then the effective \( k_s \) at A would be twice the \( k_s \) for A alone (i.e. \( k_s = 0.5 \)) if structural damping and mass alone were considered. However, the cylinder B would almost certainly experience undefined amounts of fluid damping and these could increase the effective \( k_s \) at A, thus exceeding \( k_{sc} \) and suppressing excitation for certain \( G/d \). By this reasoning it will be seen that A could never drive B for \( h/L = 0.712 \) because the effective structural Stability Parameter at A (assuming no additional fluid damping) was already equal to \( k_{sc} \). The nature of the instabilities confirmed that vortex shedding and not buffetting was responsible for the excitation, (see 12.2(i)).
12.6.3 Second normal mode in-line (s.n.m.). Cylinders VII.2.

Cylinders VII.2, used in these tests were tested in the one water level of \( h/L = 0.712 \), when uncoupled and also when coupled with each of the three coupling members. The isolated cylinder tests in the fundamental and second normal modes in-line (Chapter 9) identified similarities between the exciting mechanisms of the corresponding instability regions. The present wake interference tests (no coupling member) for the second normal mode were arranged to determine whether similarities existed in the fundamental and second normal modes of two cylinders coupled only by the fluid between them. The additional tests with the three coupling members, were devised to provide additional basic information in the second normal mode of in-line motion. Figures 12.15, 12.16, 12.17 summarise the results from the tests with the coupled and uncoupled cylinders.

For all spacings \( G/d \), the downstream cylinder B was never excited in the second normal mode in the uncoupled condition. However, it was observed that at \( G/d = 1.5 \), B was exhibiting large amplitude 'figure of eight' motion at the fundamental frequency whilst A oscillated in the first instability region of the s.n.m. in-line. In common with the fundamental mode results, the maximum amplitudes of A remained sensibly constant for all \( G/d > 1.5 \). At \( G/d = 1.5 \) the second instability region for fundamental motion in-line was excited at B, followed by two fundamental mode peaks of \( \pm 0.31d \) at \( V/nd = 12 \). A maximum amplitude of second normal mode motion was recorded at A, when \( V/fd = 2.2 \) and \( G/d = 1.5 \). For all \( G/d \) above this value, only one fundamental mode peak was observed at B, and the motion was exclusively in-line. The \( V_{rm} \) at which fundamental mode maxima were recorded varied from \( V_{rm} = 8.5 \) at \( G/d = 2.5 \), to \( V_{rm} = 6.4 \) at \( G/d = 6 \).

The Reduced Velocities \( V/fd \) of the maximum amplitudes in the s.n.m. at A were restricted to a fairly narrow band of \( 1.9 < V/fd < 2.1 \).
Coupling the cylinders at $G/d \leq 1.5$ resulted in second normal mode oscillations in the first instability region for all three coupling members; fundamental mode oscillations were not excited. For $G/d = 2.5$, only fundamental mode oscillations were excited in-line when the cylinders were coupled by the $\frac{1}{4}''$ Darvic and rigid brass members. Reference to the graph of fig. 12.18 showed that these couplings reduced the fundamental mode $k_s$ to 1.5 compared with $k_s = 2.0$ for the 1/32'' Darvic member. It was reasoned that for the downstream cylinder B, the limiting $k_s$ for the second instability region of the fundamental mode of B was probably located between 1.5 and 2.0 (assuming both cylinders contributed equally to the excitation and that one was not being driven by the other). This should be compared with $k_{sc} \approx 1.2$ for the isolated cylinder. For $G/d \geq 3.5$ only instability in the second normal mode was recorded. The first instability region of this mode was excited at $V/fd = 1.3$ for the bent coupled with the rigid brass member. With the two Darvic coupling members, excitation was limited to the second instability region of the s.n.m.

12.6.4 Discussion of results for 12.6.3.

It was shown that when two identical, uncoupled cylinders were separated by various distances in-line, ($1.25 \leq G/d \leq 6$) in flowing water, the downstream cylinder was not excited in the first instability region of the second normal mode except for $G/d \leq 1.5$. Similarly, in the fundamental mode tests, the downstream cylinder was never excited in the first instability region for $G/d \geq 1.5$ and the explanation tentatively proposed was given in the discussion of those tests (12.6.2). Large amplitude fundamental mode oscillations in-line were recorded on the downstream cylinder B when A was oscillating in the second normal mode in-line. Rigidly coupling the cylinders produced oscillations in the first instability region of the s.n.m.; this effect was not observed when flexible couplings were used. The notion of A driving B when the sum of the Stability Parameters was considerably less than $k_{sc}$ (as outlined in 12.6.2) could
only be applied if the hydrodynamic damping was zero, as the sum of the Stability Parameters was approximately unity (the Critical $k_s$ for the isolated cylinder $A$ in the s.n.m. was $k_{sc} = 1.2$.) The flexible and rigid couplings theoretically should have produced similar Stability Parameters for the bent, although the logarithmic decrement (and thus $k_s$) with the Darvic coupling was approximately 20% higher than with the rigid coupling. Possibly, the rigid coupling achieved a more efficient transmission of energy between the two cylinders. The shapes of the instability curves were similar to those of the fundamental mode and it was concluded that the excitation undoubtedly was due to vortex shedding. (See 12.2(i).)

12.6.5 Fundamental mode cross-flow oscillations.

The lead filled cylinders were examined in one depth of water ($h/L = 0.712$) in the fundamental mode for uncoupled and coupled conditions. Figs. 12.19, 12.20, 12.21, show the summarised and 'as recorded' results of this section. The behaviour of the uncoupled cylinders conformed to a consistent pattern for all $G/d < 6$. The upstream cylinder $A$ reached its maximum amplitudes at $V_{rm} = 6.4$ and gradually decreased to zero amplitude at $V_r = 8$. Maximum amplitudes of $B$ occurred at $V_{rm} = 7.2$ (except for the one case of $G/d = 1.25$ when the maximum amplitude coincided with $V_{rm} = 7.7$); the cylinder continued to oscillate with a fairly uniform amplitude of slightly less than the maximum, up to $V_r = 12$, at which point the amplitude decreased abruptly to zero. For $G/d = 6$, the upstream cylinder $A$ reached a peak amplitude at $V_{rm} = 5.5$; the maximum for $B$ occurred at $V_{rm} = 6.4$ followed by a rapid decrease to one half the maximum amplitude at $V_r = 8.8$.

Coupling the cylinders produced consistent results for all $G/d < 6$. Two peaks were recorded for the cross-flow motion of the bent; the first, at $V_r = 6.5$ was approximately 50% larger than the second at $V_r = 10$. For $G/d = 6$, the relative magnitudes of the peaks were reversed and the values of reduced velocity at which they occurred, reduced to 5.5 and 7.2 respectively.
12.6.6 Discussion of results of 12.6.5.

The amplitude of the upstream cylinder A showed substantial departure from that of the isolated cylinder when the spacing was less than 2.5 diameters, and at \( G/d = 1.5 \) the amplitude of A exceeded by 30% the amplitude of the isolated cylinder. For \( G/d \geq 2.5 \), A behaved as an isolated cylinder.

Nagai & Kurata (137) noted that the wake interaction effects between stationary, rigid cylinders spaced at various distances in-line could influence the \( C_d \) of the upstream and downstream cylinders for all \( G/d \leq 15 \). However, they recorded a wake interference reduction in \( C_d \) of only 8% at the upstream cylinder for \( G/d = 1.5 \). The downstream cylinder of the present tests oscillated with a maximum amplitude 50% greater than the isolated cylinder value when \( G/d = 1.5 \), and for \( G/d \geq 2.5 \) the maximum amplitudes were consistently 20% higher than that of the isolated cylinder. In reference (137) it was shown that the \( C_d \) of the downstream cylinder experienced considerable reductions for \( G/d \leq 15 \), and even at \( G/d = 15 \), reductions of up to 40% were recorded. Unfortunately, those authors did not comment on the corresponding variations in the lift (cross-flow) force coefficients.

