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COMPUTER SIMULATION OF THE TAKEOFF IN SPRINGBOARD DIVING

by

Pui Wah Kong

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

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ABSTRACT

Computer simulation of the takeoff in springboard diving
Pui Wah Kong, Loughborough University, 2005

A computer simulation model of a springboard and a diver was developed to investigate diving takeoff techniques in the forward and the reverse groups. The springboard model incorporated vertical, horizontal and rotational movements based on experimental data. The diver was modelled as an eight-segment link system with torque generators acting at the metatarsal-phalangeal, ankle, knee, hip and shoulder joints. Wobbling masses were included within the trunk, thigh and shank segments to allow for soft tissue movement. The foot-springboard interface was represented by spring-dampers acting at the heel, ball and toes of the foot. The model was personalised to an elite diver so that simulation output could be compared with the diver's own performance. Kinematic data of diving performances from a one-metre springboard were obtained using high speed video and personalised inertia parameters were determined from anthropometric measurements. Joint torque was calculated using a torque / angle / angular velocity relationship based on the maximum voluntary torque measured using an isovelocity dynamometer. Visco-elastic parameters were determined using a subject-specific angle-driven model which matched the simulation to the performance in an optimisation process. Four dives with minimum and maximum angular momentum in the two dive groups were chosen to obtain a common set of parameters for use in the torque-driven model. In the evaluation of the torque-driven model, there was good agreement between the simulation and performance for all four dives with a mean difference of 6.3%. The model was applied to optimise for maximum dive height for each of the four dives and to optimise for maximum rotational potential in each of the two dive groups. Optimisation results suggest that changing techniques can increase the dive height by up to 2.0 cm. It was also predicted that the diver could generate rotation almost sufficient to perform a forward three and one-half somersault tuck and a reverse two and one-half somersault tuck.

Keywords: springboard, diving, takeoff, simulation, model, optimisation, torque
PUBLICATIONS AND AWARDS

Published Abstracts:


Conference Presentations:


Awards:

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Student Presentation Award, British Association of Sport and Exercise Sciences Student Conference, 2004

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DEDICATION

To Steve

and

my family
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CHAPTER 1
INTRODUCTION

1.1. Introduction

This chapter provides a general overview of the area of springboard diving and its associated research. The statement of purpose is addressed and specific research questions to be answered later in the study are presented.

A dive comprises the approach, the takeoff from platform or springboard, the airborne movement and the entry into water. In competitive diving, the three main objectives are: 1) to generate sufficient angular momentum to execute somersaults and twists; 2) to obtain height during flight and thus time in the air; and 3) to travel safely but not excessively away from the board (Miller & Munro, 1985a). The dive height, horizontal distance travelled in flight and angular momentum required to execute somersaults are all determined at the end of the takeoff phase. Once the diver is in the air, he / she can only control the speed of somersault rotation by altering body shape. It is therefore crucial to understand the mechanics of takeoff in terms of generating both linear and angular momentum.

In springboard diving, the takeoff can be subdivided into the board depression and recoil phases separated by the point of maximum board depression. Energy is stored in the springboard during depression effected by a large vertical landing velocity at touchdown and an active leg push and armswing. To initiate rotation, divers generate angular momentum using hip flexion (forward and inward group) or hyper-extension (backward and reverse group) during the recoil phase (Miller, 1981; Miller & Munro, 1985b). However, any joint flexion / hyper-extension during this phase will absorb energy causing a loss in dive height (Sanders & Wilson, 1988). Harper (1966) stated that 'there seems to be an optimal placement of the movement in every dive for maximal lift on the dive'. It appears that the compromise between height, rotation and distance has long been recognised but limited research has been done in this area.

1.2. Previous studies

Early literature on diving mainly described 'how' to perform a dive based on personal diving and coaching experience (Harlan, 1950; Hoben, 1936). There were
enormous inconsistencies and disagreement among coaches and divers on the techniques of even the same dive. Since the 1960s, scientists and coaches have started to analyse diving in terms of mechanical principles and tried to explain ‘why’ a dive should be done in a certain way (Batterman, 1968; Dyson, 1986; Rackham, 1969). Knowledge up to this stage was gained from experience, observation and theoretical suppositions.

During the past few decades, biomechanical studies of elite performances have provided useful information and given a better understanding of techniques in springboard diving (Golden, 1981; Miller, 1984; Miller & Munro, 1984, 1985a, b; Sanders & Wilson, 1988; Sanders & Gibson, 2000). Simple computer simulation models have been developed to investigate further the mechanics of diving (e.g. Miller, 1970; Sprigings, Watson, Haseganu & Derby, 1986). Statistical databases of selected variables, for example: takeoff time, dive height, knee angle, entry angle, etc. measured from elite performances, have been set up to provide a reference for coaches and divers (Sanders, 2001).

Several studies have identified the factors associated with achieving dive height in the air (Harper, 1966; Sanders & Wilson, 1988). Attention has been paid to the timing of the armswing (Sprigings et al., 1985), body configuration at touchdown (Sanders & Wilson, 1988; Sanders & Gibson, 2000) and joint movements during board depression and recoil phases (Martikkala, Oksa, Viitasalo & Luhtanen, 1995; Miller & Munro, 1984; Sanders & Wilson, 1988).

The distance that elite divers travel during flight is determined by the horizontal velocity at the last instant of takeoff ranging from 0.6 to 1.3 m/s (Miller, 1981, 1984; Miller & Munro, 1985a; Miller, 2000). Women have tended to have less board clearance (the smallest distance between the board and any body part) during flight because they have less time in the air (Miller, 1984). It has been suggested that injuries caused by striking the board are usually the result of a flawed approach (Rubin, 1999). Some common faults are recognised such as insufficient lean, incomplete knee extension, excessive hyper-extension, and rushing into the shape too early so that the mass centre of the diver is too close to the board (Barone, 1973; Fairbanks, 1964; O'Brien, 1968; Zhang, 1996).

