Moving block railway signalling

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MOVING BLOCK RAILWAY SIGNALLING

by


A Doctoral Thesis submitted in partial fulfilment
of the requirements for the award of Doctor of Philosophy of the
Loughborough University of Technology

February 1973

Supervisor: W Forsythe, M.Sc.
Department of Electronic & Electrical Engineering

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SUMMARY

An examination is made of the performance of four and five aspect fixed block signalling, theoretical pure moving block signalling, and a more practical quantised moving block system, when operating under both steady state and perturbed running conditions. For each signalling system, a number of basic geographical components, which are commonly found in a railway network, are analysed in order to determine their maximum capacity for a wide range of steady state operating conditions. An example is included of an algorithm which may be used to combine a number of these basic components to facilitate analysis of a more complex configuration.

In the investigation of perturbed operating conditions, a specific delay is imposed on a train, and, with a range of running headways, the resulting delays to subsequent trains are evaluated for each signalling system. Thus, it is possible to decide if a signalling system is stable under a given set of operating conditions. Also, if the system is stable, the total number of trains which experience some delay may be determined.

Finally, an examination is made of a line which simultaneously carries high speed trains, operating under moving block, and low speed trains, operating under fixed block signalling.
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LIST OF SYMBOLS AND DEFINITIONS OF PARAMETERS

A:Acceleration rate
ADL:Average double line length per route mile
AFT:Difference between the sum of train length and overlap and the integer number of distance quanta exceeding it
AOE:Time taken for a following train to travel a block
AOX:The square of the speed of the train when clearing an overlap
AS:Acceleration rate of low speed trains
A1:Smallest interval between high speed route trains of priority traffic in bidirectional junction
A5:Smallest interval between high speed route trains of non-priority traffic in bidirectional junction
B:Braking rate
BD2:Average headway between high speed route trains of non-priority traffic in bidirectional junction
BD4:Average headway between low speed route trains of non-priority traffic
BF:Braking rate of high speed trains
BL:Block length or fixed block signal spacing
BLL:Block length on a route signalled for reduced speed
BS:Braking rate of low speed trains
B1:Largest interval between high speed route trains of priority traffic in bidirectional junction
B2:Largest interval between high speed route trains passing through a diamond crossing which are more populous than low speed ones
CH: Capacity in coaches per hour.

CL, D: Distance separating a train from the signal in the rear of a component.

CLL: Distance separating a train on a low speed route from the signal in the rear of a component.

D: See CL.

Dn: Delay experienced by nth following train.

Do: Delay imposed upon the leading train.

DBL: Nominal inter passing loop distance.

DCL: Length of common line between loops.

DD, DD2: Increase in the distance separating two trains during the time it takes the second train to travel a block length.

DDT: Total increase in the distance separating the trains.

DES: The distance by which three block lengths exceeds the braking distance of the train.

DIS: The distance over which a low speed train accelerates prior to clearing a component.

DK: The minimum distance separating two trains operating under moving block when they are both at rest.

DL: Speed restriction length.

DOS: The distance by which one block length exceeds the braking distance of the train.

DPG: The distance initially separating two trains before the first begins to reduce speed because of the perturbation.

DPL: Length of a passing loop.

DQ: Distance between quantised moving block signals.

DST: Leading train waiting period.

Di: Distance which leading has travelled while following
train accelerates through a number of block lengths to reach the double yellow-speed.

E: Reduction in headway occurring before a train reattains full speed.

FHH(M): Interval between two trains stopping at a station in the loop.

FLH(M): Headway required between a stopping train and a non-stopping train.

FTD: Total delay imposed upon the leading train.

F1: Excess time taken by a train in covering a distance equal to the sum of the train length and the overlap, when braking from the treble yellow speed.

F2: Excess time taken by a train in covering a distance equal to the sum of one block length, a train length and an overlap when braking from the treble yellow speed.

H, HD: Minimum distance (front to front) between two trains.

H(I): Headways between high speed trains stopping at stations.

HA: Distance between a high speed route train and a component.

HB: Distance between a low speed route train and a component.

HD: See H

HF: Running headway between trains of a high speed convoy.

HFM: Minimum headway between trains of a high speed convoy.

HLS: Limiting value of headway below which trains observe multiple treble yellow aspects.

HLT: Limiting value of headway below which trains observe
multiple double yellow aspects.

HS: Running headway between trains of a low speed group.

HSM: Minimum headway between trains of a low speed group.

HW: Straight line headway.

HWC: Average headway of trains passing through a converging junction.

HWD: Average headway of trains passing through a diamond crossing.

HWR: Headway of trains passing through a speed restriction.

HWV: Average headway of trains passing through a diverging junction.

H2: Distance between a train and a component.

INN: Truncated quotient of the numbers of high and low speed trains in the priority flow of a bidirectional junction.

IRB: Truncated quotient of the numbers of high and low speed trains passing through a component.

K: Number of complete block lengths allowed between low speed trains.

KA, KI: Minimum number of distance quanta exceeding the sum of a train length and an overlap.

KD, KG: Number of distance quanta exceeding the distance required to brake through an integer number of distance quanta in excess of running speed.

KE: Minimum number of distance quanta exceeding a double overlap.

KF: Total number of block lengths through which the following train has travelled.

KG: See KD.
KGV: Maximum number of speed quanta with which following train is initially travelling.

KI: See KA.

KK: Number of block lengths over which a train must accelerate before attaining a speed in excess of double yellow speed.

KL: Total number of block lengths through which the leading train has travelled.

KNG: Updated number of blocks separating two trains.

KPG: Initial number of blocks separating two trains.

KS(I): Codified list of aspects observed by following train driver.

KT: Minimum number of distance quanta which exceeds the sum of AFT and DIS.

KV, KVG: Minimum number of speed quanta in excess of running speed.

M: Number of trains counting from front of convoy or group.

n; Number of following train.

NC: Number of coaches per train.

NCAP: Line speed capacity of a multi aspect signalling system.

ND1: Number of high speed priority flow trains in bidirectional junction.

ND2: Number of high speed non-priority flow trains in bidirectional junction.

ND3: Number of low speed priority flow trains in bidirectional junction.
ND4: Number of low speed non-priority flow trains in bidirectional junction.

NF: Number of high speed trains per convoy.

NPS: Number of stations between consecutive passing loops.

NS: Number of low speed trains in inter-convoy group.

NW, NX, NY, NZ: Numbers of trains travelling in the non-priority direction which may be fitted into intervals in the priority traffic.

N1: Number of trains travelling along high speed route of component.

N2: Number of trains travelling along low speed route of component.

OL: Overlap length.

OM: Distance equivalent of moving block safety margin at line speed.

ROA: Smallest interval between trains on high speed route of component.

ROB: Largest interval between trains on high speed route of component.

RTOA: Smallest interval between trains on low speed route of component.

RTOB: Largest interval between trains on low speed route of component.

RTOB4: Smallest usable interval between non-priority trains using diamond crossing.

RTOC4: Largest usable interval between non-priority trains using diamond crossing.
S: Minimum distance separating tail of one train from front of the next.

SD1: Average headway between priority trains in bidirectional junction.

SD2: Average headway between non-priority trains in bidirectional junction.

SLH: Interval between a high speed train clearing a component and a low speed train approaching the end of the speed restriction.

SLL: Interval between one low speed train clearing a component and another approaching the end of the speed restriction.

SPG: Initial headway between trains entering a perturbation.

ST: Total delay encountered by a following train.

STAFT, TAFT: Time taken for a train to move from the point where it clears an overlap to the next signal.

SUF, SUG, SUH: Sums of intervals between low speed trains stopping at stations.

SUP(I): Times for which trains remain stationary at signals.

T: Reduction in headway between low speed and high speed route trains after the low speed train clears the component.

t: A small increment of time.

tp: Time interval during which a train is delayed by an amount equal to the slack.

TA: Time taken for a train to reach exit of passing loop after resuming its journey.
TAFT: See STAFT.
TAL: Total time which a train spends in a passing loop.
TAS: Time for which a train is stationary in a loop for passing reasons.
TA1: Smallest interval between low speed route trains of priority traffic in a bidirectional junction.
TA3: Smallest interval between low speed route trains passing through a diamond crossing which are more populous than high speed ones.
TB: Time which elapses between the leading train clearing an overlap and the following train next passing a signal. Also the time taken by a train to come to rest after entering a passing loop.
TBX, TXX: Interval between the final high speed train in a convoy clearing the points at exit of a passing loop, and the first low speed train beginning to move.
TB1: Largest interval between low speed route trains of priority traffic in bidirectional junction.
TB3: Largest interval between low speed route trains passing through a diamond crossing which are more populous than high speed ones.
TB4: Largest interval between low speed route trains of the non-priority traffic.
TD: Period after which a second train comes to rest if the first remains stationary.
TDIF: Minimum leading train waiting period which causes the first following train to observe a red aspect.
TH: Trains per hour.
THH: Interval between two trains on the high speed route of a component.

THL: Interval between a train on the low speed route and one on the high speed.

TIC, TICF: Inter-convoy time.

TL: Train length.

TLAT: Time taken for a train to travel from a signal to the joint where it clears the overlap associated with it.

TLF: Length of high speed trains.

TLH: Interval between a train on the high speed route and one on the low speed route of a component.

TLL: Interval between two trains on the low speed route of a component.

TLS: Length of low speed trains.

TM, TNS: Minimum time which a train takes in travelling from a stationary position in a passing loop to the equivalent position in the next.

TOE: Total time taken for a following train to travel a number of block lengths.

TOT: Time taken by a train to negotiate a passing loop.

TPC: Points change time.

TRB: Quotient of the numbers of high and low speed trains passing through a junction.

TSF: Time taken for a second train to travel KK block lengths.

TSP: Time taken by a train in travelling from the point where it initially comes to rest in a passing loop to
the equivalent position in the next.

TST: Station stopping time.

TT: Time taken by second train to travel through (KK+1)th block.

TT(K): Overlap clearing times.

TT2: Time taken by second train to travel through (KK+2)th block and subsequent blocks.

TU: Time which elapses between the instant when the first low speed trains begins to move and when it clears the loop exit points.

TX: Period for which the leading low speed train continues at full speed after the second high speed convoy passes the inter-convoy group in the rear.

TXX: See TBS.

TZ: Time taken for the leading train to clear the overlap of a signal at which it has halted.

T1: Excess time taken in travelling through a block length at treble yellow speed instead of running speed.

T2: Excess time taken in travelling through a block length at double yellow speed instead of running speed.

V: Running speed.

VA: Speed of leading train.

VB: Speed of following train.

VE: Maximum speed of first train when second begins to move.

VEN: Speed of train when braking at point a distance equal to the sum of train length and an overlap from where it comes to rest.
VF: Running speed of high speed trains.
VI(t): Instantaneous speed as a function of time.
VID, VIN: Intermediate speeds reached at critical points in a block.
VL: Limit speed of a restriction.
VLI, VUM: Maximum speed which a train may attain by exit of block.
VM: Line speed.
VMI: Speed at which maximum capacity is obtained under moving block.
VMX: Maximum speed possible if acceleration is maintained through an entire block length.
VON: Overlap clearing speed.
VOW: Instantaneous speed when clearing the overlap in the rear of the one for which the clearing time is being calculated.
VOX: Instantaneous speed when clearing the overlap for which the clearing time is being calculated.
VPR: Propogation velocity of delays under moving block.
VQ: Difference between adjacent speed quantisation levels.
VS: Running speed of low speed trains.
 VSF, VSF2: Double yellow speed.
 VSFK: Multi yellow speed.
 VSFL, VSFL2: Double yellow speed for speed restricted lines.
 VSF1; Treble yellow speed for speed restricted lines.
VSF1; Treble yellow speed.
DEFINITIONS

Steady State Capacity

The steady state capacity may be defined as the maximum number of trains of a given length, which may be passed through a network per unit time, if all trains of the same class have the same speed distance profile.

Speed Turnouts

The speed turnouts of a railway component may be defined
as the ratio of the maximum speeds with which trains may pass over the component when following the various alternative routes.

Traffic Volumes

The traffic volumes are defined as the ratio of the numbers of trains which are following the alternative routes. It is assumed that the chronological order of the trains is such that with traffic volumes other than 50:50 a train on the less populous route does not directly follow another on the same route.

Stability of Traffic Flow

A traffic flow is regarded as stable if the delays experienced by following trains are progressively less than the delay imposed upon the leading train.

Capacity of Multi Route Components

The capacity of a railway component having more than two ports (i.e. a component which has more than one entry route and/or more than one exit route) is defined as the maximum traffic flow consistent with the specified operating conditions which may pass along the single exit route, or the sum of such flows if there is more than one exit route.
CHAPTER 1: INTRODUCTION

The increasing tendency for people who work in large cities to live at a considerable distance from their place of employment is, among other contributory factors, raising the demand for the provision of transport facilities. It may be that, in the future, the high level of traffic congestion which is being evidenced on the roads will force an increasing proportion of the total traffic onto the railways. Thus it is important for railway operators to be able to determine what is the maximum quantity of persons or goods which their railways are able to transport between any two specific points. The ability of a railway to transport this quantity of traffic depends upon the performance characteristics of the rolling stock used, and upon the signalling system employed.

A particular signalling system may become unsatisfactory for two reasons:

(1) It may be that the potential traffic exceeds the maximum volume which the system is able to accommodate.

(2) Alternatively the system may become unsatisfactory before this point is reached if small perturbations from the timetabled schedule result in increasing delays to all subsequent traffic.

The most advanced signalling system which has been widely implemented on railway networks is multi-aspect coloured light fixed block signalling with normally up to four aspects. It has been suggested that, should this system become inadequate for either of the reasons outlined above, then it might be superseded by some form of moving block signalling. It is the purpose of this thesis to examine
both fixed block and moving block signalling according to two basic criteria. These are the maximum volume of traffic which the railway is able to carry under steady state timetabled conditions, and the performance of the system when unavoidably perturbed from the desired schedule.

Previous examinations of the steady state performance of moving block signalling (e.g. references 1 and 2), have tended to be rather limited in the geographical features to which they apply. These usually seem to be straight lines and stations at which all the trains are required to stop for the same length of time. Although these items are included in this study for the sake of completeness, other commonly found items such as junctions, speed restrictions and crossings are also dealt with.

The criterion which is used to compare the unperturbed scheduled performance of the various signalling systems is the steady state capacity. This may be defined as the maximum number of trains of a given length, which may be passed through the network per unit time, if all trains of the same class have the same speed distance profile. The word 'class', as used in this definition, may have different meanings depending upon the geographical configuration which is being examined. Thus, under some circumstances, a 'class' of trains might include all those which follow the same route, while, under other conditions, a class of trains might include only those which make a scheduled halt at a specific point. The steady state capacity is defined in this way to ensure that it is not time dependent.

In examining the perturbed performance of the various signalling systems, it is important to determine if a specific externally imposed delay to a train results in delays to subsequent trains which
are equal to, or greater than, the original delay. In addition, if the delays to the following trains tend to decrease, it is important to determine the total number of trains which experience some sort of delay.

Let us suppose that a number of trains have come to rest at consecutive signals along a line which is operating under a four aspect fixed block system. The leading train moves off and accelerates smoothly up to its running speed. It is suggested, in reference 3, that, if the following trains do not begin to move before their drivers observe a double yellow aspect, the resulting headways between the trains, when they have attained running speed, will be smaller than if they move off when a single yellow aspect is first displayed. These two alternative modes of operation are examined to determine whether or not the introduction of double yellow starting is likely to produce a significant improvement in the performance of fixed block signalling under perturbed conditions. However, any conclusions which are drawn in this respect do not necessarily invalidate the equivalent section of reference 3, because the operating conditions in these two cases are rather different.

There are two alternative techniques which may be employed to determine the steady state capacity of any specific geographical configuration operating under a particular signalling system. Either the area involved may be simulated by some mathematical model of the system, or an analysis of the operation may be made. For more complex configurations, the simulation technique may be easier to implement initially, because it becomes increasingly difficult to envisage all the possible operating alternatives in the analysis approach. However, the simulation method suffers from one rather important disadvantage
which results from its basic modus operandi. Among the variables which are specified before a simulation is commenced are the times at which the various trains involved enter the simulated area. Unfortunately, since these times directly depend upon the headway between the trains, the minimum practical headways, and thus the capacity of the network, may only be found by a trial and error method. Thus, the simulation technique tends to require very large amounts of computation time compared with the equivalent analysis.

It may be argued that the analysis approach is more difficult to implement initially, and that this becomes an increasingly great problem as the geographical configuration which is being studied becomes more complex. However, it should be noted that the very process of analysis tends to develop a better understanding of why a configuration, under certain operating conditions, behaves in a particular way. This may not be easily gained from a simulation approach. Also, it may be possible greatly to reduce the problems associated with the analysis of complex configurations, if appropriate algorithms can be found to produce suitable combinations of the basic components which are dealt with subsequently. An example of such an algorithm is the one which is used to analyse a bidirectional junction, by combining the analyses of a converging junction, a diverging junction, and a diamond crossing. This avoids the need to start every new analysis from first principles, which would otherwise make this technique tedious and impractical. The technique might be further extended if any subsequent combining algorithms are designed to cascade the combinations already in existence, as far as this is possible. It is considered that, even with large and highly complex geographical areas, this method is still preferable to the simulation technique.
If we are to examine moving block signalling systems in relation to the existing fixed block signalling, it is important to understand what is meant by the term 'moving block'. There are a number of possible theoretical moving block signalling systems. The simplest is called moving space block, in which the minimum distance separating the tail of one train from the front of the following train, \( S \), is given by

\[
S = \frac{V_m^2}{2 \times B}
\]

where \( V_m \) is the maximum line speed,

and \( B \) is the minimum braking rate of the trains.

Since the distance separating the trains is independent of the instantaneous running speed at which they are travelling, the only information which the following train requires under this system is the position of the leading train in relation to its own position. When the trains are travelling at line speed, the distance separating them is just sufficient to allow the following train to brake to rest without colliding with the tail of the leading train if the latter comes to a sudden halt resulting from a derailment. However, for lower running speeds, the separating distance is larger than the minimum required to avoid compounding collisions.

Another type of moving block signalling is moving time block, in which the interval between two trains passing any point along the line is always constant and independent of running speed. In moving time block the distance separating the trains is given by

\[
S = \frac{V_m \times V_b}{2 \times B}
\]

where \( V_b \) is the running speed of the following train.

When the speed of the following train is equal to the line speed
moving space block and moving time block give identical spacing between the trains. However, at other running speeds moving time block gives a greater line capacity which is almost independent of speed, provided that the train length remains small compared with the distance separating the trains. It should be noted that this reduction in the headway between the trains for running speeds less than line speed can only be achieved if the speed of the second train is accurately measured.

However, if the necessary information about the relative position of the trains and the speed of the following train is readily available, then another type of moving block may be implemented. This is called pure moving block, and, under this system, the minimum spacing between the trains is given by

\[ S = \frac{VB^2}{2 \times B} \]

Thus, under pure moving block, the distance separating the trains is always the minimum required to avoid compounding a collision, if the leading train comes to a sudden halt (irrespective of the running speed). Thus, pure moving block gives the minimum headway which would normally be considered satisfactory between passenger or crew carrying trains. This is therefore the type of moving block which is subsequently examined in detail.

If the speed of the leading train, \( V_A \), may be measured and communicated to the following train, relative moving block may be employed. Under this system, the spacing between the trains is given by

\[ S = \frac{(VB^2 - VA^2)}{(2 \times B)} \]

provided that \( VB \) is greater than \( VA \), and, if not, then
Because relative moving block only precludes the compounding of collisions if no train decelerates at a higher rate than B, it can only be considered satisfactory for trains which do not carry any personnel.

There is also the possibility of producing a hybrid moving block system in which the criterion for separating the trains might change according to the conditions. For example, the separation might be given by the moving space block formula when both trains are travelling at full running speed. However, when approaching some fixed target such as a station, the separation could be given by the pure moving block formula. It is difficult to envisage what extra advantage would be derived from such a system, in return for the additional complexity involved.

All these theoretical moving block systems require continuous communication between the trains if they are to be implemented. In practice, however, the best that can be achieved is frequent, but intermittent, communication of the necessary information. Thus, a practical system for pure moving block signalling is also examined in detail. This system is called quantised moving block, because the tail of the leading train is only detected as it passes certain discrete points, and the speed of the following train is detected as being below certain discrete levels.

The advent of British Rail's Advanced Passenger Train which is designed to run at very high speed on the general railway network has brought with it special scheduling problems. If the railway is to carry conventional traffic as well as these very high speed trains, passing loops must be provided at regular intervals in order that the very high speed traffic may proceed unimpeded. From an economic
standpoint it is essential that these passing loops should be made as infrequent as possible, because of the high capital costs involved. It is considered that a possible solution to this problem is to run the very high speed trains in convoys (or flights) and to allow the conventional traffic to travel between these high speed flights. Initially the number of high speed trains is likely to be relatively small, while the cost of each of these trains is relatively very high. Thus, it might be possible to justify the extra cost of installing the necessary equipment which would be required to enable these trains to run according to moving block principles.

Therefore, an examination is made of a line carrying two types of train simultaneously. The first type of train has a very high running speed and runs according to moving block criteria. The second type of train has a more conventional running speed and runs according to fixed block criteria, and stops periodically in passing loops to allow the high speed trains to progress unimpeded. The optimal spacing of the passing loops may be determined in order to accommodate certain traffic requirements. Alternatively the maximum possible traffic flow, for a specified spacing between the passing loops, may be evaluated. In addition, a method of tolerancing the headways between the trains, so that the system is immune to specified levels of perturbation affecting both the high speed and the conventional trains, is demonstrated.

In practice, the conventional traffic may be required to make frequent stops at stations along the route. The effect of these stations is considered, and a comparison is made between stations situated on the common line, and stations in the passing loops. The high speed traffic may also be required to stop at stations, although such stations
will be far less frequent than those at which the low speed trains stop. The necessary berthing arrangements for the high speed convoy are examined in order that the minimum time may be taken for the flight to make its stop and resume its journey.
Since fixed block signalling is commonly used on railways throughout the world, it is an obvious and necessary standard with which to compare other types of signalling. As with other signalling systems, it is considered here from two viewpoints, namely steady state capacity, and immunity to delays.

The basic type of fixed block considered is four aspect coloured light signalling, since this is a widely implemented advanced form of signalling. In addition, a parallel analysis of five aspect fixed block is included in order to demonstrate the advantages (or otherwise) which might result from the addition of one extra caution aspect, the signalling concept remaining unaltered.

2.1 Steady State Capacity

The study of the capacity of fixed block signalling may itself be divided into two parts. Firstly there are certain simple unique components which are considered in isolation from all other components. Although several of the basic geographical components found on railway networks are dealt with, it is not suggested that this is an exhaustive list. Secondly, some of these components are combined to form a more complex geographical feature, namely, a bi-directional junction. This is included to demonstrate that the capacity of more complex railway networks may be examined by using a suitable combination of these basic components.

No further examples of this combination approach are given here for two closely related reasons. Firstly, the more complex such a
combination of components becomes, the more rare will be its occurrence in practical railway networks. Secondly, the number of independent parameters affecting the capacity increases rapidly with the number of basic components involved, and hence the possible variations in operating conditions becomes vast.

2.1.1 Basic Components

Six components are considered: a straight line, a speed restricted curve, a simple converging junction, a simple diverging junction, a diamond crossing, and a station. Four and five aspect signalling are dealt with separately.

2.1.1a Four Aspect Signalling

(i) A Straight Line

The simplest possible component of a railway is the indefinitely long straight line, the steady state analysis of which is both simple and well known. The distance between signals (called the block length - BL) is equal, in the case of four aspect fixed block signalling, to half the maximum braking distance. Thus

$$BL = \frac{VM^2}{4 \times B}$$

where VM is the maximum line speed, and B is the minimum braking rate of the trains.

Let us suppose that the train shown in Figure 1 is stationary, and that a second train approaches from the rear at line speed, VM. As it passes the signal showing the double yellow aspect it has to commence braking so as not to collide with the first train. If it passes
the signal showing a single yellow aspect with a speed of VSF, then this speed is given by

$$V_{SF} = \sqrt{V_{M}^2 - 2 \times B \times BL}$$

Now, if the running speed of the second train is no longer VM, but some lower speed V, then, for values of V which are greater than VSF, some braking is necessary before the single yellow signal is reached. However, for V ≤ VSF braking does not need to commence until after a signal displaying a single yellow aspect is passed. Thus, under these conditions, a double yellow aspect has the same meaning as a green aspect, i.e. running speed may be maintained up to the next signal. Thus, for V > VSF, the minimum distance (H), between the front of a train and the front of one following it (see Figure 2), will be given by:

$$H = 3 \times BL + OL + TL$$

where OL is the overlap, and TL is the train length ignoring the sighting distance for simplicity. Thus, if we define headway (HW) as the time interval between successive trains passing a given point, then

$$HW = \frac{H}{V} = \frac{(3 \times BL + OL + BL)}{V}$$

If, on the other hand, V ≤ VSF, then the distance between the trains may be one block less, i.e.

$$H = 2 \times BL + OL + TL$$

and hence under these conditions

$$HW = \frac{(2 \times BL + OL + TL)}{V}$$

The capacity of the line in trains per hour will be given by

$$TH = \frac{3600}{HW}$$
if $HW$ is measured in seconds. However, we are not interested in the number of trains that can be passed down a line, but in the number of passengers, or the volume of goods, which may be moved. Thus, a more useful expression for the capacity of a line is the maximum coaches per hour, which will be given by

$$CH = NC \times TH$$

where $NC$ is the number of coaches per train.

A graph of the capacity of a straight line as a function of running speed is given in Figure 3. Note that the discontinuity occurs at the speed $VSF$, and that the line speed $VM$ is 100mph. For any running speed, $V$, less than this value, a higher capacity can be achieved if the line is resignalled for this speed, i.e. if the signal spacing is given by

$$BL = V^2/(4 \times B)$$

In practice some effort is made to place the signals at intervals which correspond to the speed at which traffic is travelling over the line, but there is obviously a practical limit to the effectiveness of this policy. The capacity of a line with four aspect fixed block signalling and a line speed equal to the running speed of the trains is also shown in Figure 3. It may be noted that, for a running speed of 40mph, the capacity of a line signalled for this speed is about 250% of that of a line signalled for 100mph.

(ii) A speed restricted curve

Let us consider a length of track $DL$, along which the maximum speed of any train is limited to $VL$. In order to find the steady state capacity of this component, it is necessary to find the minimum
headway between trains, such that successive trains possess identical speed-distance profiles. In practice this means that the spacing of the trains should be sufficient to obviate the need for any braking, other than that required to reduce the speed of the train to $VL$ at the start of the restriction.

It is evident that the required headway is equal to the appropriate straight line headway, for any running speed $V$ which is less than or equal to the limit speed $VL$. For values of $V$ greater than $VL$, a larger headway will be required. As two trains move through the speed restriction, there will be a critical instant such that, if the headway between the trains is sufficient at that moment, it will of necessity be sufficient at all other times. In Appendix A this critical instant is shown to occur when the following train is about to commence braking for the restriction.

Let us consider this critical situation. In order that there is no interaction between the trains, the distance separating the front of one train and the front of the following trains is given by

$$HD = HW \times V$$

The minimum headway required for this speed restriction is equal to the time taken for the second train to move from its present position to the position occupied by the first train. In order to determine this time, firstly it is necessary to find the quantity $W$, which is given by

$$W = HD - (V^2 - VL^2) \times (A + B)/(2 \times A \times B) - DL$$

(where $A$ is the train acceleration rate)

and is the amount by which the straight line minimum distance separation exceeds the total length of the route over which a train passes at
reduced speeds. If $W$ is positive, this means that the first train will have already regained its running speed, after passing through the restriction, at the critical instant. Thus, the required headway for the trains, when running through the restriction, will be given by

$$H_{WR} = \frac{(V - VL) \times (A + B)}{(A \times B)} + \frac{DL}{VL} + \frac{W}{V}$$

If on the other hand $W$ is negative, then, at the critical instant, the first train will not have regained its running speed. Thus it is necessary to evaluate another quantity $W_2$, which is the amount by which the straight line minimum distance separation exceeds the distance between the points $A$ and $C$ in Figure 140. This is given by

$$W_2 = HD - \frac{(V^2 - VL^2)}{(2 \times B)} - DL$$

If $W_2$ is positive, this means that the first train has passed through the speed restriction, and is accelerating at the critical instant. Thus, the required headway is given by

$$H_{WR} = \frac{(V - VL)}{B} + \frac{DL}{VL} + \frac{(VIN - VL)}{A}$$

where $VIN$ is the instantaneous speed of the first train at this point, and is given by

$$VIN = \sqrt{V_2^2 + 2 \times A \times W}$$

(remember that, in this instance, $W$ is negative)

If $W_2$ is negative, the first train is still in the speed restriction at the critical instant, and the necessary headway is given by

$$H_{WR} = \frac{(V - VL)}{B} + \frac{(DL + W_2)}{VL}$$

The capacity of the speed restriction may be found from the minimum possible headway, $H_{WR}$, in the same way that the capacity of the straight line was obtained from $H_W$, in the previous section.
A graph of the capacity of a speed restricted curve is shown as a function of running speed in Figure 4, for various values of limit speed, VL. It should be noted that the speed restriction only has an effect on the capacity at speeds which are above the limit speed. The size of the discontinuity in capacity at the speed VSF tends to be reduced as the limit speed, VL, is reduced, and the capacity above this speed becomes less dependent upon running speed.

A graph, showing the effect upon capacity of varying the speed restriction length, is given in Figure 5. It should be noted that increasing the length of the restriction has the effect of further reducing the capacity for running speeds above the limit. However, if the restriction is of such a length that, at the critical instant, the leading train is still in the restriction, then further increases in the restriction length do not produce any additional reduction in the capacity at that running speed.

(iii) A simple converging junction

Let us examine the simple converging junction shown in Figure 6, and let us assume in the first instance that the running speed of the trains is equal to line speed VM, and that the junction has speed turnouts of 1:1. In order to evaluate the capacity of the junction, it is necessary to consider some trains approaching the junction from both routes. We then have to determine the minimum interval which elapses between one train occupying the position in which train 1 is shown, and the next train taking up that position. This depends upon which route the trains have traversed while approaching the junction, and thus we need to find four time periods measured from the exit of the junction.
Please note that

(a) Capacity is a function of train length, is the number of coaches per train.

(b) The capacity of a straight line increases with increasing train length (with other parameters remaining constant), but the rate of increase diminishes.

(c) Unless otherwise stated, trains may be assumed to be comprised of ten coaches and have a length of 660 ft.
FIGURE 4
Speed Restricted Curves, 1200 ft. long
A: - Limit Speed 33 m.p.h.
B: - .. 50 ..
C: - .. 67 ..
D: - .. 100 ..
FIGURE 5: Speed Restricted Curves, Limit Speed 33 m.p.h.

A: - Restriction Length 600 ft.
B: - " " 1200 "
C: - " " 2400 "

FIGURE 6: Route: A, Route: B

Train 2
Train 1

Route: A
THH: the interval between two trains from route A
TLL: the interval between two trains from route B
THL: the interval between a train from route B and one from route A
TLH: the interval between a train from route A one from route B.

In the simplified case which we are considering, THH and TLL are both equal to the straight line headway HW. In Figure 6, train 1 is shown to have just cleared the first overlap beyond the junction. This is the first possible moment that the points can begin to move to reset the route and allow the passage of train 2. A fixed period of time, TPC, must elapse before this process is complete and the signal S2 may be set to yellow (if signal S4 is still at red). Thus, a period of TPC elapses after the situation shown in Figure 6, before the signal S1 is cleared to green, which will be the earliest time at which train 2 may arrive at this signal. Therefore, train 2 must be a distance D given by

\[ D = TPC \times V \]

from signal S1 in the situation shown in Figure 6, and hence the headway between train 1 and train 2 must be equal to THL and TLH (since speed turnout is 1:1), so that

\[ THL = TLH = HW + TPC \]

For any other running speed V, the same expression may be applied, provided that the appropriate value of HW is used.

If N1 and N2 are the percentages of the approaching traffic which traverse routes A and B respectively, the average headway, HNC, between the trains leaving the junction, provided that N2 > N1, is given by
\[ \text{HMC} = \frac{(N_2 - N_1) \times TLL + N_1 \times (Tll + TLH)}{N_1 + N_2} \]

Otherwise

\[ \text{HMC} = \frac{(N_1 - N_2) \times THH + N_2 \times (Tll + TLH)}{N_1 + N_2} \]

This value of headway may then be used to obtain the capacity of the component, as in previous sections.

Let us now consider the effects on the junction of having speed turnouts other than 1:1, the low speed route being route B (in Figure 6), and having a limit speed of VL. It is useful to distinguish between two basic classifications of this type of junction. The entire length of route B may have a line speed of VL, in which case the junction will be referred to as a converging junction of high and low speed routes. Alternatively, the speed restriction of VL may apply over a relatively short length on either side of the points, in which case the junction will be referred to as a converging junction of two high speed routes.

The value of THH is always unchanged at its former value of HW, since the running conditions on route A are unaltered, and this is also the value of TLL, for \( V \leq VL \).

However, for a junction of high and low speed routes, the interval TLL is equal to either the straight line headway HW, or the headway evaluated for a speed of VL, whichever is the greater. On the other hand, for a junction of two high speed routes, the low speed length of track near the junction must be treated as a speed restriction, so that the analysis of the previous section may be used to evaluate the required value of TLL.

If once again we consider the situation shown in Figure 6, the interval THL must be at least (HW + TPC) to ensure that the points have time to reset before train 2 reaches signal S1. However, from the
moment illustrated in Figure 6, train 2 may, under some conditions, reduce the headway between itself and train 1. This is because train 1 may not be travelling with a speed of \( V \) at the instant shown. Thus, we find the distance \( \text{DIS} \) over which train 1 has already been accelerating, provided it has not already attained running speed. This is given by:

\[
\text{DIS} = \text{BL} + \text{OL} + \text{TL} - \text{DL}
\]

Then the instantaneous speed, \( \text{VIN} \), of train 1 is given by

\[
\text{VIN} = \sqrt{\text{VL}^2 + 2 \times A \times \text{DIS}}
\]

The final headway between train 1 and train 2 will be less than \( \text{THL} \) by an amount \( T \) given by:

\[
T = (V - 2 \times \text{VIN} + \text{VIN}^2/V)/(2 \times A)
\]

It should be appreciated that the final headway must not be allowed to fall below \( \text{HW} \), so that provided \( T \leq \text{TPC} \), then \( \text{THL} = \text{HW} + \text{TPC} \) but if \( T > \text{TPC} \), then \( \text{THL} = \text{HW} + T \).

It is evident that, for a junction of two high speed routes when the running speed \( V \) is equal to or less than \( \text{VL} \), the value of the interval \( \text{TLH} \) is given by

\[
\text{TLH} = \text{HW} + \text{TPC}
\]

However, if \( V > \text{VL} \), we must consider the situation in more detail. In Figure 7 the instant illustrated is that at which the points have just completed changing from route A to route B. It should be noted that, although in the diagram the distance \( (\text{TPC} \times V) \) is shown as longer than a block length, this is not necessarily the case. Now, just prior to the instant shown, signal S4 displays a red aspect, signal S3 a yellow aspect, and signal S2 a double yellow aspect. If \( V > \text{VSF} \), then train 2 may not have passed signal S2 previously, and hence it
may at best assume position A. Alternatively, if \( V < V_{SF} \), train 2 may pass a double yellow (i.e. signal S2) with impunity, but not a single yellow. Hence, under these conditions, it may take up position B. If train 2 takes up position A, then \( CL = BL \), but if it takes up position B, \( CL = 2 \times BL \).