Zdravkovich (100) recorded instabilities in air tests for spacings up to \( G/d = 2.5 \) but was unable to detect sustained oscillations for \( G/d > 2.5 \). In the twin tower tests of Whitbread & Wootton (133) the upstream tower A was fixed and the downstream tower (B) monitored for sustained oscillations; excitation occurred for all \( G/d \leq 6.7 \). It is not known for certain why the results of Zdravkovich should differ so markedly from those of (133) and the present tests. The damping of the twin towers (133) was of the same order as the cylinders used in the tests of Chapter 12.6.5 (\( \delta_s = 0.065 \)), but the damping of Zdravkovich's rig was nearly four times higher (\( \delta_s = 0.24 \)), and it was assumed that this relatively high damping accounted for the exceptional effects observed in his tests.

Whitbread & Wootton (133) recorded maximum response for \( G/d \geq 3 \) which agreed tolerably well with the present results.
(fig. 12.19). The individual peak amplitudes of (133) in the spacing range $G/d \geq 1.75$ coincided with $V_{rm} = 7.2$, a value which was almost identical with the results of the present tests.

Coupling the cylinders VII. 2 created two instability regions; for $1.5 < G/d < 5$, the peaks of these regions coincided with $V_{rm} = 6.5$ and $V_{rm} = 10$; the first peak was in agreement with the recordings made with the solitary cylinder but there was no obvious explanation for the second peak. For $G/d = 6$, the two peaks were recorded at $V_{rm} = 5.5$, and $V_{rm} = 7.2$.

In the coupled cylinder tests of Zdravkovich, oscillations were not excited for all $G/d \leq 5$, and he concluded that relative motion between the cylinders was a prerequisite for the initiation of instability. The results of this Chapter did not support his premise.
MODIFIED FULL COUPLED PROGRAM.....TIP EFFECTS INCLUDED.

FUNDAMENTAL MODE

NATURAL FREQUENCY = 6.5994 Hz

DETERMINANT = 7.09774 -9
E11 = 94,500 LBF.FT2
E22 = 114820 LBF.
PILE MASS/UNIT LENGTH = 0.151 LBF./FT
PILE OUTER DIAMETER = 1.055 INCHES

DECK THICKNESS = 0.250 INCHES
WATER LEVEL = 26.00 INCHES
PILE HOLLOW

----------------------------------

ROTARY SPRING STIFFNESS = INFINITELY STIFF

VERTICAL PILE 0 TO 5

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EQUIVALENT MASS/UNIT LENGTH = 0.3521 SLUGS/FT.
## FUNDAMENTAL MODE

**WATER LEVEL** = 26.00 INCHES  
**DEPTH RATIO** = 0.712  
**STABILITY PARAMETER** = 0.34797  

**ROTARY SPRING STIFFNESS** = INFINITELY STIFF

### HORIZONTAL DECK 5 TO 10

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**STABILITY PARAMETER** = 0.34797  

**ROTARY SPRING STIFFNESS** = INFINITELY STIFF

### VERTICAL PILE 10 TO 15

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Examples of computer output using the programs developed for this application.
Fig. 12.1 One of the base blocks used in the coupled cylinder tests. Surface details only.

Fig. 12.2 The arrangement of the cylinders used in the wake interaction tests.
Fig. 12.3  Experimental and calculated results for the proving stage of the computer program of the coupled cylinders configurations. Fundamental mode in-line. Cylinders VII. I
Fig. 12.4 The calculated mode shapes of the coupled cylinders with different coupling members. Fundamental mode in-line.
Fig. 12.5 Proving the computer program for the second normal mode in-line. Cylinders VII.2
Fig. 12.6 The calculated mode shapes of the coupled cylinders VII.2 with different coupling members. Second normal mode in-line.
Fundamental mode in line
first instability region
uncoupled cylinders

- h/L = 0.712
- h/L = 0.822

Fig. 12.7 Experimental results, first instability region in-line. Uncoupled cylinders VII. 1.

Fig. 12.8 Experimental results, second instability region in-line. Uncoupled cylinders VII. 1.
Fig. 12.9  Experimental results. Coupled cylinders VII.1 oscillating in-line.

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<td>○</td>
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h/L = 0.72
Experimental results, 'as recorded' fundamental mode in-line.
Fig. 12.11 Experimental results, 'as recorded', fundamental mode in-line.

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G/d = 2.75

Fig. 12.12 Experimental results, 'as recorded', fundamental mode in-line. Downstream cylinder uncoupled.
Fig. 12.13  Flow visualisation of the uncoupled cylinders. G/d = 1.0.

Fig. 12.14  Flow visualisation of the uncoupled cylinders. G/d = 3.
Fig. 12.15  Experimental results. Uncoupled cylinders VII.2 oscillating in-line in the fundamental and second normal modes.
**Fig. 12.16** Experimental results. Coupled cylinders VII.2 oscillating in-line in the fundamental and second normal modes.

**Fig. 12.17** Experimental results. Coupled cylinders oscillating in-line in the second normal mode only.
Fig. 12.18 The variation of Stability Parameter with types of coupling. Fundamental and second normal modes in-line. Cylinders VII.1, VII.2
Fig. 12.19 Summary of the results of tests in the cross-flow direction. Cylinders VII.2.
Fig. 12.20  Experimental results, 'as recorded' cross-flow: cylinders VII.2 uncoupled.
Fig. 12.21 Experimental results, "as recorded", cross-flow cylinders VII.2 coupled.
13. Conclusions and Suggestions for further research.

13.1 Conclusions.

The objectives of the research programme described in this thesis were to examine the flow-induced oscillations of cylinders in water, to define the conditions under which sustained excitation occurred, and to establish the laws of similarity governing the hydroelastic modelling of structures. Additionally, a study was to be made of the added mass and damping of cylinders oscillating freely in still water. A total of thirteen different surface piercing cylinders were examined over a wide range of experimental variables; their results and those from an analysis by a transfer matrix method led to the several conclusions discussed below. Considerable progress was made in satisfying all the objectives, although there are areas in which further research must be completed, and these are detailed in the second half of this Chapter.

In-line oscillations were excited when the Stability Parameter \( k_s \) < 1.2, and when the Reduced Velocity \( V_r \) > 1.2. The frequency term in \( V_r \) refers to each of the first three normal modes and it is postulated that this \( V_r \) condition applies to all normal modes in-line. The experimental results showed that the in-line oscillations were confined to two distinct instability regions \( 1.2 < V_r < 2.5 \), \( 2.7 < V_r < 3.7 \); between these two regions the cylinders were considered stationary as the amplitudes of their excited motions were small \((y/d \simeq 0.02)\) and randomly ordered. The photographic study of Chapter 9.4 illustrated the vortex shedding appropriate to these two instability regions, the first was identified by symmetric vortex shedding and the second by the shedding of alternate vortices.

