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Defective number module or impaired access? Numerical magnitude processing in first graders with mathematical difficulties

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ABSTRACT

This study examined numerical magnitude processing in first graders with severe and mild forms of mathematical difficulties, children with mathematics learning disabilities (MLD) and children with low achievement (LA) in mathematics, respectively. Twenty children with MLD, twenty-one children with LA and forty-one regular achievers completed a numerical magnitude comparison task and an approximate addition task, which were presented in a symbolic and a non-symbolic (dot arrays) format. Children with MLD and LA were impaired on tasks that involved the access of numerical magnitude information from symbolic representations, with the LA children showing a less severe performance pattern than children with MLD. They showed no deficits in accessing magnitude from underlying nonsymbolic magnitude representations. Our findings indicate that this performance pattern occurs in children from first grade on and generalizes beyond numerical magnitude comparison tasks. These findings shed light on the types of interventions that may help children who struggle with learning mathematics.

Keywords: mathematical difficulties; first grade; magnitude representation; comparison; approximate addition
INTRODUCTION

Mathematics learning represents a stumbling block for many children in primary school. In order to devise appropriate interventions and in view of the fact that mathematical abilities are crucial to life success in modern Western societies (Ancker & Kaufman, 2007; Finnie & Meng, 2001), we need to have good understanding of the cognitive deficits underlying children’s poor achievement in mathematics. One source of these deficits may be in the types of numerical representations that underlie mathematics learning (Ansari & Karmiloff-Smith, 2002; Butterworth, 1999; Dehaene, 1997; Wilson & Dehaene, 2007). Indeed, studies have demonstrated that children with mathematical difficulties have particular impairments in understanding and processing numerical magnitudes (De Smedt, Reynvoet, et al., 2009; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008; Landerl, Bevan, & Butterworth, 2004; Landerl, Fussenegger, Moll, & Willburger, 2009; Passolunghi & Siegel, 2004; Rousselle & Noël, 2007). Two accounts for these impairments have been put forward (Rousselle & Noël, 2007; see also Wilson & Dehaene, 2007). The **defective number module hypothesis** (Butterworth, 2005) proposes that a highly specific deficit of an innate capacity to understand and represent quantities leads to difficulties in learning number and arithmetic. The **access deficit hypothesis** (Rousselle & Noël, 2007) states that mathematical difficulties originate from impairments in accessing numerical meaning, i.e. their quantity, from symbols rather than from difficulties in processing numerosity per se. To disentangle between both hypotheses, performance should be compared on numerical tasks with and without a symbolic processing requirement. If children with mathematical difficulties perform more poorly on both types of tasks, this favours the **defective number module hypothesis**; if they perform more poorly on symbolic tasks but not on non-symbolic tasks, this supports the **access deficit hypothesis**. Specifying the locus of this impairment provides a crucial building block for developing appropriate
intervention, which should then either focus on the representation of quantity or on the mapping between symbols and the quantities they represent.

To date, findings remain inconclusive and studies supporting both the defective number module hypothesis (Landerl et al., 2009) and the access deficit hypothesis (Iuculano, Tang, Hall, & Butterworth, 2008; Rousselle & Noël, 2007) have been reported. The present study aimed to contrast both hypotheses and to extend previous findings in two important ways. First, the aforementioned studies focused on children in second- to fourth grade. Difficulties in processing non-symbolic representations of quantity might have occurred in early life, but may be compensated in the early years of schooling. In other words, it might not be possible to detect difficulties in non-symbolic quantity processing at older ages, such as in the reported studies, and therefore we investigated younger children with mathematical difficulties. Second, the available studies investigated the understanding and processing of quantities only by one type of task, i.e. numerical magnitude comparison. While this task is considered to be a classic indicator of children’s understanding of numerical magnitudes, performance patterns should also generalize to other symbolic and non-symbolic tasks that measure the understanding of numerical magnitudes, such as approximate addition (e.g., Barth, Beckmann, & Spelke, 2008; Gilmore, McCarthy, & Spelke, 2007). To the best of our knowledge, there are no studies that have compared performance on symbolic and non-symbolic approximate addition tasks in children with mathematical difficulties. In the remainder of this introduction, we first review the available evidence that the ability to understand and manipulate numerical magnitudes is related to individual differences in mathematics. Next, we evaluate the studies that have examined the defective number module hypothesis and the access deficit hypothesis and finally, we present the specific aims of our study.
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Understanding Numerical Magnitudes and Mathematics Development

There exists consistent evidence that infants and young children are able to understand and manipulate numerical magnitude information by means of non-symbolic representations. For example, six-month-old infants are able to discriminate between large sets of dots on the basis of numerosity (Xu & Spelke, 2000; see Feigenson, Dehaene, & Spelke, 2004 for a review) and five-year-olds who had not yet been taught formal arithmetic can compare, add and subtract non-symbolic numerosities, i.e. dot arrays or sequences of sounds (Barth et al., 2008). These non-symbolic representations are characterized by an effect of ratio or distance: When the numerical difference or distance between the two sets that need to be compared, added or subtracted, is small or the ratio between them approaches 1, performance on these tasks is slower and less accurate than when the distance is large or ratio is small. This effect is assumed to arise from overlapping internal representations of numerical magnitudes: Magnitudes that are closer to each other have a larger representational overlap and are more difficult to compare than do magnitudes that are further apart (for a review, see Noël, Rousselle & Mussolin, 2005). Thus, the size of this distance- or ratio-effect provides an indicator for the distinctness or preciseness of representations of numerical magnitudes and these effects are known to decrease over development (Halberda & Feigenson, 2008; Holloway & Ansari, 2008).