The maximum speed, \( VIN \), which train 2 may be doing at the instant illustrated in Figure 7 is given by

\[
VIN = \sqrt{VL^2 + 2 \times B \times CL}
\]

Now, if \( SLH \) is the period of time between train 1 clearing the overlap associated with signal S5, and train 2 coming to the end of the restriction (when it is about to accelerate), then if \( VIN < V \), \( SLH \) is given by

\[
SLH = \frac{(VIN - VL)}{B} + TPC + \frac{DL}{VL}
\]

But if \( V < VIN \),

\[
SLH = \frac{(V - VL)}{B} + \frac{(CL - (V^2 - VL^2)/(2 \times B))}{V} + \frac{DL}{VL} + TPC
\]

The instantaneous speed, \( VON \), at which train 2 is travelling when it clears the overlap associated with signal S5 is given by

\[
VON = \sqrt{VL^2 + 2 \times A \times DIS}
\]

where \( DIS \) is given above.

If \( V \geq VON \),

\[
TLH = SLH + \frac{(VON - VL)}{A}
\]

But if \( V < VON \),

\[
TLH = SLH + \frac{(V - VL)}{A} + \frac{(DIS - (V^2 - VL^2)/(2 \times A))}{V}
\]

However, for a junction of high and low speed routes, the above expressions for \( TLH \) are not applicable. Instead, the block length on the low speed route B is not equal to that on route A, but is given by
\[ \text{BLL} = \frac{\text{VL}^2}{4 \times B} \]

This gives rise to a new "double yellow speed", VSFL, which is given by

\[ \text{VSFL} = \sqrt{\frac{\text{VL}^2}{4 \times B} - 2 \times B \times \text{BLL}} \]

If \( V < \text{VSFL} \), then, again referring to Figure 7, train 2 may take up position B, and hence \( \text{CLL} = \text{BLL} \). But if \( V > \text{VSFL} \), then train 2 must take up position A, and \( \text{CLL} = 2 \times \text{BLL} \). Then for \( V \leq \text{VL} \),

\[ \text{TLH} = \frac{(\text{CLL} + \text{BLL} + \text{OL} + \text{TL})}{V} + \text{TPC} \]

Otherwise, \( \text{SLH} \), as defined above, is given by

\[ \text{SLH} = \frac{(\text{CLL} + \text{DL})}{\text{VL}} + \text{TPC} \]

With the speed \( \text{VON} \), at which train 2 clears the overlap associated with signal S5, obtained exactly as before, if \( V < \text{VON} \),

\[ \text{TLH} = \text{SLH} + \frac{(V - \text{VL})}{A} + \frac{(\text{DIS} - (V^2 - \text{VL}^2))/(2 \times A)}{V} \]

but if \( V > \text{VON} \),

\[ \text{TLH} = \text{SLH} + \frac{(\text{VON} - \text{VL})}{A} \]

The average headway between the trains, \( \text{HWC} \), may be obtained from the four quantities \( \text{THH}, \text{TLL}, \text{THL}, \) and \( \text{TLH} \) as with the junction with 1:1 turnouts, and hence from this the capacity of the junction may be evaluated.

A graph of the capacity of a converging junction with speed turnouts of 1:1 is shown plotted against traffic volumes in Figure 8. As expected this function is symmetrical about traffic volumes of 50:50. It should be noted that, at very low running speed, the capacity is almost independent of traffic volumes. This is because
Definition of capacity of multi route components

The capacity of a railway component having more than two ports (is a component which has more than one entry route and/or more than one exit route) is defined as the maximum traffic flow consistent with the specified operating conditions which may pass along the exit route, or the sum of such flows if there is more than one exit route.

Definition of speed turnouts

The speed turnouts of a railway component may be defined as the ratio of the maximum speeds with which trains
may pass over the component when following the various alternative routes.

Significance of traffic mix

Since the chronological order of trains passing through a component may affect the capacity of the component, it is assumed in all multi-route components that, with traffic volumes other than 50:50, a train on the less populous route does not directly follow another on the same route.
the straight line headways for these speeds are very large compared with the points change time. However, as the running speed increases, the straight line headways decrease, and the effect of the points change time becomes more marked. For any given running speed, the capacity is a minimum for 50:50 traffic volumes, since this requires the largest number of points changes. It should also be noted that, for a given ratio of traffic volumes, the capacity, as a function of running speed, exhibits the familiar discontinuity at the speed VSF.

The capacity of a converging junction of two high speed routes, with speed turnouts of 100:33, is shown in Figure 9. It is evident that the effect of the speed restriction upon the traffic entering the junction on route B (Figures 6 and 7) is to increase THL, TLH and TLL from the equivalent values for speed turnouts of 1:1, although this only applies for running speeds greater than VL (in this case 33 m.p.h.). The increase in THL and TLH enlarges the difference between these quantities and TNN. Thus the capacity for traffic volumes of 50:50 is decreased, while for 100:0 it remains unaltered. The increase in TLL is responsible for much of the decrease of capacity in the range of traffic volumes where there is a majority of slow traffic. This decrease is so marked that, for running speeds of 50 m.p.h. and above, the minimum capacity no longer occurs at 50:50 traffic volumes, but at 0:100 (all traffic approaching junction along route B, when the junction acts as a simple speed restriction).

The equivalent graph, for a junction of high and low speed routes, is given in Figure 10. It may be seen that the symmetry about traffic volumes of 50:50 is present in this case, which can only be true if TLL is equal to TNN. Thus, we may deduce that the capacity of a line
signalled for 33mph equals or exceeds that of a line signalled for 100mph at all running speeds up to 33mph. Also, the capacity of the 33mph line operating at line speed, equals or exceeds the capacity of the 100mph line operating at any running speed above 33mph. Careful examination of Figure 3 shows this to be the case.

It should be noted that, with 50:50 traffic volumes and running speeds of 70mph, 90mph and 100mph, the capacity is lower than with speed turnouts of 1:1, thus making the variation with traffic volumes greater at this speed. However, at running speeds of 40mph, 50mph and 80mph the reverse is true. This must be due entirely to variation in THL and TLH. It may be seen that, as the difference between V and VL increases, THL must tend to increase, since more time is required for the slow train to accelerate up to V. However, TLH is less than with speed turnouts of 1:1, since, although the slow trains move through the junction more slowly, they are much nearer to the junction when the points change is complete (due to a much shorter block length on route B). A discontinuity in the value of TLH occurs at a speed of VSF (72mph) in the case of the junction with speed turnouts 1:1, due to the fact that, for running speeds below this figure, the "route B" trains may be one block length closer to the junction at the completion of a points change, than is possible for running speeds above VSF.

(iv) A simple diverging junction

Let us consider the diverging junction shown in Figure 11. The diagram illustrates the moment when the points have just completed
changing route, from route B to route A, after the passage of train 1. As with the converging junction, which has already been described, we need to evaluate the four time periods THH, THL, TLH and TLH. From these values, the average headway, between trains approaching the junction, HW, and hence the capacity, may be evaluated exactly as before.

The points change begins when train 1 clears the overlap associated with signal S5. Hence, when this is completed, it will have travelled a further distance D which depends upon the type of junction under examination (two high speed routes or high and low speed routes), the speed turnouts, etc. However, it is not necessary, for the purposes of this analysis, to determine the value of D since we know that, after a period TPC, the points change is complete. During this period, signal S4 continues to display a red aspect, while signals S3, S2 and S1 display single yellow, double yellow, and green aspects respectively. Thus, at the completion of the points change, train 2 may be in position A or position B, depending upon whether the running speed, V, is greater than, or less than, VSF.

When the points change is complete, signals S3 and S4 change to green aspects. This will also apply to signal S2, if train 2 is in position A. Therefore, train 2 proceeds at running speed, V, and a period of HW (the straight line headway for that speed) later, it clears the overlap associated with signal S6. If the train, which is following train 2, is travelling along route B, the points may begin to change again at this instant. Train 2 has thus reached the equivalent position to that occupied by train 1 when it cleared the overlap of signal S5. Hence, for all cases:
\[ \text{THL} = \text{HW} + \text{TPC} \]

Also, it is evident that, for all cases, the value of THH is equal to the straight line headway, HW.

Now, let us consider the situation illustrated in Figure 1.2. Train 1 has just cleared the overlap associated with signal S6, and the points are about to change. If \( V \leq VSF \), train 2 may be in position B, which is a distance \( D \) from signal S3, so that it arrives at that signal at the moment that the points change is completed, i.e. TPC later. If \( V > VSF \), train 2 may be in position A, which is a corresponding distance from signal S2. If CL is the distance between the position which train 2 occupies, when the points change is completed, and the start of the speed limit VL at the signal S4, then

\[
\begin{align*}
\text{CL} &= \text{BL} \quad \text{if } V \leq VSF \\
\text{or} \quad \text{CL} &= 2 \times \text{BL} \quad \text{if } V > VSF
\end{align*}
\]

Let SLH be the time which elapses between the instant shown in Figure 12 and the moment when train 2 clears the end of the speed limit (this will always be a distance DL from signal S4, even if DL is a very large nominal value, as in the case of a junction of high and low speed routes).

Then,

\[
\text{SLH} = \text{TPC} + \frac{(V - VL)}{B} + \frac{(\text{CL} - (V^2 - VL^2)/(2 \times B))}{V} + \frac{DL}{VL}
\]

Unless, of course, \( V \leq VL \), in which case we may say immediately that

\[
\text{TLH} = \text{HW} + \text{TPC}
\]

However, if \( V > VL \), and if DIS is the distance over which train 2 has been accelerating, from the end of the speed restriction until the
time it clears the overlap of signal S5, then

\[ \text{DIS} = BL + OL + TL - DL \]

If DIS is negative, train 1 is still in the speed restriction, which certainly will be the case with the junction of high and low speed routes. Under these conditions

\[ \text{TLH} = \text{SLH} + \frac{\text{DIS}}{\text{VL}} \]

(Remember that DIS is negative.)

However, if DIS is positive, it is necessary to find the speed \( V_{IN} \), which is the maximum speed which train 2 might attain at the moment it clears the overlap of signal S5. This is given by:

\[ V_{IN} = \sqrt{\text{VL}^2 + 2 \times A \times \text{DIS}} \]

If \( V \geq V_{IN} \), train 2 will still be accelerating as it clears the overlap, and

\[ \text{TLH} = \text{SLH} + \frac{(V_{IN} - \text{VL})}{A} \]

Alternatively, if \( V < V_{IN} \), the train is once again travelling with its running speed, V, and

\[ \text{TLH} = \text{SLH} + \frac{(V - \text{VL})}{A} + \frac{(\text{DIS} - (V^2 - \text{VL}^2))/(2 \times A)}{V} \]

Finally, let us consider the value of the period TLL. If a junction of two high speed routes is being analysed, TLL is equal to the headway required by a speed restriction of appropriate length. Alternatively, if we are examining a junction of high and low speed routes, TLL must be made equal to the largest of the following three quantities:

(a) the straight line headway for a running speed of V, i.e. equal to \( \text{THH} \);
(b) the headway required when braking the trains for a very long speed restriction of value VL;

(c) the headway required on route B, when remote from the junction, which is given by

\[ S_{LL} = \frac{(3 \times B_{LL} + O_{L} + T_{L})}{V_{L}} \]

for \( V \geq V_{L} \),
or

\[ S_{LL} = \frac{(3 \times B_{LL} + O_{L} + T_{L})}{V} \]

for \( V_{L} > V > V_{SFL} \),
or

\[ S_{LL} = \frac{(2 \times B_{LL} + O_{L} + T_{L})}{V} \]

otherwise, where \( B_{LL} \) and \( V_{SFL} \) are given by

\[ B_{LL} = \frac{V_{L}^2}{4 \times B} \]

and

\[ V_{SFL} = \sqrt{V_{L}^2 - 2 \times B \times B_{LL}} \]

For speed turnouts of 1:1, the converging and diverging unidirectional junctions have identical values of \( T_{HI}, T_{HL}, T_{LH} \) and \( T_{LL} \). Hence it is evident that the capacity of the diverging junction is exactly equal to that of the converging junction for equivalent operating conditions. Hence Figure 8 applies to the diverging junction just as much as to the converging junction.

A graph of the capacity of a diverging junction of two high speed routes is given in Figure 13. Observation of this graph shows that the junction behaves in a similar way to the converging junction of two high speed routes (compare with Figure 9). The main difference
between the two cases, is that the capacity of the diverging junction for high running speeds, tends to be slightly greater than that of the converging junction, especially at traffic volumes of 50:50. Such differences arise from the need for a train traversing the slow route to brake on the line common to both routes when approaching a diverging junction, as opposed to the need for some acceleration on the common line when leaving a converging junction. This will have the most marked effect when the differences between running speed, \( V \), and limit speed, \( V_L \), are greatest, i.e. when long periods of braking and acceleration are involved.

The equivalent capacity graph, for a diverging junction of high and low speed routes, is shown in Figure 14. When a large majority of the traffic using the junction traverses the slow route, the capacity is considerably less than that indicated in Figure 13. This may be explained in terms of an increase in \( T_{LL} \), due to the speed limit being of far greater length in the junction of high and low speed routes. With 50:50 traffic volumes and equivalent running speeds, the capacity in Figure 14 is slightly less than in Figure 13, indicating an increase in \( T_{HL} \). Whereas, in the junction of two high speed routes, some acceleration may be possible by a train on route B before clearing the overlap of signal S5 (Figure 11), this may certainly not be allowed in the junction of high and low speed routes. Thus, a train will tend to take longer to clear the junction and hence \( T_{HL} \) will be increased.
(v) A diamond crossing

In examining the performance of a diamond crossing, reference will be made to the one shown in Figure 15. In the diagram, train 1 has just cleared the overlap of signal S3, and hence the signal S2, which protects the crossing on route B, is cleared at this instant from a red aspect to a green aspect. Simultaneously, signals S4 and S5, which were displaying single yellow and double yellow aspects respectively, are also cleared to green aspects. At the same time, train 2 will take up position A or position B, depending upon whether its running speed V is greater than or less than VSF.

If there is no speed restriction for the trains on route B, i.e. if the speed turnouts are 1:1, then the headway required between the trains, HWD, is equal to the straight line headway, HW, since this is the time it takes train 2 to move from the position which it occupies at the instant of Figure 15 to the point where it clears the overlap of signal S1. Thus the capacity of a crossing with 1:1 speed turnouts is equal to the straight line capacity and independent of the traffic volumes.

Let us examine a crossing of two high speed routes, i.e. consider a speed restriction, VL, of length DL on route B starting at signal S2. If V ≤ VL, then the crossing is equivalent to one having speed turnouts of 1:1, which has already been dealt with. However, if V > VL, then we make CL equal to the distance which separates train 2 from the start of the restriction at signal S2, at the moment when that signal is cleared to a green aspect, i.e.

if V ≤ VSF,

\[ CL = BL \]
and if $V > V_{SF}$

$$CL = 2 \times BL$$

Again, it is necessary to determine the length of the intervals $THH$, $THL$, $TLH$ and $TLL$, measuring them between the arrival of the appropriate trains at the crossing itself (C in Figure 15). If we assume that $DL/2$ is the distance of this point from the signal in the rear on both routes, then

$$TLH = \frac{BL + OL + TL - DL/2 + CL - (V^2 - VL^2)/(2 \times B)}{V} + \frac{DL}{2 \times VL} + \frac{(V - VL)}{B}$$

which is the time taken for a train on route A to move from C to clear the overlap of signal $S3$, added to the time taken for a train on route B to travel from either position A or position B to the point C.

The interval $THH$, between two trains on route A, will be the straight line headway $HW$ for the running speed $V$ in all cases. The maximum speed, $VIN$, which a train on route B can attain by the time it has passed through the speed restriction and cleared the overlap of signal $S1$ is

$$VIN = \sqrt{VL^2 + 2 \times A \times (BL + OL + TL - DL)}$$

Now, if $VIN \leq V$, the train is still accelerating when it clears the signal $S1$ overlap. Hence, $SHL$, the time taken for the train to move to this point from C, is given by

$$SHL = \frac{DL}{2 \times VL} + \frac{(VIN - VL)}{A}$$

However, if $VIN > V$, then

$$SHL = \frac{DL}{2 \times VL} + \frac{(V - VL)}{A} + \frac{(BL + OL + TL - DL - (V^2 - VL^2)/(2 \times A))}{V}$$
The interval $TH_L$ is given by

$$TH_L = S_H + (CL + DL/2)/V$$

The value of $TLL$ is found by considering a speed restriction of the appropriate length. The intervals $TH_L$ and $TLL$ may, in theory, take values less than the straight line headway, $H_W$, but otherwise the values of $TH_H$, $TH_L$, $TL_H$ and $TLL$ are used exactly as before to obtain the average headway, $H_W$, and hence the capacity of the crossing.

Now let us consider that we have a crossing of high and low speed routes, i.e. that route $B$ is signalled for a lower line speed $VL$.

The block length on route $B$ is given by

$$BLL = VL^2/(4 \times B)$$

and there will be a lower double yellow speed on this route given by

$$VSFL = \sqrt{VL^2 - 2 \times B \times BLL}$$

In all cases the value of $TH_H$ will be the straight line headway, $H_W$.

If $V > VL$

$$TL_H = (BL + OL + TL - DL/2)/V + (2 \times BLL + DL/2)/VL$$

since, in this case, the train on route $B$ must be at signal $S_5$, when the overlap of signal $S_3$ is cleared.

$$TH_L = (BLL + OL + TL - DL/2)/VL + (CL + DL/2)/V$$

where $CL$ is as defined previously, and

$$TLL = (3 \times BLL + OL + TL)/VL$$

since the trains on route $B$ will be running at their line speed, $VL$.

Alternatively, if $V < VL$
let \( \text{CLL} = \text{BLL} \) if \( V \leq \text{VSFL} \)
or

\[ \text{CLL} = 2 \times \text{BLL} \] otherwise.

Then,

\[ \text{THL} = \frac{\text{BLL} + \text{OL} + \text{TL} + \text{CL}}{V} \]
\[ \text{TLH} = \frac{\text{BL} + \text{OL} + \text{TL} + \text{CLL}}{V} \]
and \( \text{TLL} = \frac{\text{BLL} + \text{OL} + \text{TL} + \text{CLL}}{V} \)

In the case of the crossing of high and low speed routes, there is no need to put a minimum value of straight line headway, \( \text{HW} \), on \( \text{THL} \), \( \text{TLH} \) and \( \text{TLL} \). The four intervals are used to obtain the capacity in the same way as before.

A graph of the capacity of a diamond crossing of two high speed routes is given in Figure 16. It may be seen that the capacity of the crossing, for traffic volumes of 0:100 and 100:0, is equal to that of the converging and diverging junctions of two high speed routes under equivalent operating conditions. However, the capacity, at other values of the traffic volume ratio, will tend to be greater than either of the junctions, since no points change time is involved in this case.

At running speeds below the limit speed, \( \text{VL} \) (33 mph), the capacity of the crossing is independent of traffic volumes, as in the case of a crossing having speed turnouts of 1:1.

The capacity of a diamond crossing of high and low speed routes is shown plotted against traffic volumes in Figure 17. For all running speeds above the limit speed, the capacity of the crossing with traffic volumes of 0:100 is constant. This is because all the traffic will be on the "slow" route, which will be operating at its line speed.
FIGURE 16
Diamond Crossing, 4 of Two High Speed Routes
Speed Turnouts 100:33

[Graph showing capacity in coaches per hour vs. fast to slow traffic volumes]
(VL). Hence the capacity will be that of a line operating at, and signalled for, a speed of VL (please compare with the curve of resig-nalled four aspect fixed block in Figure 3). At running speeds less than VL, the capacity is reduced because the route is no longer operating at line speed. The values of capacity, for traffic volumes of 100:0 are identical with those of a crossing of two high speed routes, since under these conditions both may be regarded as a straight line with maximum speed VM.

(vi) An isolated station

In examining the steady state performance of this component, we shall make reference to Figure 18. In order to keep this analysis reasonably general, it is necessary to consider a situation in which a variable percentage of the total traffic approaching the station may be required to make a stop at it.

If the variables THH, THL, TLH, TLL, N1 and N2 are redefined in the way explained below, they may be used to give the average headway, HWS, using the expressions which previously gave the headway for the converging junction, HWC.

THH is the interval between two stopping trains
THL is the interval between a non-stopping train and a stopping train
TLH is the interval between a stopping train and a non-stopping train
TLL is the interval between two non-stopping trains
$N_1$ is the percentage of stopping trains

$N_2$ is the percentage of non-stopping trains.

If two trains do not stop at the station, the headway required between them is the straight line headway, $H_{NL}$, and hence this is the value of $T_{HL}$ in all cases. If a stopping train is following a non-stopping train, the earliest moment when the station berth becomes clear for the stopping train to approach it is shown in Figure 18, i.e. this is when train 1 has just cleared the overlap of signal S4. At that moment train 2 will be in position A, if its running speed, $V$, is greater than $V_{SF}$, or position B, if not. If $CL$ is the distance separating train 2 and the station berth at that instant, then if

$$V > V_{SF}, \quad \text{let } CL = 3 \times BL$$

and if

$$V \leq V_{SF}, \quad \text{let } CL = 2 \times BL$$

If train 2 is a stopping train, then it comes to rest in the station a period $SHL$ after the instant shown in Figure 18, where

$$SHL = \frac{V}{2 \times B} + \frac{CL}{V}$$

The maximum speed, $VIN$, which train 2 may have attained by the time it clears the overlap of signal S4 (i.e. when in the position of train 1 in Figure 18), is given by

$$VIN = \sqrt{2 \times A \times (OL + TL)}$$

If $VIN < V$, train 2 is still accelerating when it clears the overlap of signal S4, and, if $TST$ is the station stop time, $T_{HL}$ is given by

$$T_{HL} = SHL + TST + VIN/A$$
This is because THL is the time it takes for train 2 to travel from its own position at the instant illustrated in Figure 18 to the position which is occupied by train 1.

If VIN > V, train 2 will already have attained its full running speed of V, and hence THL is given by

\[ THL = V \times (A + B)/(2 \times A \times B) + (CL + OL + TL)/V + TST \]

In determining the value of THL, it has not been necessary to make any assumptions about the speed-distance profile of the leading train. Thus we may say that this headway would be equally applicable if the leading train had previously stopped at the station. Hence THL will, under all conditions, be equal to THL.

Now, let us suppose that, in Figure 18, train 1 is a stopping train, but train 2 does not stop. If train 1 has attained its running speed by the time it clears the overlap of signal S4, (i.e. if VIN ≥ V), then train 2 will be in position A, if V > VSF, or position B otherwise, since this is the minimum straight line separation, and the headway will not change subsequently. Thus, if

\[ VIN ≥ V, \quad THL = HW \]

However, if VIN < V, then train 2 may not have advanced as far as position A, because some decrease in distance separation will occur until train 1 attains the speed of V. Hence TLH is given by

\[ TLH = HW + V/(2 \times A) - VIN \times (2 \times V - VIN)/(2 \times A \times V) \]

The values of THH, THL, TLH and TLL, obtained from the expressions outlined above, may be used directly to obtain the average headway, and need not be checked against any minimum value.
The capacity of an isolated station is shown plotted against the ratio of stopping to non-stopping traffic in Figure 19. As the percentage of traffic stopping at the station is increased, the capacity will decline, especially at higher running speeds, where the extra headway required for a stop at the station is large compared with the straight line headway. Suppose we measure the time taken for a train to travel between two points, one each side of the station, and both situated a large distance from it. Suppose, additionally, that the time taken for a train to travel between these two points without stopping at the station, is compared with the time if a stop is included. At high running speeds, the difference in the two times will be greater than at lower running speeds, since longer periods of braking and acceleration will be required at higher speeds.

In Figure 20, the effect of varying the station stop time is demonstrated. It may be observed that, when a large percentage of the total traffic is stopping at the station, the length of the station stop time does not have a very marked effect on capacity. This is due to the station stop time being only a relatively small part of the total time, by which a train is delayed in making a stop at the station.

2.1.1b Five Aspect Signalling

(i) A Straight Line

For five aspect fixed block signalling, the block length, Bl, is shorter than for four aspects, and is given by
BL = \( VM^2 / (6 \times B) \)

If \( VSF_1 \) and \( VSF_2 \) are the maximum speeds at which trains may be travelling when they encounter double yellow and single yellow aspects respectively, these speeds are given by

\[
VSF_1 = \sqrt{VM^2 - 2 \times B \times BL}
\]

and

\[
VSF_2 = \sqrt{VM^2 - 4 \times B \times BL}
\]

As demonstrated in Figure 21, the minimum distance separating two trains may take three different values depending upon the running speed, \( V \). If \( V \leq VSF_2 \), a train may observe a double yellow aspect without having to brake subsequently. Hence, under these conditions, the straight line headway, \( HW \), is given by

\[
HW = (2 \times BL + OL + TL) / V
\]

If \( VSF_2 < V \leq VSF_1 \), a train may observe a triple yellow aspect with impunity, but the observation of a double yellow will precipitate braking. Under these circumstances

\[
HW = (3 \times BL + OL + TL) / V
\]

If \( VSF_1 < V \), a train may not continue at its running speed, unless it continues to encounter green aspects. Hence

\[
HW = (4 \times BL + OL + TL) / V
\]

At this point, reference could usefully be made to Appendix B, which describes a method whereby the capacity of a straight line signalled for fixed block of any number of aspects may be easily evaluated.

In Figure 22, the capacity of a straight line signalled for five
FIGURE: 21

Train 2
Position: A

Train 2
Position: B

Train 2
Position: C

Train 1

FIGURE: 22
A Straight Line

Capacity in Coaches per Hour

Running Speed in MPH
aspect fixed block is shown plotted against running speed, along with the equivalent capacity when the line is resignalled for running speed, and the four aspect fixed block capacity. It is evident that a change from four to five aspects would give some increase in straight line capacity over most of the range of running speeds, the exception being a range from about 59 to 73mph. Over the range 83mph to 100mph, the increase in capacity obtained from the change would be about 12.5%, while over the remaining speed ranges it would be as high as 37.5%.

Perhaps it should be pointed out here that the use of a triple yellow aspect could add to the problems of train drivers, who might find it difficult to distinguish between triple and double yellow aspects, especially at locations where the signals, relating to several lines, are mounted on the same gantry.

(ii) A speed restricted curve

When dealing with four aspect fixed block signalling, reference was made to Appendix A. This shows that, if two trains are passing through a speed restriction, there is a critical instant when the following train has to commence braking for the restriction, and that, if the distance separating the trains is sufficient at this critical instant, it is also sufficient at all other times. These arguments apply equally well to five aspect signalling, and hence the same equations for HNR may be used for five aspect signalling, provided that

\[ HD = HW \times V \]

where \( HW \) is made the five aspect fixed block straight line headway.
The effect of changing the limit speed, VL, is shown in Figure 23, the restriction length, DL, being maintained constant. As with four aspect fixed block signalling, as the limit speed is decreased, the sizes of the discontinuities in capacity (at running speeds of VSF1 and VSF2) are reduced and the slope of the function at other speeds above VL is reduced.

The effect of maintaining the limit speed constant and varying the restriction length is shown in Figure 24. From comparison with Figure 5, it may be seen that this produces very similar results to those observed for four aspect signalling. Also shown in Figure 24 is the capacity of four aspect fixed block for a restriction of 2400ft. Comparison with the equivalent five aspect curve shows that the relationship between the two is similar to that for the straight line which was described in the previous section. In particular, it should be noted that once again the five aspect system shows no advantage, over four aspect signalling, in the speed range 59mph to 73mph.

(iii) A simple converging junction

In Figure 25 the equivalent instant to that shown in Figure 7 is illustrated for five aspect signalling. It is evident that, as in the case of the straight line, a third possible position for train 2 has been introduced. However, the analysis, previously described for four aspect signalling, is applicable, provided that the appropriate values of HW are used and that
FIGURE 23
Speed Restricted Curves, 1200 ft. long
Various Limit Speeds

FIGURE 24
Speed Restricted Curves, with Limit Speed 33 mph
A: - Restriction Length 600 ft.
B: - " 1200 " Five Aspects
C: - " 2400 "
D: - " 400 " Four Aspects
\[
\begin{align*}
\text{CL} &= \text{BL} \quad \text{if } V < \text{VSF2} \\
\text{CL} &= 2 \times \text{BL} \quad \text{if } \text{VSF2} \leq V < \text{VSF1} \\
\text{or } \text{CL} &= 3 \times \text{BL} \quad \text{otherwise.}
\end{align*}
\]

Similarly,
\[
\begin{align*}
\text{CLL} &= \text{BLL} \quad \text{if } V < \text{VSFL2} \\
\text{CLL} &= 2 \times \text{BLL} \quad \text{if } \text{VSFL2} \leq V < \text{VSFL1} \\
\text{or } \text{CLL} &= 3 \times \text{BLL} \quad \text{otherwise}
\end{align*}
\]

where
\[
\begin{align*}
\text{BLL} &= \frac{V_L^2}{(6 \times B)} \\
\text{VSFL1} &= \sqrt{V_L^2 - 2 \times B \times \text{BLL}} \\
\text{and} & \\
\text{VSFL2} &= \sqrt{V_L^2 - 4 \times B \times \text{BLL}}
\end{align*}
\]

A graph of the capacity of this junction with speed turnouts of 1:1 is given in Figure 26. Like the equivalent four aspect graph (Figure 8), this is symmetrical about 50:50 traffic volumes, and becomes increasingly dependent upon traffic volumes as the running speed is increased. For a given ratio of traffic volumes, the capacity, as a function of running speed, will exhibit the two discontinuities characteristic of five aspect signalling at speeds of \text{VSF1} and \text{VSF2}. Comparison of Figure 26 and Figure 8, reveals that once again the five aspect signalling has varying amounts of extra capacity, except for running speeds in the range 59mph to 73mph, where the capacities of the two systems are identical.

The capacity of a converging junction of two high speed routes with speed turnouts of 100:33 is shown in Figure 27. As was the case with four aspect signalling, the effect of a speed restriction near
the junction upon traffic approaching on route B (Figure 25), is to make the minimum capacity, for any given running speed of 50mph and over, occur at traffic volumes of 0:100, instead of 50:50. It is interesting to note that when there is a majority of slow traffic, the running speed which gives the highest capacity is 50mph. This is the highest running speed (in steps of 10mph) for which a train may encounter a double yellow (i.e. be in the closest allowable position to the junction when the points change is complete). Thus, for lower running speeds (40mph), a train will take longer to negotiate the junction, because for at least some of the time it is travelling more slowly. At higher running speeds the distance which a train has to travel while occupying the junction is substantially increased, which negates any advantage accruing from a higher average speed. Similar comments might be equally well applied to a running speed of 70mph in the equivalent case under four aspect signalling.

The corresponding capacity graph for a junction of high and low speed routes, is given in Figure 28. The symmetry of this graph about 50:50 traffic volumes indicates that, as with four aspect fixed block, the value of TLL is equal to TII-I. Thus, the capacity of a line signalled at 33mph equals or exceeds the capacity of a 100mph line for running speeds up to and including 33mph. Also, the capacity at 33mph is at least equal to that of the 100mph line, operating at any running speed above 33mph.

With traffic volumes of 50:50, the capacity is lower than that of a junction with 1:1 speed turnouts, for running speeds of 80, 90 and 100mph. However, for running speeds of 40mph and 60mph the capacity
FIGURE 27 Converging Junction of Two High Speed Routes
Speed Turnouts 100:33

FIGURE 28 Converging Junction of High and Low Speed Routes
Speed Turnouts 100:33
is higher whereas it remains substantially unaltered at speeds of 50mph and 70mph. As with the four aspect case, this may be explained in terms of the values of THL and TLH. As the difference between V and VL increases, there is an increase in the value of THL. At running speeds near to VL this will be balanced by a decrease in TLH, due to shorter block lengths on the slow line (route B in Figure 25). Because of two discontinuities in the value of TLH for the junction with speed turnouts of 1:1 (occurring at 59mph and 83mph), the decrease in TLH is also of similar size to the increase in THL for speeds just above these values, which explains the increase in capacity at 60mph, and the smaller decrease at 90mph.

(iv) A simple diverging junction

As in the case of the converging junction, relatively minor modifications to the analysis of this component for four aspect signalling make it applicable to five aspect fixed block. If the values of BL, BLL, VSFL1, VSFL2, CL and CLL are obtained as in the converging junction under five aspect signalling, then

\[
SLL = \frac{(4 \times BL + OL + TL)}{VL} \quad \text{if } V \geq VL \\
SLL = \frac{(4 \times BL + OL + TL)}{V} \quad \text{if } VSFL1 \leq V < VL \\
SLL = \frac{(3 \times BL + OL + TL)}{V} \quad \text{if } VSFL2 \leq V < VSFL1 \\
\text{or } SLL = \frac{(2 \times BL + OL + TL)}{V} \quad \text{otherwise}
\]

For speed turnouts of 1:1 Figure 26 applies to the diverging junction as well as the converging junction, for exactly the same
reasons that Figure 8 applies to both junctions with four aspect fixed block.

The capacity of a diverging junction of two high speed routes, with speed turnouts of 100:33, is shown in Figure 29. It is evident, from a comparison with Figure 27, that the performance of this junction is very similar to that of the converging junction of two high speed routes. The capacity of the diverging junction is slightly greater than that of the converging junction for high running speeds, especially for traffic volumes of 50:50. This directly parallels the four aspect signalling case. Thus, for any given traffic volumes, the capacity of the junction operating under five aspect fixed block will be greater than when under four aspects, for all running speeds with the exception of the range 59mph to 73mph, i.e. the relationship between the two is similar to that already examined in the case of a straight line.

The capacity of a diverging junction of high and low speed routes, with speed turnouts of 100:33, is shown in Figure 30. As in the equivalent junction operating under four aspect fixed block, the capacity, with traffic volumes in the range 0:100 to 50:50, is considerably less than that of a converging junction of two high speed routes. This is due to an increase in the interval TLL, made necessary by the greater length of the speed restriction. There is also a small increase in THL, resulting from a train, on the slow route, taking longer to negotiate the junction.
(v) A diamond crossing

The analysis of a diamond crossing for four aspect fixed block may be modified for five aspects. As with the diverging junction, the values of BL, BLL, VSFL1, VSFL2, CL and CLL are obtained in the manner described for the converging junction. In addition, for the crossing of high and low speed routes

\[ TLH = \left( BL + OL + TL - DL/2 \right)/V + (3 \times BLL + DL/2)/VL \]

and \[ TLL = (4 \times BLL + OL + TL)/VL \]

The capacity of a diamond crossing with speed turnouts of 1:1 is independent of traffic volumes and equal to that of a straight line. A graph of the capacity, of a crossing of two high speed routes with speed turnouts of 100:33, is given in Figure 31. At traffic volumes of 0:100 and 100:0 the capacity is identical with that of the converging and diverging junctions of two high speed routes. However, because in the case of the crossing, no time is required for the route to be reset, at other traffic volumes the capacity, for equivalent conditions, will tend to be higher than that of either of the junctions. For the same reason the curves have no slope discontinuity at traffic volumes of 50:50. When four and five aspect signalling is compared for this crossing, the characteristic relationship described in detail for the straight line is again evident.

The equivalent graph, for a crossing of high and low speed routes, is given in Figure 32. For traffic volumes of 0:100 the capacity is constant for all running speeds above VL, and equals that of a line resignalled for a speed of VL. Thus, uncharacteristically for running
speeds in the range 59mph to 73mph, the five aspect system possesses
a higher capacity than four aspect signalling, at all traffic volumes
except 100:0. However, comparison of Figures 17 and 32 shows that
there is great similarity in the behaviour of this component under the
two systems.