In Chapters 9.2, 9.3, thermister probes were employed to detect the dominant vortex frequencies in the wakes of the cylinders oscillating in-line. The first instability region was characterised by vortex frequencies predicted from the stationary cylinder Strouhal number \( S = 0.2 \), and in the second region, the vortex shedding frequencies
were locked in to one half the cylinder natural frequencies. The photographic study demonstrated the way in which the symmetrically shed vortices coalesced to form a stable alternate vortex street downstream from the oscillating cylinder; the dominant frequencies of these alternate vortex streets were equal to the stationary cylinder Strouhal vortex shedding frequencies. The distances downstream from the cylinder at which the alternate streets were formed were observed to be functions of \( V_r \), and at \( V_r \approx 2.4 \) the alternate vortices actually formed at the cylinder. The symmetric vortices were shed at the rate of one vortex pair per cycle of motion, and, during the first instability regions the frequencies of excited oscillations were equal to the frequencies of the cylinders in still water. It was concluded that although the symmetric vortex shedding process and the wake exhibited generally dissimilar frequencies, there was evidently a strong feedback from the wake as the thermister probes, located one diameter downstream from the cylinder, detected only the dominant Strouhal wake frequencies. The ratio of the cylinder natural frequencies to the stationary cylinder vortex shedding frequencies obviously varied with velocity through the velocity term in the definition of \( S \). When these frequencies were in simple proportions such as 4, 3.5, 3, 2.5, intermittent forced cross-flow oscillations were recorded at the stationary cylinder Strouhal frequencies, indicating that the fluctuating pressure field of the wake occasionally enveloped not only the thermister probes, but also the cylinders themselves. In one test on a very slender and lightly damped cylinder (Chapter 9.5), a remarkable recording was made of sustained, second normal mode cross flow oscillations throughout the first instability region of the cylinder's third normal mode in-line. The frequencies excited in these two directions were related consistently in a 1:3 ratio. The research programme did not result in a satisfactory conclusion concerning the mechanism by which the wake pressure field caused, or supported these cross-flow oscillations in the presence of the in-line oscillations excited by symmetric vortex shedding.
In the second instability regions in-line, the vortices were shed from alternate sides of the cylinder at its natural frequency and forced, small amplitude cross flow oscillations were recorded at one half the cylinder's natural frequency. The dominant wake frequencies measured at the thermister probes were equal to the cylinder natural frequencies and the generated vortex street arrangements were conserved for many diameters downstream. These observations were consistent with frequency synchronisation or lock-in in the in-line direction. The alternate vortices were formed at the cylinder and the frequency of the forced cross-flow motion indicated that within this second instability region, the strength of the feedback mechanism from the distant wake was insufficient to overcome the synchronised vortex effects local to the cylinder. The 2:1 relationship between the frequencies of the in-line and cross-flow components of the alternate vortex shedding were similar to the stationary cylinder measurements of (28), (29), (31), (32). For each in-line test the maximum amplitudes of the oscillations in the two instability regions were approximately equal. The overall maximum amplitudes were \( \pm 0.15 \) diameters for the fundamental, second and third normal modes when \( k_s \leq 0.4 \).

Sustained, cross-flow oscillations at the cylinders' natural frequencies were excited when \( 4.5 < V_r < 9 \); the frequency term in \( V_r \) refers to each of the frequencies in the first and second normal modes. Photography and vortex counting showed that the cross-flow oscillations were identified by alternate vortex shedding. Maximum amplitudes of \( \pm 1.6 \) diameters were recorded in the cross-flow direction compared with the maximum \( \pm 0.15 \) diameters in-line. Similarly the Critical Stability Parameters for the two directions also differed by an order of magnitude; \( k_{sc} = 17 \) for the cross-flow oscillations and \( k_{sc} = 1.2 \) in-line, although the first instability region in-line could be suppressed by a slightly lower \( k_{sc} \) of 1.0.

The experimental results of Chapter 9 and the analytical section of Chapter 11 showed that the Stability Parameter, \( k_s \) and the Reduced Velocity \( V_r \) were the criteria through which data from various
sources should be correlated. The form of $k_s$, developed in Chapter 3, resulted from redistributing the mass and energy over the water depth $h$; the equivalent cylinder defined by $k_s$ was of length $h$ and thus the maximum amplitudes of oscillations for the fundamental free ended mode were those measured at the water surface. When compared on this basis, the results of the in-line tests were well ordered with respect to both variables, and in those comparisons where $k_s$ and $V_r$ were identical, the associated amplitudes were in almost exact agreement; this was concluded to testify the validity of the form of $k_s$ postulated in this thesis. In order to reproduce approximately the free surface effects of surface piercing cylinders, the Froude number should also be considered. The bow waves were genuine Froude (i.e. gravity) effects although the cavities forming downstream of the cylinders were apparently functions of absolute velocities, and therefore not uniquely related to the Froude numbers. Chapter 9.3.6 emphasised the difficulties of reproducing free surface phenomena with models; the general conclusion was that models accurately represented the unventilated cavities but underestimated the phenomena when the cavities were ventilated.

Chapter 10 showed the way in which correctly designed hydroelastic models of large marine piles could be used to yield both qualitatively and quantitatively comparable results at unscaled Reynolds numbers. This was one of the most important conclusions to emerge from the research programme and it emphasised the major differences between stationary and instationary cylinders exposed to fluid flow as detailed in Chapter 2. The two hydroelastic models of Chapter 10, designed to geometric scales of 1:30 and 1:27 were excited to oscillate in the in-line direction in the Re range up to $4 \times 10^4$. They quantitatively reproduced the in-line flow excited behaviour of the full scale piles in the Immingham tests (112) and where Reynolds numbers of $1.6 \times 10^6$ were encountered. This confirmation of the use of reduced scale hydroelastic models was considered a significant contribution to the evaluation of preliminary designs of future offshore structures through dynamic
model tests. Such tests could result in the detection of 'on station' operational difficulties caused by tidal and density currents. They may also be used to determine the possibility of dangerous flow induced oscillations developing during the (frequently) long tows from the construction yards to the offshore oil or gas fields. However, in the tests of Chapter 9.5, a limiting Reynolds number was determined, below which in-line oscillations would not occur; this obviously places a restriction on the geometric scales selected for modelling. It is appreciated that the validity of hydroelastic modelling has been confirmed only for structures in steady currents and an extensive programme of research is necessary before their application to wave forces can be made with confidence.

The photographic study of Chapter 9.4 illustrated the interaction arising between an oscillating cylinder and the flow past it. An examination of the flow patterns around, and the separation points on the cylinder indicated that maximum fluctuating force coefficients of $C'_d = 0.86$ were possible in the symmetric shedding region. In the analytical section of Chapter 11 it was shown that the maximum recorded amplitude in-line ($y/d = 0.15$) represented $C'_d = 0.69$. However, the force coefficients were amplitude dependent and $C'_d = 0.69$ possibly represented the uniformly distributed equivalent of $C'_d = 0.86$ at the maximum amplitudes and the pseudostationary cylinder $C'_d = 0.08$ near the base of the cylinder. In the second instability region (alternate vortex shedding), a maximum value of $C'_d = 0.44$ was recorded. The reasoning of Chapter 11 led to the conclusion that the oscillatory velocity of the cylinder relative to the mean flow velocity was a factor differentiating the two regions in-line. In the first instability region, the maximum oscillatory velocity was equal to one half the mean flow velocity and the vortices were shed each time the cylinder accelerated into the flow direction. This was concluded to be the reason for the symmetric vortices closely resembling the starting vortices formed from bluff bodies accelerated relative to the surrounding fluid.

In the analysis of the cross-flow results, a maximum fluctuating force coefficient of $C'_L = 0.78$ was deduced; this was similar
to the values measured by other researchers (46), (59), (62), (63). In common with the conclusions from the in-line tests mentioned above, the ratio of the resultant velocity vector of oscillatory motion to the mean velocity of the flowing water apparently governed the maximum amplitudes of cross-flow motion, and at these maxima, the ratio was approximately 2.0.

The added mass of water per unit length of the cylinders oscillating freely in still water, in all tests could be represented by the ideal added mass function derived from potential flow theory (Chapter 8). Flow excited frequencies initially were equal to the still water frequencies and from such cases it was concluded that streaming flow did not influence the added mass function, which in still water was also shown to be independent of frequency, mode shape and free surface effects. The flowing water and still water effects were confirmed for the first three normal modes in-line and for the first two modes in the cross-flow direction.