Over the course of development, children develop the ability to represent magnitudes in a symbolic way, first through the use of counting words, later on, when they start to learn mathematics in primary school, by using Arabic numerals. These symbolic representations are characterized by similar effects of distance and ratio (Gilmore et al., 2007; Holloway & Ansari, 2009; Sekuler & Mierkiewicz, 1977). Successful mathematics development requires children to map these symbolic representations onto pre-existing non-symbolic representations of magnitudes (Lipton & Spelke, 2005; Mundy & Gilmore, 2009). Through
this integrated knowledge network formal numerical symbols acquire their meaning (Griffin, 2002).

Individual differences in mathematics achievement have been related to both symbolic (Durand, Hulme, Larkin, & Snowling, 2005; De Smedt, Ghesquière, & Verschaffel, 2009; Holloway & Ansari, 2009) and non-symbolic (Halberda, Mazzocco, & Feigenson, 2008; Mundy & Gilmore, 2009) numerical magnitude comparison tasks and to number line estimation tasks (Booth & Siegler, 2006; Ramani & Siegler, 2008), which are all considered to be reliable indicators of children’s understanding of numerical magnitudes (Laski & Siegler, 2007). Most interestingly, Booth and Siegler (2008) demonstrated that children’s representations of magnitude, as measured by a number line estimation task, were uniquely predictive of their learning of answers to novel addition problems and of the errors on them. These authors argued that a good understanding of numerical magnitudes narrows down the number of potential answers to a problem and improves the quality of errors.

These behavioral data fit nicely with cognitive neuroimaging studies that have shown that the intraparietal sulcus, which is dedicated to the processing of numerical magnitudes in children (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Temple & Posner, 1998) and adults (for a review see Ansari, 2008; Dehaene, Piazza, Pinel, & Cohen, 2003), is consistently active during mathematical tasks (e.g., Dehaene et al., 2003; Rivera, Reiss, Eckert, & Menon, 2005). Thus, there is also neural evidence to suggest that the processing of numerical magnitudes is important for higher-level mathematics, such as arithmetic. Moreover, structural (Isaacs, Edmonds, Lucas, & Gadian, 2001; Rotzer et al., 2008) and functional (Mussolin et al., 2010; Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007) abnormalities in those areas of the brain that are dedicated to the processing of numerical magnitudes have been reported in children with difficulties in mathematics.
The Defective Number Module Hypothesis vs. the Access Deficit Hypothesis

There exists considerable evidence that representations of magnitude are impaired in children with mathematical difficulties (De Smedt, Reynvoet, et al., 2009; Geary et al., 2007, 2008; Iuculano et al., 2008; Jordan, Kaplan, Olah, & Locuniak, 2006; Landerl et al., 2004, 2009; Passolunghi & Siegel, 2004; Rousselle & Noël, 2007). However, the majority of these studies only relied on tasks with a symbolic processing requirement and do not allow us to clarify whether difficulties in mathematics result from difficulties in representing numerical magnitudes, as postulated in the defective number module hypothesis, or from difficulties in the ability to access numerical magnitudes from formal symbols, such as Arabic numerals, as assumed in the access deficit hypothesis.

Rousselle and Noël (2007) were the first to contrast both hypotheses by examining performance on symbolic and non-symbolic numerical magnitude comparison in children with mathematical difficulties. They showed that these children differed from controls on symbolic but not on non-symbolic numerical magnitude comparison, favouring the access deficit hypothesis. This pattern was further replicated by Iuculano et al. (2008) and Landerl and Kölle (2009). Findings by Holloway and Ansari (2009), who found that symbolic but not non-symbolic numerical magnitude comparison correlated with individual differences in mathematics achievement in typically developing children, are consistent with this. By contrast, Landerl et al. (2009) recently showed that children with mathematical difficulties performed more poorly on both symbolic and non-symbolic comparison tasks, thereby providing evidence for the defective number module hypothesis. Thus, it remains unclear whether children with mathematical difficulties have difficulties in accessing number magnitude from symbols rather than in processing numerosity itself.

It should be noted that previous studies that have contrasted performance on symbolic and non-symbolic magnitude comparison tasks in children with mathematical difficulties only
considered children who were already in second grade or older. Difficulties in processing non-symbolic representations might have occurred in early life, but may be compensated in the early years of schooling, which might explain why the available studies reported symbolic rather than non-symbolic processing difficulties. To the best of our knowledge, there are no studies that have examined both symbolic and non-symbolic magnitude processing in children younger than second grade. Therefore, we examined children who were in the middle of first grade, an age at which difficulties in non-symbolic representations might be more likely to be detected.

Furthermore, previous studies only used a numerical magnitude comparison task, which involved the comparison of two numbers or dot arrays to examine the defective number module hypothesis and the access deficit hypothesis. The difficulties of children with mathematical difficulties should, however, be generalizable across different types of tasks that measure children’s representation of numerical magnitudes and children’s access to these representations. It remains to be determined whether evidence in favour of one of both hypotheses remains when performance on different tasks is considered. Importantly, these tasks should be able to be administered with both symbolic and non-symbolic stimuli. An example of such a task is approximate addition, which has been successfully used with non-symbolic (Barth, La Mont, Lipton, & Spelke, 2005) and symbolic (Gilmore et al., 2007) stimuli in children from the age of five. This task may also be particularly informative because it includes larger (i.e. multi-digit) numerosities than numerical magnitude comparison tasks and captures the accuracy rather than the speed with which numerical representations are available. Iuculano et al. (2008) were the first to investigate non-symbolic approximate addition in children with mathematical difficulties and found no group differences between children with mathematical difficulties and regular achievers. To the best of our knowledge, there are no studies available that have examined symbolic approximate addition in children...
with mathematical difficulties.