(vi) An isolated station

The analysis of a station for four aspect signalling may be used
for five aspects, if the following modifications are incorporated:

\[
\begin{align*}
CL &= 2 \times BL \quad \text{if } V < VSF2 \\
CL &= 3 \times BL \quad \text{if } VSF2 \leq V < VSF1 \\
or & \quad CL = 4 \times BL \quad \text{otherwise}
\end{align*}
\]

The capacity of a station is shown plotted against the ratio of
stopping to non-stopping traffic in Figure 33. It is evident that when
a majority of the traffic is stopping at the station the capacity
does not vary greatly with running speed, except at very low speeds.
This is because the straight line headway (i.e. the headway between
two non-stopping trains) only becomes significant at these low running
speeds when compared with the headway between two stopping trains.
Comparison between Figure 33 and Figure 19, reveals that this effect
is more marked with the five aspect system than with four aspect
fixed block.

The effect of varying the station stop time while maintaining
the running speed constant is shown in Figure 34. It may be seen
that this is very similar to the equivalent four aspect graph (Figure
20), with the stop time not having a pronounced effect on capacity.

2.1.2 The bidirectional Junction

The analysis of the steady state capacity may be extended to cover bidirectional junctions (see Figure 142) by interrelating three of the basic components already examined. These are a converging junction, a diverging junction, and a diamond crossing. The combination of these components is achieved by considering the size of the intervals between trains of a given route to see if any 'gap' is sufficient to permit the entry of a train (or trains) on another route. The particular algorithm, which is used for this purpose, is common to all signalling systems, and is described in detail in Appendix C.

For any basic component the algorithm requires the sizes of four intervals ROA, ROB, RTOA and RTOB, which are defined as the smallest and largest intervals between trains on the high speed route, and the smallest and largest intervals on the low speed route. A simplification is made by assuming that a train on the less populous route in either direction of flow does not immediately follow another on the same route. Thus, all intervals between trains following exactly the same route through one of the basic components are equal to one of four intervals defined above.

For fixed block signalling, the intervals ROA, ROB, RTOA and RTOB are obtained in exactly the same manner for each of the three basic components involved, and for both four and five aspect systems. If the variables $N_1$, $N_2$, $TH$, $TL$, $THL$ and $TLL$ are as defined previously,
and if \( N_1 = N_2 \), i.e. if there are equal numbers of trains on the fast and slow routes of any given component, then

\[
\text{ROA, ROB, RTOA, RTOB} = \text{THL} + \text{TLH}
\]

However, if there are more trains on the fast route than on the slow route, i.e. if \( N_1 > N_2 \), then

\[
\begin{align*}
\text{ROA} &= \text{THH} \\
\text{ROB} &= \text{THL} + \text{TLH}
\end{align*}
\]

If \( TRB = N_1/N_2 \), and IRB is equal to \( TRB \) truncated to an integer value, then

\[
\text{RTOA} = \text{THL} + \text{TLH} + (\text{IRB} - 1) \times \text{THH}
\]

If \( TRB = \text{IRB} \),

\[
\text{RTOB} = \text{RTOA}.
\]

But if \( TRB > \text{IRB} \),

\[
\text{RTOB} = \text{RTOA} + \text{THH}
\]

Alternatively, if there are more trains on the slow route, than on the fast route, i.e. if \( N_1 < N_2 \), then

\[
\begin{align*}
\text{RTOA} &= \text{TLL} \\
\text{RTOB} &= \text{THL} + \text{TLH}
\end{align*}
\]

If this time \( TRB = N_2/N_1 \), and IRB is again equal to \( TRB \) truncated to an integer value, then

\[
\begin{align*}
\text{ROA} &= \text{THL} + \text{TLH} + (\text{IRB} - 1) \times \text{TLL} \\
\text{ROB} &= \text{ROA}
\end{align*}
\]
But if $TRB > IRB$

$$ROB = ROA + TLL.$$ 

2.1.2a Four aspect signalling

It has already been mentioned that the number of independent parameters which affect the capacity of a bidirectional junction is considerably greater than the number relating to any of the basic components which have so far been considered. For this reason, no attempt is made here to present a comprehensive survey of the operating performance of the junction. However, the graphical diagrams, given in Figures 35 to 39 inclusive, do give some indication of the variation of capacity with some of the basic parameters.

In Figure 35 the capacity of a right hand bidirectional junction, with speed turnouts of 100:100 and converging traffic running at a maximum is shown as a function of running speed and train length, for traffic volumes of 10:90. It will be noted that the variation of junction capacity with train length, for a specific value of running speed, is almost linear, and also that, for a given train length, the variation with running speed demonstrates the characteristic discontinuity at approximately 70% of line speed.

The capacity of the same junction is shown in Figure 36 as a function of the running speed and the traffic volumes of the priority (converging) trains, for a fixed train length of 660ft. It is evident that, for a given running speed, the variation of the capacity with traffic volumes displays several discontinuities. A small change in the length of an interval between two trains negotiating the diamond crossing in the
FIGURE 36
Right Hand Bidirectional Junction
with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum
Train Length 660 ft.
FIGURE 37
Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts of 100:33
Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE 39
Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts of 100:33
Converging Traffic Running at a Maximum with Various Fast:Slow Traffic Volumes
Train Length 660 ft.
priority direction, may make little difference to the average headway between trains of the priority flow. However, it is possibly sufficient to change the number of trains which may be fitted into this interval (naturally by an integer number of trains). For this reason, it may cause a very considerable change in the non-priority traffic flow, and hence a discontinuity in the total capacity of the junction. As one might expect for a junction with speed turnouts of 100:100, for any specific running speed, the capacity appears to be a symmetrical function of the priority flow traffic volumes about the ratio 50:50. The capacity at traffic volumes of 50:50 is somewhat greater than that at ratios close to this value, the traffic volumes of the non-priority flow being constant at 50:50 over this limited range. The intervals available are better utilised at priority flow traffic volumes of 50:50 since, at other ratios in the range, they are larger than the minimum required for actual number of trains passing, but not large enough for any extra trains to be inserted.

In Figure 37 the capacity of a right hand bidirectional junction of two high speed routes is given for speed turnouts of 100:33, as a function of running speed and priority flow traffic volumes. It will be evident that the effect of the speed restriction in the immediate vicinity of the junction is to reduce the capacity in very much the same way as it did for the simple converging and diverging junctions. In fact, for some values of traffic volumes, the total capacity of the junction is unaffected by any conflicts at the diamond crossing and is equal to the sum of the capacities of the two simple junctions operating under appropriate conditions. Three examples of this are
given in more detail in Figure 39 for traffic volumes of 0:100, 50:50 and 100:0. It is worth noting that, for traffic volumes of 100:0, all traffic runs down the high speed routes, and hence the capacity is always twice that of a straight line signalled for 100mph. At traffic volumes of 0:100, all traffic will travel the restricted speed routes, and the capacity is twice that of a speed restriction of the appropriate limit speed and length.

In Figure 38 the equivalent capacity graph to Figure 37 is given for a right hand bidirectional junction of high and low speed routes. The capacity, for priority flow traffic volumes of 100:0, will be equal to that of the junction of two high speed routes, and also, of course, that of a junction with speed turnouts of 1:1. However, when the majority of the priority flow travels along the low speed route, the capacity is increased, especially for the lower running speeds. This is due to the diamond crossing being able to pass large quantities of trains on the low speed route. Another discontinuity in capacity occurs at a running speed equal to VSFL (i.e. the speed at which there is a discontinuity in the capacity of line signalled for the limit speed, VL). In practice, the capacity of an isolated junction of this type may never exceed that of two parallel straight lines, since all traffic negotiating the junction will have to travel along such a pair of lines.

2.1.2b *Five aspect signalling*

A comparable set of diagrams to those just discussed under four aspect fixed block signalling are included in Figures 40 to 44 inclusive.
FIGURE 41
Right Hand Bidirectional Junction
with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum
Train Length: 660 ft.
FIGURE: 42
Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts of 100:33. Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE 44: Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts 100:33 Converging Traffic Running at a Maximum with Various Fast:Slow Traffic Volumes Train Length 660 ft.

FIGURE 45: Delays to Following Trains Leading Train Waiting Period 0 sec Running Speed 100 mph Train Length 660 ft. for Various Running Headways
In conjunction with Figures 35 to 39, these give a somewhat limited comparison between the two signalling systems, but, seeing that the possible range of operating conditions is so very large, an exhaustive comparison would involve a very large number of such graphs. For this reason, any comparisons drawn here are in very general terms.

From comparison of the corresponding graphs (e.g. Figure 35 and Figure 40), it will be seen that the capacity of the junction operating under five aspect signalling displays the same relationship to the capacity under four aspects as we have come to expect when considering the more basic components, e.g. the straight line. Hence, the five aspect system again shows an advantage in capacity of some 37.5% for running speeds up to the first discontinuity (about 59mph). For running speeds in the range 59mph to 73mph, i.e. up to the discontinuity of the four aspect case, there is not a significant difference in the capacity of the two systems, while, for speeds between 73mph and 83mph, a similar advantage is shown by the five aspect case, as for low running speeds. Over the remainder of the range the five aspect system holds an advantage of some 12.5%.

All this is very much as expected, since each basic component involved in the bidirectional junction has shown a similar relationship, and the process, whereby these components are interrelated, is independent of the number of aspects.

2.2 Immunity from Delays

As was mentioned in the introduction, it has been suggested (reference 3) that perturbations from timetabled conditions would be
eliminated sooner, if, when a train has come to rest at a signal displaying a red aspect, it does not resume its journey when the signal aspect changes to single yellow, as would normally be the case. Instead, it was suggested that it should wait until a double yellow aspect is displayed. This procedure of "double yellow starting" is examined in parallel with the standard method of "single yellow starting", for both four and five aspect fixed block signalling.

The basis for comparison of the perturbed running performance of the various signalling systems and operating procedures is an infinitely long straight line, with an unlimited number of trains at equal headways on it. With the leading train being subjected to an imposed delay, the behaviour of the subsequent trains is examined, to see how many of the trains experience a delay, and what are the magnitudes of any such delays.

There is not a simple analytical expression for the delays encountered by the subsequent trains in the case of fixed block signalling. Instead, the method used considers the pattern of signal aspects observed by the driver of a train, and from this derives the delay which would be produced. The procedure is described in some detail for single yellow starting with four aspect signalling, but only the differences between this and double yellow starting, and five aspect signalling, are emphasised in subsequent sections.

2.2.1 Single Yellow Starting

This is the simplest procedure, since it involves the train moving off at the earliest possible moment, i.e. when the signal
displaying a red aspect, at which the train is standing, changes to single yellow.

2.2.1a Four Aspect Signalling

In order to determine the aspects displayed by the signals as a train passes them, it is necessary to know the times at which the preceding train has cleared the overlaps associated with these signals. Thus, it is necessary to calculate these clearing times for the leading train, i.e. the train which has imposed upon it an initial delay, which consists of braking from its running speed of \( V \) to come to rest at a signal, waiting there for a period of \( DST \), and then accelerating back to running speed once again. Therefore, the total delay imposed upon the leading train is given by

\[
FTD = (A + B) \times \frac{V}{(2 \times A \times B)} + DST
\]

where \( A \) and \( B \) are as defined previously. Let us place the overlap clearing times in an array \( TT(K) \). Prior to the perturbation, the train is travelling with a speed of \( V \), and since the overlaps are a distance \( BL \) apart, the first few elements in the array will be given by

\[
TT(1) = \frac{BL}{V} \\
TT(2) = TT(1) + \frac{BL}{V} \\
TT(3) = TT(2) + \frac{BL}{V}
\]

If \( TT(4) \) is the time at which the train clears the overlap associated with the signal in the rear of the block where braking commences, and if \( DIS - OL - TL = 0 \), then
\[ TT(4) = TT(3) + \frac{BL}{V} \]

and

\[ VOX = VOW = V \]

where

\[ DIS = BL - \frac{V^2}{2 \times B} \text{ if } V < VSF \]

or

\[ DIS = 2 \times BL - \frac{V^2}{2 \times B} \text{ otherwise,} \]

and where VOX and VOW are the instantaneous speeds of the train, when clearing the overlap for which the clearing time is being determined, and the one in the rear of this, respectively.

But, if \( DIS < (OL + TL) \), then VOW is still equal to V, but VOX is given by

\[ VOX = \sqrt{VOW^2 - 2 \times B \times (OL + TL - DIS)} \]

while, if \( DIS > (OL + TL) \)

\[ VOX = \sqrt{VOW^2 - 2 \times B \times (BL + OL + TL - DIS)} \]

In both these cases \( TT(4) \) will then be given by

\[ TT(4) = TT(3) + \frac{BL}{V} + \frac{(VOW - VOX)/B - (VOW^2 - VOX^2)/(2 \times B \times VOW)}{2 \times B \times VOW} \]

When considering the clearing time of the next overlap, the speed, previously referred to as VOX, becomes VOW, since, by definition, the latter is the instantaneous speed of the train when clearing the overlap in the rear of the one for which the clearing time is being calculated. Let AOX be given by

\[ AOX = VOW^2 - 2 \times B \times BL \]

Then, provided that AOX is positive,
\[ TT(5) = TT(4) + \frac{(VOW - VOX)}{B} \]

where \( VOX = \sqrt{AOX} \). By the time the train clears the next overlap, it will have come to rest, and will be accelerating back to line speed. The identical situation exists one block earlier if previously \( AOX \) is found to be negative.

Under these conditions

\[ VOX = \sqrt{2 \times A \times (OL + TL)} \]

and, provided that the train regains its running speed before clearing the overlap, i.e. if \( VOX > V \), then

\[ TT(N+1) = TT(N) + \frac{VOW}{B} + DST + \frac{V}{2 \times A} + \frac{(OL + TL)}{V} \]

where \( TT(N) \) is the last clearing time which has already been calculated. All subsequent overlap clearing times are then given by

\[ TT(N+1) = TT(N) + \frac{BL}{V} \]

However, if \( VOX < V \), then the train is still accelerating when it clears the overlap associated with the signal at which it came to rest. Hence,

\[ TT(N+1) = TT(N) + \frac{VOW}{B} + DST + \frac{VOX}{A} \]

If, when calculating the next clearing time, \( VOW \) is again made equal to \( VOX \), and if \( N \) is suitably incremented, then

\[ VOX = \sqrt{VOW^2 + 2 \times A \times BL} \]

and, provided that \( VOX < V \),

\[ TT(N+1) = TT(N) + \frac{(VOX - VOW)}{A} \]

which may be repeated for subsequent blocks until \( VOX > V \).

Under this condition,
TT(N+1) = TT(N) + (V - VOW)/A + (BL - (V^2 - VOW^2)/(2 x A))/V
and all subsequent clearing times are given by

TT(N + 1) = TT(N) + BL/V

The overlap clearing times of the leading train may then be used to determine the aspects which the driver of the second train observes as he passes down the line. If we suppose that the headway between the trains is initially SPG, then the distance between them (tail to tail), before the leading train commences braking, is given by

DPG = SPG x V

Let us suppose that the tail of the leading train has just cleared an overlap. The number of complete block lengths, KPG, between the signal associated with this overlap, and the following train, is the minimum integer value which fulfills the condition

DPG < (KPG + 1) x BL - OL - TL

The time which elapses between the leading train clearing an overlap and the following train next passing a signal, is given by

TB = (DPG - KPG x BL - OL - TL)/V

Now, if we store the aspects which the driver of the second train observes in an array KS(I), and if the following code is used

A red aspect is represented by the integer 0
A single yellow aspect by the integer 1
A double yellow aspect by the integer 2
and a green aspect by the integer 3
Then, since only headways equal to or greater than the steady state minimum straight line headway are sensibly considered,

\[ KS(1) = KPG \]

if \( KPG \leq 3 \), and \( KS(1) = 3 \) for \( KPG > 3 \).

In order to determine the aspects of subsequent signals, and the total delay to the train, it is necessary to count the number (KL) of blocks which the leading train has travelled through, and the number (KF), which the following train has traversed during the same period. Thus, when the second train is considered to be at the next signal, KF must be incremented, and the time taken to traverse the block (AOE) determined together with the instantaneous speed of the train at that signal, VUX (where the speed at the start of the block is VUW). The maximum speed (VLI), at which the train may be travelling at the end of the block, is given by

\[ VLI = 0.0 \text{ if } KS(I - 1) \leq 1 \]

or \( VLI = VSF \) if \( KS(I - 1) = 2 \), and if \( VSF < V \)

or \( VLI = V \) otherwise,

where \( KS(I - 1) \) is the signal aspect observed at the entry to the block under consideration. If \( VUW = VLI \), then AOE and VUX are given by

\[ VUX = VUW \]

and \( AOE = BL/VUW \)

except in the case when \( VUW = VLI = 0.0 \), where AOE is given by

\[ AOE = \sqrt{2 \times (A + B) \times BL/(A \times B)} \]
provided that the train does not have to wait at the signal at the end of the block, i.e. if
\[ \text{TOE + AOE - TT(KL + 1)} \geq 0 \]
where TOE is the total time taken for the following train to reach the entry to the block under consideration, i.e. it is initially set equal to TB.

If \( \text{TOE + AOE - TT(KL + 1)} < 0 \), then let
\[ \text{SUP(I)} = \text{TT(KL + 1)} - \text{TOE} - \text{AOE} \]
where the array SUP(I) stores the waiting times at signals. The time taken to travel through the block is then given by
\[ \text{AOE} = \text{TT(KL + 1)} - \text{TOE} \]

If it is initially found that \( \text{VUW} < \text{VLI} \), and also \( \text{VUW} < \text{V} \), then a period of acceleration is necessary for at least some of the time the train is moving through the block. The maximum speed, which the train could attain by accelerating throughout the entire journey through the block, is given by
\[ \text{VMX} = \sqrt{\text{VUW}^2 + 2 \times \text{A} \times \text{BL}} \]
Thus, if \( \text{VMX} \leq \text{VLI} \), then
\[ \text{VUX} = \text{VMX} \]
and
\[ \text{AOE} = \frac{(\text{VIX} - \text{VUW})}{\text{A}} \]
Alternatively, if \( \text{VMX} > \text{VLI} \), then
\[ \text{VUX} = \text{VLI} \]
and
\[ \text{AOE} = \frac{(\text{VUX} - \text{VUW})}{\text{A}} + \frac{(\text{BL} - (\text{VUX}^2 - \text{VUW}^2)/(2 \times \text{A})}{/ \text{VUW}} \]
However, if it is found that $V_{UW} > V_{LI}$, then a period of braking is necessary before the train reaches the signal at the end of the block. Thus, if $K_S(I - 1) \geq 2$, then

$$V_{UX} = V_{LI}$$

and

$$AOE = \frac{(V_{UW} - V_{LI})}{B} + \frac{BL - (V_{UW}^2 - V_{LI}^2)/(2 \times B)}{V_{UW}}$$

If $K_S(I - 1) = 1$, it is again necessary to check to see if the train has to wait at the end of the block. As previously, no waiting is necessary if

$$TOE + AOE - TT(KL + 1) > 0$$

If a wait is required, it is allowed for in the same way as described above.

When $AOE$ and $V_{UX}$ have been determined, then $AOE$ should be added to $TOE$ to make it the time taken for the second train to reach the entry of the next block. Then a check is made to determine if the leading train has cleared any overlaps during the interval $AOE$. Thus, if necessary, $KL$ is incremented until a value is found such that

$$TT(KL + 1) > TOE$$

Then the number of complete blocks between the following train and the signal displaying a red aspect in the rear of the leading train is given by

$$K_{NG} = K_{PG} + KL - K_F$$

Thus, the signal aspect, displayed by the signal at the end of the block, will be given by
\[ KS(I) = KNG \quad \text{if} \quad KNG \leq 3 \]
\[ \text{and} \quad KD(I) = 3 \quad \text{otherwise}. \]

The process described above is then repeated, in order to fill the array \( KS(I) \) with the pattern of the signal aspects observed by the second train. Also, the total delay, which is encountered by the second train, is given by

\[ ST = TOE - TB - KF \times BL/V \]

using the final values of \( TOE \) and \( KF \), when both trains have regained their running speed after the perturbation.

If the second train experiences some delay, it is necessary to determine the aspects observed by the driver of the third train, in order to see if this train also encounters a delay. This may be achieved by using the procedure just described above, if the overlap clearing times of the second train are known. The method by which these clearing times may be obtained from the aspects observed is described below.

Two time intervals are used in the following procedure which have not been referred to previously. These are \( TLAT \), which is the time taken for the train to travel from a signal to the point where its tail clears the overlap associated with that signal, and \( TAPF \), which is the time taken for the train to travel from the point where it clears an overlap to the next signal. When the train is travelling at running speed these intervals are given by

\[ TAFI = (BL - OL - TL)/V \]
\[ \text{and} \quad TLAT = (OL + TL)/V \]
In order to calculate the time taken by a train between clearing one overlap and clearing the next it is necessary to add the value of TLAT, pertaining to one block, and the value of TAFT for the block in the rear. In order to calculate the values of TLAT and TAFT for only one block at a time the value of TAFT, for the block in the rear, is referred to as STAFT. Thus, after initially setting

\[ TT(1) = \frac{BL}{V} \]

the following general formula is used to determine all subsequent clearing times.

\[ TT(I + 1) = TT(I) + STAFT + TLAT \]

Initially STAFT is equal to TAFT, and these quantities, together with TLAT, are given by the expressions quoted above for a train travelling at running speed. Thus, in order to determine the overlap clearing times of the train, it is necessary to calculate TLAT and TAFT for any block length given the signal aspect displayed at the entry to the block.

If \( V_{W} \) is the instantaneous speed of the train when entering the block, and similarly, if the maximum speed at which the train may be travelling when it reaches the end of the block is \( V_{LI} \), then, provided that \( V_{W} \neq 0 \), if \( V_{W} = V_{LI} \)

\[ TLAT = \frac{(OL + TL)}{V_{W}} \]

and \[ TAFT = \frac{BL}{V_{W}} - TLAT \]

The value of \( V_{LI} \) is dependent upon the signal aspect observed in exactly the same way as described above in the determination of the aspects encountered by the second train.
If $V_{UW} = VLI = 0.0$, the maximum speed attained by the train during the course of its journey through the block is given by
\[ V_{ID} = \sqrt{2 \times A \times B \times BL / (A + B)} \]
When the train clears the overlap associated with the signal at the entry to the block it is still accelerating if
\[ V_{ID} > \sqrt{2 \times A \times (OL + TL)} \]
in which case
\[ TLAT = \frac{\sqrt{2 \times (OL + TL)}}{A} \]
and \[ TAFT = \frac{\sqrt{2 \times (A + B) \times BL / (A \times B)}}{A} - TLAT \]
Alternatively, if the train is braking when it clears the overlap
\[ TAFT = \frac{\sqrt{2 \times (BL - OL - TL)}}{B} \]
and \[ TLAT = \frac{\sqrt{2 \times (A + B) \times BL / (A \times B)}}{B} - TAFT \]
However, in both cases, any period during which the train is stationary at the signal at the end of the block while a red aspect is displayed must be added to TAFT. Any such waiting period has already been calculated during the determination of the aspects, and is stored in the array element $SUP(I)$.

If it is initially found that $V_{UW} < VLI$, and also $V_{UW} < V$, then some acceleration will be necessary in order for the train to be travelling at VLI (or at a speed closer to it that $V_{UW}$), when leaving the block. The maximum speed at which the train can be travelling when it clears the overlap is given by
\[ V_{UP} = \sqrt{V_{UW}^2 + 2 \times A \times (OL + TL)} \]
If $V_{UP} > VLI$, the train will have already completed its acceleration when it clears the overlap. Thus
\[ TLAT = (VLI - V_{UW})/A + (OL + TL - (VLI^2 - V_{UW}^2)/(2 \times A))/VLI \]
and \[ TAFT = (OL - OL - TL)/VLI \]
But, if $V_{UP} < V_{LI}$, the train must continue accelerating after it has cleared the overlap. Under this condition

$$TLAT = \frac{(V_{UP} - V_{UW})}{A}$$

The maximum speed at which the train may be travelling when it reaches the end of the block is given by

$$V_{UM} = \sqrt{V_{UW}^2 + 2 \times A \times BL}$$

If $V_{UM} < V_{LI}$, the train continues to accelerate throughout the entire length of the block. Thus,

$$TAFT = \frac{(V_{UM} - V_{UP})}{A}$$

Alternatively, if $V_{UM} > V_{LI}$, then

$$TAFT = \frac{(V_{LI} - V_{UW})}{A} + \frac{(BL - (V_{LI}^2 - V_{UW}^2)/(2 \times A))}{V_{LI} - TLAT}$$

However, if initially it is found that $V_{UW} > V_{LI}$, some braking is required, in order that the train may leave the block with a speed of $V_{LI}$. The distance over which the train may maintain the entry speed, $V_{UW}$, is given by

$$DIS = BL - (V_{UW}^2 - V_{LI}^2)/(2 \times B)$$

If $DIS > (OL + TL)$ the train has not commenced braking when it clears the overlap, and hence

$$TLAT = \frac{(OL + TL)}{V_{UW}}$$

and

$$TAFT = \frac{(V_{UW} - V_{LI})}{B} + \frac{(BL - OL - TL - (V_{UW}^2 - V_{LI}^2)/(2 \times B))}{V_{UW}}$$

whereas, if $DIS < (OL + TL)$, the instantaneous speed of the train when it clears the overlap is less than $V_{UW}$, Thus

$$TLAT = \frac{DIS}{V_{UW}} + \frac{(V_{UW} - \sqrt{V_{UW}^2 - 2 \times B \times (OL + TL - DIS)})}{B}$$

and

$$TAFT = \frac{(V_{UW} - V_{LI})}{B} + \frac{DIS}{V_{UW} - TLAT}$$
If the value of VLI is zero, it is necessary to add SUP(I), (which is the time the train waits at the signal at the end of the block), to the value of TAFT.

The overlap clearing times of the second train, obtained in this way, may be used in another iteration of the aspect determination procedure, in order to give the aspects and the delay encountered by the third train. Thus the process may be repeated until it is found either that the Nth train in the series does not suffer any delay, or that the delays to subsequent trains are fully propagated, or perhaps increase.

The delay encountered by the following trains is given in Figure 45 for various values of running headway, but a constant running speed of 100 mph. With headways of 177 seconds and more the delay to the leading train does not cause the train following it to encounter any restrictive aspects, and hence it experiences no delay. This results from the fact that, for a running headway greater than 177 seconds, the delay encountered by the leading train does not exceed the slack (defined as the difference between the actual running headway, and the steady state minimum headway). As the running headway is decreased from 177 seconds the following train begins to encounter restrictive aspects. Let us suppose that the actual running headway is slightly less than this limiting value, so that the following train encounters just one double yellow aspect. This requires the train to brake such that it is travelling with a speed of VSF at the next signal, where the train driver observes a green aspect. The train may then commence accelerating back to running
speed. Such a procedure would delay the train by approximately 10 seconds, which is therefore the minimum delay which any following train encounters. In fact the delay experienced by a train tends to equal certain discrete levels, which correspond to the different combinations of restrictive aspects. Thus there is a tendency for a train to be delayed by more than the difference between the delay to the preceding train and the slack. Therefore, the headway between the two trains, when they attain their running speeds subsequent to the delay, may itself have a small degree of slackness, and thus be able to absorb small quantities of further delay without propagation.

With running headways from 177 seconds down to about 135 seconds, the second train experiences various amounts of delay corresponding to different numbers of double yellow aspects. However, for headways less than this, the second train encounters a single yellow aspect, and brakes to an instantaneous stand at the next signal. Since Figure 45 applies to the case where the leading train waiting period is zero, this ensures that at least the full delay is propagated to the second train, and hence to the third etc. Thus an unstable condition is created, whereby an infinitely large number of trains are affected by the original imposed delay. It is interesting to observe that the delay which is propagated with a running headway of 130 seconds is larger than for smaller headways of 110 and 120 seconds. This is due to the second train encountering more than one double yellow aspect before observing the single yellow aspect, whereupon the increased delay is fully propagated to subsequent trains. As the running headway is further reduced towards the steady state

*Please see definitions
minimum value, a situation is reached where the delays continue to increase, as shown in Figure 45 for headways of 30 and 100 seconds.

A further possibility has been observed for some combinations of running speed and headway, which is not evident in Figure 45. It occurs in the unstable region, the propagated delays increasing and decreasing, within certain limits, in an oscillatory manner. The number of trains involved in one "cycle" appears to vary from two, to as many as eight.

The number of following trains affected by an initial imposed delay is obviously an important factor when assessing the acceptability of a value of running headway. Therefore it is shown plotted against headway in Figure 46 for the same initial delay, running speed, and train length as Figure 45. The equivalent graphs, to Figure 46 for leading train waiting periods of 30 and 60 seconds, are shown in Figures 47 and 48. A comparison of Figures 46, 47 and 48 shows that increasing the leading train waiting period from 0 to 30 seconds increases the range of headways over which there is stable delay propagation, while the headway at which limit of stability occurs is almost unchanged. Further raising the leading train waiting period to 60 seconds increases the value of headway corresponding to the limit of stability as well as the headway at which the second train is just affected, such that the headway range, for stable delay propagation, is slightly decreased. Thus, for any given leading train waiting period, running speed, and train length, three values of headway are of particular interest. These are the minimum steady state headway, the headway corresponding to the limit of stability, and the headway
FIGURE 46 Leading Train Waiting Period 0 sec
Running Speed 100 mph
Train Length 660 ft.

Minimum Steady State Headway

FIGURE 47 Leading Train Waiting Period 30 sec
Running Speed 100 mph
Train Length 660 ft.

Minimum Steady State Headway
above which there is no delay propagation at all.

In Figure 49 these three critical values are shown plotted against running speed, for a fixed value of leading train waiting period and train length, while, in Figure 50, the running speed is kept constant and the train length varied. There is a discontinuity in the minimum steady state headway, and the limit of delay propagation, at the speed VSF. At running speeds above this value, the observation of a double yellow aspect causes a train to be delayed, whereas this is not the case for lower running speeds. For running speeds less than VSF the limit of stability and the limit of delay propagation coincide. This is to be expected for a leading train waiting period of zero, since a single yellow, and not a double yellow, is the least restrictive aspect which produces a delay. As is described above, the observation of a single yellow aspect causes the full delay to be propagated and hence gives rise to instability. It is evident from an examination of Figure 50 that train length does not have much effect on the perturbed performance of the system, only producing a small linear change in the minimum steady state headway, and thus corresponding changes in the limits of stability and propagation.

The curves shown in Figure 49 may be used to give an indication of the type of performance which might be expected from the system for any combination of headway and speed. If the point corresponding to the chosen combination lies below the minimum steady state headway curve, then such a combination is not practical, even in the steady state. If the point lies between the curves for minimum steady state headway and the limit of stability, the headway is adequate while
FIGURE 48  Leading Train Waiting Period 60 sec
Running Speed 100 mph
Train Length 660 ft.

FIGURE 49  Leading Train Waiting Period 0 sec
Train Length 660 ft.

Minimum Steady State Headway
Limit of Delay Propagation
Limit of Stability
steady state conditions prevail, but the situation may become unstable with any delays to trains building up, or, at best, being fully propagated. If the point lies between the limit of stability and the limit of delay propagation, the perturbed performance is stable, with the delays rapidly being eliminated. For points above the limit of propagation, the imposed delay does not produce any effect upon other trains. It should be noted, however, that all these curves only apply for one value of the imposed delay.

The corresponding graphs to Figure 49 are given in Figures 51 and 52 for leading train waiting periods of 30 seconds and 60 seconds respectively. Naturally, the minimum steady state headway is unaffected by the size of the imposed delay. It is also evident, from examining Figures 51 and 52, that when the leading train waiting period is not zero, the limit of stability does not coincide with the limit of delay propagation, even for running speeds less than VSF. The observation of a single yellow aspect still produces the same delay for a given running speed, but this is no longer necessarily as large as the imposed delay. Thus there is a range of headways giving stable delay propagation for all running speeds, with positive values of leading train waiting periods. Increasing the leading train waiting period to 60 seconds enlarges this range for lower running speeds, while it remains substantially unaltered above 70 mph.

In practice it is evidently desirable to ensure that the system never becomes unstable. However, since the limit of stability depends upon the magnitude of the initial delay, this requires some estimate of the largest likely perturbation. Also, it should be noted that a
reduction in running speed might also lead to the situation becoming unstable.

2.2.1b Five Aspect Signalling

The procedure for determining the delay encountered by following trains after the imposition of a delay on the leading train, which was described for four aspect signalling, requires the following modifications to make it applicable to five aspects.

When calculating the overlap clearing times of the leading train it is important to appreciate that it may be necessary for the train to brake over a distance of up to three block lengths. Thus the length of DIS is given by

\[ DIS = BL - \frac{V^2}{2 \times B}, \text{ if } V < VSF_2 \]

or

\[ DIS = 2 \times BL - \frac{V^2}{2 \times B}, \text{ if } VSF_2 \leq V < VSF_1 \]

or

\[ DIS = 3 \times BL - \frac{V^2}{2 \times B}, \text{ otherwise.} \]

There is also a slight difference after the calculation of \( TT(4) \). When considering the next overlap clearing time, AOX is again given by

\[ AOX = VOW^2 - 2 \times B \times BL \]

where \( VOW \) is the instantaneous speed of the train when clearing the previous overlap, in exactly the same way as was described for four aspects. Provided that \( AOX \) is positive, the next clearing time is given by

\[ TT(N + 1) = TT(N) + \frac{(VOW - \sqrt{AOX})}{B} \]
where \( N = 4 \) in this case. The same formulae for AOX and \( TT(N + 1) \) may be applied to the following overlap clearing time, when \( N = 5 \), provided that AOX is again found to be positive. When AOX becomes negative, the train will have come to rest and will be accelerating back to line speed. Thus, from that point onward, the calculation of the leading train overlap clearing times becomes identical to the four aspect case.

The code which is used to represent signal aspects requires the following modification.

A treble yellow aspect is represented by the integer 3, and a green aspect is now represented by the integer 4. The first aspect observed by the second train is given by

\[
KS(1) = KPG, \quad \text{if } KPG \leq 4
\]

and

\[
KS(1) = 4, \quad \text{if } KPG > 4
\]

and, similarly, subsequent signal aspects are found using \( KNG \) instead of \( KPG \). The determination of aspects and overlap clearing times of following trains may then proceed exactly as for four aspects, provided that \( VLI \) is found in the following way:

\[
VLI = 0.0 \quad \text{if } KS(I - 1) \leq 1
\]

\[
VLI = VSF2 \quad \text{if } KS(I - 1) = 2, \quad \text{and if } VSF2 \leq V
\]

\[
VLI = VSF1 \quad \text{if } KS(I - 1) = 3, \quad \text{and if } VSF1 \leq V
\]

or

\[
VLI = V \quad \text{otherwise.}
\]

The delay encountered by following trains is shown for values of headway, in Figure 53. It is apparent that for a headway of 110 seconds, there are oscillations in the amplitude of the propagated
FIGURE 52
Leading Train Waiting Period 60 sec
Train Length 660 ft.

- Limit of Delay Propagation
- Limit of Stability
- Minimum Steady State Headway

Running Speed in M.P.H. vs. Headway in Seconds

FIGURE 53
Delays to Following Trains
Leading Train Waiting Period 0 sec
Running Speed 100 mph
Train Length 660 ft.
Various Running Headways

Delay in Seconds vs. Number of Following Train

- 171 sec
- 120 sec
- 110 sec
- 100 sec
- 90 sec
- 80 sec
- 70 sec
- 60 sec
- 50 sec
- 40 sec
- 30 sec
- 20 sec
- 10 sec
- 0 sec
delay. As mentioned previously, this phenomenon is observed, under certain combinations of headway and running speed, for four aspect signalling, and thus should not be regarded as a function of the number of aspects of the system.