It was once believed that takeoffs for all dives were the same and that somersault was generated at the top of flight by head movement (Billingsley, 1965; Hoben, 1936). Great efforts have been made by scientists and coaches to clarify that somersault rotation must be initiated during the takeoff when the diver is in contact with the board (e.g. Batterman, 1968; Eaves, 1969; Rackham, 1969). Later kinematic and kinetic studies have shown that
rotation is initiated by torques acting about the mass centre and that the direction of rotation depends on the position of the mass centre relative to the point of force application (Miller, 1983; Miller, Hennig, Pizzimenti, Jones & Nelson, 1989; Miller, Jones, Pizzimenti, Hennig & Nelson, 1990).

Within each dive group, researchers have identified the critical factors responsible for generating angular momentum as the rotation requirement increases. Stroup and Bushnell (1969) calculated the theoretical relationship between rotation and lean angle at takeoff in terms of energy partition based on a fixed amount of energy gained at the takeoff. Some descriptive studies observed that takeoff angle increases with rotational requirement (Swain, cited in Adrian & Cooper 1989; Golden, 1981; Hamill, Golden, Ricard & Williams, 1985; Miller, 1974). Good divers are characterized by less flexion (forward and inward group) and less hyper-extension (backward and reverse group) in the hips during the recoil phase (Sanders & Wilson, 1988; Sanders & Gibson, 2000). It is clear that increasing joint flexion and/or hyper-extension is required as the rotational requirement increases but to what extent the flexion and/or hyper-extension should increase for optimal performance has yet to be examined.

In short, there is a general understanding of takeoff techniques to achieve good height and initiate rotation. As the somersault requirement increases, there is a compromise between dive height and rotation. The horizontal velocity required for board clearance depends on the vertical velocity which, in turn, is closely related to the rotation requirement. It is agreed that the diver should aim for maximum dive height while generating sufficient angular momentum for rotation and keeping a safe distance away from the springboard. The inter-relationship between dive height, board clearance distance and somersault rotation, however, is not well established.

1.3. Statement of purpose

It is the intention of the present study to understand the mechanics of takeoff in springboard diving in terms of generating both linear and angular momentum. To facilitate this a computer simulation model of a diver and a springboard will be developed. The model, after satisfactory evaluation, will be used to investigate takeoff techniques and to optimise diving performance.
1.4. Research questions

Q1. For a specific dive with fixed rotational requirement, what is the optimal takeoff technique to obtain maximum height while generating sufficient angular momentum and travelling safely away from the springboard?

According to the laws of mechanics, the mass centre of the diver travels in a parabolic path in the air. To increase the height, there will be a decrease in distance travelled. During the springboard depression phase, energy is stored in the springboard and returned to the diver during the recoil phase during which the diver flexes / hyper-extends the joints to generate rotation. The more the diver flexes / hyper-extends the joints, the more energy is lost in gaining height (Sanders & Wilson, 1988). Thus, it is clear that increasing dive height will lead to a compromise in distance and rotation.

Q2. What is the maximum rotation the diver can perform in the forward and reverse groups without an increase in strength?

It has been shown that as the rotational requirement increases, there is an increase in forward lean and hip flexion for the forward group and hip hyper-extension for the reverse group. The increase in flexion / hyper-extension generates more angular momentum but at the same time reduces dive height. The loss in height reduces the time for somersault rotation and therefore the increase in angular momentum may not produce maximum rotation. In terms of distance, forward lean facilitates rotation but decreases height in the forward group. For the reverse group, hyper-extension facilitates backward rotation but may bring the diver’s mass centre too close to the springboard.

1.5. Chapter organisation

Chapter 2 reviews the literature on diving, springboard characteristics, simulation models and techniques of investigation. The pros and cons of different research techniques are discussed and limitations of previous research are addressed.

Chapter 3 presents the development of the springboard and the diver model. This includes an innovative method to model the springboard with vertical, horizontal and rotational movements and features two diver models: angle-driven and torque-driven.
Chapter 4 describes in detail how subject-specific model parameters and kinematic data are determined for input to the torque-driven model. Model parameters including body segmental inertia, strength, wobbling mass, visco-elastic and springboard parameters are determined either directly from experiment or indirectly using a subject-specific angle-driven model.

Chapter 5 outlines the procedure for the evaluation of the torque-driven model. This includes using an objective score to compare simulation outputs with an elite diver’s performance. Results of the evaluation indicate the ability of the model to reproduce realistic human movement.

Chapter 6 applies the model to optimise diving takeoff techniques in the forward and the reverse dive groups. Answers to the specific research questions addressed in Chapter 1 are discussed using results from the optimisations for maximum dive height and rotation.

Chapter 7 summaries the main findings of the study regarding the optimal takeoff techniques for maximum dive height and rotation. It also discusses the limitations of the present study with suggestions for future research.
CHAPTER 2
REVIEW OF LITERATURE

2.1. Overview of diving research

Early diving literature mainly described 'how' to do a dive based on personal sporting and coaching experience (Harlan, 1950; Hoben, 1936). There were enormous inconsistencies and disagreement among coaches and divers on the techniques of even the same dive. Later on, scientists and coaches started to analyse diving in terms of mechanical principles and tried to explain 'why' a dive should be done in certain ways (Batterman, 1968; Dyson, 1986; Rackham, 1969). Knowledge up to this stage was gained from experience, observation and theoretical assumption.