The Present Study

The present study aimed to contrast the defective number module hypothesis and the access deficit hypothesis in children with mathematical difficulties at an earlier age and with a wider set of tasks than previously reported. To achieve this, all children in the current study were in the middle of first grade. These children had only recently been introduced to symbolic numbers in school. At this stage, difficulties in non-symbolic representations might be more likely to be detected. All children completed a numerical magnitude comparison task and an approximate addition task, which were presented in a symbolic and a non-symbolic (i.e. dot arrays) format. If mathematical difficulties result from a failure to represent numerical magnitudes, as proposed in the defective number module hypothesis, children with mathematical difficulties should perform more poorly than regular achievers on both symbolic and non-symbolic tasks. If mathematical difficulties originate from a difficulty in accessing magnitude information from symbols, children with mathematical difficulties should perform more poorly on the symbolic but not on the non-symbolic tasks.

It should be emphasized that children’s performance on the symbolic tasks might be affected by their number identification skills or symbolic knowledge. Therefore, we administered a number identification task as a control to find out whether group differences are due to differences in knowledge of the symbolic system rather than to differences in accessing numerical magnitude from symbols.

Much of the existing research has identified children as having mathematical difficulties if their performance on a standardized mathematics achievement test is below the 25th percentile (Swanson & Jerman, 2006). This cut-off score for mathematical difficulties is rather lenient and might result in a sample of children with potentially severe and potentially mild forms of mathematical difficulties, which both may show different cognitive profiles.
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(Geary et al., 2007, 2008; Murphy, Mazzocco, Hanich, & Early, 2007). Against this background, we divided our sample of children with mathematical difficulties into children with low achievement (LA) in mathematics, if they had math achievement scores between the 16th and 25th percentile, and children with mathematics learning disabilities (MLD), if they had math achievement scores below the 16th percentile. We wanted to investigate whether performance patterns on the administered numerical tasks differed between children with severe (MLD) and mild (LA) forms of mathematical difficulties.

METHOD

Participants

Participants were 82 first graders (34 boys, 48 girls) with a mean age of 6 years and 8 months (SD = 4 months), which were recruited from a larger sample of 290 children from 11 regular primary schools. All schools were located in provincial towns in the middle of Flanders (Belgium) and had a dominantly middle to high-income school population. None of these children had a developmental disorder or mental retardation and none of the children had repeated a grade.

All 290 children completed the curriculum-based standardized general mathematics achievement test Math up to Ten (Dudal, 1999) as a screening measure. From this sample, we selected children with mathematical difficulties, i.e. those children whose performance was below the 25th percentile on this standardized screening measure. The 25th percentile cut-off has been commonly used to select children with mathematical difficulties (Swanson & Jerman, 2006). This group of children with mathematical difficulties was divided into the LA-group (math achievement scores between the 16th and 25th percentile; \(n = 21\)), and the MLD-group (math achievement scores below the 16th percentile; \(n = 20\)). For each child with mathematical difficulties, we selected from the total sample a child from the same school that performed above the 35th percentile on the screening measure. This yielded a sample of 41
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typically achieving (TA) children who were matched in terms of educational environment to the children with mathematical difficulties.

Table 1 shows the descriptive statistics of the three achievement groups. The groups did not differ in the numbers of boys and girls, $\chi^2(2, N = 82) = 0.99, p = .61$, or in age, $F(2,81) = 1.01, p = .37, \eta_p^2 = .02$. The three groups differed in their performance on the general standardized mathematics achievement test $F(2,81) = 247.85, p < .01, \eta_p^2 = .86$: Children with MLD performed significantly lower than children with LA, $t(39) = -8.70, p < .01, d = -2.51$, who in turn performed more poorly than the TA children, $t(60) = -11.92, p < .01, d = -4.33$.

Procedure

Children first completed the group-administered standardized mathematics achievement test. One month later, the experimental tasks were administered individually in a quiet room at the children’s school. All children were tested in the middle of first grade at which point they were receiving regular instruction in the number domain up to 10 but had not received any formal instruction in the number domain above 10.

Materials

*General Mathematics Achievement Test*

Mathematics achievement was assessed using a curriculum-based general standardized achievement test for mathematics, Math up to 10 (Dudal, 1999). This test involved the number domain 1 to 10 and consisted of 45 items, covering number writing (e.g., writing numbers to dictation), counting (e.g., counting the number of objects; draw a number of dots equal to the presented number), number knowledge (e.g., putting numbers on a number line; splitting a number into two parts) and simple arithmetic (e.g., single-digit addition and subtraction). The test used both symbolic and non-symbolic formats. It should be emphasized that this test contained neither items measuring approximate arithmetic nor items that required
the explicit comparison of numbers or quantities as in the numerical magnitude comparison tasks. Cronbach’s alpha of this test was .88 for the current sample.