If Figure 53 is compared with Figure 45, which is the equivalent graph for four aspects, it is evident that they show great similarity. The limit of delay propagation is reduced slightly because of the decrease in the minimum steady state headway. However, the range of headways for which the system is stable (i.e. the full imposed delay is not propagated), is larger than for four aspects. This is more immediately apparent if reference is made to Figure 54, which is the five aspect equivalent to Figure 46. The explanation for the increase in the range of stable headways is to be found in the fact that the observation of a double yellow results in a somewhat larger delay to a train than under four aspects. This can result in a following train being able to avoid encountering a single yellow aspect, so that the total delay which the train encounters is reduced.

In Figure 55 the minimum steady state headway, the limit of stability and the limit of delay propagation are shown plotted against running speed, for a leading train waiting period of zero. The minimum steady state headway and the limit of delay propagation display the two discontinuities which are characteristic of five aspect signalling. It should be noted that the limit of stability coincides with the limit of delay propagation for running speeds below VSF2. At these lower running speeds the observation of a single yellow aspect is required to produce any delay at all to a following train, but the
FIGURE 54 Leading Train Waiting Period 6 sec
Running Speed 100 mph
Train Length 660 ft.

Minimum Steady State Headway

FIGURE 55 Leading Train Waiting Period 0 sec
Train Length 660 ft.

Limit of Delay Propagation
Limit of Stability
Minimum Steady State Headway
magnitude of the delay, which results from encountering this aspect, is equal to that of the imposed delay. Let us compare Figure 55 with the equivalent graph for four aspect signalling, which is Figure 49. The range of running speeds is increased, over which it is possible to achieve stable propagation of delays, because VSF2 is a lower speed than VSF. It should also be noted that the range of headways which produce instability is reduced for all running speeds, since the limit of propagation is closer to the minimum steady state headway for running speeds below 20 mph.

The corresponding graphs to Figure 55 are given for leading train waiting periods of 30 seconds and 60 seconds, in Figures 56 and 57 respectively. It is evident that the effect of increasing the leading train waiting period is very similar to that observed with four aspects. For a 30 second leading train waiting period, the limit of stability and the limit of propagation do not coincide for any value of running speed, while further increasing the waiting period to 60 seconds greatly increases the headway range over which some stable delay propagation is possible, this being particularly the case for running speeds less than 40 mph.

2.2.2 Double Yellow Starting

This procedure only differs from single yellow starting when a train has come to rest at a signal displaying a red aspect. In double yellow starting the train does not resume its journey when the signal aspect changes from red to single yellow, but waits until a double yellow aspect is displayed.
2.2.2a Four Aspect Signalling

The procedure for determining the delay encountered by following trains and resulting from an imposed delay on the leading train as described for single yellow starting on four aspects, requires very little modification to make it applicable to double yellow starting. The only differences occur after a following train has come to rest at a signal displaying a red aspect. Thus, if at any signal waiting is necessary, i.e. if

\[ \text{TOE} + \text{AOE} - \text{TT} (KL + 1) < 0 \]

then, for double yellow starting, the waiting time is given by

\[ \text{SUP}(I) = \text{TT}(KL + 2) - \text{TOE} - \text{AOE} \]

and the time taken for the train to travel through the block is given by

\[ \text{AOE} = \text{TT}(KL + 2) - \text{TOE} \]

If the double yellow starting performance is compared with that of single yellow starting for any given train length and running speed, it is evident that the minimum steady state headway and the limit of delay propagation are identical in both cases. It also appears that, for the range of initial imposed delay considered, the limit of stability is also unaffected by the starting procedure. For a leading train waiting period of zero, a train length of 660ft, and a running speed of 100 mph, (which are the conditions pertaining to Figure 45), there is no difference in performance between double yellow and single yellow starting for any of the running headways shown in that diagram. However, for a running headway of 70 seconds,
the delays encountered by following trains under the two procedures, are illustrated in Figure 58.

It is apparent that in both cases the delays encountered by subsequent trains are larger than the imposed delay, but that the actual time, by which a specific train is delayed, depends upon the starting procedure in use. It is interesting to note that for some trains the delays produced by double yellow starting are greater than those produced by single yellow starting. In such cases, any advantage, derived from encountering less restrictive aspects when the train has resumed its journey after stopping at a red aspect, is not sufficient to compensate for the extra period for which it remains stationary after the red aspect has cleared to single yellow. For other following trains (from the eighth onwards), the double yellow starting procedure does evidence the advantage expected of it. The example given in Figure 58 is not entirely typical of those conditions under which the performances of single yellow and double yellow starting diverge, because, in most cases, there is an advantage shown by double yellow starting for all following trains. However, it does seem to be a characteristic of headways, for which there is a difference between single yellow and double yellow starting, that they are close to the minimum steady state headway, and are in the region of instability. Thus, the only advantage of double yellow starting would seem to be that the rate of increase of the delays is decreased. The fact that the performance is still unsatisfactory because the instability still persists, suggests that the procedure is of little practical value.
FIGURE 58
Leading Train Waiting Period 0 sec
Running Headway 70 sec
Running Speed 100 mph
Train Length 660 ft.

Delay in Seconds

Number of Following Train
However, the minimum leading train waiting period which causes the second train to encounter a red aspect can be determined for specified values of train length, running speed, and running headway. Let us consider a situation in which the headway between the trains is large enough for the leading train to come to rest before the second train has reached the signal displaying a double yellow aspect. If the leading train remains stationary, the second train comes to rest at the red signal in the rear of the leading train a period of \( TD \) after the leading train came to rest, which is given by

\[
TD = HS - BL/V
\]

where \( HS \) is the running headway.

However, if the leading train has restarted but has not cleared the overlap associated with the signal at which it waits, the second train still encounters the red aspect. Thus, the minimum leading train waiting period, which causes the second train to observe a red aspect, is given by

\[
TDIF = HS - BL/V - TZ
\]

where \( TZ \) is the time taken for the leading train to clear the overlap associated with a signal at which it has halted, and is given by

\[
TZ = \frac{VZ}{A} \quad \text{if } VZ \leq V
\]

where \( VZ = \sqrt{2 \times A \times (OL + TL)} \)

or \( TZ = \frac{V}{2 \times A} + \frac{(OL + TL)}{V} \) otherwise.

Now, if \( V < VSF \), then \( TDIF \) is given by the expression quoted above, provided that the leading train has not commenced braking when it
clears the overlap associated with the signal immediately in the rear of the one at which it comes to rest, i.e. if \( \text{DOS} \geq (\text{OL} + \text{TL}) \), where \( \text{DOS} = \text{BL} - \frac{\text{V}^2}{2 \times \text{B}} \).

However, if \( \text{DOS} < (\text{OL} + \text{TL}) \), the second train might encounter a single yellow aspect before the leading train clears this overlap, if the headway was close to the minimum steady state value. Thus, the above expression for \( \text{TDIF} \) only applies of \( \text{HS} > \text{HL} \), where

\[
\text{HL} = \frac{\text{HW} + \text{V}/(2 \times \text{B}) - \text{VIN} \times (2 \times \text{V} - \text{VIN})/(2 \times \text{B} \times \text{V})}{(2 \times \text{B} \times \text{V})}
\]

in which \( \text{HW} \) is the steady state minimum headway, and

\[
\text{VIN} = \sqrt{2 \times \text{B}} \times (\text{BL} - \text{OL} - \text{TL})
\]

If \( \text{HS} \leq \text{HL} \),

\[
\text{TDIF} = \frac{\text{HS} - \text{BL}/\text{V} - \text{TZ} + \text{V} \times (\text{A} + \text{B})/(2 \times \text{A} \times \text{B})}{(2 \times \text{A} \times \text{B})}
\]

Alternatively, if it is initially found that \( \text{V} \geq \text{VSF} \), then provided that \( \text{HS} > \text{HL} \), where \( \text{HL} \) is as given above,

\[
\text{TDIF} = \frac{\text{HS} - \text{BL}/\text{V} - \text{TZ}}{2 \times \text{A} \times \text{B}}
\]

But, if \( \text{HS} \leq \text{HL} \), then the second train will encounter more than one double yellow aspect, and

\[
\text{TDIF} = \frac{\text{HS} - \text{BL}/\text{V} - \text{TZ} + \text{BL} \times (\text{V} - \text{VSF})/(\text{V} \times \text{VSF})}{2 \times \text{A} \times \text{B}}
\]

if \( \text{DIS} \geq (\text{OL} + \text{TL}) \), where \( \text{DIS} \) is given by

\[
\text{DIS} = 2 \times \text{BL} - \frac{\text{V}^2}{2 \times \text{B}}
\]

i.e. if the leading train has not commenced braking before it clears the overlap associated with the signal at which it observes a double yellow aspect.

However, if \( \text{DIS} < (\text{OL} + \text{TL}) \), then the above expressions only
apply if \( HS > HLT \),

where

\[
HLT = \frac{HW + V/(2 \times B) - VON \times (2 \times V - VON)/(2 \times B \times V)}{}
\]

and

\[
VON = \sqrt{2} \times B \times (2 \times BL - OL - TL)
\]

if

\[
HS \leq HLT,
\]

then

\[
TDIF = \frac{HS - BL/V - TZ + 2 \times BL \times (V - VSF)/(V \times VSF)}{}
\]

The performance of single yellow and double yellow starting, only differs if a following train comes to rest at a signal displaying a red aspect. Thus, TDIF is also the minimum leading train waiting period required to produce such a difference. This is shown as a function of running headway for various running speeds in Figures 59 and 60.

These graphs may be used to determine if, for a given running speed, and leading train waiting period, there is a range of running headways over which the performances of single yellow and double yellow starting diverge. They also show that there may be headways slightly larger than the minimum steady state headway which are also less than this range, because the following trains encounter a larger number of restrictive aspects producing so much delay earlier on, that they never encounter a signal displaying a red aspect.
FIGURE 59 Train Length 660 ft. Various Running Speeds

Minimum Waiting Period for a Difference in Single & Double Yellow Starting - in Seconds

Running Headway as a Percentage of the Minimum Steady State Value

100 106 116 124 132 140

FIGURE 60 Train Length 660 ft. Various Running Speeds

Minimum Waiting Period for a Difference in Single & Double Yellow Starting - in Seconds

Running Headway as a Percentage of the Minimum Steady State Value

100 106 116 124 132 140
2.2.2b Five Aspect Signalling

The procedure used to determine the delay encountered by following trains for single yellow starting on five aspects may be made applicable to double yellow starting by identical modifications to those described for double yellow starting under four aspects.

As might be anticipated, double yellow starting produces a very similar effect on five aspect signalling to that observed on four aspects. The only differences in performance occur at values of running headway in the unstable region, and, when differences do occur, the effect of double yellow starting is to reduce the rate at which delays build up for subsequent trains.

The process for the determination of the minimum leading train waiting period which results in the second train observing a red aspect, and which was described for four aspects, may be applied directly to five aspects for running speeds less than $V_{SF1}$, provided that $V_{SF2}$ is used instead of $V_{SF}$. It also applies if $V > V_{SF1}$, but

$$DES \geq (OL + TL)$$

where

$$DES = 3 \times BL - V^2/(2 \times B)$$

However, if $DES < (OL + TL)$, the previously described expressions only apply if $HS > HLS$

where

$$HLS = HW + V/(2 \times B) - VEN \times (2 \times V - VEN)/(2 \times B \times V)$$

and

$$VEN = \sqrt{2} \times B \times (3 \times BL - OL - TL)$$

Now, if $HS < HLS$, and if $Fl < (OL + TL)/V$,

where

$$Fl = (V_{SF1} - \sqrt{V_{SF1}^2 - 2 \times B \times (OL + TL)})/B - BL/V_{SF1}$$

then $TDIF$ is given by
TDIF = HS - BL/V - TZ + T1 + 2 x T2

where T1 = BL x (V - VSF1)/(V x VSF1)
and T2 = BL x (V - VSF2)/(V x VSF2)

But if F1 ≥ (OL + TL)/V > F2, where

\[ F2 = \frac{(VSF1 - \sqrt{VSF1^2 - 2 \times B \times (BL + OL + TL)})}{B} - BL/VSF1 \]

then TDIF is given by

TDIF = HS - BL/V - TZ + 2 x T1 + T2

Alternatively, if F2 ≥ (OL + TL)/V, then TDIF is given by

TDIF = HS - BL/V - TZ + 3 x T1

The minimum leading train waiting period, which is required to produce a difference in the performance of single and double yellow starting, is shown as a function of running headway in Figures 61 and 62 for various values of running speed. If these are compared with the equivalent graphs for four aspect signalling (Figures 59 and 60) it is evident that, for a given running speed and leading train waiting period, the range of headways over which the performances of single and double yellow starting differ, may be either larger or smaller. It should be noted, however, that this range of headways again lies within the unstable region, and thus the double yellow starting procedure still seems to be of little practical value. However, in order to make any definite general conclusions, it is necessary to consider a situation in which very large leading train waiting periods and headways are employed.
CHAPTER 3: PURE MOVING BLOCK SIGNALLING

The signalling system examined in this chapter is essentially a theoretical one, in which a train is continuously supplied with accurate information of the position of the nearest obstacle on the track ahead of it, relative to its own position. This obstacle can be of different types, e.g. it may be a preceding train, which itself may be moving or stationary, or it could be a junction with the points set for another route. The speed of the train is constantly checked and adjusted by braking or acceleration if necessary, so that it is always possible for the train to be brought to rest without colliding with the obstacle.

It should be noted that this is not suggested as a practical system, but represents a theoretical idea. Other forms of moving block have been suggested in the past, and are outlined in principle in Chapter 1 and reference 4. Moving space block, in which the distance separating a train from its predecessor (or another closer obstacle) must be at least equal to a service braking distance, has the advantage that the speed of the train need not be measured. However, the distances separating trains under similar running conditions can be considerably greater than under pure moving block, and thus the capacity is lower. Relative moving block, in which the speed of the obstacle is required, must be discounted for passenger traffic, since, if a train is brought to a sudden halt (perhaps resulting from collision or derailment), the following train will not be able to avoid hitting it.
3.1 Steady State Capacity

In order to obtain a direct comparison of pure moving block and fixed block signalling, an analysis of the capacity of the same basic components which were examined in Chapter 2 is made for moving block. Also, the same algorithm is used to combine these components to give a bi-directional junction.

3.1.1 Basic Components

(i) A Straight Line

If two trains are travelling along an indefinitely long straight line with a constant speed of \( V \), then the minimum distance separating the tail of the leading train and the front of the following train must be at least equal to

\[
\frac{V^2}{2 \times B}
\]

in order to fulfil the basic moving block requirement.

However, it is obviously desirable to add some sort of safety margin to the separation in order that, if the first train comes to a sudden dead stop, the following train has a period of time to determine that braking is necessary, and to initiate the application of the brakes. Since the required period is independent of running speed, the extra distance separating the trains, which provides this safety margin, is given by

\[
V \times \frac{OM}{VM}
\]

where \( OM \) is the distance equivalent of the safety interval for
line speed VM.

When, for any reason, two trains are brought to rest, it is necessary that there remains a short minimum distance separating them which could be as small as a few feet. This distance, DK, must also be added to the distance separating the trains when they are moving. Thus the total distance between the tail of one train and the front of the following train is given by

$$DK + V \times (OM/VM + V/(2 \times B))$$

Therefore the minimum straight line headway is given by

$$HW = (TL + DK)/V + OM/VM + V/(2 \times B)$$

If HW is measured in seconds, the capacity of the line in coaches per hour is obtained in the same way as with fixed block signalling, i.e. CH is given by

$$CH = 3600 \times NC/HW$$

where NC is the number of coaches per train.

The capacity of a straight line under moving block signalling is shown, as a function of running speed, in Figure 63. Also shown in this diagram are the capacity of four aspect fixed block on a line signalled for 100 mph, and the capacity of lines resignalled at all running speeds for both four and five aspect fixed block. It is evident that the moving block curve has a similar shape to the curves for resignalled line, but that, at all running speeds, it gives increases in capacity ranging from 40% to 70% over resignalled four aspect fixed block. Naturally
the increase in capacity over a line signalled at 100 mph for four aspects is much greater for all speeds other than line speed, and can be as high as 400% at speeds close to 30 mph. It is also interesting to note that, whereas at 100 mph a change from four to five aspects would give an increase in capacity of approximately 30% of that resulting from a change to moving block, for lines signalled at 20 mph this has fallen to under 3%. The disproportionate advantage, shown by moving block at the lower end of the speed range, is derived from the use of a constant time (and thus variable distance) safety margin, as opposed to the fixed length overlap used in both four and five aspect fixed block systems.

In moving block the maximum capacity can be achieved between 30 and 40 mph, since, above this speed, the dominant factor in the distance separating the trains is the braking distance which increases as the square of the speed, and hence causes an increase in minimum headway despite the fact that the train is travelling faster. At lower running speeds, the train length is the more dominant factor, and the capacity thus increases with increasing speed. Thus the maximum capacity is approximately 50% greater than the capacity at line speed, whereas under four aspect fixed block the maximum capacity is obtained at a speed just below the discontinuity, although this is very similar to the capacity at line speed. Therefore, the moving block system would be more resilient to any temporary reduction in line speed which might be required, since, under
moving block, the line speed capacity is attainable down to running speeds of about 12 mph, whereas for any running speed below 70 mph the line speed capacity is quite unattainable under four aspect fixed block.

Since the headway is a simple function of running speed, it is fairly easy to determine the speed at which maximum capacity occurs. This speed is given by

$$VM_1 = \sqrt{2} \times B \times (TL + DK)$$

(ii) A speed restricted curve

In Chapter 2 reference was made to Appendix A. This shows that if the distance between two trains which are passing through a speed restriction is sufficient at a certain critical instant, then it will also be sufficient at all other times. The critical instant is shown to be the point when the second train must commence braking in order to be travelling with a speed of VL (the limit speed) when it enters the restriction. The arguments advanced in Appendix A apply equally well to moving block as to fixed block, and hence, in order to determine the necessary minimum headway for trains passing through a restriction, the critical instant is examined in detail.

As in Chapter 2, a restriction with a length of DL and a speed limit of VL is considered, and reference is made to Figure 140. Naturally, the headway required between the trains when passing through the speed restriction (HWR) is equal to the straight line headway, provided that V < VL. However, if V > VL,
and for a practical speed restriction having a limit speed greater than zero, at the critical instant (i.e., when the second train reaches the point A in Figure 140), the leading train has already passed the point B, and hence is either in the speed restriction between points B and C, or it has passed the point C and may be accelerating between C and D, or it may once again be travelling at running speed V beyond the point D.

If $HD$ is the distance between the tail of one train and the tail of the one following it when the trains are travelling along a straight line, then $HD$ is given by

$$HD = HW \times V$$

where $HW$ is the minimum straight line headway.

In the case of the speed restriction, the distance between the positions occupied by two trains at the critical instant must also be equal to $HD$. Thus, if, when the following train reaches the point A, the leading train is between points B and C, i.e. if

$$HD < DL + (V^2 - VL^2)/(2 \times B)$$

then the necessary headway is given by

$$HWR = (HD - (V^2 - VL^2)/(2 \times B))/VL + (V - VL)/B$$

However, if $HD > DL + (V^2 - VL^2)/(2 \times B)$, then the leading train has passed the point C when the following train reaches the point A. The leading train will not have passed the point D by the critical instant, if
and hence, under these conditions, it is travelling with an instantaneous speed of $V_{IN}$, which is given by

$$V_{IN} = \sqrt{V_{L}^2 + 2 \times A \times \left( \frac{HD - DL - (V^2 - V_{L}^2)}{2 \times B} \right)}$$

and the required headway is given by

$$HWR = \frac{(V - VL)/B + DL/VL + (V_{IN} - VL)/A}{1}$$

Alternatively, if it is found that

$$HD > DL + \frac{(V^2 - V_{L}^2) \times (A + B)}{(2 \times A \times B)}$$

then the leading train has passed the point $D$ when the following train reaches the point $A$. Under these circumstances, the required headway is given by

$$HWR = \frac{\left( HD - DL - \frac{(V^2 - V_{L}^2) \times (A + B)}{2 \times A \times B} \right) / V + DL/VL + \frac{(V - VL) \times (A + B)}{(A \times B)}}{1}$$

The capacity of a speed restricted curve is shown in Figure 64 for various values of limit speed. It is evident that a speed restriction of 67 mph produces a considerable reduction from the straight line value, especially with running speeds close to line speed. In fact, with a running speed of 100 mph the capacity is only 75% of the straight line value. Reducing the limit speed produces further reductions in capacity, to such an extent that, with a limit of 33 mph the capacity for a running speed of 100 mph is only approximately 52.5% of the straight line capacity. It is interesting to note that the capacity curve for a speed limit of 33 mph becomes almost
parallel to the straight line curve for running speeds above 65 mph. Thus, over this range of running speed the actual loss in capacity due to the restriction remains constant at approximately 380 coaches per hour. For running speeds in this range the leading train is still in the speed restriction at the critical instant when the following train commences braking. Thus the dominant factor in the headway becomes the period during which the train is braking, from running speed $V$ to limit speed $V_L$. Since this time increases almost linearly with $V$ for small values of $V_L$, it varies in a similar way to the dominant term in straight line headway over this speed range. This term, as already mentioned, is the time taken to travel the braking distance.

(iii) A simple converging junction

The problem of finding the capacity of a simple converging junction operating under moving block signalling may be reduced as before to finding the length of the four time intervals

THH: the interval between two trains from the high speed route;

TLL: the interval between two trains from the low speed route;

THL: the interval between a train from the low speed route and one from the high speed route; and

TLH: the interval between a train from the high speed route and one from the low speed route.
These four quantities are used to obtain the average
headway, $H_{MC}$, between trains passing through the junction, and
hence the capacity, by means of the method described in Chapter
2 for this type of junction operating under four aspect fixed
block signalling.

In the following description of the method whereby the
four intervals $TH_H$, $TLL$, $TH_L$ and $TL_H$ may be evaluated, reference
is made to the junction shown in Figure 65. For a junction with
speed turnouts of 1:1 and for other junctions when the running
speed is less than the limit speed, the interval between two
trains which approach the junction on the same route must have
a minimum value equal to the straight line headway.

Thus,

$$TH_H = TLL = MW$$

Alternatively, if two trains which approached from different
routes are passing through the junction, then the points may
commence changing their route setting when the tail of the first
train clears the point $PC$, which, like the points $PA$ and $PB$,
is a distance $OL$ from the junction at $PJ$. Let us suppose that
this first train approaches the junction along route $B$, so that,
when the points change is completed after a period $TPC$, the
second train is approaching along route $A$ and has reached position
$A$ (in Figure 65). During the period in which the points are
changing, train 1 travels a distance $DPC$ which is equal to
$(TPC \times V)$, and hence, when the points have reset to route $A$, the
situation existing is that shown in Figure 65, except that there
is no train on route B.

Just prior to the completion of the points change, train 2
must not be so close to the point PA that it cannot be brought
to rest at that point. Thus, the distance separating train 2
from PA when the points change is completed is given by

\[ HA = DK + V \times \left( \frac{OM}{VM} + \frac{V}{2 \times B} \right) \]

Thus the total time which elapses between the completion of the
points change and the instant when the tail of train 2 clears
the point PC is given by the expression

\[ HW + (2 \times OL)/V \]

Hence the interval, THL, between the instant when the tail of
train 1 clears the point PC and the time when train 2 occupies
the same position, is given by

\[ THL = HW + TPC + (2 \times OL)/V \]

Since the junction being considered has speed turnouts of 1:1, the
interval TLH is equal to THL.

As with fixed block signalling, there is a need for two
categories of converging junctions which have speed turnouts
other than 1:1, which are defined in the same way as in Chapter
2. For a junction of two high speed routes, i.e. one where the
speed limit imposed on trains travelling through the low speed
side lasts for only a short distance on each side of the points
the interval between two trains on the high speed route is
unaffected by the speed limit. Thus

\[ THL = HW \]
However, TLL is found from considering the junction as a speed restriction having the appropriate speed limit and length. Wherefore

\[ TLL = M \times R \]

Once again referring to Figure 65, there is no difference from the junction with speed turnouts of 1:1 in the time which must elapse between the tail of a train which approaches along route B, clearing the point PC, and the following train, which approaches along route A, being able to take up the same position. However, when train 1 clears the point PC it is not travelling at running speed, \( V \), but at the limit speed, \( V_L \), and therefore the distance separating the trains is subsequently reduced, since the speed of train 2 is at all times equal to \( V \). It is necessary to ensure that the headway between the trains does not ultimately fall below the straight line value, \( HW \).

Thus if \( E \) is the amount by which the headway between the trains is reduced during the period before train 1 attains running speed and after passing through the junction, then it is given by

\[ E = \frac{(V - V_L)^2}{2 \times A \times V} \]

If \( E \leq \frac{(TPC + (2 \times OL)/V)}{} \), then the interval which must be allowed between the trains to enable train 2 to approach the junction at full speed is sufficient to ensure that the headway does not subsequently fall below \( HW \), and \( THL \) is given by

\[ THL = HW + TPC + (2 \times OL)/V \]

But, if \( E > \frac{(TPC + (2 \times OL)/V}{\} \), then
\[
\text{THL} = \text{HW} + E
\]

In order to find the interval TLH let us suppose that train 1 approaches the junction along route A, so that when it is in the position shown the points have just reset for route B, and that train 2 is in position B. Train 2 must still be travelling at running speed, \(V\), and hence the distance separating the train and the point PB is given by

\[
\text{HB} = \text{DK} + V \times \left(\frac{OM}{VM} + \frac{V}{2 \times B}\right)
\]

During the course of part of its journey over this distance, train 2 must brake from the speed of \(V\) so that it is travelling with a speed of \(VL\) when it reaches the point PB. Thus the time taken for this journey is given by the expression

\[
\frac{\text{HB}}{V} + \frac{(V - VL)^2}{(2 \times B \times V)}
\]

Train 2 then proceeds with a speed of \(VL\) until its tail clears the point PC, which takes a further period of

\[
\frac{(2 \times OL + TL)}{VL}
\]

Thus the total time which elapses between train 1 clearing the point PC and train 2 reaching the identical position is given by

\[
\text{TLH} = \text{HW} + \text{TPC} + \frac{(2 \times OL)}{VL} + \frac{(V - VL)}{(VL)} \times \left(\frac{TL}{V \times VL} + \frac{(V - VL)}{(2 \times B \times V)}\right)
\]

In the case of the other category of junction which has speed turnouts other than 1:1, the limit speed, \(VL\), is also the line speed over the complete length of the low speed route. The interval which is necessary between two trains on the high
speed route is again given by

$$THH = HW$$

The minimum headway required between two trains travelling along the low speed route before they reach the junction is given by

$$HML = (DK + TL)/VL + OM/VM + VL/(2 \times B)$$

During the period in which the first train is accelerating while the second train is not, the actual distance between the trains increases, while the minimum requirement does not, since this is a function of the second train. Thus, this intermediate period does not affect the headway requirement. However, as the second train begins to accelerate, the minimum separating distance required also increases, reaching a maximum when the second train reaches running speed, V. Thus the interval between two low speed trains, TLL, is equal to either HML or HW, whichever is the greater of the two.

The case of a high speed train following a low speed one is not affected by the length over which the speed limit applies on the approach side of the junction. Thus the interval THL may be found in exactly the same way for a junction of high and low speed routes as was described above for a junction of two high speed routes.

If train 1, in Figure 65, approaches the junction along the high speed route A, at the instant when the points complete their change to set up for route B, train 2 is in position B, and travelling with a speed of VL. Thus the distance HB separating the train from the point PB is given by
\[ HB = DIK + VL \times \left( \frac{OM}{VM} + \frac{VL}{(2 \times B)} \right) \]

Train 2 travels over this distance at a constant speed of \( VL \), and, before its tail clears the point PC, it travels a further distance of \( (2 \times OL + TL) \) at the same speed. Thus the interval \( TLH \) is given by

\[ TL = HWL + TPC + (2 \times OL)/VL \]

where \( HWL \) is as defined above.

The capacity of a converging junction with speed turnouts of 1:1 is shown as a function of traffic volumes in Figure 66 for various values of running speed. It is evident that this function is symmetrical about traffic volumes of 50:50, at which there is a minimum capacity available for any value of running speed. If the curves shown in Figure 66 are compared with the equivalent ones for four aspect fixed block, which are given in Figure 8, it becomes apparent that the capacity available under moving block is invariably greater, although the actual increase obtained varies from 20% to 900%. Moving block shows its largest advantage at lower running speeds, which is due to the relatively small braking distance at such speeds being very much smaller than the two block lengths, which must separate the trains under fixed block. It is also evident that, for any given running speed, the variation of capacity with traffic volumes is much greater with moving block, thus causing the largest increases in capacity to occur at traffic volumes of 0:100 and 100:0. There are two contributory factors to this effect. One of these arises from the same period being allowed under both systems.
FIGURE: 65
Converging Junction
Speed Turnouts 1:1

Capacity in Coaches per Hour

Fast: Slow Traffic Volumes
for the points to change their route setting, since, in the case of moving block, this represents a larger percentage of the straight line headway. The other is the fixed length safety margin which is allowed on each side of the junction, under moving block, since this has to be added to the straight line headway, whereas under fixed block no equivalent additional separation is required.

The equivalent curves for a converging junction of two high speed routes having speed turnouts of 100:33 are shown in Figure 67. Naturally, for running speeds less than the limit speed, the available capacity remains unaffected. For higher running speeds the curves are no longer symmetrical about 50:50 traffic volumes, since the headway necessary between two trains passing through the low speed side of the junction may be considerably greater than the straight line values required on the high speed side. The capacity is also lower at traffic volumes of 50:50, since additional headway is required between a train on the low speed route and one on the high speed route. This is because of the existence of a speed differential between them after the slow train has passed through the junction. The largest decreases in capacity are observed for the higher running speeds, because as the running speed increases the difference between it and the speed limit increases, thus requiring a greater additional inter-train spacing for the following train to absorb during the leading train's acceleration period. If reference is made to the equivalent curves for four aspect fixed block (Figure 9), it is evident that the effect of the speed limit
upon capacity is very similar under both signalling systems.

In Figure 68 the equivalent curves are given for a converging junction of high and low speed routes. The capacity at traffic volumes of 0:100 is equal to that at 100:0, since the capacity of the low speed route approaching the junction is very high for its line speed of \( V_L \). Thus the headway required between two trains passing through the low speed side of the junction is determined by what is necessary after the trains have negotiated the junction, and attained a full running speed of \( V \). For running speeds greater than the limit speed, \( V_L \), the capacity for traffic volumes of 50:50 is less than with speed turnouts of 1:1, but greater than that available with the junction of two high speed routes. The interval, \( T_{HL} \), between a train on the low speed route and one on the high speed route, must allow for a subsequent reduction, due to the speed differential between the trains, in exactly the same way as with the junction of two high speed routes. However, in this case, there is also a reduction in the interval \( T_{HL} \) between a train on the high speed route and one on the low speed route, because the low speed train may be much closer to the junction when the points change is complete since it is already travelling at the limit speed. This reduction in \( T_{HL} \) partially compensates for the increase in \( T_{HL} \), so that the capacity is greater than that of a junction of two high speed routes. Considerable similarities to the equivalent fixed block case may again be observed, if reference is made to Figure 10.
(iv) **A simple diverging junction**

As with the converging junction, the capacity of a diverging junction may be found from the intervals $T_HH$, $T_LL$, $T_HL$ and $T_LH$, using the expressions given in Chapter 2 for a converging junction operating under four aspects.

For a diverging junction with speed turnouts of 1:1, the intervals between trains which are following the same route, must have a minimum value equal to the straight line headway, i.e.

$$ T_{HH} = T_{LL} = HW $$

However, if two trains are following different routes, then, referring to Figure 69, when the points complete their route change, the distance separating train 2 from the point PC must be given by

$$ H_2 = DK + V \times (OM/VM + V/(2 \times B)) $$

if train 2 is to be able to approach the junction without having to brake. Thus the intervals between the trains are given by

$$ T_{HL} = T_{LH} = HW + T_{PC} + (2 \times OL)/V $$

For a junction of two high speed routes the interval required between two trains on the high speed route is not affected by the speed limits and hence is again equal to the straight line headway, i.e.

$$ T_{HH} = HW $$

As with the converging junction, the interval between two trains
which are both passing through the low speed side of the junction is equal to the minimum headway required for a speed restriction having appropriate values of length and limit speed. Thus TLL is given by

$$TLL = \frac{HW}{V}$$

The interval which must elapse between a train clearing the junction as it moves along route B and the following train clearing the junction while travelling along route A, remains unaffected by the introduction of the speed limit. Thus it is given by

$$THL = HW + TPC + \frac{2 \times OL}{V}$$

However, if one train is travelling along the high speed route, while the following train travels along the low speed route, then, when the points complete their change of setting, train 1 occupies the position A (as shown in Figure 69). In order that train 2 does not need to brake prematurely, the distance H2 is again given by the expression quoted for the junction with speed turnouts of 1:1. Train 2 continues to travel at its running speed for a time, but it must brake for a period prior to reaching the point PC, since, at that point, it must have a speed of VL. The train then moves through the junction with a speed of VL until its tail clears the point PB. Thus, the interval TLH is given by

$$TLH = HW + TPC + \frac{2 \times OL}{VL} + (V - VL) \times \left(\frac{TL}{V \times VL} + \frac{(V - VL)}{(2 \times B \times V)}\right)$$
The analysis of a diverging junction of high and low speed routes is very similar to the one described above. The intervals THH, THL and TLH are all determined in exactly the same way as described for the junction of two high speed routes. However, in determining the interval TLL, the length of the speed restriction must be considered to be infinite.

For a diverging junction with speed turnouts of 1:1 the values of the intervals THH, TLL and TLH are exactly the same as for an equivalent converging junction. Thus the capacity is also identical, so that Figure 66 applies equally to converging and diverging junctions having speed turnouts of 1:1.

The capacity of a diverging junction of two high speed routes is given as a function of traffic volumes in Figure 70, for speed turnouts of 100:33. From a comparison with the equivalent graphs for a converging junction (Figure 67), it is evident that there is great similarity in the performance of the two junctions. This is to be expected, since the intervals THH, TLL and TLH are obtained from identical expressions. In fact, the interval THL, and thus the capacity, only differs from that of the converging junction when there is a large disparity between the running speed and the limit speed. Under these circumstances, extra headway must be allowed in the case of the converging junction, because of the reduction in the distance separating the trains, which occurs during the period that the low speed train is accelerating after passing through the junction. Thus the capacity of the diverging junction is greater than that of the converging junction for traffic volumes other than 0:100.
FIGURE: 69 Diverging Junction of Two High Speed Routes

FIGURE: 70 Diverging Junction of Two High Speed Routes
Speed Turnouts 100:33
Various Running Speeds

Capacity in Coaches per Hour

Fast: Slow Traffic Volumes
and 100:0 and running speeds greater than approximately 74 mph. It is apparent that the capacity is greater than that of the converging junction for running speeds of 80, 90 and 100 mph, this being especially noticeable for traffic volumes of 50:50.

The corresponding curves for a junction of high and low speed routes are given in Figure 71. The capacity is very similar to that available with the junction of two high speed routes, except that, for running speeds greater than the limit speed, VL, and traffic flows having a majority of low speed trains, it is slightly lower. This is due to the increased length of the restriction.

From a comparison with the equivalent graphs for four aspect fixed block signalling, it may be observed that the curves, which apply to a diverging junction of two high speed routes, have a relationship to one another, which is similar to that described for a converging junction of two high speed routes. However, the difference between the capacity of a junction of high and low speed routes and that of a junction of two high speed routes is very much less, under moving block, than under four aspect fixed block.