During the past few decades, diving research has tended to change from qualitative to quantitative. Biomechanical studies of elite performances have provided useful information and given a better understanding of techniques in both springboard diving (Golden, 1981; Miller, 1984; Miller & Munro, 1984, 1985a, b; Sanders & Wilson, 1988; Sanders & Gibson, 2000) and platform diving (Hamill, Ricard & Golden, 1986; Miller, Hennig, Pizzimenti, Jones & Nelson, 1989; Miller, Jones, Pizzimenti, Hennig & Nelson, 1990; Murtaugh & Miller, 2001). Simple computer simulation models have been developed to further investigate the mechanics of diving (eg. Miller, 1970; Sprigings, Watson, Haseganu & Derby, 1986). Statistical databases of selected variables, for example, takeoff time, dive height, knee angle, entry angle, etc., measured from elite performances have been set up to provide a reference for coaches and divers (Sanders, 2001).

Dives can be classified into twisting and non-twisting. Various studies have focused on twisting dives (eg. Frohlich, 1979; Liu & Nelson, 1985) and a knowledge of twisting somersaults has been gained through mathematical modelling and simulation (Yeadon, 1984, 1993a, b, c, d, e). The focus of this thesis was on non-twisting dive groups and therefore the review will only cover the forward, inward, backward and reverse groups.

2.2. The takeoff

2.2.1. Introduction

A dive is composed of the takeoff, the airborne movement and the entry. Of all the phases, the takeoff holds the key to the success or failure of the performance (Miller,
In the backward and inward group, divers adopt a standing takeoff, which includes an initial activation of springboard movement and the final push. In the forward and reverse group, a running approach is usually adopted to obtain more height. The running takeoff consists of the approach, the hurdle and the takeoff. The takeoff can be further divided into a depression phase and a recoil phase, separated by the point of maximal board depression.

Figure 2.1. The takeoff can be further divided into a depression phase and a recoil phase, separated by the point of maximal board depression.

Miller (1985a) identified three main objectives in springboard diving: 1) to generate sufficient angular momentum to execute somersaults and twists; 2) to obtain height and thus time in the air; and 3) to travel safely away from the board. The dive height, horizontal distance travelled and angular momentum required to execute somersaults are all determined at the last instant of takeoff. Once the diver is in the air, he / she can only control the speed of rotation by altering body shape. It is, therefore, crucial to understand the mechanics of takeoff in terms of generating both linear and angular momentum.

2.2.2. Height

Dive height attained has both direct and indirect influence on the dive score (McCormick, 1982). Height itself is a factor that judges take into account when judging a dive. Moreover, greater height allows more time for rotation, for adopting a good shape and for preparing for entry in an unrushed manner (Sanders, 2001).
Factors associated with achieving dive height have been identified in biomechanical studies. It has been suggested that the diver should have a high hurdle giving high vertical velocity at touchdown (Harper, 1966; Sanders & Wilson, 1988). He / she should catch the board when it has oscillated for 2.25 - 2.5 cycles at which point it is going down (Miller, Osborne & Jones, 1998). The optimal flexion in hips and knees at touchdown is highly dependent on strength such that further flexion following the touchdown is minimised to avoid energy absorption (Sanders & Wilson, 1988; Sanders & Gibson, 2000). A late armswing should be initiated just before the touchdown (Sprigings, Paquette & Watson, 1987) such that the touchdown coincides with the beginning of upward acceleration of arms (Miller & Munro, 1984; Sprigings & Watson, 1985). A vigorous extension at hips, knees and ankles should follow to depress the board to a maximum extent (Martikkala, Oksa, Viitasalo & Luhtanen, 1995; Miller & Munro, 1984; Sanders & Wilson, 1988). During the recoil phase, divers should seek minimal hip flexion (forward and inward group) or hyper-extension (backward and reverse group) while still generating sufficient angular momentum (Sanders & Wilson, 1988).

Recently, new techniques were employed attempting to increase dive height by introducing a period of flight preceding the hurdle step. Miller, Zecevic and Taylor (2002) investigated the new hurdle pre-flight techniques and compared them to the traditional walking or running approach. It was concluded that the costs of the new techniques might outweigh their potential benefits. Although the trend of adopting this new technique is not clear at present, this study has provided much information on the kinematics of takeoff.

In short, the takeoff involves the utilization of energy between the diver and the springboard. Energy is stored in the springboard during depression effected by active leg push, armswing acceleration and large landing velocity from a high hurdle. During the recoil phase, the energy stored in the springboard is returned to the diver. Any joint flexion during the recoil phase will absorb or waste energy and therefore should be minimised.

2.2.3. Distance

The distance that the diver travels during flight is determined by the horizontal velocity at the last instant of takeoff. In standing dives, the diver builds up most of the horizontal velocity during the takeoff whilst in running dives, he / she changes the forward momentum brought along from the approach and the hurdle (Miller, 2000a). Longer hurdles are associated with greater horizontal velocity during flight (Miller, 1984). Divers with longer and faster hurdles tended to reduce their horizontal velocity during
takeoff whilst those with shorter and slower hurdles tended to increase their horizontal velocity during takeoff (Miller, 1984, 2000a). The horizontal velocity at touchdown across different dive groups was consistent in some divers (Miller, 1984; Miller & Munro, 1985a) but fluctuating in others (Miller, 1984; Newton & Greenwood, 1993). At the last instant of takeoff, the horizontal velocity ranged from 0.6 - 1.3 m/s (Miller, 1981, 1984; Miller & Munro, 1985a; Miller, 2000a). Women tend to have less board clearance (the smallest distance between the board and any body part) during flight because they had less time in air (Miller, 1984).

Some studies have examined the pattern of horizontal velocity changes during the takeoff across different dive groups (Miller, 1981; Miller & Munro, 1985a). It was found that there was an initial reduction of horizontal velocity during the depression phase in dives from the reverse and inward groups. In dives from the forward and backward groups, a general increase in horizontal velocity from the beginning of the takeoff until near the end of recoil was observed.