**Experimental Tasks**

All experimental tasks were presented using the E-prime 1.0 software (Schneider, Eschmann, & Zuccolotto, 2002). They were all administered with a 17-inch notebook computer. Children were instructed to perform both accurately and quickly, except in the approximate addition tasks on which they were asked to be only accurate. Each trial was initiated by the experimenter with a control key. In the numerical magnitude comparison and approximate addition tasks, children were required to select the larger of two response alternatives, one displayed on the left and one displayed on the right, by pressing a key on the side on the larger one. Key presses were made on an external computer keyboard that was put in front of the notebook and was connected to it. The left response key, labelled with a blue sticker was ‘d’; the right response key, labelled with a yellow sticker was ‘k’. Each task was preceded by three practice trials to familiarize the child with the key assignments. Answers and reaction times were recorded by the notebook. In number identification, responses were verbal. When the child responded, the experimenter, who was seated next to the child, immediately pressed the spacebar of the external keyboard connected to the notebook to register reaction time. After the registration of the reaction time, the child’s answer was entered on the keyboard by the experimenter. We decided to use this approach rather than a voice-key. Even though the data were collected in a quiet room, the use of a voice key at school might be problematic because random noise – albeit subtle – from adjacent classrooms is inevitable, which might have resulted in losing a lot of trials. Two practice trials were included to make children familiar with task administration.

**Numerical Magnitude Comparison**

*Symbolic comparison.* A classic symbolic numerical magnitude comparison task was
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administered (Sekuler & Mierkiewicz, 1977). In this task, children indicated the numerically larger of two simultaneously presented numbers—one displayed on the left and one displayed on the right—on the computer screen. Stimuli comprised all combinations of the numbers 1 to 9, yielding 72 trials. A trial started with a 200-ms fixation cross in the centre of the computer screen accompanied by a beep of 440 Hz. After 1000 ms, the stimulus appeared in the centre of the screen and remained visible until the child responded. The position of the largest number was counterbalanced.

Non-symbolic comparison. Children indicated the larger of two simultaneously presented arrays of dots—one displayed on the left and one displayed on the right—on the computer screen. Stimuli comprised the same combinations of numerosities as in the symbolic number comparison task, yielding 72 trials. The stimuli were generated by means of the MATLAB script provided by Piazza et al. (2004) and were controlled for non-numerical parameters, i.e. individual dot size, total occupied area, and density. Dot size, array size, and density were positively correlated with number on half of the trials and negatively correlated with number on the other half. This ensured that children could not reliably use these non-numerical cues or perceptual features to make a correct decision. A trial started with a 200-ms fixation cross in the centre of the computer screen accompanied by a beep of 440 Hz. After 1000 ms, the stimulus appeared in the centre of the screen and disappeared after 840ms, in order to avoid counting. The position of the largest numerosity was counterbalanced.

Approximate Addition

Symbolic approximate addition. This task was similar to the one used by Gilmore et al. (2007). In this task, two characters appeared and were named on the screen. On a trial, the experimenter stated, for example, “Tommy has five candies” and a blue coloured box displaying the appropriate Arabic numeral appeared below the character on the left. Next, a second blue coloured box displaying an Arabic numeral appeared below the same character
and the experimenter stated “and he gets five more”. Finally, a red coloured box displaying an Arabic numeral appeared below the other character and the experimenter stated “Smogg has fifty candies” and asked “Who has more?”. Twenty-four problems were administered. They presented large numbers in the range 5 to 58, selected such that the sum was greater than the comparison number on half of the trials. The sum differed from the comparison number by ratios of 4:7, 4:6 or 4:5 on 8 trials each. Three control problems, which were not analysed, involved small numbers and familiar sums, to ensure children understood the task. All problems of this task were presented both visually and verbally by the experimenter. No speeded instruction was given. The children received no feedback, but general encouragement throughout.

Non-symbolic approximate addition. Similar to the approximate symbolic addition task, two characters appeared and were named on the screen. On a trial, the experimenter stated, for example, “Daniel has some marbles and puts them in a box” and an array of blue dots fell down behind a grey occluder below the character. Next, a second array of blue dots fell down behind the same grey occluder and the experimenter stated “and he puts some more into his box. Now, all Daniel’s marbles are in his box”. Finally, an array of red dots appeared below the other character and the experimenter stated “Look. Polluto has also some marbles” and asked “Can you tell me who has more?”. The stimuli comprised the same numerosities as in the symbolic approximate addition task. The stimuli were controlled for non-numerical parameters in the same way as in the non-symbolic comparison task. No speeded instruction was given. The children received no feedback, but general encouragement throughout. This task was based on the non-symbolic approximate addition task developed by Barth et al. (2005).

Number Identification

Each of the numbers 0 to 19 was presented on the computer screen and the child was
asked to name each number as fast as possible. The task consisted of two blocks: the first block contained the numbers 0 to 9 in pseudo-random order and the second block involved the numbers 10 to 19 in pseudo-random order.

RESULTS

Before we turn to the comparison and approximate addition tasks, performance on the number reading task is considered. Although children with MLD ($M = 0.98, SD = 0.04$), children with LA ($M = 0.94, SD = 0.08$) and TA children ($M = 0.99, SD = 0.02$) were highly accurate in number identification, a significant group difference emerged $F(2,81) = 9.54, p < .01, \eta_p^2 = .19$: children with LA performed more poorly than children with MLD, $t(39) = -2.61, p = .03, d = -0.62$, and TA children $t(60) = -4.36, p < .01, d = -1.13$, but the latter two groups did not differ, $t(59) = 1.30, p = .40, d = -0.63$. This difference was entirely due to differences in reading multidigit numbers (children with MLD: $M = 0.96, SD = 0.07$; children with LA: $M = 0.90, SD = 0.13$; TA children: $M = 0.99, SD = 0.03$), whereas the mean accuracy for naming single-digit numbers was equal and at ceiling in all groups ($M = 0.99$).