The hydrodynamic damping forces exerted on a cantilever in still water were deduced from the records of transient motion in the fundamental mode, (Chapter 8). Throughout the range of variables used in these tests, the damping was viscous and could be calculated by the application of Rayleigh's element damping theory. However, from the reasoning of Chapter 12, it was concluded that the theory applied only to unseparated flow conditions and that when vortex-excited oscillations were induced, the theory necessarily became invalid.

Wake interaction effects between two cylinders in-line were studied in Chapter 12. The recordings showed that these effects were sufficient to suppress excitation of the first instability region of the downstream cylinder for G/d > 1.5, in both the fundamental and second normal modes. Photography suggested that this was a consequence of interactions between the symmetric and alternate vortices as described in the discussion of Chapter 12. Coupling the cylinders altered the mode shapes and the Stability Parameters in-line; the stiffer coupling
members caused greatest reductions in $k_s$ and thus increased the probability of flow excitation. The conclusion from these tests was that increasing the stiffness of the structure and thus its static stability, could decrease the dynamic stability.

In the cross-flow mode, coupling the cylinders slightly increased the Stability Parameters by the effects of the concentrated end masses of the coupling members, thereby contributing little to the instability in this mode. In general, coupling the cylinders increased the degrees of instability through improved phase relationships between the vortex shedding from the two cylinders.

This 'wake interaction' part of the research programme was designed as an introduction to the problems related to closely spaced cylinders in flowing water. The generality of the conclusions outlined in Chapter 12 should be qualified by an additional series of tests with cylinders of various diameters, mass densities and structural dampings.

13.2 Suggestions for further research.

The work described in this thesis has resulted in several definite conclusions concerning the mechanisms of instability and the factors governing the amplitudes of these. However, certain aspects are still in need of completely satisfactory explanations and these are enumerated below:

(i) The transfer matrix method of analysis should be developed to include the modified Van der Pol equation, as suggested from the cross-flow work of Griffin, Skop & Koopman (131). This would include an analysis of the hydrodynamic damping during the cross-flow excitation. The Van der Pol equation should also be incorporated in the in-line analysis, and a convenient point is that at which excitation occurs. In figure 9.2, the limit amplitude was reached after approximately 40 cycles of motion, and during this period the negative logarithmic decrement remained sensibly constant at the still water hydrodynamic log decrement $\delta_h$. 
The generality of this result should be confirmed and, if possible, used in the modified analysis.

(ii) Further tests are needed to define the conditions determining the Reduced Velocities at which maximum oscillations are excited (i.e. $V_{rm}$). $V_{rm}$ apparently was a complex function of $k_s$, $m_r$, $h/d$, surface roughness and turbulence. The variations in $V_{rm}$, and thus the energy available there were responsible for the variations in the oscillatory amplitudes of excitation between cylinders of identical Stability Parameters.

(iii) Although the first instability region of in-line oscillations was investigated in considerable detail, phenomena were recorded in this thesis research programme for which there are currently no completely satisfactory explanations. In particular the wake feedback to the cylinder was manifested in small, forced oscillations of the cylinder at the distant wake frequencies throughout the constant frequency first instability region of in-line motion. As an extension of these phenomena sustained oscillations in the second normal mode of the cross-flow direction were recorded when $V_r \leq 5$ in that direction and throughout the first instability region of the third normal mode in-line (Chapter 9.5). The natural frequencies in the two directions were related by a ratio of $1:3$. In the experimental results of Chapters 9.2, 9.3 it was noted that the cylinder natural frequency ($n$) and the dominant wake frequency ($f_v$) apparently changed in steps to preserve simple ratios of $n/f_v$ such as 4, 3.5, 3, 2.5 and it is thought that these two effects could be related. Further research could incorporate tests with a splitter plate in the wake of the cylinder oscillating in-line. Supplementary tests by the writer have shown that a splitter plate mounted immediately downstream of the cylinder has the effect of extending the first instability region and suppressing the second. In this extended first instability region in-line, the forced cross-flow motion was absent, thus implying that the feedback mechanism of the
wake was associated with a fluctuating circulation around the
cylinder circumference.

(iv) The hydroelastic modelling criteria developed in this thesis was
applied to cylinders in steady currents and further research in
wave conditions would establish similar criteria for that medium.
Some of the complications were mentioned in Chapter 4 and
these included the inertia and drag coefficients, variations in
wave orbital velocities (superposed on steady current flow),
cylinder roughness and the ratio of wave frequencies to cylinder
natural frequencies. It is felt that this is one of the more impor-
tant areas of applied research, particularly in view of the rapid
expansion of the offshore industries into the North Sea. There,
wave effects almost certainly will dwarf the steady currents in
locations away from shelving bed profiles near coastlines.
However, long period waves may approach steady flow conditions
and the governing criteria could be similar to those developed
from the tests in exclusively steady flow conditions.
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APPENDIX 1. TRANSFER MATRIX ANALYSIS

A1.1 Introduction.

In the transfer matrix method of analysis, the structure under investigation was divided into a convenient number of elements; the mass of each element was considered concentrated at the mid point of the element and supported by massless structural springs. An energy balance along the element resulted in a series of equations relating conditions at one end to those at the other. By progressing from element to element, the whole structure could be related to some arbitrary reference point (usually a point of fixity). Suitable mathematical transformations were used to rearrange these equations in matrix form for systematic processing by a digital computer. The application of boundary conditions yielded information such as structural natural frequencies, and deflections, slopes, shear forces and bending moments at salient points in the structure. The term 'transfer matrix' arose from the process of transferring conditions at one point to those at another by a transfer matrix embracing the two points.

The response of a cylinder to periodic forcing was calculated using extended and partitioned transfer matrices which, in general, had at least twice the number of elements contained within the transfer matrices describing undamped, free oscillation.

A1.2 Definitions and coordinate system.

There are three main matrix expressions employed in this Chapter, viz:

(i) State Vector \( \{Z\} \);

this is a column vector the elements of which are the displacements and corresponding forces at any position in the structure.

(ii) Field Transfer Matrix \( [F.T.] \);

this is a square matrix transferring conditions from one
State Vector to those at the next through a massless elastic field.

(iii) Point Transfer Matrix $[P]$:
this is a square matrix transferring displacements and forces across a discontinuity in shear forces or bending moments.

The coordinate system used throughout this thesis is shown below in Fig. A1.1:

![Coordinate System Diagram](image)

**Fig. A1-1** Co-ordinate system

A1.3 **Formulation of the State Vector and Field Transfer Matrix** of a beam element.

The State Vector could be arranged in any consistent form and the form adopted here was based on that developed by Postel & Leckie (141), i.e. displacements were assembled in the top half of the vector and the corresponding forces occupied the lower half.

Consider an arbitrary element of a beam element undergoing transverse free oscillations (i.e. no damping); the forces and displacements will be as shown in fig. A1.2 (note superscripts $l$ and $r$ refer to left and right faces of the point considered).
Fig. A1-2 An elastic massless beam element.