(v) **A diamond crossing**

In examining the capacity of a diamond crossing, reference is made to the one shown in Figure 72. The intervals TIIH, TLL, THL and TIIH may be used to determine the capacity by the same method that is employed for the converging and diverging
FIGURE: 71 Diverging Junction of High and Low Speed Routes

Speed Turnouts 100:33
Various Running Speeds

FIGURE: 72

Train: 2
PA
PB
PC
PD
H2
DL/2
Train: 1
Route: A
Route: B
junctions. These intervals are measured between the times, at which the tails of appropriate trains, clear the overlap points (PC and PD) beyond the crossing.

Let us initially consider a crossing which has speed turnouts of 1:1. The headway required between any two trains on the same route is not affected by the presence of the crossing, so that it is equal to the straight line headway, hence

\[ THH = TLL = HW \]

If we have two trains, the first of which travels along route B and the second along route A, then train 1 must have reached the position shown in Figure 72 before the crossing is clear for train 2 to approach at full speed. In order that train 2 may maintain its running speed throughout, the distance which separates it from the point PA must be given by

\[ HZ = DK + V \times (\frac{OM}{VM} + \frac{V}{2 \times B}) \]

Thus the time taken for train 2 to travel from the position shown in Figure 72 to the place where its tail just clears the point PC is given by

\[ THL = TLH = HW + DL/V \]

For a crossing of two high speed routes which has speed turnouts other than 1:1, the interval between two trains, which are both travelling along the high speed route, is not influenced by the presence of the speed restriction on the other route. Thus

\[ THH = HW \]
The interval between two trains on the low speed route is found from consideration of a speed restriction of the appropriate length and limit speed, hence

\[ T_{LL} = HWR \]

The interval between a train on route B clearing the point PD and one on route A clearing the point PC is unaffected by the existence of the speed restriction, since it is determined by the time taken for the high speed train to travel the distance to the clearing point. Thus THL is given by

\[ THL = HW + DL/V \]

When the tail of a train on route A clears the point PC the closest which a following train on route B can be to the point PB is given by

\[ H2 = DK + V \times \left( \frac{OM}{VM} + \frac{V}{(2 \times B)} \right) \]

For part of its journey over this distance the train must be reducing its speed from V to VL. Hence the journey to the point PB takes a time given by the expression

\[ \frac{DK}{V} + \frac{OM}{VM} + \frac{V}{(2 \times B)} + \frac{(V - VL)^2}{(2 \times B \times V)} \]

From the point PB the train proceeds with a speed of VL until its tail clears the point PD. Thus TLH is given by

\[ TLH = HW + DL/V + (V - VL) \times \left( \frac{TL}{V \times VL} + \frac{(V - VL)}{(2 \times B \times V)} \right) \]

If we have a crossing of high and low speed routes then the speed limit is considered to act as the line speed of the entire low speed route, instead of applying for only a relatively short
length in the vicinity of the crossing. The intervals \( T_{LL} \) and \( T_{HL} \) are both obtained from the expressions which are given above for the crossing of two high speed routes. The speed of a train on the low speed route does not exceed the limit speed, \( VL \), for any part of its journey. Thus the minimum interval between two trains on this route is given by

\[
T_{LL} = \frac{(T_L + D_K)}{VL} + \frac{OM}{VM} + \frac{VL}{(2 \times B)}
\]

When the tail of a train on route A clears the point PC the closest which a following train on route B can be to the point PB is given by

\[
H_2 = D_K + VL \times \left(\frac{OM}{VM} + \frac{VL}{(2 \times B)}\right)
\]

The train travels over this distance with a constant speed of \( VL \) and maintains the same speed until its tail clears the point PD. Hence the interval \( TL_H \) is given by

\[
TL_H = \frac{(T_L + D_K)}{VL} + \frac{OM}{VM} + \frac{VL}{(2 \times B)} + \frac{DL}{VL}
\]

The capacity of a diamond crossing with speed turnouts of 1:1 is shown in Figure 73, as a function of traffic volumes. The capacity at traffic volumes of 0:100 and 100:0 is identical to that of a converging or a diverging junction operating under the same conditions, since all three components are behaving like a straight line in these circumstances. However, the capacity of the crossing is greater than that of the junctions for all other values of traffic volumes, with the largest difference occurring at traffic volumes of 50:50. This is due
FIGURE 73 Diamond Crossing
Speed Turnouts 1:1
Various Running Speeds

FIGURE 74 Diamond Crossing of Two High Speed Routes
Speed Turnouts 100:33
Various Running Speeds
to zero time being required for changes in route setting in the case of the crossing.

The capacity of a crossing of two high speed routes with speed turnouts of 100:33 is given in Figure 74. From a comparison with Figure 70 it is evident that the introduction of the speed limit has a very similar effect on the capacity of the crossing as the same speed limit has on the capacity of a diverging junction. Thus for running speeds above the limit speed and traffic volumes in the range of 0:100 to 50:50 the capacity is considerably less than for a crossing with speed turnouts of 1:1, because of the increased headway required between two trains which are both following the low speed route, due to the effect of the speed restriction. The capacity at traffic volumes of 50:50 is also less than that available with speed turnouts of 1:1, since the value of TLH must be increased. This results from a train on the low speed route taking longer to move through the junction, because for part of its journey it is running at reduced speed.

The equivalent capacity curves for a crossing of high and low speed routes are shown in Figure 75. It should be noted that for running speeds greater than the limit speed and traffic volumes of 0:100 the capacity is independent of the actual value of the running speed. This is due to the crossing acting as a straight line with a line speed of $VL$, which has an appropriately large value of capacity (see Figure 63). The capacity, at traffic volumes of 50:50, is slightly less than that obtained from a crossing with speed turnouts of 1:1, but greater than that of a crossing of two high speed routes. For
a crossing of high and low speed routes the interval TLH is less than it is for a crossing of two high speed routes because the train on the low speed route may be much closer to the crossing when the train on the other route clears it, and this more than compensates for the lower average speed of approach. However, the interval TLH is greater than it is for a crossing with speed turnouts of 1:1, because the smaller time taken for the train to reach the point PB (Figure 72), which is \( DL/2 \) in the rear of the crossing, is compensated by the longer time taken to move through the more immediate vicinity of the crossing.

If the graphs of diamond crossing capacity are compared with the equivalent ones for four aspect fixed block (Figure 16 and 17), it may be observed that, whereas the curves applying to a crossing of two high speed routes have a relationship to one another which is similar to that observed for a converging junction of two high speed routes, this is not the case with either a crossing having speed turnouts of 1:1, or one of high and low speed routes. Unlike the equivalent crossing operating under four aspect fixed block, the capacity of the crossing with speed turnouts of 1:1 is not independent of traffic volumes. This is because of the need, in moving block, to introduce an additional fixed length overlap, which must be cleared by a train on one route before the crossing is regarded as unoccupied as far as a train approaching on the other route is concerned. No equivalent additional overlap is required under fixed block. The same reason causes the minimum capacity for any given running speed to occur at traffic volumes of 50:50 in the case of a
crossing of high and low speed routes. The increase in headway, which is necessary to accommodate this overlap, is so large that, for traffic volumes of 50:50 and a running speed of 100 mph, there is very little difference in the capacity of moving block and four aspect fixed block.

(vi) **An isolated station**

The intervals THH, THL, TLH and TLL may be used to give the average headway required between trains in the same way as with other components, providing that the following definitions are used.

- THH is the interval between two stopping trains;
- THL is the interval between a non-stopping train and a stopping train;
- TLH is the interval between a stopping train and a non-stopping train;
- TLL is the interval between two non-stopping trains;
- N1 is the percentage of stopping trains;
- N2 is the percentage of non-stopping trains.

If two trains do not stop at the station the minimum interval required between them is equal to the straight line headway, i.e.

\[ TLL = HW \]

If a stopping train is following a non-stopping train, the minimum interval required between them at a point a long
distance in the rear of the station is also equal to the straight line headway, since the separating distance increases subsequently after the stopping train begins braking for the station. Thus

\[ T_{II} = H_{\text{W}} \]

If a non-stopping train is following a stopping train the interval required between them when they are both travelling at running speed before reaching the station must be sufficiently large to ensure that the headway does not fall below the straight line value at any time subsequently. Thus the interval \( T_{LI} \) must exceed the straight line headway, \( H_{\text{W}} \), by the difference between the time taken by the stopping train in braking for, stopping at, and accelerating away from the station, and the time in which the non-stopping train travels the same distance. Hence \( T_{LI} \) is given by

\[ T_{LI} = H_{\text{W}} + T_{ST} + V \times (A + B)/(2 \times A \times B) \]

In order to determine the interval required between two stopping trains reference is made to Figure 76. In order that train 2 does not have to commence braking prematurely the distance separating the trains must not fall below a value \( H_D \), which is given by

\[ H_D = H_{\text{W}} \times V \]

before it reaches the position shown in Figure 76. Subsequently it must commence braking in order to come to rest in the station. Since the speed of train 1 never exceeds that of train 2 before this instant, if the spacing is equal to \( H_D \) at the instant shown, it is also adequate at all other times.
FIGURE 75: Diamond Crossing of High and Low Speed Routes
Speed Turnouts 100:33
Various Running Speeds

FIGURE 76
In order that the distance separating the trains is IID when train 2 commences braking, train 1 must have travelled a distance given by the expression
\[
(DK + TL + (OM \times V)/VM)
\]
from the station. If train 1 is still accelerating at the end of this distance, then \((DK + TL + (OM \times V)/VM) < V^2/(2 \times A)\) and the interval \(THH\) is given by
\[
THH = V/B + TST + \sqrt{2 \times (DK + TL + (OM \times V)/VM)}
\]
However, if train 1 has already attained running speed, then
\[
THH = V \times (A + B) / (A \times B) + TST + (DK + TL + (OM \times V)/VM - V^2(2 \times B))/V
\]

The capacity of an isolated station is shown, as a function of the ratio of stopping to non-stopping traffic, in Figure 77. It is evident that the capacity declines rapidly, as the percentage of the traffic stopping at the station is increased. The percentage decrease in capacity between the case where no traffic stops at the station, and the case where all the traffic stops, is greatest at higher running speeds, due to the longer periods of braking and acceleration producing a larger amount by which a train is delayed in making a stop.

If this graph is compared with the equivalent one for four aspect fixed block (Figure 19), it may be seen that, for any given running speed, a variation in the percentage of traffic stopping at a station has a more marked effect under moving block than under fixed block. Thus for a very high percentage of stopping traffic the moving block signalling shows a much smaller increase in capacity over fixed block than when there is
a low percentage stopping. This is because the interval required between two trains, which are both stopping at an isolated station, is not significantly different under the two signalling systems especially at high running speeds. The period taken by a train to brake from running speed to rest, wait at the station, and accelerate away until the station berth is clear for the following train to approach, is the dominating factor contributing to the necessary headway. This is almost identical under the two signalling systems for high running speeds. For lower running speeds the leading train does not have to travel as far, under moving block, before the station berth becomes clear for the following train. Since the braking time is also decreased at lower running speeds, while the time taken to travel the length of a fixed 'block' is increased, the difference in the capacity available for large percentages of stopping traffic is much greater under moving block as opposed to fixed block, than at high running speeds.

The effect on capacity of varying the station stop time for a constant value of running speed is shown in Figure 78. In exactly the same way as with fixed block, the length of the stop produces relatively little variation in capacity, provided that it remains a comparatively small part of the total time, by which a train is delayed in making a stop at a station.

### 3.1.2 The Bidirectional Junction

An analysis of the steady state capacity of a bidirectional junction may be made by inter-relating the three basic components:
FIGURE 77 Isolated Station
Stop Time 30 sec
Various Running Speeds

Ratio of Stopping to Non-stopping Traffic
Capacity in Coaches per Hour

FIGURE 78 Isolated Station
Various Stop Times
Running Speed 100 mph

Ratio of Stopping to Non-stopping Traffic
Capacity in Coaches per Hour
a converging junction, a diverging junction and a diamond crossing, the analysis of which has already been described. The combination of these components is achieved by means of the algorithm which was used for fixed block signalling, and which is described in Appendix C. The intervals ROA, ROB, RTOA and RTOB which the algorithm uses are obtained for each component in turn, from the intervals TIIFI, THL, TLH and TLL, by the same method used for fixed block, and described previously in section 2.1.2.

An equivalent series of graphs to those which were given for a bidirectional junction operating under four aspect fixed block in Figures 35 to 39 are given, for moving block operation, in Figures 79 to 83 inclusive. In Figure 79 the capacity of a right-hand bidirectional junction with speed turnouts of 1:1 and converging traffic running at a maximum is given as a function of running speed and train length, for fast:slow traffic volumes of 10:90. This shows that the variation of junction capacity with train length for any specific value of running speed is almost linear, although there is a tendency for the rate of increase of capacity to be gradually reduced as the train is lengthened. For a given train length, the variation of capacity with running speed follows the form characteristic of moving block (compare with Figure 63). Therefore, if Figure 79 is compared with the corresponding diagram for four aspect fixed block signalling (Figure 35), it is evident that the variation of capacity with train length is of a similar form in both cases, whereas the variation with running speed under moving block has a relationship to that obtained with fixed block, which is similar to that observed in the case of a straight line.
FIGURE 79
Right Hand Bidirectional Junction with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum with Traffic Volumes of 10:90
FIGURE : 80
Right Hand Bidirectional Junction with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE 80
Right Hand Bidirectional Junction
with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum
Train Length 560 ft.
FIGURE: 82
Right Hand Bidirectional Junction of High and Low Speed Routes with Speed Turnouts of 100:33
Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE 83 Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts 100:33 Converging Traffic Running at a Maximum with Various Fast:Slow Traffic Volumes Train Length 650 ft.

Running Speed in M.P.H.

Capacity in Coaches per Hour

FIGURE 84 Delays to Following Trains Leading Train Waiting Period 0 sec Running Speed 100 mph Train Length 650 ft. Various Running Headways

Delays in Seconds

Number of Following Train
The capacity of the same junction is shown in Figure 80 as a function of running speed and the traffic volumes of the priority flow, which, in this case, applies to those trains which travel through the converging junction. It is apparent that, for a given running speed, the variation of capacity with traffic volumes displays similar discontinuities to those observed in the corresponding graph for four aspect fixed block (Figure 36). These are a characteristic of the algorithm combining the components, rather than the signalling system, and their cause has already been indicated in Chapter 2. The variation with traffic volumes has a similar form to that observed under fixed block, but the difference between the capacity attainable at traffic volumes of 50:50 and that attainable at 100:0 is considerably greater. This is to be expected, since the capacity of the separate basic components also shows a larger variation with traffic volumes, when operating under moving block.

The capacity of a right-hand junction of two high speed routes is given in Figure 81, for speed turnouts of 100:33. It is evident that, for traffic volumes of 0:100 when all trains are negotiating the low speed lines, the junction capacity is always twice that of a speed restriction of appropriate length and limit speed. Thus, for running speeds less than the limit speed, the capacity is symmetrical about traffic volumes of 50:50, while at higher speeds, this is not the case. In this respect the effect, which the speed limit has upon capacity, is very similar to its effect under fixed block signalling (see Figure 37). However, in the moving block case, there is a small range of traffic volumes close to the value 50:50, for which, at running speeds above the limit speed, the capacity is significantly higher than for
values of traffic volumes just outside this range. For traffic volumes within the range, the intervals between those trains of the priority converging flow which are using the diamond crossing are not sufficiently large for any non-priority traffic to use the crossing. Thus all the traffic passing through the diverging junction follows the high speed route, so that the number of trains per hour, which make up the non-priority flow, may rise to a level only limited by the straight line capacity. For priority flow traffic volumes outside this limited high capacity range, some diverging traffic is able to pass through the diamond crossing, and the total non-priority flow is thus limited by the capacity of the diverging junction, operating under the appropriate prevailing conditions.

The variation of capacity with running speed is shown in more detail for three values of priority flow traffic volumes in Figure 83. The discontinuity in the capacity of the junction, which occurs at the limit speed for converging traffic volumes of 50:50, is caused by a sudden change in the diverging traffic volumes, from 50:50 at lower running speeds, to 100:0 at higher ones.

The equivalent capacity graph to Figure 81 is given, for a junction of high and low speed routes, in Figure 82. As with the equivalent four aspect fixed block case, the capacity, for converging traffic volumes of 0:100, is very high, because the diamond crossing is able to pass large numbers of trains along the low speed route. However, as with fixed block, the capacity of the line along which trains must approach the diverging junction will, in practice, prove an additional limitation.
3.2 Immunity to Delays

It has been suggested in the past that moving time block gives a more satisfactory performance than pure moving block when a convoy of trains running at straight line headways passes through a speed restriction. Since this is a form of perturbed operation, it is felt appropriate that moving time block should be examined in this section as well as pure moving block, according to the criteria already established for fixed block signalling in Chapter 2. It should be noted that it is not suggested that any conclusions which are postulated below, having regard to the use of moving time block, affect in any way the desirability of its application to the particular situation outlined above.

3.2.1 Pure Moving Block

If a simulation is made of an infinitely long straight line, having on it an unlimited number of trains at equal headways, and if the leading train has a delay imposed upon it, then it is found that the final headway, between any two trains which have both experienced some delay, is equal to the minimum straight line value. Thus the delay encountered by any following train may be determined from the following simple analytical expression

\[ D_n = D_0 - n \times (H - H_W) \]

where
- \( D_0 \) is the original delay imposed on the leading train;
- \( D_n \) is the delay experienced by the \( n^{th} \) following train;
- \( H \) is the running headway;
- \( H_W \) is the minimum straight line headway for the particular
value of running speed.

Thus there is no need for a complicated iterative procedure, like that used in the case of fixed block signalling, to determine the delay encountered by the following trains.

The delay experienced by following trains is given in Figure 84, for various values of running headway, but a constant running speed of 100 mph, and a leading train waiting period of zero. The full delay is propagated to an infinite number of trains if the running headway is equal to the minimum steady state value. However, for all headways greater than this, subsequent trains encounter progressively smaller delays, and thus a finite (although possibly large) number of trains are affected. It should be noted that this process absorbs all the slack between any two trains which both experience some delay. Thus, any further delay would be fully propagated to the last train which was affected by the original delay, and only then would the delays begin to decrease.

The number of trains affected by the same initial imposed delay is shown as a function of headway in Figure 85. It is evident that there is no sudden change from a finite to an infinite number of trains being affected. Instead, the graph becomes asymptotic to the vertical at the minimum steady state headway. Thus the limit of stable delay propagation, as defined in Chapter 2, coincides with the minimum steady state headway, since any propagation of delays, which might occur with a practical running headway, is certain to be stable, i.e. the delays are gradually reduced to zero.

The effect of increasing the leading train waiting period to 30 seconds may be seen by comparing Figures 86 and 87 with Figures 84 and
FIGURE 85 Leading Train Waiting Period 0 sec
Running Speed 100 mph
Train Length 660 ft.

Number of Trains Delayed

Minimum Steady State Headway

Running Headway in Seconds

FIGURE 86 Delays to Following Trains
Leading Train Waiting Period 30 sec
Running Speed 100 mph
Train Length 660 ft.
Various Running Headways

Delay in Seconds

Number of Following Train
85 respectively. It is apparent that the magnitude of the leading train waiting period does not change the rate at which the propagated delays decrease for a given running headway. This is because the slope of the lines in Figures 84 and 85 are entirely dependent upon the slack present in the system, which itself is equal to the difference between the actual running headway and the minimum steady state headway.

However, for headways at which there is some propagation of delays, an increase in the leading train waiting period produces an equal increase in the delay encountered by any train which was already affected. As shown in Figure 87, the total number of trains affected at any given headway may also be increased.

The limit of delay propagation for various leading train waiting periods, together with the minimum steady state headway are shown as functions of running speed, in Figure 88. It should be noted that increasing the leading train waiting period has a similar effect upon the limit of delay propagation to that observed for fixed block, viz: it is raised by a constant amount throughout the speed range. However, under moving block, the limit of stability always coincides with the minimum steady state headway, so that it is unaffected by any change in the leading train waiting period. This is very important since it means that, for any running headway above the minimum steady state value, the system is always stable irrespective of the size of the initial delay. This would be a significant improvement upon the performance of fixed block signalling, under which instability can occur if the initial delay becomes very large. A comparison between Figure 88 and the equivalent graphs for fixed block also shows that, for lower running speeds, there are some headways which are impractical.
FIGURE 87 Leading Train Waiting Period 30 sec
Running Speed 100 mph
Train Length 660 ft.

FIGURE 88 Limit of Delay Propagation for Various Leading Train Waiting Periods
Train Length 660 ft.
even in the steady state, under fixed block, but produce no delay propagation at all for small imposed delays under moving block.

The graphs shown in Figure 84 may be normalised so that they apply to all values of initial imposed delay, and all values of train length, running speed, and any other factor which influences the value of the minimum straight line headway, $HW$. This may be achieved by re-writing the equation for the delay to following trains as

$$(D_0 - D_n) = (H - HW) \times n$$

Thus the difference in the delays to the leading train and the $n^{th}$ following train, $(D_0 - D_n)$, is plotted against the number of the following train, for various values of slack $(H - HW)$, to give the normalised graphs shown in Figure 89.

It has been suggested elsewhere that the propagation velocity of delays may be a useful parameter in assessing the performance of a signalling system. Appendix D shows how this may be determined for moving block signalling.

3.2.2 Moving Time Block

In moving time block signalling, the minimum steady state straight line headway is given by

$$HW = \frac{(TL + DK)}{V} + \frac{OM}{VM} + \frac{VM}{(2 \times B)}$$

which is exactly equivalent to the pure moving block case, for a running speed equal to the line speed. Also, it is found that the performance of moving time block is identical with that of pure moving block, under the straight line perturbance conditions described in the previous section. The delay to the $n^{th}$ following train is again given
by the expression

\[ D_n = D_0 - n \times (H - HW) \]

although, of course, HW is given by the moving time block expression which is quoted above. Since, for all running speeds other than line speed, the minimum steady state straight line headway required under moving time block is larger than that required under pure moving block, the slack, in the system for a specific running headway, is smaller under moving time block. This results in larger delays being encountered by a specific following train, and a larger number of trains being affected under an equivalent set of conditions.

Because moving time block and pure moving block are identical at line speed, and since Figure 84 to 87 refer to a running speed of 100 mph, which is the line speed, these graphs apply equally well to moving time block. However, the equivalent graph to Figure 88, which shows the limit of delay propagation as a function of running speed, is given for moving time block in Figure 90. It is apparent that these two graphs are very similar, any differences in the limit of delay propagation, for a specific leading train waiting period, being caused by differences in the minimum steady state headway. This would indicate no particular advantage for moving time block, but, in fact, the converse appears to be true, especially for running speeds below 20 mph. At these speeds, there are headways which are impractical under steady state moving time block operation, but which are not only practical, but do not produce any delay propagation with a leading train waiting period of zero, when operating under pure moving block.

It should be noted that the normalised delay graphs given in Figure 89 are equally applicable to moving time block and pure moving
block, provided that the appropriate value of the steady state minimum straight line headway is used.
FIGURE 89 Normalised Delays to Following Trains
Various Values of Slack (H - H').

FIGURE 90 Limit of Delay Propagation for
Various Leading Train Waiting Periods
under Moving Time Block
Train Length 660 ft.
CHAPTER 4: QUANTISED MOVING BLOCK SIGNALLING

This chapter is closely related to the preceding two, because it examines quantised moving block signalling according to the criteria used for fixed block and pure moving block signalling. Quantised moving block is worthy of consideration at this point, since it may be regarded as one possible method of producing a practical system to implement the theoretical concept dealt with in Chapter 3. It has already been shown that the introduction of pure moving block signalling gives substantially greater steady state capacity to a range of basic components, which are commonly found in a railway network, for the majority of possible operating conditions. In addition, it has been established that pure moving block has a performance, under perturbed conditions, which is inherently far more stable than that of fixed block. Thus it is important to establish to what extent these theoretical advantages would be realised in a practical system.

In the quantised moving block system, the information which is presented to the train driver is updated at discrete points along the track and is given in the form of a maximum safe speed at which the train may be travelling when it reaches the next point. The speed information is also presented in discrete levels.

4.1 Steady State Capacity

An analysis of the steady state capacity is made for each of the basic components which were examined for fixed block and pure moving block signalling. The bidirectional junction is also investigated by
using the same algorithm to combine three of these components.

4.1.1 Basic Components

(i) A Straight Line

The point at which information is supplied to a train driver when operating under quantised moving block signalling, may be regarded as a type of fixed block signal. However, a specific maximum speed at which the train must be travelling when it reaches the next signal is communicated directly to the driver, instead of being implied in the aspect of a fixed block signal. As with fixed block, there is an overlap associated with each signal, which must be cleared by the tail of one train before a second train is permitted to approach that signal. However, whereas in fixed block the overlap length is usually short, compared with distance between signals, under quantised moving block, the converse can be true. Thus on the straight line signalled for quantised moving block, which is shown in Figure 91, the signal Q3, associated with the clearing point C3, is not the one immediately in the rear of that point.

The distance between signals, DQ, may be chosen arbitrarily although the cost of the installation will increase as DQ is reduced. If the quantity VQ is the difference between adjacent speed quantisation levels, then all of the speeds, which a signal may indicate, are integer multiples of VQ. The value of VQ may be set to some nominal figure which, like that of DQ, may be influenced by cost considerations, although reducing VQ would
probably not prove as expensive as reducing the size of \( DQ \).

Let us examine more closely the situation shown in Figure 91. If train 1 remains stationary with its tail just clearing the point C3, as shown in the diagram, then train 2 has to brake to rest at the signal Q3. If the maximum speed, \( V_{UP} \), indicated by a certain signal, Q1, is such that train 2 may proceed at running speed \( V \) as far as the next signal Q2, then \( V_{UP} \) must be greater than or equal to \( V \), and is given by

\[
V_{UP} = KV \times VQ
\]

where \( KV \) is an integer. If \( V_{UP} \) is the lowest speed which may be indicated by a signal, and at the same time does not require a train to travel at less than running speed, then \( KV \) must fulfil the condition

\[
(KV - 1) \times VQ < V \leq KV \times VQ
\]

Because train 2 may be travelling with a maximum speed of \( V_{UP} \) when it reaches signal Q2, this signal must be a minimum distance of \( (KD \times DQ) \) from the signal Q3 where it could be required to come to rest, where \( KD \) is the smallest integer which fulfils the condition

\[
KD \times DQ \geq V_{UP}^2/(2 \times B)
\]

Thus the minimum straight line headway between train 1 and train 2 is given by

\[
HW = \frac{(KD + 1) \times DQ + OL + TL}{V}
\]

The capacity of a straight line operating under quantised
moving block is shown as a function of running speed in Figure 92, together with the capacity under pure moving block and four aspect fixed block. It is apparent that, for the levels of distance and speed quantisation shown (which correspond to 100 metres and 10 kilometres per hour), the capacity of quantised moving block falls approximately halfway between that of pure moving block and four aspect fixed block for most values of running speed. However, for running speeds less than 30 mph, the quantised moving block does not do as well, proportionately. This is due to the use of a fixed distance overlap, in the quantised moving block, as opposed to the fixed time overlap, used with pure moving block. This difference in overlap length is more marked at lower speeds, while at higher running speeds overlap length makes a smaller percentage contribution to total inter-train headway.

Figure 93 shows the effect on the capacity of a straight line of increasing the quantisation distance, DQ, from 100 metres to 1 kilometre, while keeping the speed quantisation constant. Evidently, decreasing the quantisation distance produces an increase in capacity, together with an increase in the number of discontinuities. In practice, a compromise must be found, since a decrease in the quantisation distance requires a proportionate increase in the trackside equipment, and hence the cost of installation.

The effect of varying the speed quantisation while maintaining the distance quantisation constant is shown in
Figure 94. It is evident that, as the number of speed quantisation levels is decreased, the capacity is decreased at some running speeds, while for other running speeds it remains constant. The number of discontinuities, in the graph of capacity, is directly related to the number of speed quantisation levels. Since increasing the number of speed quantisation levels is probably not as expensive as decreasing the quantisation distance, it is desirable to have the largest number of speed quantisation levels, which is practical.

(ii) A Speed Restricted Curve

The analysis of this component operating under four aspect fixed block, which was described in detail in Chapter 2, may be applied to quantised moving block, provided that in the expression

\[ \text{HD} = \text{HW} \times V \]

the quantised moving block straight line headway is used as the value of HW.

The capacity of a speed restricted curve is shown, as a function of running speed for various values of the limit speed, in Figure 95. It is evident that a speed restriction of 33 mph reduces the capacity of the line to approximately 50% of the straight line value, for running speeds close to line speed. If this graph is compared with the equivalent curves for four aspect fixed block, and pure moving block (Figures 4 and 64), it is apparent that, under quantised moving block, the overall
FIGURE 93: Capacity of a Straight Line
Speed Quantisation 9 ft/sec (5 mph)
Various Distance Quantisation Levels

FIGURE 94: Capacity of a Straight Line
Distance Quantisation 328 ft.
A: Speed Quantisation 48 mph
B: 24 mph
C: 12 mph
D: 6 mph
effect of the speed restriction is very similar to that of pure moving block. However, as with fixed block, the discontinuities at speeds above the limit speed, are still present although reduced in magnitude.

The effect of varying the length of the speed restriction is shown in Figure 96. It is evident that, as with fixed block (see Figure 5), increasing the length of the restriction produces a further reduction in the capacity. However, if the speed restriction is of such a length that, at the critical instant when the second train commences braking, the first is still in the restriction, then further increases in the length of the restriction have no effect upon capacity.

(iii) A Simple Converging Junction

In the analysis of the capacity of this component, repeated reference is made to Figure 97. Also, the four intervals THH, THL, TLH and TLL, which have been used in previous chapters, are again evaluated. The capacity may be obtained from these intervals. by the method described for this component when operating under four aspect fixed block.

For a junction with speed turnouts of 1:1 the headway required, between two trains which are both following the same route, is equal to the straight line headway, i.e.

\[ \text{THH} = \text{TLL} = \text{HW} \]

However, if the trains are following different routes, then the
FIGURE 95  Speed Restricted Curves, 1200 ft. long  
Distance Quantisation 328 ft.  
Speed Quantisation 6 mph  
Various Limit Speeds

FIGURE 96  Speed Restricted Curves of Various Lengths  
Distance Quantisation 328 ft.  
Speed Quantisation 6 mph  
Limit Speed 33 mph
points begin to change their route setting when the tail of train 1 clears the point C3, which is the clearing point associated with signal Q3. After a period TPC, this change is complete, and the route is clear for train 2. If signal Q2 is the place train 2 would come to rest, if the points were permanently set to route A, then signal Q1 is the nearest point to signal Q2 which train 2 may be travelling past when the points change is complete. This is because signal Q1, which is a distance of \((KD \times DQ)\) from signal Q2 (where KD is obtained, for the running speed involved, by the method described for a straight line), is the first one to be reached by train 2 which displays a speed indication lower than the running speed prior to the points change being complete. Signals Q2 and Q3 must be a minimum distance of \((2 \times OL)\) apart, thus KE is the minimum integer value to fulfil the condition

\[
KE \times DQ \geq 2 \times OL
\]

Therefore the interval between two trains which are travelling on different routes is given by

\[
THL = TLH = HW + TPC + (KE - 1) \times DQ/V
\]

For a junction of two highspeed routes with speed turnouts other than 1:1, and a limit speed less than running speed, the headway between two trains, which are both travelling along the high speed route, is not affected by the presence of the speed restriction, and thus

\[
THH = HW
\]
However, the minimum headway between two trains which are both travelling along the low speed route, is equal to that required for a speed restriction of the appropriate length and limit speed, hence

\[ TLL = HR \]

If a train, which is travelling along the low speed route, is followed by one travelling along the high speed route, then the headway between them must have a minimum value given by

\[ THL = HW + TPC + (KE - 1) \times DQ/V \]

as in the case of the junction with speed turnouts of 1:1, in order that the points shall have time to change. However, when the first train clears the point C3, it may be travelling with a speed less than the running speed, V. Thus the distance separating the trains is subsequently reduced and, since this must not be allowed to fall below the straight line value, the length of THL, given by the expression quoted above, may not be adequate in all conditions. Thus it is necessary to find the instantaneous speed of the first train as it clears the point C3, which is given by

\[ VIN = \sqrt{VL^2 + 2 \times A \times DIS} \]

where DIS is the distance over which acceleration has already taken place, and is given by

\[ DIS = KE \times DQ + OL + TL - DL \]

The final headway which exists between the trains is less than the interval THL by an amount given by
\[ T = \frac{(V - 2 \times VIN - VIN^2/V)}{(2 \times A)} \]

Thus it is not necessary for the value of THL to be larger than that given by the expression which is quoted above, unless

\[ T > TPC + (KE - 1) \times DQ \]

in which case, THL is given by

\[ THL = HW + T \]

If a train, travelling along the high speed route (route A), is followed by one travelling along the low speed route, then, at the instant that the points change is complete, train 2 must again be at signal Q1, if premature braking is not to be necessary. However, since there is some speed reduction necessary before the train reaches signal Q2, the time taken for train 2 to reach the end of the speed restriction, is given by

\[ SIB = \frac{(V - VL)}{B} + \frac{(KD \times DQ - (V^2 - VL^2)/(2 \times B))}{V} + TPC + DL/VL \]

If the instantaneous speed, VIN, at which the train is travelling when it clears the point C3, is less than the running speed, V, then the interval, TLH, is given by

\[ TLH = SLH + (VIN - VL)/A \]

However, if the acceleration period is already complete when the train clears C3, then TLH is given by

\[ TLH = SLH + \frac{(V - VL)}{A} + \frac{(DIS - (V^2 - VL^2)/(2 \times A))}{V} \]

For a junction of high and low speed routes with speed turnouts other than 1:1, the interval, THH, between two trains which are both travelling along the high speed route, is again equal to
the minimum straight line headway, HW.

When the running speed, V, exceeds the limit speed, VL, the interval, between two trains which are both travelling along the low speed route, must have a minimum value equal to the straight line headway, HW, for the running speed, V, which ensures sufficient spacing after both trains have negotiated the junction. However, if the minimum headway, which must exist between trains as they approach the junction, is greater than HW, then the interval TLL is equal to this headway, which is given by

\[ TLL = \frac{(KG + 1) \times DQ + OL + TL}{VL} \]

where KG is the minimum integer value to fulfil the condition

\[ KG \times DQ \geq (KVG \times VQ)^2 / (2 \times B) \]

and KVG is the integer which fulfils the condition

\[ (KVG - 1) \times VQ < V \leq KVG \times VQ \]

The interval between a train on the low speed route and one on the high speed route is not affected by the history of the low speed train prior to the instant when it clears the point C3. Thus the interval TIL is obtained in exactly the same way as for a junction of two high speed routes.

If a train, travelling along the high speed route, is followed by one travelling along the low speed route, then, at the instant when the points change is complete, train 2 may be somewhat nearer to signal Q2 than is possible with the junction of two high speed routes, because it is already travelling
with a speed of $VL$. Hence, the minimum distance which must separate train 2 from signal Q2 when the points change is complete is equal to $(KG \times DQ)$, where $KG$ is defined above. Thus the time, $SLH$, which elapses between the beginning of the points change and the instant when train 2 reaches the end of the speed restriction, is given by

$$SLH = \frac{(KG \times DQ + DL)}{VL + TPC}$$

This value of $SLH$ may be used to find $TLH$, by substituting it in the expressions quoted above for the junction of two high speed routes.