Group differences in the speed of number identification were also observed, $F(2,81) = 6.31, p < .01, \eta_p^2 = .14$. TA children ($M = 1219 ms, SD = 290$) were significantly faster than children with MLD ($M = 1573 ms, SD = 425$; $t(59) = -2.89, p = .01, d = -1.06$) and children with LA ($M = 1569 ms, SD = 674$; $t(60) = -2.91, p = .01, d = -0.78$), whereas the latter two groups did not differ from each other, $t(39) = 0.03, p = .99, d = 0.01$. Therefore, performance on the number identification task was additionally considered in subsequent analyses.

The mean accuracy and speed on the administered experimental tasks per group are displayed in Figures 1, 2 and 3. The mean reaction times are based on the correct responses only. Repeated measures analyses of variance (ANOVA) were calculated to examine group differences on the administered tasks. All $p$-values were corrected by means of the Huynh-Feldt procedure to correct for non-homogeneous data. Tukey-Kramer adjustments were used
for post-hoc tests. Partial eta-squared was computed as a measure of effect size.

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Group differences on this task were evaluated by means of a repeated measures ANOVA with task (symbolic vs. non-symbolic) and the numerical distance as within-subject factors and group as a between-subject factor on children’s reaction time and accuracy.

With regard to reaction time, there was a main effect of task ($F(1, 79) = 44.30, p < .01, \eta_p^2 = .36$), showing faster reaction times on the non-symbolic than on the symbolic comparison task. There was a main effect of distance ($F(7, 553) = 40.22, p < .01, \eta_p^2 = .34, \varepsilon = 0.78$), indicating that reaction time decreased when distance increased. A significant task × distance interaction ($F(7, 553) = 2.47, p = .03, \eta_p^2 = .03, \varepsilon = 0.80$) indicated that the effect of distance was different in the non-symbolic than in the symbolic task. There was no main effect of group ($F(2, 79) = 2.50, p = .09, \eta_p^2 = .06$). Crucially, a significant group × task interaction emerged ($F(2, 79) = 5.12, p < .01, \eta_p^2 = .11$) (Figure 1). Post-hoc $t$-tests revealed that on the symbolic task children with MLD were significantly slower than TA children ($t(59) = 3.33, p < .01, d = 0.88$). Children with LA did not differ from TA children ($t(60) = 1.22, p = .45, d = 0.37$) or from the children with MLD ($t(39) = -1.87, p = .16, d = -0.55$); in contrast, there were no group differences on the non-symbolic task ($t < 1$). There was no interaction between group and distance ($F(14, 553) = 1.81, p = .06, \eta_p^2 = .04, \varepsilon = 0.78$) and no interaction between group, task and distance ($F(14, 553) = 1.43, p = .15, \eta_p^2 = .03$).

To evaluate whether these findings were explained by individual differences in number identification, we repeated the analysis with number identification speed as a covariate. Because the comparison task only involved single digits, we controlled for the reading speed of single digits. After controlling for this variable, the group × task interaction was significant ($F(1, 78) = 3.69, p = .03, \eta_p^2 = .09$) and the group difference between children with MLD and TA children remained ($t(59) = 2.42, p = .04, d = 0.88$).
We also evaluated whether the group differences on the symbolic magnitude comparison task could be explained by performance on the nonsymbolic magnitude comparison task. We therefore re-examined this group difference by additionally controlling for performance on the non-symbolic task and number identification. Findings revealed that the group difference on symbolic numerical magnitude comparison speed remained significant \((F(2,77) = 4.41, p = .02, \eta_p^2 = .10)\). Post-hoc t-tests revealed that children with MLD were significantly slower than TA children \((t(59) = 2.97, p < .01, d = 0.63)\). Children with LA did not differ from regular achievers \((t(60) = 1.23, p = .22, d = 0.29)\) and from the children with MLD \((t(39) = -1.61, p = .11, d = -0.27)\).

The overall accuracy on the numerical magnitude comparison tasks was high. There was a main effect of task \((F(1,79) = 26.82, p < .01, \eta_p^2 = .25)\), indicating that more errors were made on the non-symbolic \((M = 0.87, SD = 0.07)\) than on the symbolic \((M = 0.91, SD = 0.05)\) comparison task (Figure 2). There was a main effect of distance \((F(7,553) = 51.54, p < .01, \eta_p^2 = .39, \varepsilon = 0.90)\), showing that accuracy increased with increasing distance. This effect was more pronounced in the non-symbolic than in the symbolic task, as shown by a significant task × distance interaction \((F(7,553) = 5.16, p < .01, \eta_p^2 = .06, \varepsilon = 0.90)\). There was no significant main effect of group \((F(2,79) = 2.36, p = .10, \eta_p^2 = .06)\). The effects of task \((F(2,79) = 1.86, p = .16, \eta_p^2 = .05, \varepsilon = 0.90)\), distance \((F(14,553) = 1.57, p = .09, \eta_p^2 = .04)\) and task × distance \((F(14,553) = 0.57, p = .87, \eta_p^2 = .01, \varepsilon = 0.90)\) were not affected by group membership. To evaluate the effect of accuracy on the reaction time data of the numerical magnitude comparison tasks, we re-calculated the abovementioned ANOVAs on reaction time data additionally controlling for the accuracy. Findings revealed that the observed group differences in reaction time remained when accuracy was additionally controlled for.

**Approximate Addition**

We first checked whether there were children in the sample that did not perform above
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chance level (= 0.50). Four TA children and one child with MLD performed below chance on the symbolic approximate addition task; seven TA children, three children with LA and five children with MLD performed below chance on the nonsymbolic approximate addition task. These children were excluded from subsequent analyses. As shown in Figure 3, performance on these tasks was much less accurate than the numerical magnitude comparison tasks (Figure 2), which is consistent with previous research (Barth et al., 2006; Gilmore et al., 2007).