Diving books have emphasized proper body posture and essential lean from the board for safety (Barone, 1973; Batterman, 1968; Fairbanks, 1964; Hoben, 1936; O'Brien, 1992; O'Brien, 1968). Olympic champion Greg Louganis (1995) hit the springboard and platform many times throughout his diving life. Two fatal head injuries have occurred in the history of competitive diving where, in both cases, divers were attempting a reverse three and one-half somersaults tuck from the 10-metre platform (Miller, 2000a; Rubin, 1999). In a questionnaire study of senior competitive divers, there were 26 springboard injuries of which 17 occurred during the depression phase and nine were hitting the board during flight (Mizel, Marymount, Decker, Elly & Rubin, 1996). This study also reported that the reverse group was most commonly associated with injury. It has been suggested that injuries caused by striking the board are usually the result of a flawed approach (Rubin, 1999). Some common faults are recognised as insufficient lean, incomplete knee extension, too much hyper-extension, and rushing into shape too early such that the mass centre of the diver is too close to the board (Barone, 1973; Fairbanks, 1964; O'Brien, 1968; Zhang, 1996).

Barone (1973) states that the diver should enter the water at about two to three feet from the one-metre springboard. It should be noted that the horizontal velocity required for board clearance depends on the vertical velocity which, in turns, is closely related to the rotation requirement. Thus, it appears that the horizontal velocity required for board clearance varies among different dive groups, rotational requirement and individual divers.
2.2.4. Rotation

2.2.4.1. How to generate rotation?

It was once believed that takeoffs for all dives were the same and that somersault was generated at the top of flight by head movement (Billingsley, 1965; Hoben, 1936). Great efforts have been made by scientists and coaches to clarify that rotation must be initiated during the takeoff when the diver is in contact with the board (Batterman, 1968; Eaves, 1969; Rackham, 1969). Mechanical principles were applied to explain how and why certain techniques should be employed to generate rotation. Eaves (1969) summarised six methods:

i) **Topple.** A toppling dive begins with the diver in a standing position on the edge of the board and fall off without any active push. This is sometimes referred as 'lean' or 'overbalancing' (Rackham, 1969). The mechanics of toppling techniques were investigated in terms of conservation of angular momentum by Page (1974) and later corrected by Wilson (1977). This method is only used for line-ups practice and platform armstand dives.

ii) **Run and stop.** Eaves (1969) states that the linear kinetic energy from the approach is transferred to rotational energy when the feet are anchored to the board by friction. Applying the principle of conservation of energy to this situation should, however, be viewed with caution. Even if energy lost as heat, sound and in damping can be ignored, active muscular contraction of the diver contributes to increasing the energy of the system. This 'run and stop' approach can better be understood as utilizing the braking force to promote forward rotation.

iii) **Hip-bent.** The 'hip-bent' method is also known as 'eccentric leg thrust' (Rackham, 1969). This principle assumes that the reaction force from the springboard goes from the feet through the hips even though Eaves (1969) has admitted that the magnitude and direction of the force is unknown. Rackham (1969) introduced another 'eccentric springboard thrust' principle, arguing that the line of thrust was at right angles to the springboard. Similar assumptions have been made by Dyson (1986). This principle explains that a torque is generated since the mass centre of the diver does not pass through the line of reaction force. Despite the unjustified assumptions, the 'eccentric springboard thrust' principle has another strong limitation in that it can only explain dives in the forward and backward groups.

iv) **Momentum transfer.** 'Momentum transfer' is also known as 'jerk' (Rackham, 1969) which means that movement of body parts during the takeoff are transferred to the
whole body during flight. For example, the forward and downward movement of the arms and upper body have been transferred to whole body forward rotation, as seen in the forward and inward groups. Later quantitative studies (Miller, 1981; Miller & Munro, 1985a) on segmental contributions to angular momentum have provided sound evidence for this principle.

v) Lean. Eaves (1969) states that when the line of springboard thrust no longer passes through the mass centre of the diver, the motion will be both translation and rotation. This method shares similar principles with the ‘hip-bent’ method but cannot explain the mechanics of dives in the reverse and inward groups.

vi) Contrary motion. ‘Contrary motion’ applies the conservation of angular momentum when the diver is in the air. This method has important application to twisting dives but not a great deal to somersaulting dives.

The above methods explain different dive groups with different forces and mechanical principles. It is clear that those theories are incomplete and thus fail to give a full understanding of the mechanics of generating rotations in the different dive groups. By the late 1970s, Frohlich (1979) proposed a better theory that resolved the reaction force from the springboard into a vertical and a horizontal component. Thereafter, kinematic and kinetic studies have shown that the direction of torques generated by the vertical and horizontal forces depends on the position of the mass centre. (Miller, 1983; Miller et al., 1989; Miller et al., 1990).

In the forward and backward group, it is the torque about the centre of mass produced by the vertical springboard reaction force that is almost entirely responsible for building up the required angular momentum. This torque promotes rotation in the required direction whilst the torque of the horizontal force, on the other hand, contributes to rotation in the opposite direction.

In the reverse and inward group, the mass centre moves away from the board during the recoil phase. During this period, the torque of the horizontal force promotes required rotation while that of the vertical force retards rotation. It was observed that better divers tended to maintain their mass centre closer to the fulcrum (Miller, 1981). This results in a smaller retarding torque of the vertical force due to a shorter moment arm. Although placing the mass centre closer to the board is effective in producing rotation, safety concerns should be emphasized (Miller, 2000a) especially in executing dives from the reverse group.
In platform diving, a force plate can be used to measure the direction and the magnitude of the reaction force during the takeoff (Hamill, Golden, Ricard & Williams, 1985; Miller et al., 1989; Miller et al., 1990). However, to our knowledge, this technique has only been used once (Bergmaier, Wettstein & Wartenweiler, 1971) in springboard diving due to practicality. Instead, the inverse dynamic method is often employed to calculate the springboard reaction force from video images (Miller, 1983; Sanders & Wilson, 1987) despite the possible errors resulting from digitisation.