For equilibrium, the sum of the vertical and horizontal forces must be zero, and the moments about point i-1 must also be zero. These conditions resulted in the establishment of the following equations:

\[
\begin{align*}
\frac{x_i - x_{i-1}}{r} - L_i \frac{M_{i-1}}{EA_i} &= 0 \\
N_i &= N_{i-1}^r \\
V_i &= V_{i-1}^r \\
M_i^l - M_{i-1}^r - V_i L_i &= 0 \\
-y_i^1 + y_{i-1}^r - L_i s_{i-1}^r - L_i^2 M_{i-1}^r / 2EI_i - L_i^3 V_{i-1}^r / 6EI_i &= 0 \\
s_i^1 - s_{i-1}^r - L_i M_{i-1}^r / EI_i - L_i^2 V_{i-1}^r / 2EI_i &= 0
\end{align*}
\]

\[\ldots (A1.1)\]
The equations (A1.1) were rearranged to suit the assembly of the two State Vectors at points \( i \) and \( i-1 \):

\[
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & \text{L/EA} \\
0 & 1 & L & L^2/2EI & L^3/6EI & 0 \\
0 & 0 & 1 & L/2EI & L^2/2EI & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x^r \\
y^r \\
M^r \\
V^r \\
N^r
\end{bmatrix}
= 
\begin{bmatrix}
x^l \\
y^l \\
M^l \\
V^l \\
N^l
\end{bmatrix}
\quad \ldots (A1.2)
\]

\[
\left\{Z\right\}^1_i = \left[FT\right]_i \left\{Z\right\}^r_{i-1}
\quad \ldots (A1.3)
\]

A1.4 Formation of the Point Transfer Matrix [\( P \)]

A concentrated mass introduced a discontinuity in shear force, and the Point Transfer Matrix transferred conditions from the left to the right face of the mass. It was assumed that the mass was rigid, so that deflections, slopes and bending moments were continuous across the mass; thus

\[
\begin{align*}
x^r_i &= x^l_i \\
y^r_i &= y^l_i \\
M^r_i &= M^l_i \\
N^r_i &= N^l_i \\
V^r_i &= V^l_i - m_i w^2 y_i
\end{align*}
\quad \ldots (A1.4)
\]

When arranged in the assembly consistent with (A1.2) the equations of (A1.4) related the left and right faces of the State Vector at point \( i \), as shown below.
\[
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & m_i w^2 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
y \\
s \\
v \\
N_i
\end{pmatrix}^r = \begin{pmatrix}
x \\
y \\
s \\
v \\
N_i
\end{pmatrix}^1 

\ldots (A1.5)

\{Z\}^r_i = [P] \cdot \{Z\}^1_i 

\ldots (A1.6)

(For a linear spring of stiffness S attached to the concentrated mass, the element in the (5,2) position of [P] would be modified to read \((m_i w^2 - S)\).)

A1.5 Formation of the Point Transfer Matrix \([R]\) for a rotary spring.

A deflected rotary spring imposed a restoring bending moment on the element equal and opposite to the bending moment across the element:

Fig. A1-3 A point rotary spring.
A bending moment $M$ deflected the spring $R$ by a rotation $s$ and the matrix $[R]$ was defined by

$$\{Z\}^r_i = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1/R & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \{Z\}^1_i \quad \ldots \quad (A1.7)$$

$$\{Z\}^r_i = [R]_i \{Z\}^1_i \quad \ldots \quad (A1.8)$$

For the hydroelastic tests of Chapter 10, this form of matrix was used to simulate the influence of the capping beam, and by incorporating a similar rotary spring at the base it was possible to calculate the variations of frequencies and normal mode shapes imposed by imperfectly encastre base conditions.

A1.6 Corner transformations of axes.

Consider the forces, moments and deflections acting at the upper left corner of the bent shown in figure A1.5, and detailed in fig. A1.4.
An inspection of the figure above resulted in the following transformations:

\[
\begin{align*}
x^r &= y^1 \\
y^r &= -x^1 \\
s^r &= s^1
\end{align*}
\quad M^r &= M^1 \\
V^r &= -N^1 \\
N^r &= V^1 \quad \ldots \quad (A1.9)
\]

The corner matrix \([C]\) was formed by writing equations (A1.9) into an assembly consistent with the State Vectors on each side of the corner, whence:

\[
[C] = \begin{bmatrix}
0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix} \quad \ldots \quad (A1.10)
\]

A1.7 Transfer matrix analysis of a bent.

In this section, the transfer matrix analysis is applied to one of the test configurations used in the coupled cylinder tests of Chapter 12. The example chosen, although more complex than a single cantilever was easily modified to accommodate that more simple configuration, thus demonstrating the comparative versatility of the transfer matrix technique.

A1.7.1 Establishing the overall matrices for the bent.

The figure A1.5 shows the bent represented by an idealised equivalent, divided into a series of contiguous lumped springs and masses. For ease of explanation, only five masses per member have been shown; for the thesis calculations, use was made of Myklestad's (142) criteria of a minimum of six masses for the correct definitions.
of normal mode shapes and not less than four for frequencies.

Fig. A1-5  The bent analysed by the transfer matrix method

Applying the equation (A1.3) to the first massless field

\[ \{ Z \}_{1}^{1} = \left[ FT \right]_{10} \cdot \{ Z \}_{0} \]  \hspace{1cm} \text{(A1.11)}

and at the concentrated mass 1, from (A1.6):

\[ \{ Z \}_{1}^{r} = \left[ P \right]_{1} \cdot \{ Z \}_{1}^{1} \]  \hspace{1cm} \text{(A1.12)}

From (A1.11) and (A1.12) the relationship between the right face of 1 and zero State Vector \( Z_{0} \) was formed:

\[ \{ Z \}_{1}^{r} = \left[ P \right]_{1} \cdot \left[ FT \right]_{10} \cdot \{ Z \}_{0} \]  \hspace{1cm} \text{(A1.13)}

i.e. \[ \{ Z \}_{1}^{r} = \left[ P \right]_{1} \cdot \left[ FT \right]_{10} \cdot \{ Z \}_{0} \]
By carrying out a similar process for all the masses up to the left
of the corner 5 of the bent, the overall relationship was constructed
for 5 to 0:

\[
\{Z\}_5^1 = [FT]_{54}^1 [P]_{43}^1 [FT]_{43}^1 [P]_{32}^1 [FT]_{32}^1 \{Z\}_0 \quad \ldots \quad (A1.14)
\]

or \[
\{Z\}_5^1 = U_5 \{Z\}_0 \quad \ldots \quad (A1.15)
\]

where $U_5$ is the overall matrix relating points 5 and 0.

Similarly, the two ends of the horizontal member 5 to 10
were represented by an overall matrix $U_{10}$

\[
i.e., \quad \{Z\}_{10}^1 = U_{10} \{Z\}_5^r \quad \ldots \quad (A1.16)
\]

Finally, the overall matrix $U_{15}$ of the vertical member 10 to 15
was also established:

\[
\{Z\}_{15} = U_{15} \{Z\}_{10}^r \quad \ldots \quad (A1.17)
\]

However, two corner transformations were necessary
before points 15 and 0 could be related by one overall matrix. The
two corner matrices were identical and were used to relate $\{Z\}_5^r$
and $\{Z\}_{15}^1$, $\{Z\}_{10}^r$ and $\{Z\}_{15}^1$. These corner matrices were developed
in section A1.6.

The overall transfer matrix for the bent was formed by
starting at point 0, and progressing through to point 15 with the
inclusion of the two corner matrices, i.e.

\[
\{Z\}_{15} = U_{15} [C] U_{10} [C] U_5 \{Z\}_0 \quad \ldots \quad (A1.18)
\]
Equation (A1.18) was simplified further by replacing the five matrices with the overall matrix for the bent $U_B$

$$\{Z\}_15 = U_B \{Z\}_o$$

\[\ldots \text{ (A1.19)}\]

**A1.7.2 Frequency determinant for the bent.**

Equation (A1.19) effectively described the conditions at both ends of the bent, and by writing that equation out in more detail the following was obtained:

$$\begin{pmatrix} x \\ -y \\ s \\ M \\ V \\ N \end{pmatrix}_{15} = \begin{pmatrix} \_ \\ \_ \\ \_ \\ U_B \\ \_ \\ \_ \end{pmatrix} \begin{pmatrix} x \\ -y \\ s \\ M \\ V \\ N \end{pmatrix}_o$$

\[\ldots \text{ (A1.20)}\]

The vertical members of the bent were perfectly encastre at points 0 and 15, thus:

$$x_{15} = x_o = 0$$

$$y_{15} = y_o = 0$$

$$s_{15} = s_o = 0$$

Substitution of these boundary conditions into (A1.20) reduced $U_B$ from a 6 x 6 matrix to a 3 x 3 sub-matrix $U'_B$, i.e.

$$\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \_ \\ \_ \\ \_ \\ U'_B \\ \_ \\ \_ \end{pmatrix} \begin{pmatrix} M \\ V \\ N \end{pmatrix}_o$$

\[\ldots \text{ (A1.21)}\]

For a non-trivial solution, the determinant ($\Delta$) of $U'_B$ must disappear for all eigen frequencies. Fig. A1.6 shows a graph of $\Delta$ versus frequency ($w$) where the curve intersected the horizontal axis (i.e. $\Delta = 0$). These
Fig. A1-6 Location of natural frequencies for the unforced, undamped cantilever.