A repeated measures ANOVA with task (symbolic vs. non-symbolic) and ratio (4:5, 4:6, 4:7) as within-subject factors and group as a between-subjects factor on children’s accuracy was performed. Children solved the symbolic task more accurately than the non-symbolic task ($F(1,60) = 6.72, p = .01, \eta^2_p = .10$). There was a main effect of ratio ($F(2,120) = 66.65, p < .01, \eta^2_p = .53, \epsilon = 1.01$), showing that accuracy decreased when ratio approached 1. A significant ratio × task interaction ($F(2,120) = 10.94, p < .01, \eta^2_p = .15, \epsilon = 0.91$) indicated that performance on the symbolic task was more accurate than on the non-symbolic task for the ratios 4:6 and 4:7 ($p < .01$) but not for ratio 4:5 ($p = .66$) (Figure 5). There was a main effect of group ($F(2,60) = 7.09, p < .01, \eta^2_p = .19$) and the group × task interaction was also significant ($F(2,60) = 5.05, p < .01, \eta^2_p = .05$) (Figure 3). Post-hoc $t$-tests revealed that TA children performed significantly more accurately on the symbolic tasks than children with LA ($t(47) = 4.11, p < .01, d = 1.19$) and children with MLD ($t(43) = 3.50, p < .01, d = 1.14$), but the latter two groups did not differ ($t(30) = 0.26, p = .96, d = 0.10$). In contrast, on the non-symbolic tasks, no significant group differences were observed (TA vs. MLD: $t(43) = 1.72, p = .20, d = .58$; TA vs. LA: $t(47) = 0.80, p = .71, d = 0.23$ ; MLD vs. LA: $t(30) = -0.90, p = .64, d = -0.35$). Group membership affected neither the effect of ratio ($F(4,120) = 0.76, p = .55, \eta^2_p = .02, \epsilon = 1.01$) nor the ratio × task interaction ($F(4,120) = 1.64, p = .18, \eta^2_p = .05, \epsilon = 0.91$).

To evaluate whether these findings were due to individual differences in number identification, we also repeated this analysis with number identification accuracy as a
covariate. When controlling for this variable, the group × task interaction remained \(F(2,59) = 4.46, p = .02, \eta^2_p = .13\). Post-hoc \(t\)-test confirmed that TA children performed significantly more accurately on the symbolic task than children with LA \((t(47) = 3.69, p < .01, d = 1.21)\) and children with MLD \((t(43) = 3.47, p < .01, d = 1.14)\), but the latter two groups did not differ \((t(30) = 0.28, p = .96, d = 0.13)\). Again, no group differences on the non-symbolic task were observed \((TA \text{ vs. MLD: } t(43) = 1.71, p = .21, d = 0.58; TA \text{ vs. LA: } t(47) = 0.64, p = .80, d = 0.21; MLD \text{ vs. LA: } t(30) = -0.88, p = .65, d = -0.38)\).

We additionally examined whether the observed group differences in symbolic approximate addition could be explained by performance on the nonsymbolic approximate addition task. We therefore re-examined the group difference on the symbolic task by additionally controlling for performance on the non-symbolic task and number identification. Findings revealed that the group difference on symbolic approximate addition remained significant \(F(2,58) = 8.50, p < .01, \eta^2_p = .23\). Post-hoc \(t\)-tests showed that TA children performed significantly more accurately than children with LA \((t(47) = 3.63, p < .01, d = 0.97)\) and children with MLD \((t(43) = 3.00, p < .01, d = 1.15)\), but the latter two groups did not differ \((t(30) = -0.57, p = .84, d = -0.24)\).

**DISCUSSION**

Understanding the nature of the impairments underlying mathematical difficulties is a necessary prerequisite to designing appropriate interventions. We have shown that children with mathematical difficulties, who are at the earliest stages of learning mathematics, have impairments in the ability to access numerical magnitude information from symbolic representations. This deficit was consistent across simple comparison and more complex approximate arithmetic tasks.

**Defective Number Module vs. Access Deficit**

Our findings are in line with previous evidence that children with mathematical
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difficulties have particular problems on tasks that tap into the understanding of numerical magnitudes. Two explanations for these impairments have been proposed (Rousselle & Noël, 2007). According to the *defective number module hypothesis* (Butterworth, 2005), difficulties in mathematics originate from a specific deficit in the ability to represent numerical magnitudes. According to the *access deficit hypothesis* (Rousselle & Noël, 2007), difficulties in mathematics arise from problems in accessing numerical meaning from symbols. To date, the locus of the impairment remains unclear as evidence in favour of the *defective number module hypothesis* (Landerl et al., 2009) and the *access deficit hypothesis* (Iuculano et al., 2008; Landerl & Kölle, 2009; Rousselle & Noël, 2007) has been reported. By examining both hypotheses at an earlier age and with a wider set of tasks than previously reported, we have shown that children with mathematical difficulties were impaired on symbolic tasks, while no group differences were found on non-symbolic tasks. Moreover, impairments on a symbolic task remained when performance on the non-symbolic version of that task was additionally controlled for. These data favour the *access deficit* hypothesis and suggest that the access to representations of magnitude from symbolic numbers rather than the representation of magnitude per se is impaired. Our findings replicate earlier observations on numerical magnitude comparison tasks by Rousselle and Noël (2007), Iuculano et al. (2008), and Landerl and Kölle (2009), and extend these studies by showing that these performance patterns are already present in first grade and generalize to other tasks that measure the understanding of numerical magnitudes, such as approximate addition.