2.2.4.2. Increasing rotation requirement

Within each dive group, it is interesting to investigate how to generate the amount of angular momentum needed for increasing rotational requirement.

In the forward group, it has been suggested that an increase in forward lean at takeoff is needed as rotational requirement increases (Fairbanks, 1964; O'Brien, 1992). Some coaches emphasize bringing the head, arms and upper body down faster and further (Batterman, 1968; Still & Carter, 1979; Zhang, 1996) while others stress a harder push (Batterman, 1968; Zhang, 1996). Similar techniques are employed to perform dives from the inward group.

In the backward group, Hobden (1936) states that the more vigorously the head is ‘thrown back’, the quicker the body rotates. Other have argued that the head should not be ‘driven back’ (Barone, 1973; Batterman, 1968; O'Brien, 1992; Zhang, 1996) and increasing rotation should be achieved by a faster armswing (Batterman, 1968), a stronger push (Batterman, 1968; Fairbanks, 1964; O'Brien, 1968; Zhang, 1996), increasing hip hyper-extension (O'Brien, 1992; Zhang, 1996) and ‘pulling into shape’ earlier (Zhang, 1996). Similar techniques are applied in the reverse group, with an additional emphasis of pushing the hips forward.

Among the different techniques used to increase rotation, researchers have tried to identify the critical factors responsible for the most increase in angular momentum. Stroup and Bushnell (1969) calculated the theoretical relationship between rotation and takeoff angle in terms of energy partition based on a fixed amount of energy gained at the takeoff. They suggested that the amount of rotation was dependent on the magnitude and direction of the takeoff angle. Some descriptive studies observed that the takeoff angle increased with rotational requirement (Golden, 1981; Hamill, 1985; Miller, 1974; Swain, cited in Adrian, 1989).
Experimentally, Hamill et al. (1985) found that platform divers imparted maximum ground reaction force at the takeoff phase for the forward dive, one and one-half somersault and two and one-half somersaults pike. The divers increased angular momentum by decreasing the trunk angle to the horizontal and increasing the angular velocity of the trunk at takeoff. In springboard diving, Miller (1974) analysed the takeoff of dives from the forward and reverse groups. It was observed that forward lean at touchdown, maximum board depression and the last contact was greater in forward two and one-half somersault pike than in forward dive straight. A larger degree of back arch was also observed in reverse multiple somersaults than in reverse dive. Golden (1981) closely examined the joint kinematics during the takeoff in dives from the forward and inward group. He concluded that the most visible and systematic changes to increase rotation were at the hip and shoulder joint. With each pike somersault increment, there was an average 20° increase in hip flexion and 20° to 25° increase in shoulder extension at the last instant of the takeoff.

More recently, Xu and Zhang (1996) have demonstrated the different body configurations at the last instant of takeoff in each dive group. They make clear the concept that the same method of initiating rotation should be used but to a greater extent as rotational requirement increases. In the forward and inward groups, this is characterized by increased forward lean and hip flexion. In the backward and reverse groups, there is increased hip hyper-extension and final knee flexion. It has also been shown that there is little if any angular momentum before the takeoff (Miller, 1981; Miller & Munro, 1985a; Miller, 2000b; Sanders & Wilson, 1987) and that angular momentum is built up mainly during the recoil phase of takeoff (Miller, 1981; Miller & Munro, 1985b).

To summarise, there is a general understanding of the different techniques employed in initiating rotation in different dive groups. Within each dive group, it has also been demonstrated that increasing joint flexion and / or extension is required as rotational requirement increases. However, the extent to which different joint flexion and / or extension could affect the resultant angular momentum has yet to be examined.

2.2.5. Inter-relationship

A compromise between gaining height, rotation and distance has long been recognised. Harper (1966) states that ‘there seems to be an optimal placement of the movement in every dive for maximal lift on the dive’. Theoretical calculations (Stroup &
Bushnell, 1969) and biomechanical studies (Miller & Munro, 1984; Sanders & Wilson, 1988) have provided evidence that dive height is reduced as somersault rotations increase.

Good divers are characterized by less flexion (forward and inward group) and less hyper-extension (backward and reverse group) in the hips during the recoil phase (Sanders & Wilson, 1988; Sanders & Gibson, 2000). More than optimal joint flexion / extension would reduce height gained and distance travelled. Divers should seek maximum height while still generating sufficient angular momentum and travelling safely away from the springboard.

Figgen (1989) developed a computer simulation model of the flight phase of a dive. Based on theoretical model inputs, it was found that there could be different combinations of vertical velocity, angular momentum and angle at takeoff to perform a good dive. Xu and Zhang (1996) and Xu (2000) suggested that the compromise in rotation, height and distance was highly dependent on an individual diver’s physical characteristics and styles. However, little is known about optimising the takeoff technique in terms of height, distance and angular momentum, and how technique varies across dive groups and individual divers.

2.2.6. Summary

It is well understood that both linear and angular momentum are generated during the takeoff. As the rotation requirement increases, the diver must increase the degree of joint flexion / extension to generate more angular momentum but to what extent the flexion / extension should increase for optimal performance has yet to be investigated. It appears that the compromise between gaining height, distance and rotation has long been recognised but little is known about the optimal takeoff techniques in terms of generating both linear and angular momentum.

2.3. The springboard
2.3.1. Physical characteristics

The springboard has evolved from Duraflex (tapered at one end) to Maxiflex A (tapered at both ends), and then Maxiflex B with an addition of 171 perforations at a region of 0.7 m from the free end. The manufacturing company (Duraflex International Corporation) once altered the Maxiflex B to 225 perforations extending back 0.8 m but soon changed to the current model of 189 perforations (Figure 2.2) extending back 0.7 m.
to avoid fatigue cracks. This Maxiflex B springboard is now used in all official competitions.