The capacity of a converging junction with speed turnouts of 1:1, is shown as a function of traffic volumes in Figure 98, for various values of running speed. It is evident that, as with other signalling systems, the function is symmetrical about traffic volumes of 50:50, for which there is minimum capacity for any given running speed. It is also evident that the capacity of the junction, when operating at a specific value of traffic volumes, varies little with running speed between 30 mph and 100 mph. This is to be expected from consideration of a straight line. It is also apparent that the variation in capacity between traffic volumes of 0:100 and 50:50 is relatively large compared with that observed for fixed block signalling, and, in this respect, is very similar to pure moving block. This is due to the additional safety margin, which is allowed on each side of the junction in both quantised moving block and pure moving block systems.
FIGURE: 97

Route: B
Train 2
(KDxDQ)
Q1

(KDxDQ)
OL
(TPGxV)
Q2

Q3
C3
Train 1

Route: A

FIGURE: 98 Converging Junction
Speed Turnouts 1:1
Distance Quantisation 328 ft.
Speed Quantisation 6 mph

Capacity in Coaches per Hour

Fast : Slow Traffic Volumes

0:100 20:80 40:60 60:40 80:20 100:0
The capacity of a converging junction of two high speed routes with speed turnouts of 100:33 is shown as a function of traffic volumes in Figure 99. As might be anticipated, at running speeds less than the limit speed, the capacity is unaffected while, at higher running speeds, the function is no longer symmetrical about traffic volumes of 50:50. The capacity, at traffic volumes of 0:100 is reduced to that of a speed restriction of appropriate length and limit speed, while, for traffic volumes of 50:50, there are also reductions in capacity due to the increased value of TLH. The latter is necessary to allow for the subsequent reduction in distance separation, resulting from the second train initially having a higher speed than the first train. The largest reductions in capacity occur at the highest running speeds, due to the longer periods of braking and acceleration which are necessary in such cases.

The equivalent capacity curves for a junction of high and low speed routes are given in Figure 100. For running speeds above the limit speed, the capacity, for traffic volumes of 0:100 is equal to that of a straight line operating at the limit speed (see Figure 92). This is because this capacity is less than that available at most higher running speeds, and TLL must be the lesser of two quantities, the capacity of a straight line operating at running speed and that at limit speed. Apart from this, the capacity of the junction of high and low speed routes, under quantised moving block, is similar, in form, to its capacity under other signalling systems. In this respect
it may be noted that the capacity for traffic volumes of 50:50 and any specific running speed, is less than that of a junction with speed turnouts of 1:1, but greater than that of a junction of two high speed routes.

(iv) A Simple Diverging Junction

The analysis of the capacity of this component may be made in terms of the intervals $THH$, $THL$, $TLH$ and $TLL$, which are subsequently used to obtain the capacity as described for a converging junction operating under four aspect fixed block.

For a diverging junction with speed turnouts of 1:1, the headway, required between two trains which are both travelling along the same route, is equal to the minimum straight line headway. Thus

$$THH = TLL = HW$$

From reference to Figure 101 it is evident that the interval required between two trains which are travelling along different routes is given by

$$THL = TLH = HW + TPC + (KE - 1) \times DQ/V$$

Thus it is evident that the capacity is identical to that of an equivalent converging junction operating under the same conditions.

For a junction of two high speed routes with speed turnouts other than 1:1, the interval between two trains on the high speed route is unaffected by the speed limit, and, therefore, is
given by

\[ T_{HI} = H \]

As with the converging junction, the interval between two trains passing through the low speed side of the junction is equal to that required by a speed restriction of the appropriate length and limit speed, so that

\[ T_{LL} = H + W \]

The interval required between a train on the low speed route and one on the high speed is also unaffected by the speed limit and thus is given by

\[ T_{HL} = H + W + P + (K \cdot E - 1) \times DQ/V \]

If a train is travelling through the high speed side of the junction, and the following one is travelling through the low speed side, then, as previously, train 2 must be a minimum distance of \((K \cdot D \times DQ)\) from the signal \(Q_2\), when the points change is complete. The following train must not be travelling with a speed in excess of the limit speed, \(V_L\), when it reaches \(Q_2\), and subsequently, it travels the length of the restriction with this speed. Thus the time taken for the train to reach the end of the speed restriction, measured from the beginning of the points change, is given by

\[ T_{SH} = (V - V_L)/B + (K \cdot D \times DQ - (V^2 - V_L^2)/(2 \times B))/V + P + D_L/V. \]

The maximum speed of the train, when it clears the overlap which allows the points to commence changing again, is given by
VIN = $\sqrt{VL^2 + 2 \times A \times DIS}$

where DIS is given by

$$DIS = KE \times DQ + OL + TL - DL$$

exactly as in the case of the converging junction. If $VIN \leq V$, then the interval $TLH$ is given by

$$TLH = SLH + (VIN - VL) / A$$

However, if $VIN > V$, then

$$TLH = SLH + (V - VL) / A + \left(\frac{DIS - (V^2 - VL^2)}{(2 \times A)}\right) / V$$

For a diverging junction of high and low speed routes, the intervals $THH$ and $THL$ are not affected by the length of the speed restriction, and thus they may be obtained by exactly the same method which was described for the junction of two high speed routes. However, the interval between two trains which are both travelling along the low speed route must be equal to that required for a speed restriction of infinite length. Also, because there is no need for a train which has just passed through the junction to accelerate back to its running speed, the interval $THL$ is given by

$$THL = \frac{(V - VL)}{B} + \left(\frac{KD \times DQ - (V^2 - VL^2)}{(2 \times B)}\right) / V + TPC$$

$$+ \frac{(OL + TL + KE \times DQ)}{VL}$$

As mentioned above, the capacity of a diverging junction with speed turnouts of 1:1 is equal to that of a corresponding converging junction for similar operating conditions. Thus the graphs shown in Figure 98 apply equally well to a diverging junction having speed turnouts of 1:1.
The capacity of a diverging junction of two high speed routes with speed turnouts 100:33 is shown as a function of traffic volumes in Figure 102. It is evident that the capacity, at traffic volumes of 0:100 and 100:0, is equal to that of the corresponding converging junction, since, for traffic volumes of 0:100 the two junctions are both equivalent to a straight line, and, for traffic volumes of 100:0, they are both equivalent to a speed restriction. For other traffic volumes the capacity of the diverging junction may exceed that of the equivalent converging junction, for running speeds greater than the limit speed. This is because the value of the interval, THL, may be smaller for the diverging junction, since there is no need to allow for a subsequent reduction in headway, resulting from the difference in the instantaneous speeds of the trains at the commencement of the points change. Such differences in the capacity of the diverging and converging junctions are more likely to occur at higher running speeds, where longer periods of acceleration are involved. Thus it may be observed from a comparison of Figures 99 and 102, that the capacity of the diverging junction is slightly greater than that of the converging, for running speeds greater than 80 mph and traffic volumes other than 0:100 and 100:0.

The capacity of an equivalent diverging junction of high and low speed routes is given in Figure 103, as a function of traffic volumes. The capacity of this junction tends to be lower than that of the junction of two high speed routes, for
Train 1

1200
Train 1
 rcxv

FIGURE 102 Diverging Junction of Two High Speed Routes with Speed Turnouts 10.32
Distance Quantisation 328 ft.

0 mph 60 mph 80 mph 100 mph 120 mph

1000 Speed Quantisation 6 mph

Vnrim vs Renninn Snanric

s

800 cu a

0

600

400

200

2080

4080

6080

8080

Fast : Slow Traffic Volumes

80/21

100/0 Fast : Slow Running Speeds

Capacity in Coaches per Hour

0

200

400

600

800

1000

1200

FIGURE 101

Diverging Junction of Two High Speed Routes with Speed Turnouts 10.32
Distance Quantisation 328 ft.

Various Running Speeds

Train 2

Train 1

Train 2

C2

C3

C2

Train 1

Train 2

C1

C2

C3

Route B

running speeds greater than the limit speed, due to the increased
length over which the restriction applies. Evidently this
difference has a maximum at traffic volumes of 0:100 for any
specific running speed, while, for traffic volumes of 100:0,
the two types of diverging junction are equivalent.

(v) **A Diamond Crossing**

The capacity of this component may be obtained from the
intervals THH, TLL, THL and TLH in the normal manner. In
the description of the method for the determination of these
intervals, which is given below, repeated reference is made
to Figure 104.

For a crossing with speed turnouts of 1:1, the interval
between two trains which are both on the same route is equal
to the straight line headway. Thus

\[ T_{HH} = T_{LL} = H_{W} \]

If we consider two trains which are travelling on different
routes, then the crossing is clear, as far as the second train
is concerned, when the leading train has reached the position
shown for 'train 1' in Figure 104. Thus the interval between
the trains is given by

\[ T_{HL} = T_{L1} = H_{W} + (K_{E} - 1) \times D_{Q}/V \]

where the integer, KE, has the same significance as previously
assigned in the section relating to a converging junction, and
\((KE \times DQ)\) is the distance between the signals Q3 and Q1, in
FIGURE 103: Diverging Junction of High and Low Speed Routes
Speed Turnouts 100:33
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Various Running Speeds

FIGURE 104
Route B
Q1
Q2
Q3
Q4
Q5
Q5
Q6
Route A
Train 2
Train 1
(KDxDQ)
For a crossing of two high speed routes which has speed turnouts other than 1:1, the interval between two trains, which are both travelling along the high speed route, is not affected by the presence of the speed restriction, and is given by

\[ T_{HH} = H_W \]

However, the interval between two trains which are both travelling along the low speed route may be found from consideration of a speed restriction of the appropriate length and limit speed, thus

\[ T_{LL} = H_{WR} \]

The interval between a train which is travelling along the low speed route, and one which is travelling along the high speed route, is not affected by the existence of the speed restriction. Thus this interval is given by

\[ T_{HL} = H_W + (K_E - 1) \times DQ/V \]

A train which is travelling along the low speed route must be a minimum distance of \((K_D \times DQ)\) from the signal Q3, at the instant when the crossing is cleared by the preceding train which is travelling on the high speed route. However, when the train reaches the signal Q3, it must be travelling with a maximum speed equal to the limit speed, \(V_L\). Thus the time taken for the train to reach the end of the speed restriction, measured from the instant when the crossing is cleared by the preceding train, is given by
\[ SLH = \frac{(V - VL)}{B} + \frac{(KD \times DQ - (V^2 - VL^2)/(2 \times B))}{V} + \frac{DL}{VL} \]

From the end of the speed restriction, the train has to travel a further distance before the crossing becomes clear, as far as the other route is concerned. This distance is given by

\[ DIS = KE \times DQ + OL + TL - DL \]

Thus the maximum instantaneous speed, with which the train may be travelling at the instant it clears the crossing, is given by

\[ VON = \sqrt{VL^2 + 2 \times A \times DIS} \]

If \( V > VON \), then the train is still accelerating when it clears the crossing, and the interval \( TLH \) is given by

\[ TLH = SLH + \frac{(VON - VL)}{A} \]

However, if \( V \leq VON \), then the train has already attained its running speed, and \( TLH \) is given by

\[ TLH = SLH + \frac{(V - VL)}{A} + \frac{(DIS - (V^2 - VL^2)/(2 \times A))}{V} \]

For a diamond crossing of high and low speed routes with speed turnouts other than 1:1, the interval between two trains which are both travelling along the high speed route is unaffected by the existence of the speed limit on the other route, so that

\[ THH = HW \]

Since the speed limit applies for the entire length of the low speed route, the headway between the two trains which are both travelling along this route is given by

\[ \text{headway} \]
\[ TLL = \frac{((KG + 1) \times DQ + OL + TL)}{VL} \]
where \( KG \) is the minimum integer value to fulfil the condition
\[ KG \times DQ \geq (KVG \times VQ)^2 / (2 \times B) \]
and \( KVG \) is the integer which fulfils the condition
\[ (KVG - 1) \times VQ < V \leq KVG \times VQ \]

The interval between a train on the low speed route and one on the high speed route is not affected by the existence of the speed restriction, and is given by
\[ THL = HW + (KE - 1) \times DQ/V \]

When a train on the high speed route clears the crossing (i.e. when its tail clears the clearing point CS), then a following train on route B must be a minimum distance of \( (KG \times DQ) \) from signal Q3. Since the following train maintains a speed of \( VL \) throughout its journey, the interval \( TLH \) is given by
\[ TLH = \frac{((KG + KE) \times DQ + OL + TL)}{VL} \]

The capacity of a diamond crossing with speed turnouts of 1:1 is shown as a function of traffic volumes in Figure 105. It is evident that, as with pure moving block signalling, the minimum capacity, obtainable for any specific running speed, occurs at traffic volumes of 50:50. The capacity of the crossing with traffic volumes of 50:50 is greater than for a junction operating under similar conditions, because there is no points change time involved in the route being reset.
The capacity of a diamond crossing of two high speed routes with speed turnouts other than 1:1 is given in Figure 106. As might be anticipated, the capacity, with traffic volumes of 0:100 is reduced to that of a speed restriction of the appropriate length and limit speed. The capacity, for traffic volumes of 50:50 is considerably less than that of a crossing with speed turnouts of 1:1, because the value of TLI is larger in this case. This is due to the longer time which is taken, by a train on the low speed route, in passing through the vicinity of the crossing.

The equivalent graphs of capacity for a crossing of high and low speed routes are given in Figure 107. It is evident that, for running speeds greater than the limit speed and traffic volumes of 0:100, the capacity is independent of the actual running speed, and equal to that of a straight line operating at a speed of VL. The capacity, with traffic volumes of 50:50, is greater, for a crossing of high and low speed routes, than for a crossing of two high speed routes, which is due to the interval TLI being smaller for the high and low speed crossing. This is so, because the following train may be much closer to the crossing when the route ahead of it becomes clear, than would be possible for a crossing of two high speed routes.

It is interesting to note that the performance of a diamond crossing, under quantised moving block signalling, is very similar to its performance under pure moving block, except that the capacity is generally lower. This is because, unlike the
FIGURE 105 Diamond Crossing with Speed Turnouts 1:1
Distance Quantisation 328 ft.
Speed Quantiser, 6 mph
Various Running Speeds

Fast: Slow Traffic Volumes

Capacity in Coaches per Hour

Fast: Slow Traffic Volumes

FIGURE 106 Diamond Crossing of Two High Speed Routes with Speed Turnouts 100:33
Distance Quantisation 328 ft.
Speed Quantiser, 6 mph
Various Running Speeds

Fast: Slow Traffic Volumes

Capacity in Coaches per Hour
position with fixed block signalling, under both pure and quantised moving block there is an additional fixed length overlap, associated with the crossing, which must be cleared by a train travelling along one route, before the other route may be considered to be unoccupied.

(vi) An Isolated Station

In the analysis of the capacity of this component, which is described below, reference is repeatedly made to the situation shown in Figure 108. If the intervals $T_{HI}$, $T_{LL}$, $T_{HL}$ and $T_{LH}$ have the definitions, which are described for this component operating under four aspect fixed block, then the capacity may be obtained from these intervals in the normal manner.

The interval between two trains which do not stop at the station is equal to the straight line headway, i.e.

$$T_{LL} = H_W$$

If a stopping train is following a non-stopping train, the station berth initially becomes clear for the second train to approach it, when the first train occupies the position shown in Figure 108, for train 1. Until this instant, train 2 must be capable of coming to rest at signal $Q_2$, which is separated from the station berth by at least an overlap length. Hence signal $Q_2$ is separated from signal $Q_3$ by a distance of $(K_I \times D_Q)$ where $K_I$ is the minimum integer value which fulfils the condition

$$K_I \times D_Q \geq O_L + T_L$$
FIGURE: 107 Diamond Crossing of High and Low Speed Routes with Speed Tunels 13.5\text{ft}
Distance Quantisation: 32\text{ft}
Speed Quantisation: 6\text{mp} \text{n}
Various Running Speeds

Capacity in Coaches per Hour

Fast: Slow Traffic Volumes

FIGURE: 108

Train2 \( Q_1 \)

Train1 \( Q_3 \)

(Q1 x DQ)

(Q1 x DQ)

(Q1 x DQ)
Since train 2 must be at signal Q1 when the berth becomes clear, and this is a minimum distance of \((KD \times DQ)\) from signal Q2 (where \(KD\) is as defined for a straight line), the total distance separating train 2 from the station berth when the latter is cleared by train 1 is given by

\[
CL = (KD + KL) \times DQ
\]

Thus train 2 comes to rest at the station a period, \(SHL\), after the instant shown in Figure 108, which is given by

\[
SHL = V/(2 \times B) + CL/V
\]

The maximum speed at which train 2 can be travelling by the time it clears the overlap C3 is given by

\[
VIN = \sqrt{2 \times A \times (OL + TL)}
\]

If \(VIN \leq V\), train 2 is still accelerating when it clears C3, and the interval \(THL\) is given by

\[
THL = SHL + TST + VIN/A
\]

However, if \(VIN > V\), train 2 has already attained its full running speed when it clears C3, and hence \(THL\) is given by

\[
THL = V \times (A + B)/(2 \times A \times B) + (CL + OL + TL)/V + TST
\]

Since no assumptions about the speed-distance curve of train 1 have been made in determining the value of \(THL\), the interval \(THL\), between two trains which stop at the station, is at all times equal to \(THL\).

The interval between a train which stops and one which does not must have a minimum value, equal to the straight line headway,
after the stopping train has regained its running speed. Thus if train 1 is travelling at a speed of $V$ when it clears $C3$, i.e. if $VIN > V$, then the interval $TLH$ is given by

$$TLH = HW$$

However, if $VIN \leq V$, then some decrease in the distance separating the trains occurs subsequently, so that

$$TLH = HW + V/(2 \times A) - VIN \times (2 \times V - VIN)/(2 \times A \times V)$$

The capacity of an isolated station is shown plotted against the ratio of stopping to non-stopping traffic, for various running speeds, in Figure 109. As with other signalling systems, the capacity of the line decreases as the percentage of stopping traffic increases. This is particularly so for high running speeds, where longer periods of braking and acceleration are needed to bring the train to rest at the station, and to return it to its running speed subsequently.

Let us compare this graph with the equivalent ones for four aspect fixed block, and pure moving block (Figures 19 and 77 respectively). It is evident that the variation in capacity, which is produced by changing the percentage of traffic which stops at the station, is more pronounced under quantised moving block than under fixed block signalling, and is similar to that observed for pure moving block. Thus, quantised moving block gives only slightly more capacity than is obtainable from four aspect fixed block when there is a high percentage of the traffic stopping at the station. This is due to the largest contributory factors to the headway between two stopping trains, being the
same under both signalling systems. These are: the time which the
train spends braking to rest, the station stop time, and the time
taken to accelerate until the station berth is clear.

The effect on capacity of varying the station stop time is shown
in Figure 110. It is evident that, as with other signalling systems,
the stop length produces relatively little variation in capacity,
provided it remains small compared with the total delay experienced
by a train while executing the stop.

4.1.2 The Bi-directional Junction

The analysis of the steady state performance of quantised moving block
may be extended to cover a bidirectional junction by inter-relating a group
of basic components, as in previous chapters. The combination of these
components is achieved by means of the algorithm which is described in
Appendix C. Similarly, the intervals ROA, ROB, RTOA and RTOB which this
algorithm uses, are obtained using the method described in detail for
fixed block in section 2.1.2.

An equivalent series of graphs to those which were given for four
aspect fixed block in Figures 35 to 39, and for pure moving block in
Figures 79 to 83, are shown in Figures 111 to 115 for quantised moving
block operation. The capacity of a right hand bidirectional junction with
speed turnouts of 1:1 and converging traffic running at a maximum, is given,
in Figure 111, as a function of running speed and train length, for priority
volumes of 10:90. It is evident that, as with other signalling systems, the
variation in capacity with train length is almost linear for any specific
value of running speed. Also, the variation of the capacity with running
speed for a specific value of train length follows the straight line form
which is characteristic of the signalling system.
FIGURE : 111
Right Hand Bidirectional Junction
with Speed Turnouts of 1:1
Converging Traffic Running at a Maximum
with Traffic Volumes of 10:90
FIGURE 113
Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts of 100:33
Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE: 114
Right Hand Bidirectional Junction of High and Low Speed Routes with Speed Turnouts of 100:33 Converging Traffic Running at a Maximum Train Length 660 ft.
FIGURE 115 Right Hand Bidirectional Junction of Two High Speed Routes with Speed Turnouts of 100:00
Converging Traffic Running at a Maximum
with Various Fast:Slow Traffic Volumes
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.

FIGURE 116 Delays to Following Trains
Leading Train Waiting Period 0 sec
Running Speed 100 mph
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.
Various Running Headways
The capacity of the same junction is shown in Figure 112, as a function of running speed and the traffic volumes of the priority (in this case the converging) flow. It is apparent that, for a specific running speed, the variation in capacity with traffic volumes displays similar discontinuities to those observed for other signalling systems operating under the same conditions. As previously stated, these are a function of the combining algorithm, and not of the signalling system employed. From Figure 112 it may be seen that the variation of capacity with traffic volumes is similar to that observed for other types of signalling, with a minimum capacity for any given running speed occurring near to, rather than at, traffic volumes of 50:50. This phenomenon was discussed in greater detail in Chapter 2.

The capacity of a right hand junction of two high speed routes with speed turnouts of 100:33 is given, in Figure 113, as a function of running speed and traffic volumes. As with other types of signalling, the capacity is symmetrical about traffic volumes of 50:50, for running speeds less than the limit speed, but for higher running speeds, this is not the case. It is evident that, for priority traffic volumes between approximately 50:50 up to 100:0, and for running speeds greater than the limit speed, but less than approximately 50mph, there is a range of operating conditions (near the centre of Figure 115) over which there are no discontinuities in the capacity in the direction of constant running speed. This is because, over this range, the algorithm finds that the intervals between those trains which use the diamond crossing and are also part of the priority converging flow, are not sufficiently large to allow any non-priority diverging trains to use the diamond crossing. Thus, all the non-
priority traffic takes the high speed route through the diverging junction, so that the capacity of the bi-directional junction is equal to the sum of the capacities of a simple converging junction, and a straight line.

The variation of capacity with running speed is shown, in greater detail, for three values of priority flow traffic volumes in Figure 115. It is apparent that, for priority flow traffic volumes of 50:50, there is an extra discontinuity in the capacity at the limit speed of 33 mph, and a very large one at a running speed of 54 mph. Between these speeds, the non-priority flow has traffic volumes of 100:0 while for all other running speeds, the value is 50:50.

The capacity of an equivalent junction of high and low speed routes is shown in Figure 114. It is evident that there is a range of operating conditions, which corresponds to that just described for the junction of high and low speed routes, and over which there are no discontinuities in capacity in the constant running speed direction. In addition, it may be seen that, for converging traffic volumes of 0:100, the capacity is very high, due to the large numbers of trains which the diamond crossing can accommodate. As with other types of signalling, in practice the maximum flow will be limited by capacity of the line, along which trains approach the diverging junction.

4.2 Immunity from Delays

The perturbed running performance of quantised moving block may be examined according to the criteria already established for other signalling systems. There is no simple analytical expression giving
the delay encountered by trains that are following a leading train on which a delay is arbitrarily imposed. Thus the method employed is basically the one used for fixed block signalling, but with suitable modifications which are outlined below.

The distance between signals, and between overlaps, is $DQ$, and hence this is used instead of $BL$, on all occasions. When calculating the overlap clearing times of the leading train, it should be appreciated that the train may have to brake over a large number of signal lengths. Thus

$$DIS = KD \times DQ - \frac{V^2}{2 \times B}$$

where $KD$ is the integer value obtained in the same way as for the steady state capacity of a straight line operating under quantised moving block. If $KA$ is the minimum integer which fulfils the condition $KA \times DQ \geq OL + TL$,

and if $AFT = KA \times DQ - OL - TL$,

then $TT(4) = TT(3) + \frac{DQ}{V}$

provided that $DQ - AFT - DIS = 0$.

Alternatively,

$$VOX = \sqrt{VOW^2 - 2 \times B \times (KT \times DQ - AFT - DIS)}$$

where $KT = 1$, if $(DQ - AFT - DIS) > 0$

and $KT = 2$, otherwise.

If $VOX \geq 0$, then $TT(4)$ is given by

$$TT(4) = TT(3) + \frac{DQ}{V} + \frac{(VOW - VOX)}{B} - \frac{(VOW^2 - VOX^2)}{2 \times B \times VOW}$$

However, if $VOX < 0$, then it should be recalculated as

$$VOX = \sqrt{2 \times A \times (DQ - AFT)}$$
and provided that this value of VOX is less than, or equal to, the running speed, V, then TT(4) is given by

\[ TT(4) = TT(3) + \frac{VQ}{B} + DST + \frac{VOX}{A} + \frac{(AFT - VQ^2/(2 \times B))}{VQ} \]

If the value of VOX exceeds the running speed, V, then

\[ TT(4) = TT(3) + \frac{VQ}{B} + DST + \frac{V}{2 \times A} + \frac{(DQ - AFT)/V + (AFT - VQ^2/(2 \times B))}{VQ} \]

Subsequently, the train may be braking through a number of block lengths, hence the clearing times are obtained from

\[ TT(N + 1) = TT(N) + \frac{(VQ - \sqrt{AOX})}{B} \]

where AOX is positive, and is given by

\[ AOX = VQ^2 - 2 \times B \times DQ \]

When AOX becomes negative, then the point is reached where the train has come to rest, and the time is given by the expressions shown above for TT(4) with VOX less than zero.

The signal information, which is observed by the following train, and which is stored in the array KS(I), is obtained in the following manner

\[ KS(1) = KGV, \quad \text{for } KGV \leq KV \]

where KGV is the largest integer value which fulfils the condition

\[ \frac{(KGV \times VQ)^2}{2 \times B} < (KPG - 1) \times DQ \]

and KV is obtained in the manner described for the steady state capacity of a straight line, operating under quantised moving block. If KGV > KV, then

\[ KS(1) = KV \]
For subsequent signals, the same expressions apply if the appropriate value of KNG is used instead of KPG. The determination of the signal indications observed by the following train then proceeds as with four aspect fixed block signalling, provided that the limiting speed, at the exit from a block, is given by

\[ V_{LI} = KS(I - 1) \times VQ \]

When calculating the overlap clearing times of the following trains from the signal information which they observe, for a train travelling at running speed, the intervals TAFT and TLAT are given by

\[ TAFT = \frac{AFT}{V} \]

and

\[ TLAT = \frac{(DQ - AFT)/V}{V} \]

Similarly, when the entry speed, \( V_{UW} \), is equal to the projected exit speed, \( V_{LI} \), then the intervals TAFT and TLAT are given by these expressions, but with \( V_{UW} \) substituted for \( V \). The exception to this rule is when \( V_{UW} = V_{LI} = 0 \), in which case

\[ TLAT = \sqrt{2} \times \frac{(DQ - AFT)/A}{A} \]

and

\[ TAFT = \sqrt{2} \times \left(\frac{A + B}{A} \times \frac{DQ/(A \times B)}{-TLAT}\right) \]

provided that the train is accelerating when the overlap is cleared, i.e. if

\[ V_{ID} > \sqrt{2} \times A \times \frac{(DQ - AFT)}{A} \]

However, if this condition is not fulfilled, then

\[ TAFT = \sqrt{2} \times \frac{AFT}{B} \]

and

\[ TLAT = \sqrt{2} \times \left(\frac{A + B}{A} \times \frac{DQ/(A \times B)}{-TAFT}\right) \]
If, initially, it is found that $V_{UW}$ is less than both $V_{LI}$ and $V$, then

\[ V_{UP} = \sqrt{V_{UW}^2 + 2 \times A \times (DQ - AFT)} \]

and, if it is also found that $V_{UP} > V_{LI}$, then $T_{LAT}$ and $T_{AFT}$ are given by

\[ T_{LAT} = \frac{(V_{LI} - V_{UW})A + (DQ - AFT - (V_{LI}^2 - V_{UW}^2))/(2 \times A)}{V_{LI}} \]

and

\[ T_{AFT} = \frac{AFT}{V_{LI}} \]

But, if $V_{UP} < V_{LI}$, the intervals may be found from the appropriate four aspect fixed block expressions, if $DQ$ is substituted for $BL$.

Alternatively, if it is found, initially, that $V_{UW} > V_{LI}$, then the distance, $DIS$, over which a speed of $V_{UW}$ may be maintained, is given by

\[ DIS = DQ - \frac{(V_{UW}^2 - V_{LI}^2)/(2 \times B)}{V_{UW}} \]

If the train has not commenced braking when it clears the overlap, i.e. if $DIS > (DQ - AFT)$, then $T_{LAT}$ and $T_{AFT}$ are given by

\[ T_{LAT} = \frac{(DQ - AFT)}{V_{UW}} \]

and

\[ T_{AFT} = \frac{(V_{UW} - V_{LI})/B + (AFT - (V_{UW}^2 - V_{LI}^2)/(2 \times B))}{V_{UW}} \]

However, if this is not the case, they are given by

\[ T_{LAT} = \frac{DIS}{V_{UW}} + \frac{(V_{UW} - \sqrt{V_{UW}^2 - 2 \times B \times (DQ - AFT - DIS)})/B}{V_{UW}} \]

and

\[ T_{AFT} = \frac{(V_{UW} - V_{LI})/B + DIS/V_{UW} - T_{LAT}}{B} \]

The delays, encountered by trains that are following a train upon which a delay is arbitrarily imposed, are given, for various values
of running headway, in Figure 116. For all the headways shown, the delays decrease in a linear manner, just as was observed for pure moving block. From a comparison with the equivalent graph for pure moving block (Figure 84), it is evident that, for a specific running headway, more trains experience some delay, under quantised moving block signalling, than under pure moving block. For example, under quantised moving block, a total of fourteen following trains encounter some delay when running at 60 second headways, whereas, under the same conditions, but with pure moving block signalling, only eight are affected. However, this is mainly due to the larger minimum steady state headway, which is required for quantised moving block, and therefore produces a lower value of slack for a given running headway. In fact, the limit of stability is very close to the minimum steady state headway, for the running speed chosen for Figure 116 (which is 100 mph). Thus, the perturbed performance of quantised moving block at this speed, for a leading train waiting period of zero, is very similar to that of a theoretical moving block system.

The number of trains affected by the initial delay is shown, in Figure 117, as a function of running headway, for a running speed of 100 mph, and a leading train waiting period of zero. It is apparent that this graph is very similar to the equivalent one for pure moving block (Figure 85), which is to be expected, since it has already been stated that the performance of quantised moving block, under these conditions, closely resembles that of pure moving block.

The effect which a variation in the leading train waiting period has upon the number of following trains which experience some delay
is shown in Figures 118 and 119. It may be seen that limit of
stability moves away from the minimum steady state headway as the
leading train waiting period is increased. For values of running
headway which are greater than the limit of stability, the performance
of the system continues to resemble that of pure moving block, whereas,
for smaller headways, an infinite number of trains experience delays
which may remain constant, or increase. Thus in this region, the
performance is more reminiscent of fixed block signalling operating
under unstable headways.

The minimum steady state headway, the limit of stable operations,
and the limit of delay propagation are shown, as functions of running
speed, in Figure 120 for a leading train waiting period of zero. It
is evident that, for running speeds below the lowest discontinuity
in the minimum steady state headway, the limit of stability coincides
with the limit of delay propagation. This is because, for such running
speeds, any signal indication which requires a train to slow down
also requires it to come to rest, whereupon the full delay is propagated.
This is very similar to the performance of fixed block signalling
under similar circumstances. As the running speed is increased, the
limit of stability moves away from the limit of delay propagation,
and towards the minimum steady state headway, virtually coinciding
with the latter above approximately 67 mph. Thus the performance
of quantised moving block compares very favourably with that of
fixed block signalling for a leading train waiting period of zero,
because, for most running speeds, the actual running headway only
needs to be a relatively small amount more than the minimum steady
FIGURE 117 Leading Train Waiting Period 0 sec
Running Speed 100 mph
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.

FIGURE 118 Leading Train Waiting Period 30 sec
Running Speed 100 mph
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.
FIGURE 119 Leading Train Waiting Period 60 sec
Running Speed 100 mph
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 650 ft.

FIGURE 120 Leading Train Waiting Period 0 sec
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.
state value, to ensure stable operation.

The effect of varying the leading train waiting period is shown in Figures 121 and 122. It may be seen that, for a leading train waiting period greater than zero, the limit of stability does not coincide with the limit of delay propagation, or the minimum steady state headway. Increasing the leading train waiting period to 30 seconds produces a large increase in the range of headways which are feasible from steady state considerations, but are impractical because they produce instability under perturbed running condition. However, increasing the leading train waiting period further to 60 seconds produces a relatively small change in the limit of stability. Thus the range of headways, over which some stable propagation of delays may occur, is greatly increased in magnitude, when the leading train waiting period is increased beyond 30 seconds. For this reason, the performance of quantised moving block, for larger initial imposed delays, becomes less like that of pure moving block and more like that of fixed block signalling. However, the quantised moving block system remains inherently more stable than fixed block, because increasing the leading train waiting period produces a proportionately smaller increase in the limit of stability.
FIGURE 121: Leading Train Waiting Period 30 sec
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.

Limit of Delay
Propagation

Limit of
Stability

Minimum Steady
State Headway

Running Speed in M.P.H.

FIGURE 122: Leading Train Waiting Period 60 sec
Distance Quantisation 328 ft.
Speed Quantisation 6 mph
Train Length 660 ft.

Limit of Delay
Propagation

Limit of
Stability

Minimum Steady
State Headway

Running Speed in M.P.H.
Special arrangements may be necessary to allow efficient use of very fast trains, such as British Rail's Advanced Passenger Train, on existing railway networks which are simultaneously carrying conventional traffic. A possible solution to this problem might be to run the high speed trains in flights (or convoys) according to moving block criteria, with the normal traffic continuing to use the fixed block signalling. An examination of this process is the subject of this chapter.

In a basic analysis the high speed traffic requirements are specified in terms of flight frequency and number of trains per flight, and, from this, the maximum possible flow rate of normal traffic is evaluated. The length and optimal positioning of the necessary passing loops is also determined.

In a subsequent section the effect is examined of adding tolerance to the headway between the trains. The aim of this is to give immunity to delays which may be experienced by both the high speed and the normal traffic. Finally, consideration is given to some of the factors involved in stopping both the normal and high speed traffic at intermediate stations.

5.1 Basic Analysis

A route is considered which has on it distinct categories of traffic which differ, not only in running speed, but also in accelerating and braking performance. Trains belonging to the first category have
have high rates of braking and acceleration, and running speeds which may be in excess of the fixed block line speed. They travel in convoys operating under moving block conditions, the convoys being spaced at regular intervals. This represents a desirable service of very high speed traffic, operating out of a major metropolitan terminus, in which a flight of trains leaves the terminus at regular intervals (perhaps hourly) with the constituent trains ultimately dividing to serve various remote cities. The second type of train has lower rates of braking and acceleration, and a running speed which may equal but not exceed the line speed. Trains of this type operate under four aspect fixed block signalling, and run between the convoys of high speed trains.

Evidently, passing loops are required at intervals along the line, in order that a convoy of high speed traffic may overtake any normal traffic immediately ahead of it. The length of the passing loops is determined by a number of factors, one of which is the behaviour of the normal traffic when it is in the loop. A possible modus operandi is that the slower traffic should proceed at its full running speed until it has been overtaken by the high speed convoy. Another possibility is that the normal traffic should be brought to rest in the passing loop until it has been overtaken. These alternatives are examined and compared below.