Fig. A1-7 Mode shapes and bending moment diagrams for the bent.
crossing points were found by an iterative process developed for this purpose and nested within the main body of the computer program.

A1.7.3 Normal mode shapes of the bent.

Equation (A1.21) connected three unknowns \((M_o', V_o', N_o')\) in three equations for the eigen frequency condition \(\Delta = 0\). By setting \(M_o = 1\), the other two unknowns \(V_o', N_o'\) were extracted by suitable manipulation. The zero State Vector \(Z_o\) of equation (A1.20) was reassembled and the normal mode shape of the bent was calculated by successive multiplication of each Point and Field Transfer Matrix as outlined in equations (A1.14) and (A1.18). The normal mode shapes were determined from the deflections \(x, y\) at each point; fig A1.7 shows typical mode shapes and bending moment diagrams for the first two normal modes of the bent. (Note the change of sign of the bending moment in the vertical legs, a strain gauge sited at the cross-over point (+ve to -ve) would detect no strain there. This demonstrates the practical assistance given by an analysis of the system prior to attaching instrumentation to the structure.)

A1.8 Transfer matrix analysis of a single cantilever in free oscillation:

Chapter A1.7 analysed a three member bent and the basic equations developed for the bent now are applied to the single cantilever.

Consider the cantilever to be the vertical member 0 to 5 of the bent in fig. A1.5, and isolated from the rest of the bent. The cantilever performed only transverse motion (i.e. there were no resultant forces parallel to the longitudinal axis) thus \(x = N = 0\) and the State Vectors reduced to \(\{-y, s, M, V\}\). Likewise, the Field and Point Transfer Matrices contained only 16 elements compared with the 36 elements of the matrices derived in the preceding sections. From equation (A1.15) we have:

\[
\begin{pmatrix}
Z \end{pmatrix}_5^1 = U_5 \begin{pmatrix}
Z \end{pmatrix}_0
\]

\(\ldots\) (A1.15)
and by including the Point Transfer Matrix for the point 5, equation (A1.15) was re-defined by:

\[
\{Z\}_5^T = U_c \{Z\}_0
\]  

(A1.22)

where \( U_c \) was the overall transfer matrix for the cantilever

Application of the boundary conditions at points 0 and 5, i.e.

\[-y_0 = s_0 = 0, \quad M_5 = V_5 = 0\]

yielded

\[
\begin{bmatrix}
0 \\
0
\end{bmatrix} = \begin{bmatrix}
U_c^T
\end{bmatrix} \begin{bmatrix}
M \\
V
\end{bmatrix}_0
\]

(A1.23)

where \( U_c^T \) was a 2 x 2 sub-matrix of \( U_c \).

The eigen frequencies were found from (A1.23) in an identical way to that employed in solving A1.21. Similarly, the normal mode shapes were calculated by putting \( M_o = 1 \) in (A1.23) with \( \Delta = 0 \), solving for \( V_o \), reassembling the zero State Vector with \( M_o \) and \( V_o \) inserted and successively multiplying each Field and Point Transfer Matrix to give State Vectors for each element.

A1.9 Transfer matrix analysis of the bent of A1.7 for the case when the horizontal member was rigid.

With a rigid horizontal coupling member, the analysis of the bent was considerably reduced, as the three member bent could be replaced by a single cantilever with an end constraint which permitted translation but not rotation (the clamped-sliding cantilever of Chapter 12). Fig. A1.8 shows the clamped-sliding single cylinder equivalent to the rigidly coupled bent.
Fig. A1-8 The mode shapes of the bent with a rigid coupling.

The Field and Point Transfer Matrices were again only 4 x 4 matrices and were identical to those of the single cylinder analysed in section A1.8. The eigen frequency was found from applying the boundary conditions at points 0 and 5,

\[ i.e. \quad -y_0 = s_0 = 0, \]
\[ s_5 = V_5 = 0 \]

- with these boundary conditions, a new sub-matrix was formed and the calculation of the normal mode shapes proceeded as described previously in A1.7, A1.8.

A1.10 Transfer matrix analysis of the forced, damped oscillations of a cantilever.

The introduction of damping into a system undergoing forced oscillations results in phase shifts between the input force and the amplitude response of the system. Thus the State Vectors
and Transfer Matrices of a forced, damped cantilever will consist of phase (real) and quadrature (imaginary) components; these complex matrices are termed the extended versions of the undamped matrices, and are written as \( \tilde{Z}, \tilde{U} \).

Consider, firstly, the unforced, damped case:

\[
\begin{align*}
\tilde{Z} &= Z^r + jZ^i \\
\tilde{U} &= U^r + jU^i
\end{align*}
\]  \\
\( \text{... (A1.24)} \)

Superscripts \( r, i \) denote real and imaginary components, and \( j \) signifies 90° phase removal.

From the general theory of transfer matrices,

\[
\begin{align*}
\tilde{Z}_i &= \tilde{U}_i \tilde{Z}_{i-1} \\
\text{i.e.} \quad Z^r_i + jZ^i_i &= (U^r_i + jU^i_i) (Z^r_{i-1} + jZ^i_{i-1})
\end{align*}
\]  \\
\( \text{... (A1.25)} \)

which, in matrix form becomes:

\[
\begin{pmatrix}
Z^r_i \\
Z^i_i
\end{pmatrix}_i =
\begin{bmatrix}
U^r_i & -U^i_i \\
U^i_i & U^r_i
\end{bmatrix}_i
\begin{pmatrix}
Z^r_i \\
Z^i_i
\end{pmatrix}_{i-1}
\]  \\
\( \text{... (A1.26)} \)

Now the introduction of a forcing term \( P \) into (A1.26) increases the local shear force and the equation was modified to:

\[
\begin{pmatrix}
Z^r_i \\
Z^i_i
\end{pmatrix}_i =
\begin{bmatrix}
U^r_i & -U^i_i & P^r_i \\
U^i_i & U^r_i & P^i_i
\end{bmatrix}_i
\begin{pmatrix}
Z^r_i \\
Z^i_i
\end{pmatrix}_{i-1}
\]  \\
\( \text{... (A1.27)} \)

In order to assist the manipulation of the matrix in (A1.27), the identity \( 1 = 1 \) was included, thus making a square matrix as shown below:
\[
\begin{bmatrix}
Z^r_v \\
Z^i_v \\
1
\end{bmatrix}
= \begin{bmatrix}
U^r_v & -U^i_v & -p^r_v \\
U^i_v & U^r_v & -p^i_v \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
Z^r_v \\
Z^i_v \\
1
\end{bmatrix}
\quad \text{(A1.28)}
\]

Having established the overall form of (A1.28) the extended transfer matrices and State Vectors were then processed by the methods developed for their more simple normal equivalents.