Figure 2.2. Maxiflex B with 189 perforations at a region of 0.7 m from the free end. (adapted from http://www.duraflexinternational.com/diving%20boards.htm, 2004)

2.3.2. Modelling the springboard

Attempts have been made to model the springboard either alone or in concert with the diver to reflect the extreme complexity of the system. It is conceptually appealing but difficult to implement modelling the board as a continuous system with the fundamental parameters distributed throughout. Instead, a numerical approximation of closed form solution can be achieved by using the finite element method (Sprigings, Stilling & Watson, 1989). Kooi and Kuipers (1994) used the continuous method to develop a lumped-parameter model. Kuipers and van de Ven (1992) examined the torsional force exerted on the springboard during unilateral contact. Since all these methods require intensive computation, there is a need for a simplified model with reasonable accuracy.

Sprigings, Stiling and Watson (1989) modelled the Duraflex board using two different approaches: the finite element analysis and a simplified mass-spring model. The mass-spring model consists of a lumped mass connected to a parallel arrangement of a linear spring and a dashpot. They compared the resulting parameters calculated from the two approaches and concluded that the linear mass-spring model could reasonably represent the dynamic behaviour of the springboard. Even though Kooi and Kuipers (1994) further proposed a single degree-of-freedom (DOF) 'bar model', the linear mass-spring model was generally accepted to represent the complex springboard system (Boda, 1992; Jiang, Li, Shu & Zhang, 2000a; Jiang, Shu & Li, 2000b; Jiang, Zhu, Li & Shen, 2001; Liu & Wu,

2.3.3. Measuring springboard parameters

In the mass-spring model, it has been shown that the effects of damping are negligible and therefore can be safely ignored (Boda, 1992; Sprigings et al., 1989). The modelling parameters required are thus stiffness and effective board mass only. The stiffness of the springboard, reflected by the spring constant $k$, can be measured experimentally by applying Hooke's Law:

$$ F = ky $$

(2.1)

where $F$ is the applied load and $y$ is the vertical board deflection. This method involves applying static loads of known weight at different positions on the board and at different fulcrum numbers. It has been found that $k$ is constant for a given condition of fulcrum position and point of load application. However, the value of $k$ varies in a non-linear fashion for changes in either fulcrum position or the point of force application. Similar results have been found in both Duraflex (Sprigings et al., 1988; Sprigings et al., 1989) and Maxiflex B (Boda, 1992; Miller & Jones, 1999; Sprigings, Stilling, Watson & Dorotich, 1990) springboards.

Once the stiffness is measured by the static loading method, the effective board mass ($m_e$) can be determined by the following formula:

$$ f = \frac{1}{2\pi} \sqrt{\frac{k}{m_e}} $$

(2.2)

where $f$ is the frequency. The test procedure consists of releasing the springboard from an initial deflection at different fulcrum settings. An accelerometer is usually employed to obtain vibration frequency. It has been observed that $m_e$ increases as the fulcrum is set further back towards the anchored end in both Duraflex (Sprigings et al., 1988; Sprigings et al., 1989) and Maxiflex B (Boda, 1992; Miller & Jones, 1999; Sprigings, Stilling, Watson & Dorotich, 1990) springboards.

The above procedures require a static loading to measure $k$ followed by a dynamic loading to calculate $m_e$. Stone (1987, stated in Miller & Jones, 1999) has developed another dynamic method with the formula:

$$ m_L = \frac{k}{4\pi^2}T^2 - m_e $$

(2.3)
where mL is the mass used for loading and T is the period of oscillation. By plotting $T^2$ against mL, both k and me can be determined in one single data collection procedure. Miller and Jones (1999) compared the values of k measured from the static and the dynamic methods. The springboard modelling parameters determined from the two methods in different studies are summarised in Table 2.1.

In Miller and Jones' (1999) study, the stiffness of the same springboard measured from the two methods resembled each other only at higher fulcrum numbers. At lower fulcrum numbers, the values of k obtained from the static method were about 500-600 N/m higher than those determined dynamically. This discrepancy was also evident when comparing Stone's (1987, cited in Miller & Jones, 1999) dynamically obtained k values with those obtained statically by Boda (1992). Miller and Jones (1999) recognised such discrepancy as measurement errors since forward fulcrum position was characterized by smaller deflection and faster oscillation, consequently resulting in greater variability. However, the reason for causing the systemic differences were not fully understood. While most elite divers set their fulcrum position at 6 or higher (where k values were in close agreement), Miller and Jones (1999) recommended that the dynamic method should be employed to reduce data collection time.

2.3.4. Limitations of the mass-spring model

The mass-spring model is appealing for its simplicity and the direct method for determining model parameters. However, it only represents the vertical behaviour of the springboard and thus the vertical reaction force acting on the diver. It should be noted that the springboard deflects in a curve, providing also a horizontal reaction force which plays an important role in the generation of angular momentum and board clearance. In addition, the springboard rotates as it defects which influences the divers' orientation. It is therefore necessary to develop a model which represents the vertical, horizontal and rotational behaviour of the springboard for subsequent use in a diver / springboard system.
Table 2.1. Summary of modelling parameters for a Maxiflex-B springboard