The present study involved two groups of children with mathematical difficulties, i.e. children with LA (if they had math achievement scores between the 16th and 25th percentile) and children with MLD (if they scored below the 16th percentile or 1SD below the age norm), that differed in terms of potential severity of their math problem and that may show different cognitive profiles (Geary et al., 2007, 2008; Murphy et al., 2007). Children with severe
mathematical difficulties, i.e. children with MLD, showed consistent group differences in the symbolic but not non-symbolic tasks, in line with the access deficit hypothesis. Children with mild forms of mathematical difficulties, i.e. children with LA, showed a similar but less severe performance pattern, with only group differences on the symbolic approximate addition task. These data are in accordance with those of Murphy et al. (2007) who showed that children with MLD performed more poorly and developed at a slower rate than their typically achieving peers whereas children with LA performed more poorly but developed at the same rate as their typically achieving peers. They also fit with longitudinal data by Geary et al. (2008), who showed that in first grade both children with LA and children with MLD performed more poorly than TA children on number line estimation whereas in second grade, children with LA performed at the same level as TA but children with MLD continued to lag behind their typically developing peers. In all, the current data converge to the conclusion that the difficulties of children with LA and MLD are mainly observed on the symbolic tasks, supporting the access deficit hypothesis.

The present study is the first to examine both non-symbolic and symbolic approximate addition in children with mathematical difficulties. The majority of the children in the present study performed above chance on these tasks; these children all showed the classic effect of ratio, as performance declined when the ratio in which the sum differed from the comparison numerosity approached 1. This is in line with earlier reports on non-symbolic (Barth et al., 2005) and symbolic (Gilmore et al., 2007) approximate addition tasks in typically developing children. Consistent with Iuculano et al. (2008), we found no group differences on the non-symbolic approximate addition task. However, our findings go beyond the previous ones by showing that group differences emerged on a symbolic version of the same approximate addition task, which remained when performance on the non-symbolic approximate addition task was additionally controlled for. This again indicates that the access to numerical
magnitude from symbolic numbers rather than the representation of numerical magnitudes per se is important in mathematical development.

It may be contended that poorer performance on the administered symbolic tasks is constrained by children with mathematical difficulties’ poorer knowledge of the symbolic system. Therefore, children were also given a number identification task to evaluate whether group differences are due to differences in knowledge of the symbolic system rather than to differences in accessing quantity from symbols. While the children with mathematical difficulties performed more slowly and less accurately on this task, this could not fully account for the group differences in the symbolic comparison and approximate addition tasks.

The precise relationship between deficits in symbolic number knowledge and in accessing magnitude information from symbolic representations remains unclear. It could be that poor knowledge of the symbolic system leads to poorer mappings between the symbols and the quantities they represent. Alternatively, weaker connections between symbols and their underlying quantities could in turn make it harder for children to learn symbolic representations in the first place. Future longitudinal studies, starting before children are introduced to symbolic representations, are needed to disentangle between both possibilities. These studies should also directly assess the quality of the mapping between symbolic representations and (non-symbolic) representations of numerical magnitudes. Mundy and Gilmore (2009) recently developed a task to directly measure this mapping ability, wherein children are shown a representation (symbolic or non-symbolic) of a target quantity and they have to select which of two alternative representations (symbolic or non-symbolic) match the target. Such a task might further shed light on the problems experienced by children with mathematical difficulties.

An alternative interpretation of the current study may be that the observed group differences are explained by recourse to general factors, such as intellectual ability, working
memory or processing speed. Although these variables were not specifically administered in the current study, the presence of group × task interactions, indicating differences on the symbolic but not on the non-symbolic tasks, might challenge this interpretation. Furthermore, additional analyses indicated that the group differences on a symbolic task remained when performance on its non-symbolic version of the task was additionally controlled for. This might suggest that general factors are not entirely accounting for the observed difficulties in symbolic performance. This is in line with Geary et al. (2008), who showed that group differences in number line estimation between children with MLD, children with LA and TA children were only partially mediated by IQ, and not mediated by differences in working memory. However, the current study used a numerical magnitude comparison task and approximation addition task and it remains to be determined to what extent the group differences observed in this study are explained by intellectual ability and working memory, an issue that should be addressed in future research.

In contrast to Landerl et al. (2009), there were no significant group differences on the non-symbolic magnitude comparison task. The task used by Landerl et al. (2009) involved larger numerosities, i.e. between 20 and 79, than the one administered in the current study, which might account for the difference between the current findings and those by Landerl et al. (2009). It could be that using larger numerosities on the non-symbolic comparison task would have revealed group differences in the current study, particularly because children with MLD in this study tended to make more errors, although not significant, than the other groups, an issue that should be addressed in future studies. Even though we found no performance differences on the non-symbolic tasks, this does not preclude the possibility that there may be abnormalities in non-symbolic processing at the neural level. For example, Price et al. (2007) found atypical activation in the right intraparietal sulcus, a key area involved in the representation and processing of numerical quantity in the brain, during the execution of a
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non-symbolic numerical magnitude comparison task in children with mathematical difficulties. These children failed to show the modulation of parietal numerical processing resources in response to smaller numerical distances, as was observed in typically developing controls. Against this background, it may be worthwhile to contrast the defective number module and access deficit hypothesis at the neural level. Neuroimaging data have the potential to reveal the use of strategies that cannot be captured by behavioral methods alone, which provides a possible example of how basic cognitive neuroscience research might aid to solve cognitive and educational questions (De Smedt et al., 2010).