Consider the single cantilever of A1.8 oscillating under the influence of a forced harmonic input, and for convenience, consider the excitation force uniformly distributed along the cantilever length. By successive multiplication, the following equation was obtained:

\[
\tilde{Z}_5 = \tilde{U}_c \tilde{Z}_0
\]

i.e.

\[
\begin{bmatrix}
-y^r_v \\
s^r_v \\
M^r_v \\
-y^i_v \\
s^i_v \\
M^i_v \\
1
\end{bmatrix}
= \tilde{U}_c
\begin{bmatrix}
-y^r_v \\
s^r_v \\
M^r_v \\
-y^i_v \\
s^i_v \\
M^i_v \\
1
\end{bmatrix}
\quad \text{(A1.29)}
\]

The boundary conditions at 0, 5 were

\[
-y^r_0 = -y^i_0 = s^r_0 = s^i_0 = 0
\]
\[
M^r_5 = M^i_5 = V^r_5 = V^i_5 = 0
\]

and substitution of these into (A1.29) gave:
\[
\begin{bmatrix}
0 \\
0 \\
0 \\
1
\end{bmatrix}
= \begin{bmatrix}
U^r_c \\
- - - - + + - - \\
U^i_c \\
- - - - U^r_c
\end{bmatrix}
\cdot
\begin{bmatrix}
M^r \\
V^r \\
M^i \\
V^i \\
1
\end{bmatrix}
\quad \cdots (A1.30)
\]

Having resolved the problem to the equation of (A1.30),
the forcing terms were extracted, yielding

\[
\begin{bmatrix}
U^r_c & -U^i_c \\
- - - - U^i_c & U^r_c
\end{bmatrix}
\cdot
\begin{bmatrix}
M^r_o \\
V^r_o \\
M^i_o \\
V^i_o \\
0
\end{bmatrix}
= \begin{bmatrix}
-p^r \\
-p^i
\end{bmatrix}
\quad \cdots (A1.31)
\]

The damped eigen frequencies were calculated from the
determinant of the submatrix \( \tilde{U} \). Fig. A1.9 shows the behaviour
of \( \Delta \) with frequency - it is seen that the eigen frequencies were
located at the points of minimum slope at the horizontal axis. The
previous iteration procedure could not be used. In its place a cubic
equation was fitted to the region of zero slope; this equation was
differentiated to yield the true location of the eigen frequencies.
The eigen frequency was replaced in (A1.31) and the unknowns of the
zero State Vector evaluated by Gaussian elimination of the equations
comprising (A1.31).

With the determination of \( M^r_o, V^r_o, M^i_o, V^i_o \), the zero
State Vector of A1.29 was re-assembled and the normal mode shapes
evaluated by successive multiplication of the extended Point and Field
Transfer Matrices.

The first two normal modes in-line and the fundamental
mode in the cross-flow direction were examined in this way.

Fig. A1.10 shows the elements of \( \tilde{U} \) for a cantilever with
lumped masses \( m_i \), hysteretic damping coefficient \( G \), hydrodynamic
damping coefficient \( C \) and forcing function of \( P \) per unit length.
Fig. A1-9 Location of natural frequencies for the forced-damped cantilever.

\[ \frac{L^2}{2E*} \quad \frac{L^3}{6E*} \quad 0 \quad 0 \quad \frac{GL^2}{2E*} \quad \frac{GL^3}{6E*} \quad 0 \]

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<tr>
<td>0</td>
<td>1</td>
<td>L/E*</td>
<td>L^2/2E*</td>
<td>0</td>
<td>0</td>
<td>GL/E*</td>
<td>GL^2/2E*</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>1</td>
<td>L</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>A</td>
<td>AL</td>
<td>BL^2/2E*</td>
<td>L+BL^3/6E*</td>
<td>CW</td>
<td>CWL</td>
<td>DL^2/2E*</td>
<td>DL^3/6E*</td>
<td>-PL</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-L^2G/2E*</td>
<td>-L^3G/6E*</td>
<td>1</td>
<td>L</td>
<td>L^2/2E*</td>
<td>L^3/6E*</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-LG/E*</td>
<td>-L^2G/2E*</td>
<td>0</td>
<td>1</td>
<td>L/E*</td>
<td>L^2/2E*</td>
<td>0</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>L</td>
<td>0</td>
</tr>
<tr>
<td>-CW</td>
<td>-CWL</td>
<td>-DL^2/2E*</td>
<td>-DL^3/6E*</td>
<td>A</td>
<td>AL</td>
<td>BL^2/2E*</td>
<td>L+BL^3/6E*</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Fig A1.10 Extended overall transfer matrix $\tilde{U}$.

\[ A = mw^2 \quad B = mw^2GCw \quad C = \text{hydrodynamic damping coefficient} \]

\[ D = G.A + CW \quad G = \text{hysteretic damping coefficient} \]

\[ E^* = E(1+jG) \]
APPENDIX 2.

ALTERNATIVE EQUATIONS FOR THE ADDED MASS INFLUENCES ON THE FUNDAMENTAL NATURAL FREQUENCIES OF CYLINDERS OSCILLATING IN STILL WATER.

In reference (143), King derived alternative, simplified equations for the fundamental mode of a cantilever oscillating freely in water. The equations were based on coefficient of added mass of unity, a result that was confirmed in the tests of (143) and of Chapter 8 of this thesis. The analysis of the cantilever and water as a two degrees of freedom system resulted in an equation defining the cylinder's fundamental natural frequency, as given below:

\[ w = \sqrt{\frac{3EI}{0.2427 \left( \frac{mL^4}{s} + \frac{mL^4}{a} \right)}} \quad \ldots \quad (A2.1) \]

The derivation of (A2.1) was based on the assumption that the second normal mode frequency was high compared with the fundamental, although the second normal mode frequency could not be inferred from the equation (A2.1). An anticipated error of 1.3% would be introduced in equation (A2.1) if the frequency ratio second normal mode to fundamental was 6 (as it was for the tests of Chapter 8).

For a cylinder with a concentrated mass \( M_t \) attached to the free end, equation (A2.1) must be modified accordingly, and (143) showed that the form of the modification was a consequence of the transcendental equation describing the cantilever's frequencies and mode shapes:

\[ \frac{1 + \cosh z \cdot \cos z}{\cosh z \cdot \sin z - \sinh z \cdot \cos z} = \frac{M_t}{mL} \cdot z \quad \ldots \quad (A2.2) \]

where \[ a = \frac{w^2m}{EI} \]

\[ z = \frac{aL}{4} \]
Now for the cantilever of uniformly distributed mass/unit length $m_s$ and concentrated mass $M_t$, the frequency was found from:

$$w = \sqrt{\frac{3EI}{L^3 (M_t + 0.2427 m_s L)}}$$  \hspace{1cm} \ldots (A2.3)

From (A2.1), (A2.3) the fundamental frequency of the cantilever in still water was defined by:

$$w = \sqrt{\frac{3EI}{M_t L^3 + 0.2427 (m_s L^4 + m_a h^4)}}$$  \hspace{1cm} \ldots (A2.4)

The contributions of the added mass term $m_a h^4$ to the frequency determined from (A2.4) are discussed in Chapter 8.
APPENDIX 3.

THE APPLICATION OF STOKES' ELEMENT DAMPING THEORY TO A CYLINDER OSCILLATING IN STILL WATER.

A3.1 Derivation of governing equation.

In his classic paper on the motion of pendula in an infinite fluid field, Stokes (72) showed that the fluid resistance forces consisted of two terms; the first was a function of acceleration and the second a function of velocity, i.e.

\[ F = \lambda_1 \frac{d^2 y}{dt^2} + \lambda_2 \frac{dy}{dt} \quad \ldots \quad (A3.1) \]

The acceleration term increases the periodic time (the added mass effect) and the velocity term causes a diminution in amplitude (the damping effect). The two effects, being 90° phase removed may be treated separately, and in the following consideration of damping forces, only \( \lambda_2 \) (dy/dt) was included in the analysis.