<table>
<thead>
<tr>
<th>source</th>
<th>board type</th>
<th>mass (kg)</th>
<th>fulcrum number</th>
<th>k (N/m) static</th>
<th>k (N/m) dynamic</th>
<th>m_e (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stone (1987, stated in</td>
<td>Duraflex*</td>
<td>53.9</td>
<td>1/2</td>
<td>7164</td>
<td></td>
<td>6.4</td>
</tr>
<tr>
<td>Miller &amp; Jones, 1999)</td>
<td></td>
<td></td>
<td>4/5</td>
<td>6213</td>
<td></td>
<td>7.9</td>
</tr>
<tr>
<td>load 0.15 m back</td>
<td></td>
<td></td>
<td>8/9</td>
<td>5424</td>
<td></td>
<td>9.0</td>
</tr>
<tr>
<td>Sprigings et al. (1990)</td>
<td>Maxiflex B</td>
<td>53.2</td>
<td>forward</td>
<td>~ 6400</td>
<td></td>
<td></td>
</tr>
<tr>
<td>load at tip</td>
<td></td>
<td></td>
<td>middle</td>
<td>~ 5900</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>back</td>
<td>~ 5000</td>
<td></td>
<td>8.0</td>
</tr>
<tr>
<td>Boda (1992)</td>
<td>Maxiflex B</td>
<td>58.2</td>
<td>1</td>
<td>8032</td>
<td></td>
<td>2.2</td>
</tr>
<tr>
<td>load at tip</td>
<td></td>
<td></td>
<td>3</td>
<td>6452</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>6250</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>7</td>
<td>5405</td>
<td></td>
<td></td>
</tr>
<tr>
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<td></td>
<td></td>
<td>9</td>
<td>5161</td>
<td></td>
<td>5.9</td>
</tr>
<tr>
<td>Miller and Jones (1999)</td>
<td>Maxiflex B-1</td>
<td>53.8</td>
<td>2</td>
<td>6004</td>
<td>5543</td>
<td>6.5</td>
</tr>
<tr>
<td>load 0.04 m back</td>
<td></td>
<td></td>
<td>4</td>
<td>5330</td>
<td>5132</td>
<td>7.0</td>
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<td></td>
<td></td>
<td>6</td>
<td>4915</td>
<td>4731</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>Maxiflex B-2</td>
<td>54.6</td>
<td>2</td>
<td>6146</td>
<td>5437</td>
<td>7.1</td>
</tr>
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<td>4907</td>
<td>4649</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>8</td>
<td>4305</td>
<td></td>
<td>8.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>10</td>
<td>3896</td>
<td>3929</td>
<td>8.3</td>
</tr>
</tbody>
</table>

*The reported m_e value suggested it was an early Maxiflex model (Miller & Jones, 1999).

2.4. Computer simulation models

2.4.1. General overview

Developing a computer simulation model includes the procedures of defining the problem, deriving mathematical equations, writing the computer program, determining input values, validating the model, and performing simulation experiments (Vaughan, 1984). The application of computer simulation models to sporting activities can provide a deeper insight into the mechanics of human movement. It can also answer the 'what if?' questions which are difficult to address by experimental studies or descriptive analysis.
This provides a safe means to investigate the optimisation of sports performance without actually having an athlete test out a new technique.

Despite all advantages and potential in using computer simulation models in human movement research, there are some limitations and drawbacks. Panjabi (1979) argued that a mathematical model was only a set of equations predicting behaviour in unknown situations and that no perfect validation was possible. Yeadon, Atha and Hales (1990) demonstrated that a model could be evaluated by taking input data from a real performance and comparing the simulation output with actual performance.

Vaughan (1984) further recognised two other drawbacks: advanced knowledge in mathematics and computer simulation is required, and that results are often difficult to translate to practicality. With the advancement in information technology and commercially available simulation package (eg. ADAMS, AUTOLEV, DADS), there has been an increased use of simulation models. There are also studies in which authors have combined rigorous mathematical procedures, careful experimental measurements and practical implications for coaches and athletes (eg. Yeadon, 1991).

2.4.2. Diving models

There are a few studies on modelling the airborne phase in diving. Miller (1970) was one of the earliest sports biomechanics researchers to use computer simulation as a research tool. She studied the airborne phase of non-twisting dives and limited her model to four segments. Figgen (1989) developed a eight-segment model to investigate pike and tuck multiple somersaulting dives. While two-dimensional (2D) models have been employed to examine somersaulting dives, Yeadon (1993a, b, c, d, e) used a three-dimensional (3D) model to study twisting dives.

Wooten and Hodgins (1996) developed a 3D dynamic model with 32 DOF to simulate platform diving. Although the authors regarded the model as satisfactory, there were obvious discrepancies when the model output was compared with actual performance. The video of the simulation did not appear natural looking as suggested by the authors. Wooten and her colleague (Wooten & Hodgins, 1996; Wooten, 1998) realised that the input parameters of the model did not account for human physiological limits. This meant that the simulated diver could perform dives using strategies that are impossible for humans. Wooten (1998) concluded that their simulation model had the potential to be useful in athletic performance if a more accurate and valid biomechanical model could be constructed.
Efforts have also been made to model the takeoff in springboard diving. Jiang and his colleagues (Jiang & Sheng, 1993; Jiang et al., 2000a; Jiang et al., 2000b; Jiang et al., 2001; Yu et al., 1997) used a two-segment model of the body and a mass-spring system to represent the springboard to obtain equations of motion together with an analytical solution. It was proposed that kinematic data could be used to determine the driving force representing knee extension. However, the inputs of the model were hypothetical values and the model was not evaluated. Any conclusion or suggestions based on the output of the model should therefore be viewed with caution.

Liu and Wu (1989) modelled the diver as a single mass and the springboard as a mass-spring system to search for the best instant for takeoff. However, these authors have made serious mistakes in generating equations of motion such that their reported values of stiffness (13428.6 N/m) and effective board mass (130 kg) are clearly incorrect (see Table 2.1). It is not entirely surprising that this model comes up with a conclusion that the diver should stamp when the board is going up, which is exactly the opposite to empirical observation (Miller et al., 1998; Sanders & Wilson, 1988).

Sprigings and Watson (1985) used a two-segment model of the diver together with a mass-spring model of the springboard to search for the optimal timing of the armswing during takeoff. It was found that the upward acceleration of the arms with respect to the shoulders should commence at the moment of board contact. Sprigings et al. (1986) later modified the model to three segments to investigate the timing of the relative force patterns of the arms, torso and legs during takeoff. The authors suggested that modification should only be made to the timing but not the already learned movement pattern. Such concern takes into account the physiological significance of the human body, which some theoretical studies omit.