Implications

There is no doubt that difficulties in accessing the numerical meaning from Arabic numerals will have a tremendous impact on the acquisition of other mathematical concepts and procedures. Without knowing that numbers represent quantities, mathematics learning runs the risk of becoming a meaningless endeavour (Griffin, 2002). For example, a good understanding of the numerical meaning of Arabic symbols might boost children’s early arithmetic development. Children initially use counting procedures when they learn to solve arithmetic problems (e.g., Geary, Bow-Thomas, & Yao, 1992; Siegler, 1996). While they first count all addends to find the solution, they gradually move to more advanced procedures such as the counting-from-larger strategy. This involves stating the larger valued addend and then counting on the number of times equal to the value of the smaller valued addend, for example in counting 8, … 9, 10 to solve 2 + 8. This advanced counting procedure, which represents an important step towards arithmetic fact development, requires the child to make a decision on the larger addend, which requires access to numerical meaning of the presented symbols. Furthermore, Booth and Siegler (2008) showed that a good access to numerical magnitude information, as measured by a number line estimation task, improves the quality of the errors on arithmetic tasks and narrows the range of candidate answers to an arithmetic problem. This
will all contribute to successful arithmetic fact development. In sum, children who have difficulties in accessing numerical meaning from symbols are running the risk of developing more immature counting strategies and may acquire more arithmetic facts without meaning, which will be more difficult to retrieve from long-term memory.

The current findings have important implications for teaching and intervention. These should provide plenty of opportunities where children learn to connect symbols and the quantities they represent in rich and meaningful ways. One mathematics program developed to teach this is *Number Worlds* developed by Griffin and her co-workers, which has been shown to yield substantial improvement in children’s early mathematical development (see Griffin, 2004). Furthermore, Ramani and Siegler (2008) showed that playing linear number board games enhances children’s numerical knowledge and understanding of quantities. As indicated by Siegler and Booth (2004), such board games are particularly suited because they provide multimodal cues to the connection between symbols and their quantities. For example, the larger the number that indicates how many squares the counter needs to be moved, the larger the distance the child has to move the counter, the larger the number of moves to be made by the child, and the larger the number of number words spoken by the child. Ramani and Siegler (2008) showed that playing linear board games is effective in enhancing numerical magnitude understanding in kindergarteners from low-income backgrounds. These findings may generalize to children with mathematical difficulties although this possibility needs further investigation.

Recently, Booth and Siegler (2008) examined first graders’ exposure to numerical magnitudes presented on a number line. They showed that providing accurate visual representations of numerical magnitude significantly improved math learning as early as first grade. This suggests that the number line is a powerful representational tool to forge connections between symbols and the quantities they represent.
Conclusion

The present study showed that children with mathematical difficulties have particular impairments in accessing numerical meaning from symbolic digits. Our findings provide the first evidence that this pattern of performance occurs in children from first grade on and generalizes beyond numerical magnitude comparison tasks.
ACKNOWLEDGEMENTS

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PA: Psychology Software Tools.


Table 1

*Descriptive Statistics of the Sample (N = 82)*

<table>
<thead>
<tr>
<th>Group</th>
<th>n</th>
<th>Sex</th>
<th>Age (years)</th>
<th>General mathematics achievement&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>MLD</td>
<td>20</td>
<td>8 boys, 12 girls</td>
<td>6.62 (0.26)</td>
<td>71.52 (9.68)</td>
</tr>
<tr>
<td>LA</td>
<td>21</td>
<td>7 boys, 14 girls</td>
<td>6.70 (0.35)</td>
<td>88.61 (2.43)</td>
</tr>
<tr>
<td>TA</td>
<td>41</td>
<td>19 boys, 22 girls</td>
<td>6.75 (0.38)</td>
<td>108.71 (5.52)</td>
</tr>
</tbody>
</table>

*Note.*<sup>a</sup> Standardized scores ($M = 100$; $SD = 15$). MLD = Mathematics Learning Disabilities; LA = Low achieving; TA = Typically achieving.
Figure Captions

**Figure 1.** Mean reaction time on the symbolic and non-symbolic magnitude comparison tasks as a function of group. MLD = Mathematics Learning Disabilities; LA = Low achieving; TA = Typically achieving. Error bars depict 1 SE of the mean.

**Figure 2.** Mean accuracy on the symbolic and non-symbolic magnitude comparison tasks as a function of group. MLD = Mathematics Learning Disabilities; LA = Low achieving; TA = Typically achieving. Error bars depict 1 SE of the mean.

**Figure 3.** Mean accuracy on the symbolic and non-symbolic approximate addition tasks as a function of group. MLD = Mathematics Learning Disabilities; LA = Low achieving; TA = Typically achieving. Error bars depict 1 SE of the mean.
Figure 1

Numerical magnitude comparison

<table>
<thead>
<tr>
<th></th>
<th>Reaction time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbolic</td>
<td></td>
</tr>
<tr>
<td>MLD</td>
<td>1500 ± 100</td>
</tr>
<tr>
<td>LA</td>
<td>1400 ± 100</td>
</tr>
<tr>
<td>TA</td>
<td>1500 ± 100</td>
</tr>
<tr>
<td>Nonsymbolic</td>
<td></td>
</tr>
<tr>
<td>MLD</td>
<td>1000 ± 100</td>
</tr>
<tr>
<td>LA</td>
<td>1100 ± 100</td>
</tr>
<tr>
<td>TA</td>
<td>1100 ± 100</td>
</tr>
</tbody>
</table>

Legend:
- MLD
- LA
- TA
Figure 2

Numerical magnitude comparison

Proportion correct

Symbolic

Nonsymbolic

MLD

LA

TA
Figure 3

Approximate addition

Proportion correct

Symbolic

Nonsymbolic

MLD
LA
TA