5.1.1 Slower Traffic Proceeds Unimpeded

Referring to the passing loop shown in Figure 123, if the low speed traffic maintains a constant running speed, the length of the
passing loop, which is the distance between A and B, is given by

\[ \text{DPL} = \text{VS} \times \text{TOT} \]

where VS is the running speed of the normal traffic, and TOT is the time taken for the train to negotiate the loop. This time is made up of a number of factors which are examined in more detail below. The time taken for the low speed train (or the first one, if there is more than one) to move from the point A, to the position 1 (shown in Figure 123), is given by

\[ \frac{(\text{OL} + \text{TLS})}{\text{VS}} \]

where TLS is the length of the low speed train. If there are NS low speed trains between consecutive flights of high speed trains, then the time which elapses between the instant when the front of the first low speed train is at the point A, and the instant when the last train is in position 1, is given by

\[ \frac{(\text{OL} + \text{TLS})}{\text{VS}} + (\text{NS} - 1) \times \left( \frac{K \times \text{BL} + \text{OL} + \text{TLS}}{\text{VS}} \right) \]

where BL is the distance between fixed block signals, and is given by

\[ \text{BL} = \frac{\text{VM}^2}{(4 \times \text{BS})} \]

where BS is the braking rate of the low speed traffic. Also,

\[ K = 3, \quad \text{if } \text{VS} \geq \text{VSF} \]

and \[ K = 2, \quad \text{otherwise} \]

where \[ \text{VSF} = \sqrt{\text{VM}^2 - 2 \times \text{BS} \times \text{BL}}. \]

The set of points at A may begin to change their route setting when the last slow train reaches position 1, and this points change
is completed after a further period of TPC. When the points change is complete the leading high speed train must be no closer than its braking distance from the point X, which is a distance OL in the rear of A. Thus the front of the leading high speed train reaches the point A an interval of

$$\frac{VF}{2 \times BF} + \frac{OL}{VF}$$

after the points change is completed, where VF is the running speed of the fast traffic, and BF is its braking rate.

The leading high speed train in the convoy must then travel from A to B, which takes a time of DPL/VF, and, subsequently, it clears the point C after a further interval of

$$\frac{(OL + TLF)}{VF}$$

where TLF is the length of a high speed train. If a high speed flight consists of NF trains, the final train in the flight clears the point C an interval of

$$\frac{(NF - 1) \times \left( \frac{VF}{2 \times BF} + \frac{OL + TLF}{VF} \right)}{VS}$$

after the first one does so. When the last train in the flight clears the point C, the points at B may begin to change their route setting in order to allow the low speed trains to leave the loop. This change is completed after a further time, TPC, when the final train in the flight has reached the position 3, and, at this instant, the leading low speed train is in position 2, as shown in Figure 123. Thus a further interval of

$$\frac{(K - 1) \times BL + OL}{VS}$$
elapses before the leading low speed train reaches the point N, which is a total time of TOT after it reached the point A. Thus the time TOT, which the slow trains spend in the passing loop, is given by

$$\text{TOT} = \frac{(K - 1) \times BL + 2 \times OL + TLS + (NS - 1) \times (K \times BL + OL + TLS)}{VS} + \frac{\left(\frac{VF}{(2 \times BF) + 2 \times OL + TLF + (NF - 1) \times \left(\frac{VF}{(2 \times BF) + OL + TLF})\right]}{VF} + 2 \times TPC + DPL/VF$$

By substitution, this may be used to determine the necessary length of the passing loop, which is given by

$$\text{DPL} = \left[\frac{(K - 1) \times BL + 2 \times OL + TLS + (NS - 1) \times (K \times BL + OL + TLS)}{VS} \times \left(\frac{VF}{(2 \times BF) + 2 \times OL + TLF + (NF - 1) \times \left(\frac{VF}{(2 \times BF) + OL + TLF})\right]}{VF} + 2 \times TPC \times VS\right]/(1 - VS/VF)$$

and which may itself be substituted back to give TOT, if this is required.

The distance between passing loops depends upon an additional independent variable, which has not been encountered until now. This is the scheduled time interval, TIC, between consecutive flights of high speed trains. Considering the situation shown in Figure 124, let us take as our time datum the instant when the leading high and low speed trains occupy positions 1F and 1S respectively. After a period of TIC a second high speed convoy occupies exactly the same position as the first one did, since this is also the position 2F. By this time, the low speed trains have reached some intermediate point, represented by position 2S, and, since they have been moving with a constant speed of VS during the entire period of TIC, position 2F is a distance of (VS \times TIC) in the rear of position 2S. This
distance separation is subsequently reduced at a rate of \((VF - VS)\), due to the differential in the running speeds of the two types of train. Thus if constant running speeds are maintained, the leading trains occupy positions 3S and 3F a period of \(VS \times TIC/(VF - VS)\) after they reach positions 2S and 2F. Positions 3S and 3F are the equivalent ones, in the second passing loop, to positions 1S and 1F in the first passing loop. Thus, the distance between position 2F and position 3F is equal to the distance between points X and Z, and is given by

\[
VF \times VS \times TIC/(VF - VS)
\]

So the length of the common line, which is the distance between X and Y, is given by

\[
DCL = VS \times VF \times TIC/(VF - VS) - DPL
\]

A measure of the cost of installation of the necessary passing loops is the average double line length per route mile, which is given by the expression

\[
ADL = DPL/(DCL + DPL)
\]

This is shown as a function of the inter-convoy time, for various values of normal traffic running speed, in Figure 125. It is evident that, for inter-convoy times which are less than 12 minutes, a system of passing loops is not able to accommodate the specified levels of traffic, and the route would require two sets of tracks for its entire length. For an inter-convoy time of 30 minutes, some common line is practical for normal traffic running speeds greater than 30 mph, but the actual value of the average double line length per route mile may
vary considerably with the running speed. Even with those running speeds which require the smallest length of double line (which are also the speeds giving the maximum straight line capacity under four aspect fixed block) the average double line length per route mile is in excess of 2000ft. With further increases in the inter-convoy time the amount of double line required continues to fall, but, even with an inter-convoy time of 60 minutes, the minimum value of average double line length per route mile is still approximately 1000ft. This would seem an unacceptably high figure, and the very large capital expenditure involved makes it imperative to find another mode of operation. An obvious possibility is to bring the normal traffic to rest in the passing loop while the high speed traffic overtakes it.

5.1.2 Slower traffic brought to rest in passing loop

If the low speed trains are braked to rest at consecutive fixed block signals, then the passing loop must have a minimum length, which is given by

\[ DPL = 2 \times OL + TLS + (NS - 1) \times BL \]

In order to determine the maximum possible distance between the passing loops, it is necessary to find the value of HS (the minimum headway between consecutive low speed trains), which is consistent with starting on double yellow from positions one block length apart.

The leading low speed train has travelled a distance of
(TLS + 2 × OL + BL) before the signal, at which the second train is standing, changes its aspect to double yellow so that this train can also begin to move. The maximum possible speed of the first train at this time is given by

\[ VE = \sqrt{2 \times AS \times (TLS + 2 \times OL + BL)} \]

where AS is the rate of acceleration of the low speed traffic. The time which elapses between the two trains beginning to move is \( \frac{VE}{AS} \), unless \( VE \) is greater than VS. In the latter case, the speed of the first train, when the second begins to move, is equal to VS, and the time between the two trains beginning to move is

\[ \left( \frac{VS}{2 \times AS} \right) + \frac{(TLS + 2 \times OL + BL)/VS}{VS} \]

If the running speed, VS, is less than, or equal to, VSF, then the second train is able to accelerate directly back to running speed without further restriction. Since the distance separating the trains is

\[ \frac{(TLS + OL + 2 \times BL)/VS}{VS} \]

at the instant that the second train begins to move, the resulting headway is given by

\[ HS = \frac{(TLS + OL + 2 \times BL)/VS + VA/(2 \times AS)}{VA/(2 \times AS)} \]

where \( VA \) is equal to \( VE \) or \( VS \), whichever is the smaller of the two. However, if \( VS > VSF \), the above expression only applies if the second train does not encounter a double yellow aspect at the entry of the first block in which it is able to attain a running speed greater than VSF. In order to determine if this is the case, it is necessary
to find an integer value, KK, which is the number of block lengths over which the second train must accelerate, before it attains a speed in excess of VSF. The value of KK is the integer which fulfils the condition

\[(KK + 1) \times BL > \frac{VSF}{AS} \geq KK \times BL\]

The time taken for the second train to travel through these KK blocks is given by

\[TSF = \sqrt{2} \times KK \times BL/AS\]

During this time, the first train has travelled a distance of

\[D1 = KK \times BL + TSF \times \sqrt{2} \times AS \times (TLS + OL + BL)\]

if it continues to accelerate throughout the entire period of TSF, i.e. if

\[VS \geq \sqrt{2} \times AS \times (TLS + OL + BL + D1)\]

However, if the first train has already attained its running speed, then the distance which the first train has travelled is given by

\[D1 = (VS^2 - VE^2)/(2 \times AS) + (TSF - (VS - VE)/AS) \times VS\]

The expression for HS, which was quoted above, applies, provided that

\[D1 \geq (KK + 1) \times BL\]

since the second train encounters a green aspect at the signal in the rear of the (KK + 1)th block if this condition is fulfilled. However, if the second train encounters a double yellow aspect at this signal, then the time it takes to travel through the (KK + 1)th block is given by
TT = \( \frac{(VSF - \sqrt{2 \times AS \times KK \times BL})}{AS} + \frac{(BL - (VSF^2 - 2 \times AS \times KK \times BL) / (2 \times AS))}{VSF} \)

The increase in the distance separating the two trains, which occurs during this time, is given by

\[ DD = TT \times VS - BL \]

provided that the first train is already travelling at the running speed, VS, i.e. if

\[ VS \leq \sqrt{2 \times AS \times (TLS + OL + D1 + BL)} \]

However, if the first train attains a speed of VS during the time that the second train is travelling through the \((KK + 1)\)th block, then

\[ \sqrt{2 \times AS \times (TLS + OL + D1 + BL)} < VS \leq \sqrt{2 \times AS \times (TLS + OL + D1 + BL)} + TT \times AS \]

and the increase in distance separation is given by

\[ DD = \left(\frac{VS^2 - 2 \times AS \times (TLS + OL + D1 + BL)}{(2 \times AS)}\right) + \frac{TT - \left(\frac{VS - \sqrt{2 \times AS \times (TLS + OL + D1 + BL)}}{AS}\right) \times VS - BL}{2} \]

Alternatively, if neither of the conditions stated above is fulfilled and the first train is still accelerating at the end of the period TT, then the distance, DD, is given by

\[ DD = \left(\sqrt{2 \times AS \times (TLS + OL + D1 + BL)}\right) \times TT + AS \times TT^2/2 - BL \]

The second train encounters a green aspect at the signal in the rear of the \((KK + 2)\)th block, if

\[ D1 + DD \geq (KK + 1) \times BL \]

and, if this is the case, the resulting headway, HS, is given by

\[ HS = \left(3 \times BL + OL + TLS + D1 + DD - (KK + 1) \times BL \right) \times (VS^2 - VSF^2) / (2 \times AS) / VS \]

+ \( (VS - VSF) / AS \).
However, if the second train encounters another double yellow aspect at this signal, it maintains a constant speed of VSF while travelling through the \((KK + 2)^{th}\) block. The maximum possible speed, with which the first train may be travelling when the second enters the \((KK + 2)^{th}\) block, is given by

\[
VW = \sqrt{2} \times AS \times (TLS + OL + D1 + BL) + TT \times AS
\]

Naturally the actual instantaneous speed of the train cannot be greater than the running speed VS. The time taken for the second train to pass through the \((KK + 2)^{th}\) block is given by

\[
TT2 = \frac{BL}{VSF}
\]

and the maximum possible speed of the first train, at the end of this time, is given by

\[
VV2 = VW + AS \times TT2
\]

The increase in the distance separating the trains, which occurs during the period \(TT2\), is given by

\[
DD2 = VS \times TT2 - BL
\]

provided that \(VW < VS\), i.e. if the first train is already travelling with a speed of VS when the second train enters the \((KK + 2)^{th}\) block. If the first train attains the running speed during the period \(TT2\), then the condition

\[
VW < VS \leq VV2
\]

is fulfilled, and the increase in the distance separation of the trains is given by

\[
DD2 = \frac{(VS^2 - VW^2)}{(2 \times AS)} + \left( TT2 - \frac{(VS - VW)}{AS} \right) \times VS - BL
\]
However, if the first train has not attained a speed of \( VS \) by the end of the period \( TT2 \), then \( DD2 \) is given by

\[
DD2 = TT2 \times (VV + AS \times TT2/2) - BL
\]

The second train encounters a green aspect at the signal in the rear of the \( (KK + 3) \)th block, if

\[
D1 + DDT > (KK + 1) \times BL
\]

where \( DDT = DD + DD2 \). If this is the case, the resulting headway, between the trains, is given by

\[
HS = \left( 3 \times BL + OL + TLS + D1 + DDT - (KK + 1) \times BL - (VS^2 - VSF^2)/(2 \times AS) \right)/VS
\]

\[
+ \frac{(VS - VSF)}{AS}
\]

If the second train encounters yet another double yellow aspect at the signal in the rear of the \( (KK + 3) \)th block, further iterations of the process described for the \( (KK + 2) \)th block may be made, until a green aspect is observed.

In order to determine the common line length, \( DCL \), it is necessary to find the distance between the points at which a train comes to rest in consecutive passing loops. If the running headway, \( HS \), is obtained as described above, then the second train comes to rest a period of \( (TDIF + TZ) \) after the first one does so, where \( TDIF \) and \( TZ \) are given by the expressions quoted in section 2.2.2a, which dealt with double yellow starting under four aspect fixed block signalling. Thus the final low speed train, in the inter-flight group, comes to rest a total time of \( (NS + 1) \times (TDIF + TZ) \) after the first train. When the final low speed train has come to rest, the points at entry to the passing loop change their route setting, taking a time of \( TPC \).
to do so. When the points change is complete, the leading high speed train is a minimum distance of \((OL + VF)/(2 \times BF)\) in the rear of the entry to the loop. Thus the tail of the final train in the high speed flight clears the overlap associated with the points at the exit of the passing loop, a period of

\[
(4 \times OL + TLS + VF^2)/(2 \times BF) + (NS - 1) \times BL + TLF)/VF + (NF - 1) \times HF
\]

after the points change is complete, where HF is the headway between the high speed trains, and is given by

\[
HF = (TLF + OL)/VF + VF/(2 \times BF)
\]

The first low speed train begins to move after a further period of TXX, which is equal to TPC or \((2 \times BL)/VF\), whichever is the greater of the two, since the train must observe a double yellow aspect before starting. Then the train accelerates up to running speed, taking a time of VS/AS to do so. Thus, when the inter-flight time, TIC, has expired after the leading low speed train first came to rest in the passing loop, this leading train has travelled a distance, from the place where it came to rest in the passing loop, which is equal to

\[
(TIC - (4 \times OL + TLS + VF^2)/(2 \times BF) + (NS - 1) \times BL + TLF)/VF - TPC
- (NF - 1) \times HF - (NS - 1) \times (TDIF + TZ) - TXX - VS/AS \times VS + VS/(2 \times AS)
\]

Let us assume that the leading low speed train is able to continue at full speed for a further period of TX before it is necessary to brake to rest in the next passing loop. The total distance between equivalent points in consecutive loops is given by

\[
DCL + DPL = \left[ TIC - (4 \times OL + TLS + VF^2)/(2 \times BF) + (NS - 1) \times BL + TLF)/VF
- TPC - (NF - 1) \times HF - (NS - 1) \times (TDIF + TZ) - TXX - VS/BS \right] \times VS
+ VS^2 \times (AS + BS)/(2 \times AS \times BS) + VS \times TX
\]
However the same distance is covered by the fast trains, at their running speed of $VF$, in a time of $(TX + VS/BS)$, so that

$$DCL + DPL = (TX + VS/BS) \times VF$$

These two expressions may be used to find $TX$, and this value substituted back to give

$$DCL = \left[ \frac{TIC - (4 \times OL + TLS + VF^2/(2 \times BF) + (NS - 1) \times BL + TLF)}{VF - TPC} \right. - (NF - 1) \times HF - (NS - 1) \times (TDIF + TZ) - TXX - VS/BS \left] \times VS/(VF - VS) + VS/BS \right. \times VF - DPL$$

The average double line length per route mile is obtained from the expression quoted in the previous section, and is shown, as a function of inter-convoy time for various values of normal traffic running speed, in Figure 126. If this is compared with the equivalent graph for the case when the normal traffic proceeds unimpeded (Figure 125), it may be seen that, for inter-convoy times greater than 30 minutes, the double line length required, when the normal traffic is brought to rest in the loop, is considerably less. This is particularly so at higher running speeds, e.g., with a running speed of 100 mph, the double line length per route mile is 350ft, and 130ft for inter-convoy times of 30 and 60 minutes respectively, whereas, if the normal traffic maintains full speed, the double line length required is 2000ft and 1000ft. Such large differences in the double line length per route mile required by the two modes of operation mean that, from financial considerations, it is essential that the normal traffic be brought to rest in the passing loop. However, it should be noted that, for normal traffic running speeds greater than
FIGURE 125 Slow Trains Proceed Unimpeded with Various Running Speeds

Average Double Line Length per Route

No. of Trains in Fast Convoy 3
No. of Trains in Slow Convoy 10
Fast Convoy Running Speed 150 mph

Inter Convoy Time in Minutes

FIGURE 126 Slow Trains Brought to a Halt from Various Running Speeds

Average Double Line Length per Route

No. of Trains in Fast Convoy 3
No. of Trains in Slow Convoy 10
Fast Convoy Running Speed 150 mph

Inter Convoy Time in Minutes
30 mph, the inter convoy time, below which any common line is not practical, tends to be greater, due to the lower speeds at which the normal trains are travelling when entering and leaving the passing loop.

The effect of changing the running speed of the high speed flights of trains is shown in Figure 127. It is evident that reducing the running speed of the fast traffic does not affect the threshold level of the inter convoy time below which any common line is impractical. However, for inter convoy times greater than this threshold, the required double line length per route mile is reduced, especially at inter convoy times which are only slightly greater than the threshold. Although the length of any given passing loop is not affected by the running speed of the high speed traffic, as the latter is decreased the distance between the loops may be increased, because the second high speed flight takes longer to reduce the distance initially separating it from the low speed trains. Thus the low speed trains can maintain their running for a longer period and can travel further before they need to enter another passing loop.

In practice, it is not always possible to situate passing loops at the optimal position for the actual, or potential, traffic requirements of the line. Therefore it is necessary to specify the nominal inter-loop distance, DBL (which is the sum of the loop length, and the common line length), and use this to determine the possible permutations of traffic which may use the line. Two alternatives present themselves, namely that the high speed traffic should be specified, and the maximum volume of normal traffic, consistent with this, should
be determined, or that the maximum volume of high speed traffic should be found which is consistent with a specified pattern of low speed traffic. This problem reduces to one of finding the maximum value of NS, or NF, whichever is not specified, for which the following condition is fulfilled.

\[ DBL \geq DPL + DCL \]

The average double line length per route mile is shown, in Figure 128, as a function of the distance between loops for various inter-loop times. It is evident that, for a distance between the passing loops of less than 5 miles, the maximum volume of low speed traffic which may be accommodated, is very large. It would require a loop longer than 5 miles for this number of trains to be spaced, at rest, at distances of one fixed block length. Thus a double line is required for the entire length of the route, although, in practice, it is possible to operate both types of traffic on a line with passing loops every 5 miles, provided that the level of normal traffic is less than the maximum indicated by this analysis. For any given distance between the passing loops, the average double line length per route mile tends to increase as the inter convoy time increases. This is because the maximum number of low speed trains which may be accommodated between consecutive flights increases with increasing inter convoy time (see Figure 129), and thus the required length of loop is also increased. For each inter convoy time there is a limiting distance between the passing loops above which it is not possible to run any low speed traffic. Thus, for example, it is not practical to run any normal traffic on a line which has passing loops that are spaced at
FIGURE 127 Slow Trains Brought to a Halt
Number of Trains in Fast Convoy 3
Number . " . Slow Group 10
Running Speed of Slow Trains 100 mph
Various Speeds of Fast Trains

Average Double Line Length per Route
(Feet)

Inter Convoy Time in Minutes

110 mph

150 mph

FIGURE 128 Number of Trains in Fast Convoy: 3
Number " " Slow Group is Maximum possible for Various Inter Fast Convoy Times
Fast Train Running Speed 150 mph
Slow " " 100 mph

Distance Between Passing Loops in Miles
distances greater than 80 miles.

The same information is shown in a different way in Figure 130, where the average double line length per route mile is given as a function of inter convoy time, for various distances between the passing loops. This is of interest, since the siting of the loops is relatively difficult to alter once the loop has been constructed. It is evident that the largest differences, in the double line required, occur at the largest values of inter-convoy time. However, for long inter convoy times, the percentage change in the maximum possible volume of normal traffic is smaller than for shorter inter convoy times. Thus, for long inter convoy times the average double line length per route mile may be kept at an economic level by a relatively small sacrifice in the maximum volume of normal traffic which may be accommodated.

The equivalent graph to Figure 130 is given, for the case when the low speed traffic is specified, in Figure 131. It may be seen that, within the practical region, the average double line length per route mile is independent of the inter convoy time, for a given distance between the passing loops. For any inter convoy time which is less than the limit of practicality, it is not possible to run any high speed traffic if the specified volume of low speed traffic is to be attained. For increased distances between the passing loops, the minimum inter convoy time, which can be used for a practical high speed service, also increases.

It is not desirable to have passenger trains standing stationary in a passing loop for long periods, unless this is totally unavoidable.
FIGURE : 129
Number of Trains in Fast Convoy 3
Running Speed of Fast Trains 150 mph
Running Speed of Slow Trains 100 mph
Various Distances Between Passing Loops

FIGURE : 130 Maximum Slow Traffic Possible Using Passing Loops Separated by Various Distances.
Number of Trains in Fast Convoy 3
Running Speed of Fast Trains 150 mph
Running Speed of Slow Trains 100 mph
A method for calculating the maximum period which any given low speed train may be required to spend in a passing loop, together with the time it is actually stationary, is given in Appendix E.

5.2 Immunity from delays by headway tolerancing

If the system described in the preceding section is implemented some unscheduled delays are certain to occur from time to time. Naturally it is desirable to keep the disruption caused by such delays to a minimum, without losing too much line capacity. A simple method of achieving this is to make the headways between trains of the same type sufficiently large that, given a certain delay to the leading train of any group of trains, the final train of the group does not experience any delay, so that subsequent groups of trains are not affected either. This principle can be applied to both high speed and low speed traffic.

Since the high speed flights are running according to pure moving block criteria, the delay to the \((n + 1)^{th}\) train in a flight is given by

\[
D_n = D_0 - n \times (H - HW)
\]

where \(D_0\) is the delay experienced by the leading train of the flight. If the flight consists of a total of \(NF\) trains, and if the final train must not experience any delay, then the slack required in the headway is given by

\[
(H - HW) = D_0 / (NF - 1)
\]

Thus, if \(HM\) is the minimum headway between the high speed trains,
as determined by the basic analysis, the toleranced headway is given by

\[ HF = HFM + \frac{D_o}{(NF - 1)} \]

A convoy of high speed traffic may be said to have a tolerance of \( XF \% \) if the running headway is such that no delay is encountered by the final train in the flight, when the leading train of the flight is delayed by this percentage of the time taken to travel from one passing loop to the next. Thus \( D_o \) is given by

\[ D_o = \frac{XF \times DBL}{(100 \times VF)} \]

and the toleranced value of the headway between the trains of the flight is given by

\[ HF = HFM + \frac{XF \times DBL}{(100 \times VF \times (NF - 1))} \]

The low speed traffic may be toleranced in a similar way, so that a tolerance of \( XS \% \) prevents any delay to the final train in a group of \( NS \) trains, when the leading train in the group is delayed by

\[ FTD = \frac{XS \times DBL}{(100 \times VS)} \]

A simple technique is used to find the minimum headway for which the number of following trains delayed is less than \((NS - 1)\). This procedure starts from the knowledge that the required value of headway must lie between the minimum value derived from the basic analyses, \( HSM \), and the limit of delay propagation \((HSM + FTD)\). The number of trains, affected by the delay at any specific headway, may be determined by use of the method described in detail in section 2.2.2a.

The effect which this tolerancing has upon the maximum number of low speed trains which may be included in an inter convoy group is
shown in Figure 132. Tolerancing the headway between the high speed trains increases the minimum inter convoy time for which it is practical to insert any low speed trains between the high speed convoys, because the time taken for a flight to overtake a single low speed train is also increased. Above this minimum inter convoy time, the slope of the graph is not changed by tolerancing the headway between the high speed trains, because this is controlled by the time which elapses between the first and the last trains of the inter convoy group coming to rest.

Tolerancing the headway between the low speed trains does not change the minimum inter convoy time for which it is practical to insert any low speed trains between the convoys. This is because the headway, HS, only becomes involved in the analysis when there is more than one low speed train between consecutive flights. When there is more than one low speed train in an inter flight group, the headway between the trains in the group has a significant effect upon the time which the group spends in the passing loop. Thus only one train per inter convoy group is possible, for a range of inter convoy times, when the headway between the low speed trains is toleranced. For larger inter convoy times, tolerancing the headway between the low speed trains tends to increase the time which elapses between the instants when the first and the last trains of a group come to rest. This decreases the rate at which the maximum number of low speed trains in a group increases with increasing inter convoy time.

The addition of tolerances to the headways between trains constituting the specified flow has a similar effect upon the maximum rate of the
FIGURE 131  Maximum Numbers of Fast Trains in Each Convoy
Number of Trains in a Slow Group 10
Various Distances Between Passing Loops

FIGURE 132  Number of Trains in a Fast Convoy 3
Running Speed of Fast Trains 150 mph
Running .. Slow .. Slow 100 mph
Distance Between Loops 40 miles

Headways Between Trains are Tolerated as Follows:
Fast Trains  Slow Trains
A: - 0%  0%
B: - 100% (16 mins.) 0%
C: - 100%  40%
D: - 100%  100% (24 mins.)
unspecified flow, irrespective of whether the high speed or the normal speed traffic is specified. Similarly, applying tolerances to the headways between trains of the unspecified flow always has a similar effect to that described above.

If, instead of pure moving block signalling, the high speed traffic is operating under quantised moving block, then the minimum headways between the trains will be somewhat greater than those indicated above. Thus the overall effect upon the maximum quantity of normal traffic which may be carried by the line would be somewhat similar to that observed when a small tolerance is added. Because the performance under perturbed conditions is very similar for quantised and pure moving block (with the high running speeds involved), the use of quantised moving block in this mixed traffic situation should allow only slightly smaller traffic flows to be accommodated.

5.3 Problems associated with stations

Two problems, relating to the introduction of stations, are dealt with in this section. The first of these concerns the positioning of stations at which the normal traffic, or some portion of it, is required to stop. Secondly, an examination is made of the alternative berthing arrangements which could be provided at an intermediate station at which the high speed flights of trains are required to stop.
5.3.1 Positioning a station for normal traffic

A station at which some proportion of the normal traffic is required to make a stop may be positioned on the common line between the passing loops, or it may be situated within a passing loop. Both of these alternatives are considered below.

5.3.1a Stations situated on the common line

Let us suppose that a specified proportion of the normal low speed traffic is required to make a stop at each of a number of stations which are situated at equal intervals along the common line between passing loops. The required headway between any two low speed trains is equal to one of the four intervals $T_{LL}$, $T_{HL}$, $T_{LH}$ and $T_{HH}$ which are defined as in the analysis of the capacity of an isolated station operating under four aspect fixed block signalling. These intervals may be evaluated as follows. If

$$\text{VIN} = \sqrt{2 \times AS \times (OL + TLS)}$$

and $CL = 2 \times BL$ if $VS \leq VSF$ or $CL = 3 \times BL$ otherwise, then

$$T_{LH} = T_{HH} = VS/BS + (CL - VS^2/(2 \times BS))/VS + TST + VIN/AS$$

and

$$T_{HL} = HSM + (VS - VIN)/AS - (VS^2 - VIN^2)/(2 \times AS \times VS)$$

provided that $VS \geq VIN$. However, if $VS < VIN$, then

$$T_{LH} = T_{HH} = VS \times (AS + BS)/(2 \times AS \times BS) + TST + (CL + OL + TLS)/VS$$

and

$$T_{HL} = HSM$$

On all occasions the interval between two non-stopping trains is given by

$$T_{LL} = HSM$$
None of these intervals must have a value which is less than $HSM$, because this is the minimum value necessary for the correct operation of the passing loops. A more detailed derivation of the expressions quoted above is given in the analysis of the capacity of a station operating under four aspect fixed block signalling.

The number of stations $NPS$ which are on the common line between consecutive passing loops, is the largest integer value which fulfils both of the following conditions.

$$DBL - DPL \geq (NPS + 1) \times VS \times \left(\frac{HSM + VS \times (AS + BS)}{2 \times AS \times BS}\right)$$

and $$DBL \geq NPS \times DPS$$

where $DPS$ is the specified nominal distance between stations. This ensures that the spacing of the stations is such that they may be regarded as isolated components.

If the number of trains in an inter flight group, which stop at the stations, is $N1$, then the number, $N2$, which do not stop is given by

$$N2 = NS - N1$$

If the majority of the trains stop at the stations, then $N1 \gg N2$, and the headway required between the trains is given by

$$HS = \frac{(N1 - N2) \times THH + N2 \times (TLH + THL)}{NS}$$

provided that $NPS \leq 1$.

However, if $NPS > 1$, then the headway is given by

$$HS = \left[\frac{(N1 - N2) \times THH + N2 \times (NPS \times (THL + TLH) - 2 \times TLL \times (NPS - 1))}{NS}\right]$$

Similarly, if the majority of the low speed trains do not stop...
at the stations, then \( N_1 < N_2 \), and \( HS \) is given by

\[
    HS = \frac{(N_2 - N_1) \times TLL + N_1 \times (THL + TLH) / NS}{N_2}
\]

provided that \( NPS \leq 1 \). However if \( NPS > 1 \), then the headway is given by

\[
    HS = \left( \frac{(N_2 - N_1) \times TLL + N_1 \times (NPS \times (THL + TLH) - 2 \times TLL \times (NPS - 1)) / NS}{N_2} \right)
\]

The value of \( HS \) which is derived from these expressions is used, in the method described in the basic analysis, to determine the maximum practical value of the number of low speed trains which may form an inter flight group. If the specified proportion of stopping to non-stopping traffic does not correspond to a practical ratio, then different inter convoy groups have different numbers of stopping trains. Under these circumstances, the headway, \( HS \), which is used to determine the value of \( NS \) must be equal to that required for the group which has the highest number of stopping trains.

The maximum number of low speed trains in an inter convoy group is shown, in Figure 133, as a function of the percentage of the normal traffic which stops at the stations. It is evident that, for any given inter convoy time, the minimum group size does not occur when 100% of the low speed traffic is stopping at the stations, as might be expected. This is because a very large interval is initially required between a stopping train and a non-stopping one, since, for each stop that the first train makes, the headway between it and the second train is reduced. Thus the headway between a stopping train and a non-stopping train depends upon the number of stations which are situated between adjacent passing loops. However, the headway
between two stopping trains does not need to be any larger when the trains stop at a number of stations than when there is only a single station involved. This is because they possess identical speed-distance curves, so that they maintain the same headway throughout the length of common line. Thus, as the number of stations is increased, the average headway between the trains remains constant, if 100% of them stop at the stations, but is increased, if only some of them stop. One would expect that the reduction, in the maximum size of the inter convoy group, would be most marked when 50% of the low speed trains stop at the stations, since the maximum number of intervals between stopping and non-stopping trains, occur under these conditions. This is confirmed by Figure 133, particularly for longer inter convoy times.

The maximum number of low speed trains in an inter convoy group is shown, as a function of the distance between passing loops, in Figure 134. It is apparent that the size of the inter convoy group, with 50% of the low speed trains stopping, is less than that with 100% stopping, except at very short distances between the passing loops. Since the number of stations between consecutive passing loops is dependent upon the distance between the loops, the difference between the intervals TLH and THH becomes larger as the inter-loop distance increases. Thus, for small distances between the loops, the difference between TLH and THH may be so small that an inter convoy group may have more trains in it, if 50% of them are stopping at the stations, than if 100% are doing so.

The effect upon the maximum number of low speed trains in an inter convoy group which is produced by varying the station stopping
FIGURE 133 Stations on the Common Line
Spaced at Intervals of 5 miles
Number of Trains in Fast Convoy 3
Distance Between Passing Loops: 40 miles
Station Stop Time 20 sec
Various Inter Fast Convoy Times

Maximum Number of Slow Trains Between Convoys

Percentage of Slow Trains Stopping at Stations

FIGURE 134 Stations on the Common Line
Spaced at Intervals of 5 miles
Number of Trains in Fast Convoy 3
Inter Fast Convoy Time 30 minutes
Station Stop Time 20 sec
Various Percentages of Slow Trains Stopping at the Stations

Maximum Number of Slow Trains Between Convoys

Distance Between Passing Loops in Miles
time, is shown in Figure 135. Naturally, if none of the trains stop at the stations, the stopping time has no effect on the number of trains in the group. As the proportion of stopping trains is increased, the station stopping time produces a larger variation in the size of the inter convoy group, the latter being diminished as the stopping time is increased.

5.3.1b A station in the passing loop

Let us consider the case in which there is one station per passing loop, and this station is situated in the place where the leading train of the inter convoy group would normally come to rest. Since the leading train of the group stops in the station berth, irrespective of whether or not it is required to make a formal stop, it should be made a stopping train, provided there are any in the group. The time, TAS, which the leading train spends stationary in the station berth for "passing" reasons, may be obtained by the method described in Appendix E. If this is not shorter than the station stopping time, then the leading train may proceed as if the station did not exist. However, if TAS < TST, then the value of TXX, as used in the basic analysis, is given by

\[
TXX = TBX + TST - TAS
\]

where TBX is the previous value assigned to TXX.

If the \( M^{th} \) train in an inter convoy group (where \( M \) is greater than one), makes a stop at the station, then the time which elapses between the instant when it leaves the place where it initially
came to rest in the loop, and its arrival at the station, is given by

\[ T_1 = \frac{(M - 1) \times BL/VS + VS \times (AS + BS)}{(2 \times AS \times BS)} \]

provided that the running speed, VS, is attained during this journey. This is the case if

\[ VS < \sqrt{2 \times AS \times BS \times (M - 1) \times BL/(AS + BS)} \]

If this condition is not fulfilled, then \( T_1 \) is given by

\[ T_1 = \frac{\sqrt{2 \times (M - 1) \times BL \times (AS + BS)}/(AS \times BS)}{AS \times BS} \]

The time which elapses between this train's departure from the station and the instant when it initially comes to rest in the next passing loop is given by

\[ T_2 = \frac{(DBL - (M - 1) \times BL)/VS + VS \times (AS + BS)/(2 \times AS \times BS)}{AS \times BS} \]

provided that the speed of VS is attained during this journey. This is the case if

\[ VS < \sqrt{2 \times AS \times BS \times (DBL - (M - 1) \times BL)/(AS + BS)} \]

However, if this condition is not fulfilled, then \( T_2 \) is given by

\[ T_2 = \frac{\sqrt{2 \times (DBL - (M - 1) \times BL) \times (AS + BS)/(AS \times BS)}}{AS \times BS} \]

Thus the total time taken by the train in travelling from the place where it initially comes to rest in one passing loop, to the equivalent place in the next loop, is given by

\[ T_{SP} = T_1 + T_2 + T_{ST} \]

The time taken by a train to travel the same distance, but without stopping at the station, is given by
\[ TNS = \frac{DBL}{VS} + VS \times \frac{(AS + BS)}{(2 \times AS \times BS)} \]

provided that the running speed is attained at some point in the journey. This is the case if

\[ VS \leq \sqrt{\frac{2 \times AS \times BS \times DBL}{(AS + BS)}} \]

However, if the running speed is not attained, the time \( TNS \) is given by

\[ TNS = \frac{\sqrt{2 \times DBL \times (AS + BS)}}{(AS \times BS)} \]

The headway required between a stopping train and a non-stopping train is given by

\[ FLH(M) = HS_M + TSP - TNS \]

which is a function of the number, \( M \), of the stopping train counted from the front of the group, since \( TSP \) varies as one progresses along the inter convoy group.