Consider an elemental length of cylinder dz at height z from the base of a cantilever oscillating in a viscous fluid; the damping force, obtained by expanding the second term on the r.h.s. of (A3.1) was given by:

\[ F_z = m_a w \left\{ 2 \sqrt{\frac{2}{E}} + \frac{2}{E} \right\} dz \frac{dy}{dt} \quad \ldots \quad (A3.2) \]

where \( E = \frac{w r^2}{v} \)

The work done per cycle (Wz) in overcoming viscous damping on the element dz was found by integrating (A3.2) with respect to the distance travelled by dz

\[ W_z = 4 \int_0^T m_a w B \ dz \left\{ \frac{dy}{dt} \right\} z \frac{dy}{dz} \quad \ldots \quad (A3.3) \]

where \( B = 2 \sqrt{\frac{2}{E}} + \frac{2}{E} \)
By assuming sinusoidal motion of each element \( dz \), the total work done in a depth \( h \) was calculated from:

\[
W_h = \int_0^h \int_0^T m_a w^3 B y_z^2 \cos^2 w t \, dz, \, dt \quad \ldots \quad (A3.4)
\]

from which:

\[
W_h = m_a w^2 B y_m^2 \pi L \int_0^h \left( \frac{y_z}{y_m} \right)^2 \frac{dz}{L} \quad \ldots \quad (A3.5)
\]

A comparison with the expression for the work done in a viscous damper (Chapter 3) indicates that the damping coefficient \( C_h \) equivalent to the work done in (A3.5) was:

\[
C_h = m_a B L w \int_0^h \left( \frac{y_z}{y_m} \right)^2 \frac{dz}{L} \quad \ldots \quad (A3.6)
\]

From equation (3.7) the logarithmic decrement resulting from \( C_h \) was written:

\[
\delta_h = m_a B L \pi \frac{w^2}{K} \int_0^h \left( \frac{y_z}{y_m} \right)^2 \frac{dz}{L} \quad \ldots \quad (A3.7)
\]

and when the integral of (A3.7) was assigned the symbol \( \Phi \) the following equation for \( \delta_h \) was obtained:

\[
\delta_h = m_a \left\{ \frac{2}{\sqrt{w r^2}} + \frac{2 \nu}{w r^2} \right\} L \pi \frac{w^2}{K} \cdot \phi \quad \ldots \quad (A3.8)
\]

From (3.24) the total logarithmic decrement recorded at the cylinder \( (\delta_t) \) would be

\[
\delta_t = \delta_h + \delta_s
\]
A3.2 Application of the derived equation to the cylinder II.

By substituting into (A3.8) the properties of the cylinder II and of the water it was seen that the second term of B was negligibly small compared with the first and (A3.8) reduced to:

$$\delta_h = 0.416 \frac{n^{3/2} \Phi}{K} \quad \ldots (A3.9)$$

The value was determined from inserting an equivalent mass into the equation (A2.4) which defined the frequency $n$. The equivalent mass $M_e$ contained an empirical coefficient $\beta$ which was deduced from the experimental results of II.

$$M_e = \beta \left\{ M_t + 0.2427L \left( m_s + m_a \frac{h}{L} \right) \right\} \quad \ldots (A3.10)$$

Substitution of $M_e$ into (A3.9) gave:

$$\delta_h = \frac{0.0105}{M_e} n^{-1/2} \Phi \quad \ldots (A3.11)$$

and from the tests with II, the coefficient $\beta$ was evaluated as 1.430 and the overall equation (A3.8) was modified to read:

$$\delta_h = m_a \left\{ \frac{2v}{w} + \frac{2v}{w} \right\} \frac{L \cdot \pi}{\sqrt{w}} \frac{\Phi}{1.43 \left\{ M_t + 0.2427(m_s + m_a) \frac{h}{L} \right\}} \quad \ldots (A3.12)$$

This equation was then applied to the other cantilevers tested in Chapter 8, and the calculated results presented in that chapter were all obtained from (A3.12).

The value of 1.43, to which the coefficient $\beta$ was equated was apparently dependent upon the way in which $M_e$ was defined; alternative definitions would have resulted in different values of $\beta$. However, the consistent agreement between experimental and calculated results incorporating $\beta = 1.43$ indicated that the combination of variables comprising $M_e$ were selected correctly. The individual
transient recordings themselves verified the predominance of viscous damping, and the consistent behaviour of the theoretical function based on \( \beta = 1.43 \) was considered a confirmation of the ability of the Stokes’ equation to assess the viscous damping in the present tests.

### A3.3 Derivation of the governing equation for the pendulum.

The deflected shape of the pendulum was defined by the straight line \( y_z = y_m \frac{z}{L} \); thus the analysis was more compact than for the cantilever.

Proceeding as shown in the previous section A3.1, the work done in a depth \( h \) was given by:

\[
W_h = m_a \pi \left( \frac{2}{\sqrt{v}} + \frac{2 v}{w^2} \right) \cdot \frac{h^3}{3L} \cdot \frac{y^2}{w^2} \cdot \frac{y_m}{w} \ldots (A3.13)
\]

and the corresponding damping coefficient \( C_h \) was defined by:

\[
C_h = \frac{m_a \pi}{r} \cdot 2 \sqrt{2v} \cdot \frac{h}{3L} \cdot \frac{y^2}{w^3/2} \ldots (A3.14)
\]

(since the second term of \( B \) in the square brackets of (A3.13) may be neglected for the pendulum.)

The hydrodynamic logarithmic decrement resulting from \( C_h \) was written

\[
\delta_h = \frac{2m_a}{3r} L \sqrt{2v} \frac{h}{L} \cdot \frac{y^2}{w^3/2} \ldots (A3.15)
\]

As in the cantilever tests, the determination of \( K \) was decided from inserting an equivalent mass \( M_e \) into the formula for the pendulum frequency in water. \( M_e \) was defined by:

\[
M_e = \beta \left\{ 0.2427L \left( m_s + m_a \frac{h}{L} \right) \right\} \ldots (A3.16)
\]

and the empirical coefficient \( \beta \) was evaluated as 0.5.
The conclusions reached concerning this value of $\beta$ were similar to those of the preceding section, in that absolute values were dependent upon the way in which $M_e$ was defined. The combination of $\beta$ and the other terms of $M_e$ were considered to be selected correctly and the consistently close agreements between the calculated and experimental results bore witness to that fact. The individual transient results contained only viscous damping effects and thus there were no means by which the definitions of $M_e$ could have masked amplitude-dependent damping effects.

The mass and stiffness distributions of the cantilevers and the pendulum were quite dissimilar and it was not surprising that this was reflected in the variations between the respective values of $\beta$ for the two cases.

The governing equation for the pendulum was modified to include $\beta$ and equation (A3.16) became:

$$\delta_h = \frac{8.24}{3r} \cdot \sqrt{2}v \left( \frac{h}{L} \right)^3 w^{3/2} \cdot \frac{1}{\{m_s + m_a \frac{h}{L}\}} \ldots (A3.17)$$

The values of $\delta_h$ and $\delta_t$ presented in Table 8.11 were calculated from (A3.17) and the condition:

$$\delta_t = \delta_h + \delta_s$$
List of publications by the writer

Technical papers


Contributions to non-academic journals


Internal (BHRA) reports

King, R.  "Flow induced vibrations". BHRA Report (1971) RR 1093

King, R.  "The added mass of cylinders". BHRA Technical Note (1971) TN1100

King, R.  "The hydrodynamic damping of natural vibrations of a cantilevered cylinder". BHRA Report (1972) RR1122
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BHRA Report (1973) RR1193

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Barking Creek tidal barrier".
BHRA Report (1973) RR1212