The interval between two trains which do not stop at the station and the interval between a non-stopping train and a stopping one are both unaffected by the existence of the station, and are given by

\[ TLL = THL = HS_M \]

However, the interval between two stopping trains is given by

\[ FHH(M) = FLH(M) \]

provided that the first train has attained running speed by the time it clears the station berth, i.e. if

\[ VS \leq \sqrt{\frac{2 \times AS \times (OL + TLS)}}{\} \]

However, if this is not the case, then \( FHH(M) \), which is also a function of \( M \), is given by

\[ FHH(M) = FLH(M) - \frac{VS}{(2 \times AS)} + \frac{VIN \times (2 \times VS - VIN)}{(2 \times AS \times VS)} \]

provided this is larger than \( HS_M \).
If, in an inter convoy group of NS low speed trains there are N which stop at the station, then the average headway required between the trains is given by

\[ HS = TLL \]

provided that \( N \leq 1 \).

If there are less stopping trains than non-stopping trains in the group, i.e. if \( (2 \times N) < NS \), then the headway is given by

\[ HS = \frac{(SUF + N \times THL + (NS - 2 \times N) \times TLL)}{(NS - 1)} \]

where \( SUF = FLH(3) + FLH(5) + FLH(7) + \ldots + FLH(2 \times N - 1) \)

However, if all of the trains in the group stop at the station then the headway is given by

\[ HS = \frac{SUG}{NS - 1} \]

where \( SUG = FHH(1) + FHH(2) + FHH(3) + \ldots + FHH(NS - 1) \)

For all other conditions, when a majority, but not all, of the trains stop at the station, and \( NS/2 \leq N < NS \), then the headway is given by

\[ HS = \frac{(SUF + (N + 1) \times THL + SUH)}{(NS - 1)} \]

where \( SUF \) is given above,

and \( SUH = FHH(2 \times N + 1) + FHH(2 \times N + 2) + \ldots + FHH(NS - 1) \)

The value of \( HS \), which is obtained from the expressions given above, is used in the basic analysis, as previously described, to give the maximum number of low speed trains comprising an inter convoy group. The latter is shown, as a function of the percentage of low speed traffic stopping at the stations, in Figure 136. It is evident
FIGURE 135 Stations on the Common Line
Spaced at Intervals of 5 miles

Number of Trains in Fast Convoy: 3
Distance Between Passing Loops: 40 miles
Inter Fast Convoy Time: 60 minutes
Various Percentages of Slow Trains Stopping at the Stations

FIGURE 136 A Station in Each Passing Loop
Number of Trains in Fast Convoy: 3
Distance Between Passing Loops: 40 miles
Station Stop Time: 20 sec
Various Inter Fast Convoy Times

Percentage of Slow Trains Stopping at Stations
that there is a tendency for the maximum number of trains in the inter convoy group to be less with 50% and 100% of the trains stopping at the station, than for some percentages between these values. In an inter convoy group, in which a majority (but not all) of the trains stop at the station, all of the N non-stopping trains are positioned alternately, with an equal number of stopping trains, in the first 2N places in the group. The remaining positions are all taken by stopping trains. Thus, when considering trains near the end of the group, the following condition is fulfilled

\[ (FHH(M) + FHH(M + 1)) < (FLH(M) + THL) \]

Thus the average headway, HS, is less than that required with 50% of the trains stopping at the station, and hence the maximum number of trains in the group may be increased. However, as the percentage of stopping trains is further increased, consecutive stopping trains may be found closer to the front of the group. As M becomes small, it is found that

\[ (FHH(M) + FHH(M + 1)) > (FLH(M) + THL) \]

so that the average headway, HS, increases once more, and the maximum number of trains in the group decreases.

Let us compare the maximum number of trains per inter convoy group, which is possible with a station in each passing loop (Figure 136), with that which is possible with stations on the common line (Figure 133). It is evident that, for longer inter convoy times, more low speed trains per group are possible with a station in each passing loop, than with stations on the common line, except
when there are very high percentages of stopping trains. However, for an inter convoy time of 30 minutes, a station in the loop permits shorter inter convoy groups, than those which are possible with stations on the common line, not only with nearly all the trains stopping at the stations, but also when just under 50% of them do so. Thus, under some operating conditions, the introduction of a station situated in the passing loop and in addition to some already on the common line, would produce a restrictive effect on the maximum number of slow trains which make up an inter convoy group. However, this would not necessarily be the case for some other operating conditions.

The maximum number of low speed trains in an inter convoy group is shown, as a function of the distance between passing loops, in Figure 137. It is evident that this relationship is very similar to that observed for the case of stations on the common line (Figure 134). However, an interesting difference is that the maximum number of trains per group continues to be greater for 100% of the trains stopping than for 50% stopping, even with a distance of ten miles between the loops.

5.3.2 Berthing the high speed convoy

In the preceding sections of this chapter the flight of high speed trains is considered to maintain a constant speed, while the low speed traffic, which is also using the line, takes the necessary avoiding action. However, it is quite probable that, at some stage of its journey, the high speed convoy is required to make a stop at an intermediate station. This might be shortly before the place where
the convoy divides into individual trains, which proceed ultimately to different destinations. It may be desirable that all the trains in the convoy stop at this intermediate station in order to spread the load of arriving passengers equally among the trains in the convoy. However, a more important reason for requiring all the fast trains to stop at the intermediate station is the creation of a service from this station to all the individual destinations of the high speed trains.

Let us suppose that two berthing positions for the high speed trains are to be provided at the intermediate station. These berths may be arranged in two ways as shown in Figure 138. If the station provides a single platform of double length, so that a high speed train can come to rest immediately behind its predecessor, prior to the latter resuming its journey, this is defined as series berthing. The alternative is the provision of two single length platforms, which is defined as parallel berthing.

If the station provides series berthing for the convoy, the leading train may enter its berth entirely unaffected by the presence of any other train. However, if the second train in the convoy is to commence braking at the latest possible moment in order to come to rest in the berth in the rear of that occupied by the leading train, then the latter must have cleared the second train's berth before its braking is commenced. Thus, when the leading train has just come to rest the second train must be a minimum distance in the rear of \((HFM \times VF)\), where \(HFM\) is the minimum straight line headway. Thus the headway required between the first and second trains in the convoy is given by
FIGURE : 137  A Station in Each Passing Loop  
Number of Trains in Fast Convoy 3  
Inter Fast Convoy Time 30 minutes  
Station Stop Time 20 sec  
Various Percentages of Slow Trains Stopping at the Stations

FIGURE : 138

Series Berthing

Parallel Berthing
The second train must have cleared the third train's berth (which is also the one which was occupied by the leading train) before the third train has to commence braking. The second train clears the third train's berth a period, TS, after starting from rest, where TS is given by

\[ TS = \sqrt{2 \times \frac{(DK + 2 \times TLF)}{AF}} \]

provided that it has not already attained its running speed of VF, i.e. if

\[ VF > \sqrt{2 \times AF \times (DK + 2 \times TLF)} \]

However, if the second train is already travelling with a speed of VF when it clears the third train's berth, then TS is given by

\[ TS = \frac{VF}{(2 \times AF)} + \frac{(DK + 2 \times TLF)}{VF} \]

Thus the headway required between the second and third trains in the convoy is given by

\[ H(2) = HFM + \frac{VF}{(2 \times BF)} + TST + TS \]

From this point onwards the third train behaves exactly as the first train, while the fourth train behaves as the second train, and so on. Thus, subsequent headways between the trains are given by the expressions

\[ H(2 \times M - 1) = H(1) \quad \text{where} \quad M = 1, 2, \ldots, \frac{NF}{2} - 1 \]

and

\[ H(2 \times M) = H(2) \]

Alternatively, if the station provides parallel berthing, the leading train of the convoy enters its berth in exactly the same way
as with series berthing. However, as soon as the leading train comes to rest, the set of points in the rear of the station berths begin to change their route setting, taking a time of TPC to complete this operation. When the points change is complete, the other station berth is clear for the second train to enter, but this train must be a minimum distance of \((HFM \times VF)\) in the rear of the points at the time. Thus the headway between first and second trains is given by

\[
H(1) = HFM + VF/(2 \times BF) + TPC + (TLF + OL)/VF
\]

If \(TS\) is the time taken by the leading train to clear its berth after it moves away from rest, then this is given by

\[
TS = \sqrt{2 \times TLF/AF}
\]

provided that

\[
VF \geq \sqrt{2 \times AF \times TLF}
\]

or \(TS = VF/(2 \times AF) + TLF/VF\) otherwise.

The third train in the convoy may enter the berth formerly occupied by the leading train, provided that the first train has already cleared this berth when the second train has entered its berth and the points have changed back to their original setting. Thus the headway between the second and third trains is given by

\[
H(2) = H(1) = HFM + VF/(2 \times BF) + TPC + (TLF + OL)/VF
\]

provided that the following condition is fulfilled

\[
(TS + TST) \leq (2 \times TPC + VF/(2 \times BF) + HFM)
\]

If this is not the case, \(H(2)\) is given by

\[
H(2) = H(1) + TS + TST - (2 \times TPC + VF/(2 \times BF) + HFM)
\]
However, the earliest moment that the second train may leave its berth after the leading train has done so is equal to \((TU + TPC)\), where \(TU\) is the time taken by the first train to clear the points at the exit of the station berths, and measured from the instant at which it moves off. This is given by

\[
TU = \frac{\sqrt{2} \times (TLF + 2 \times OL)}{AF}
\]

provided that the first train has not attained its running speed, i.e. if \(VF > \frac{\sqrt{2} \times AF \times (TLF + 2 \times OL)}{VF}\).

However, if this condition is not fulfilled, then \(TU\) is given by

\[
TU = \frac{VF}{(2 \times AF) + (TLF + 2 \times OL)}
\]

If the value of \(H(2)\), which is obtained from the expressions given above, is greater than \((TU + TPC)\), then this is the headway between the second and third trains, and between all subsequent trains in the convoy. Otherwise \(H(2)\) is given by

\[
H(2) = TU + TPC
\]

and this is also the headway between all subsequent trains.

The aggregate headway required between the first and last trains in a high speed convoy, when series and parallel berthing is employed, is shown, as a function of running speed, in Figure 139. It may be seen that, for a given length of convoy, series berthing requires shorter average headways than parallel berthing, when the running speed is low, while the reverse is the case at higher running speeds. The running speed, above which the parallel berthing procedure produces smaller headways than series berthing, depends upon the length
FIGURE 139 Comparison of Series & Parallel Berthing for Various Numbers of Trains in the Convoy
Station Stop Time 60 sec
Points Change Time 6 sec

 Aggregate Headway Between First & Last Trains in Minutes

Running Speed in M.P.H.
of the convoy. In general, this critical speed becomes lower for longer convoys. Thus, for long convoys, travelling at high speeds, parallel berthing is to be preferred, while series berthing is more appropriate to short convoys travelling at lower speeds.
The comparison of four and five aspect fixed block signalling shows that the five aspect system gives a greater steady state capacity, but only over certain speed ranges. The additional steady state capacity which is obtained by the introduction of one extra aspect tends to decrease as the number of aspects increases. Also, as the number of aspects of the signalling system increases, the number of running speeds at which there are discontinuities in capacity increases, because this is always three less than the number of aspects.

Theoretical pure moving block signalling gives a steady state capacity which is much larger than that obtained from four or five aspect fixed block. The capacity of pure moving block, as a function of running speed, has a similar form to that of a line operating under fixed block, but resignalled for the appropriate running speed. A maximum capacity is obtained with running speeds of approximately 30 to 40% of line speed under pure moving block. Thus, if it becomes necessary to impose a speed restriction, throughout the length of the line, which is greater than 20% of line speed, the capacity of the line is not necessarily reduced under pure moving block, as it would be under fixed block signalling.

The capacity of a quantised moving block signalling system depends upon the chosen levels of both distance and speed quantisation. As the quantisation of distance and speed is made more fine, the capacity is increased, but this is also accompanied by an increase in
the cost of installing the necessary equipment. Thus some compromise
is necessary. For the maximum number of speed quantisation levels
and the shortest distance between information points which has been
considered, the capacity graph obtained is somewhat similar to that
for sixteen aspect fixed block signalling. This is to be expected,
since a system of speed signalling (the maximum speed permitted at
the next signal being communicated to the driver instead of a
coloured light aspect) for sixteen aspect fixed block requires only
slightly different signal spacing to the quantised moving block
system, while the differences between adjacent possible indicated
speeds vary with speed instead of being constant.

The relationship between the capacities which may be obtained
under the various signalling systems is maintained for most of the
geographical configurations examined, including the bi-directional
junction. However, under some operating conditions, there are
exceptions to this rule.

The perturbed performance of pure moving block signalling is
excellent compared with that of four aspect fixed, because, for all
headways which are greater than the steady state minimum value, the
system remains stable with the externally imposed perturbations
tending to subside. With fixed block signalling, there is a range
of headways which are practical under steady state conditions but
become unstable when any perturbations are introduced, with the
delays encountered by subsequent trains either remaining approximately
constant, or increasing.

For the imposed delays which have been considered, the use of
double yellow starting (under which a train, which has come to rest at a signal displaying a red aspect, only resumes its journey when a double yellow aspect appears) seems to have little advantage, since its performance only differs from that of single yellow starting when the headways are such as to produce instability. Since these headways must be avoided in practical operation, double yellow starting would seem of little practical use. An important disadvantage of double yellow starting is the confusion which might result from attaching different significance to a single yellow, which is encountered when a train is in motion, from one which is observed if a train is stationary.

A quantised moving block signalling system gives a performance, under perturbed running conditions, which is superior to that of fixed block. However, as might be expected, it is inferior to that of theoretical pure moving block, marginally so under some operating conditions, while under other conditions its performance is more like that of fixed block. The superior performance of quantised moving block depends upon the use of several brake applications of relatively short duration, compared with fewer longer brake applications under fixed block. Thus the time which elapses between the driver initiating a braking sequence and the brakes actually being applied will have a more pronounced detrimental effect upon the performance of quantised moving block, than that of fixed block. This also applies to the time which elapses between the driver taking action to terminate the braking sequence, and the actual removal of the brakes. Recent developments have greatly reduced these times, thus
increasing the likely practical advantages of quantised moving block.

The performance of both fixed block and quantised moving block would be improved by the use of cab signalling. This means that the information, otherwise communicated to the train driver at each signal, is available to him within the cab, while the train is in the block in the rear of the signal. Fixed block is likely to derive greater benefits from cab signalling than is quantised moving block, in terms of capacity and immunity to delays. However, under quantised moving block, a train driver will find it much easier to understand the information communicated to him if this is available within the cab.

If a line is carrying mixed traffic which has differing performances and running speeds, the low speed traffic must be brought to rest in the passing loops, if the latter are not to be uneconomically long. It would seem preferable that trains of the same type should be formed into convoys or groups, which are scheduled to run between groups of the other type of traffic. In addition, the headways between trains of the same group should be toleranced sufficiently to absorb reasonable delays to the leading train in the group, without causing any perturbations to be propagated to subsequent groups of trains. However, since such tolerancing reduces the line capacity, a compromise must be arrived at, as to what constitutes a reasonable delay.

If the low speed traffic is required to stop periodically at stations, the capacity of the line is not always as high, if these stations are situated in the passing loops, as it would be if they
were situated on the common line between the loops. Similarly, the optimal berthing arrangement, for an intermediate station at which the high-speed traffic is required to make a stop, depends upon the actual operating conditions. Since the choice may depend upon the time between convoys, the running speed, and the number of trains in a convoy, it is important to take into account the likely future development of traffic requirements when selecting an appropriate arrangement.
APPENDIX A

Let us consider a speed restriction with a limiting speed of $V_L$, extending for a length of $D_L$, from the point $B$ to the point $C$ (Figure 140). The running speed of trains approaching this restriction is $V$, which is greater than $V_L$, but less than or equal to the line speed, $V_M$. The trains have to commence braking at the point $A$, in order to be travelling with a speed of $V_L$ at the point $B$, and will be able to resume a running speed of $V$ at the point $D$, by accelerating directly after passing the point $C$.

Let us consider two trains passing through this configuration. Before the leading train reaches the point $A$, both trains will be travelling with a speed of $V$, and the straight line headway, for a speed of $V$, will be the only necessary interval between them. When the leading train passes the point $A$, it will begin to decelerate, while the second train continues at a speed of $V$. The distance between the trains will thus decrease. However, because the second train is still travelling with a speed of $V$, the distance between trains must not be allowed to fall below a value given by

$$HD = HW \times V$$

where $HW$ is the straight line headway. This minimum distance between the front of one train and the front of the next must be maintained until the second train reaches the point $A$. The instantaneous distance between the trains will have continued to reduce from the moment that the first train passed the point $A$, unless it has also passed the point $D$ before the second reaches $A$. In this case the
separation is maintained at its instantaneous value when the first train reaches D. Thus, we may say that, if the distance between the trains is sufficient when the second train reaches the point A, it will also be sufficient at all times before this instant.

Now, let us consider what happens after the second train reaches the point A. We may assume that when the second train is at A, the first is beyond the point B, since the distance AB must be less than HD (as defined above), for all positive values of VL. Hence, when the second train is at the point A, the first will be between B and C, C and D, or beyond D.

If the first train is beyond D, it is obvious that the distance between the trains will increase from this instant on. Hence, under this condition, if the distance between the trains is sufficient when the second train is at the point A, it will also be sufficient at all times afterwards.

If the first train is between the points C and D when the second train reaches the point A, then it will be instantaneously travelling with a speed, VIN, which is greater than VL but less than V. Now, let us consider an instant a short time 't' later. The distance travelled by the second train during this time is given by

\[(V - \frac{B \times t}{2}) \times t\]

The distance travelled by the first train during the same period is

\[(VIN + \frac{A \times t}{2}) \times t\]

Thus the decrease in the distance separation during this interval is
\[
(V - \frac{B \times t}{2}) \times t - (V \text{IN} + \frac{A \times t}{2}) \times t
\]

\[
= (V - \text{VIN}) \times t - \frac{(B + A) \times t^2}{2}
\]

However, because the speed of the second train has been reduced from \(V\) to \((V - B \times t)\), the necessary minimum distance between the trains will have been reduced by

\[
\frac{V^2}{(2 \times B)} - \frac{(V - B \times t)^2}{(2 \times B)}
\]

\[
= \left[ V^2 - (V^2 - 2 \times B \times V \times t + B^2 \times t^2) \right] / (2 \times B)
\]

\[
= V \times t - \frac{B \times t^2}{2}
\]

Hence, if the distance separation between the trains is just sufficient when the second train is at the point \(A\), then a short interval 't' later, the distance separation exceeds the minimum necessary by

\[
V \times t - \frac{B \times t^2}{2} - (V - \text{VIN}) \times t + \frac{(B + A) \times t^2}{2}
\]

\[
= \text{VIN} \times t + \frac{A \times t^2}{2}
\]

Thus, in this case also, if the distance between the trains is sufficient when the second train is at the point \(A\), it will also be sufficient at all times afterwards.

Finally, let us consider the alternative that the first train is between points \(B\) and \(C\) when the second reaches the point \(A\), and, as in the last case, let us consider a small time interval 't' after this instant. As previously, the second train travels a distance of

\[
(V - \frac{B \times t}{2}) \times t
\]
during this interval. However, this time the first train moves a
distance given by

$$VL \times t$$

Thus the decrease in the distance separation of the trains is

$$(V - \frac{B \times t}{2}) \times t - VL \times t$$

The decrease in the required distance between the trains is exactly
the same as it was in the previous case, and hence, this time, the
actual separation, after the period of 't', will exceed the minimum
required by

$$V \times t - \frac{B \times t^2}{2} - (V - \frac{B \times t}{2}) \times t + VL \times t = VL \times t$$

Thus, for all real cases (positive values of VL), we may say
that sufficient separation when the second train is at A ensures
that the separation will be sufficient subsequently.

Therefore it is evident that if, when the following train is
at the point A, the headway is large enough to prevent interaction,
then no interaction will occur during the passage of the trains
through the restriction.
APPENDIX B

Let us consider a fixed block signalling system with N aspects, where N is greater than two. The block length, BL, is given by

$$BL = \frac{VM^2}{2 \times (N - 2) \times B}$$

and the minimum straight line headway, HW, at line speed is

$$HW = \frac{(N - 1) \times BL + OL + TL}{VM}$$

Thus the capacity at line speed, NCAP, is obtained by using this value of HW in the following expression.

$$NCAP = 3600 \times NC/\text{HW}$$

There will be \((N - 3)\) discontinuities in the graph of capacity against running speed. Let us consider one of these discontinuities, for example the \(K\)th one from the line speed end of the graph. This discontinuity occurs at a speed of VSFK, given by

$$VSFK = \sqrt{VM^2 - 2 \times K \times B \times BL}$$

and the straight line headway at this speed is given by

$$HW = \frac{(N - 1 - K) \times BL + OL + TL}{VSFK}$$

The headway, at a running speed, \(V\), which lies between the \(K\)th and \((K + 1)\)th discontinuities (i.e. \(V\) is slightly less than VSFK), is given by

$$HW = \frac{(N - 1 - K) \times BL + OL + TL}{V}$$

However, if \(V\) is between the \((K - 1)\)th and \(K\)th discontinuities, (i.e., \(V\) is slightly greater than VSFK), then HW is given by
Hw = \frac{(N - K) \times BL + OL + TL}{V}

Therefore, from a list of the speeds at which discontinuities occur and the capacity at these speeds, it is possible to construct the graph of capacity against running speed. An example is given, in Figure 141, for values of N of ten and sixteen.
FIGURE : 140

Direction of Motion

Speed - Distance Profile of a Speed Restriction

FIGURE : 141 Capacity of a Straight Line

Capacity in Coaches per Hour

Running Speed in M.P.H.
APPENDIX C

In describing the algorithm which determines the steady state capacity of a bi-directional junction by combining the three basic components of a converging junction, a diverging junction and a diamond crossing, reference will be made to the junction shown in Figure 142. Let us make the simplifying assumption that, in either direction, a train, on the less populous route, does not immediately follow another on the same route. Then, all intervals between trains following the same route, through any one of the basic components comprising the junction, will be equal to one of the four quantities $ROA$, $ROB$, $RTOA$ and $RTOB$ associated with that basic component. These quantities are, respectively: the smaller and larger intervals between trains on the high speed route, and the smaller and larger intervals on the low speed route. The manner in which these quantities are evaluated is described elsewhere.

There are obviously several possible alternative methods of arriving at a value for the total traffic flow through the junction shown in Figure 142 (i.e. the sum of the traffic along I - D and the traffic along C - H), depending upon which of the variables have assigned values before the process commences. A common requirement of a bi-directional junction which is situated near a major terminus is that, for certain periods during the day, the traffic in a given direction runs at as high a level as possible. Thus, in the algorithm, traffic flow in one direction is given priority, and is the maximum consistent with a specified traffic volume. Although either direction
may be specified as having priority, as an example let us assume that this is the converging traffic (i.e. the traffic flow along I - D). Therefore, since any conflicts at the crossing G will be resolved in favour of traffic running along A - I, we may determine the converging traffic flow, by considering the simple converging junction operating with specified traffic volumes. From this analysis, the following useful information will be obtained.

\[
SD1 = HMC
\]

which is the average headway between trains travelling along I - D.

\[
A1 = ROA \\
B1 = ROB \\
TA1 = RTOA \\
TB1 = RTOB
\]

where these intervals specify the sizes of the time gaps in the traffic travelling along A - I and along E - I.

Then the algorithm takes the gaps in the traffic travelling along A - I, and fits into these the maximum possible traffic which may travel along H - F, subsequently inserting the maximum traffic along C - B which is compatible with this. In order to achieve this, information is required from the component analysis of a diamond crossing, operating with traffic volumes of 60:40 and 40:60, and a diverging junction, also operating with these two traffic volumes. The useful information obtained from these analyses is as follows:

From the case of a diamond crossing with traffic volumes of
From the case of a diamond crossing with traffic volumes of 40:60,

\[ TA3 = RTOA \]

and \[ TB3 = RTOB \]

From the case of a diverging junction with traffic volumes of 40:60,

\[ TB4 = RTOB \]

and from the case of a diverging junction with traffic volumes of 60:40,

\[ AS = ROA \]

Also, as already mentioned, the traffic volumes of the converging traffic are specified. Let us suppose that these are \( ND1:ND3 \), i.e. \( ND1 \) travel along A-D and \( ND3 \) travel along E-D in every hundred trains passing I.

Firstly, a check is made to see if A-I is more populous than E-I, or vice versa. Then, if \( ND1 > 50 \), the majority of the converging traffic approaches the junction from A. Since the interval \( Al \) will then be the straight line headway for two trains running along A-D, there will be no opportunity for any train to travel along H-F during it. Hence, a train that is to pass along H-F, must do so during the larger interval \( Bl \), which must at least equal or exceed the value of \( B2 \). If \( NX \) is the minimum integer value which will cause the expression
to become negative, then this is the maximum number of trains which may pass along H - F during the period B1. If NX is zero, the diverging traffic volumes are given by

\[ ND2 : ND4 = 100:0 \]

and the average headway between the diverging traffic running along C - H is given by

\[ SD2 = A5 \]

since, under these conditions, C - B is acting as a straight line. If NX is not zero, then the average headway between trains passing along H - F will be given by

\[ BD4 = 100 \times SD1 / (ND3 \times NX) \]

since, during the time which elapses while 100 trains pass along I - D, there will be ND3 intervals of length B1 between the trains travelling along A - I.

It is necessary to know the actual intervals between the trains which travel along C - F in order to find how many trains may pass along C - B. Two such usable intervals may exist, and are given by

\[ RTOB4 = TA1 - (NX - 1) \times TA3 \]

and \[ RTOC4 = TB1 - (NX - 1) \times TA3 \]

Let NY and NZ be the maximum number of trains which travel along C - B during the intervals RTOB4 and RTOC4 respectively. Then, NY is the minimum integer value which will cause the expression
(RTOB4 - TB4 - NY × A5)
to become negative, and, similarly, NZ causes the expression
(RTOC4 - TB4 - NZ × A5)
to become negative.

If (NY + NZ) is zero, all the diverging traffic travels along
H - F (i.e. no trains travel along C - B), and hence the diverging
traffic volumes are given by

\[ ND2 : ND4 = 0 : 100 \]

while the average headway, between the diverging traffic running
along C - H, is given by

\[ SD2 = BD4 \]

However, if (NY + NZ) is not zero, the average headway between trains
passing along C - B is given by

\[ BD2 = 100 \times SD1 / (ND3 × NY + (NZ - NY) × (ND1 - INN × ND3)) \]

where INN is given by the integer division

\[ INN = ND1 / ND3 \]

The average headway of the total diverging traffic is given by

\[ SD2 = BD4 × BD2 / (BD4 + BD2) \]

and the diverging traffic volumes by

\[ ND2 : ND4 = (100 × SD2 / BD2) : (100 × (BD2 - SD2) / BD2) \]

If, initially, it is found that E - I is more populous than A - I,
i.e. ND1 < 50, then it may be possible for a train to pass along H - F
during the interval A1, as well as the interval B1. Thus it is
necessary to find NW, which is the minimum integer value which causes the expression

\[(A1 - TB3 - NW \times TA3)\]

to become negative, because this will be the maximum number of trains which may pass along H - F during the period A1. Similarly, the number of trains, NX, which travel along H - F during the larger interval, B1, is the minimum integer value which causes

\[(B1 - TB3 - NX \times TA3)\]

to become negative.

If \((NW + NX)\) is zero, all the diverging traffic must travel along C - B, hence the diverging traffic volumes are given by

\[ND2 : ND4 = 100 : 0\]

and the average headway between these trains is given by

\[SD2 = A5\]

If \((NW + NX)\) is not zero, the average headway between trains passing along H - F will be given by

\[BD4 = 100 \times SD1 / \left( ND1 \times NW + (NX - NW) \times (ND3 - INN \times ND1) \right)\]

where, in this case, INN is given by the integer division

\[INN = ND3 / ND1\]

The usable time intervals between trains travelling along C - F, in which trains may pass along C - B, are given by

\[RTOB4 = A1 - (NW - 1) \times TA3\]
and
\[RTOC4 = E1 - (Nx - 1) \times TA3\]
The number of trains NY and NZ which pass along C - B during the intervals RT0B4 and RT0C4 are found as described previously. If \((NY + NZ)\) is zero, the diverging traffic volumes are given by

\[
\text{ND2:ND4} = 0:100
\]

and the average headway of the diverging traffic is given by

\[
\text{SD2} = \text{BD4}
\]

However, if \((NY + NZ)\) is not zero, the average headway, between trains travelling along C - B is given by

\[
\text{BD2} = \frac{100 \times \text{SD1}}{\left(ND1 \times (\text{INN} + 1) - ND3\right) \times NY + (ND3 - \text{INN} \times ND1) \times NZ}
\]

where, as above, \(\text{INN}\) is given by the integer division

\[
\text{INN} = \frac{ND3}{ND1}
\]

The average headway and the traffic volumes are obtained from \(\text{BD2}\) and \(\text{BD4}\) in exactly the same way as described for the case when \(\text{ND1} > 50\).

The algorithm may be applied to the case in which the diverging traffic is given priority, by a suitable transposition of the input variables, and this also applies to left hand junctions (the convention is shown in Figure 143, from which it will be seen that the junction shown in Figure 142 is a right hand one). Further classifications will also arise, depending upon whether the component analyses used apply to two high speed routes, or high and low speed routes, but it is obvious that this will in no way influence the algorithm itself.
FIGURE 142

FIGURE 143 Convention for Bidirectional Junctions
The Propagation Velocity of delays under moving block (VPR) may be defined as the distance between the points at which successive trains are initially perturbed from steady state conditions, divided by the time which elapses between such occurrences.

It is evident that this velocity is a function of the running headway. Some cases are trivial, e.g. if the running headway, $H$, is equal to the minimum straight line headway, $HW$, then VPR is infinite in the positive direction, since, by definition, the negative direction of delay progression is the direction of train movement. Thus, when $H = HW$, $VPR = +\infty$.

For practical headways, when $H > HW$, the delay propagation velocity is finite, since there is a time interval between one train commencing to brake for its delay, and the next train beginning the same process. If the distance initially separating the trains is $(H \times V)$, where $V$ is the running speed, then the second train begins to brake after this distance has been reduced to $(HW \times V)$. Thus the reduction in the distance separating the trains is $(H - HW) \times V$, which is equivalent to a delay of $(H - HW)$ to the first train.

If we define a time interval, $t_p$, as the period during which a train is delayed by an amount equal to $(H - HW)$, then this is the period which it takes for the delay to spread from this train to the following one. During this time the following train has travelled a distance of $(t_p \times V)$, and, since it was originally a
distance of \((H \times V)\) in the rear of the point where the first train commenced braking, the delay has "travelled" a distance of \((I_i - t_p) \times V\).

Since the time taken to travel this distance is \(t_p\), \(VPR\) is given by

\[
VPR = \frac{H}{t_p} - 1 \times V
\]

Thus the problem of determining \(VPR\) is reduced to one of finding the value of \(t_p\). It is apparent that the value of \(t_p\) changes between succeeding trains running in a convoy at equal headways greater than the theoretical minimum. When the interval is considered between the last train which encounters any delay and the one following it, then \(t_p\) becomes infinite, and \(VPR = -V\).

In general, \(t_p\) is not readily obtainable from analysis, although it must fulfil the condition

\[
(H - HW) = \int_0^{t_p} (V - V(t)) dt
\]

where \(V(t)\) is the instantaneous speed, which is a function of time.

Unless the function \(V(t)\) is readily analysed, which is the case for the leading train but, in general, is not so for any other, \(t_p\) may not be easily evaluated, except from a simulation. If a simulation is undertaken, the information required from it consists of the times at which successive trains commence braking. Thus, if the \(n^{th}\) train commences braking at a time \(t(n)\), etc., then

\[
t_p = t(n) - t(n - 1)
\]

for the interval between the \((n - 1)^{th}\) and \(n^{th}\) trains.

Thus, the delay propagation velocity is given by

\[
VPR = \left[\frac{H}{t(n) - t(n - 1)} - 1\right] \times V
\]
APPENDIX E

Passing loop waiting times

The minimum time which a train takes to travel from a stationary position on one passing loop to an equivalent position in the next loop is given by

\[ TM = \frac{DBL}{VS} + VS \times \frac{(AS + BS)}{2 \times AS \times BS} \]

provided that it is able to attain the running speed, VS, during the course of the journey. This is the case, if the condition

\[ DBL > VS^2 \times \frac{(AS + BS)}{2 \times AS \times BS} \]

is fulfilled. However, if the train has to commence braking before its acceleration is complete, then \( TM \) is given by

\[ TM = \sqrt{2} \times DBL \times \frac{(AS + BS)}{(AS \times BS)} \]

The time which elapses between the instant when the leading low speed train comes to rest in one passing loop, and the instant when it comes to rest in the next loop, is equal to \( TICF + DBL/VF \). Thus, the maximum time for which the train is stationary is given by

\[ TAS = TICF + DPL/VF - TM \]

The total time which a train spends in a passing loop depends upon the distances, within the loop, over which a train is braking and accelerating. Thus this time can vary between one train and another in the same inter-flight group. In order to be general, let us consider the \( M^{th} \) train in the group. The distance between the place where this train comes to rest, and the points at the exit
of the passing loop, is equal to \((OL + (M - 1) \times BL)\). The time taken by the train to cover this distance is given by

\[
TA = \frac{(OL + (M - 1) \times BL)}{VS + VS/(2 \times AS)}
\]

provided that the train has attained its running speed by the time it leaves the passing loop, i.e. if \(VA \geq VS\) where \(VA\) is given by

\[
VA = \sqrt{2 \times AS \times (OL + (M - 1) \times BL)}
\]

If the train has not attained a speed of \(VS\) when it leaves the loop, the time \(TA\) is given by

\[
TA = \frac{VA}{AS}
\]

The distance between the entry to the loop and the place where the \(M^{th}\) train comes to rest is equal to

\[
OL + TLS + (NS - M) \times BL
\]

The time which the train takes to cover this distance is given by

\[
TB = \frac{(OL + TLS + (NS - M) \times BL)}{VS + VS/(2 \times BS)}
\]

provided that the train has not commenced braking before it enters the loop, i.e. if \(VB \geq VS\), where \(VB\) is given by

\[
VB = \sqrt{2 \times BS \times (OL + TLS + (NS - M) \times BL)}
\]

If this is not the case, the time, \(TB\), is given by

\[
TB = \frac{VB}{BS}
\]

Thus the total time which a train spends in the passing loop is given by

\[
TAL = TAS + TA + TB
\]
A graph showing a typical variation of the total time which a train spends in a passing loop is given in Figure 144. It may be seen that the train in the middle of the group spends longest time in the loop. This is because trains 5 to 8, inclusive, do not do any braking or acceleration on the common line. Therefore the average speed at which these trains travel the length of the passing loop is lower than that of trains near the beginning of the group, which do some accelerating on the common line, and that of trains near the end of the group, which do some braking on the common line. However, it is evident that such variations in the times which these trains spend in a passing loop represent only a small percentage (approximately 3% in this case) of the actual times themselves, whereas the trains spend a relatively large percentage of the time in the loop (about 70%) at rest.
FIGURE 14: Time Spent in Passing Loops
Number of Trains in a Fast Convoy 3
Number ... Slow Group 10
Inter Fast Convoy Time 30 minutes
Distance Between Passing Loops 60 miles
All Slow Trains are Stationary in Loop for 983.5 seconds
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