Boda (1992) attempted to model the springboard and the diver as an oscillating system. Oscillation frequency of individual divers during a standing backward takeoff on a springboard and on a force plate were collected for statistical analysis. With the objective of gaining maximal height, a regression equation was derived to predict the optimal fulcrum number based on preferred fulcrum number, oscillation rate on land and body weight. Whilst this equation may provide some guidelines for novice divers, it may not be as useful to elite divers without taking into account any strength parameters.

Most recently, Sprigings and Miller (2002) modelled the diver as a planar five-segment linked system with torque generator at each joint, and the springboard as mass-spring element with no damping. The model was used to optimise the takeoff technique in
dives from the reverse group. The simulation result suggested that controlled knee extension commenced late in the board depression phase was the best. This was, however, contrary to the fact that most divers begin knee extension early in the depression phase. Since the input variables used in the simulation were not determined from experiment, the model may produce results which exceed human limit. Nevertheless, this model was the pioneer work in optimising diving takeoff techniques using a torque-driven simulation model.

All of the above studies use a linear mass-spring model to represent the springboard and take maximum height as the ultimate objective. The question of how to define the objective for springboard diving performance has been raised by Kooi and Kuipers (1994) a decade ago. Even though it is agreed that there is a compromise between height, distance and angular momentum during takeoff, it is surprising that no studies have attempted to optimise the takeoff techniques in terms of generating both linear and angular momentum.

2.4.3. Non-diving takeoff models

Models of takeoff in other sports can provide some insight and better understanding for springboard diving takeoffs. Hatze’s (1981) model for long jumping consisting of 17 segments with 42 DOF and 46 muscle groups was one of the most comprehensive models that had been used. Despite the time-consuming procedures in gathering input data, this model was apparently successful in optimising performance and providing practical implications. Anderson and Pandy (1999) developed a 10-segment, 3D model actuated by 54 muscles with 23 DOF. Quantitative comparisons between model and experiment indicated that the model could reproduce the kinematic, kinetic and muscle-coordination patterns during vertical jumping (Pandy & Anderson, 2000). During the last decade, there has been a growing development of similar models for jumping with the objective of maximizing jump height (eg. Fujii, 1989; Spagele, Kistner & Gollhofer, 1999).

Blajer and Czaplicki (2001) presented a 2D simulation model of front and back somersaults on the trampoline. The trampolinist was modelled as a seven-segment rigid multi-body system and the trampoline bed was modelled as weightless canvas of measured stiffness and damping characteristics. The model was evaluated by comparing the simulation with performance of the trampolinist. While the simulation replicated the actual performance closely during the flight phase, the authors suggested that a better accuracy of measurement of bed deflection during the support phase was needed. Since the support
phase on the trampoline is similar to a springboard diving takeoff, it is predicted that there may be difficulties in accurately modelling the springboard.

Yeadon and King (2002) developed a 5-segment torque-driven model of tumbling takeoff using subject-specific parameters as model inputs (King & Yeadon, 2002). This model has been used to investigate the coping of perturbations to layout somersault performance (King & Yeadon, 2003) and to optimise somersault rotation (King & Yeadon, 2004).

2.4.4. Wobbling mass models

Most biomechanical models of the human body are composed of rigid segments. Body segments, however, are not rigid especially during motions such as landings and impacts which have high accelerations. Therefore, it is necessary to take into account the varying composition of the body, namely the rigid skeletal part and the soft components like tendons, muscles and organs.

Back in the 1970s, Cavagna (1970) conducted a study to examine the elastic bounce of the human body. Subjects performed a small vertical jump and landed on the balls of the feet with straight legs and contracting calf muscles. The force-extension curve of the elastic structure was similar to that of the series elastic elements of an isolated muscle. Nigg and Liu (1999) used a lumped mass-spring-damper system to investigate the impact force in running with different shoe properties. In their model, the wobbling mass was attached to the rigid segment through spring-dampers representing the elastic properties of a muscle-tendon unit.

It has been demonstrated that wobbling mass models reproduce the ground reaction force (GRF) during the initial phase of landing better than rigid body models. Gruber et al. (1998) developed a three-segment model including a rigid part and a soft part in each segment to study a drop jump from a height of 0.4 m with a heel landing. Compared to a rigid model, this wobbling mass model reproduced the GRF more closely during the early impact phase of 20 ms. Similarly, Pain (1999) presented a four-segment wobbling mass model to simulate landing of a drop from 0.43 m which successfully reproduced the vertical GRF for the first 80 ms of landing.

In the studies of Gruber et al. (1998) and Pain (1999), the calculated joint torques and forces using a forward dynamics rigid model were much larger than those calculated using the wobbling mass model. Gruber et al. (1998) concluded that the large joint forces and torques obtained from inverse dynamics were probably the result of the inappropriate
use of rigid segment models. This imposes a challenge on the common practice of using inverse dynamics to estimate internal joint forces and torques in relation to injuries.

More recently, Yue and colleagues (2001; 2002) investigated the effects of wobbling mass during whole-body vibration. Their studies showed that including wobbling masses reduced the total internal load compared with a rigid mass model. In contrast to Gruber et al.'s (1998) argument that internal joint forces and torques are likely overestimated, their studies indicate that the partial internal load on a certain body part may actually be larger than the total load due to phase differences among partial loads.

To date, some work has been done on the effect of wobbling masses during an impact on a rigid surface. Little is known about the impact on a compliant surface such as a springboard. With a springboard being less stiff, it can be speculated that the impact between the body and the springboard would not be as high as in the case of the ground.

2.4.5. Summary

The use of computer simulation models in sports biomechanics research has become more and more popular with the advance of information technology. The choice of equations and the complexity of the model depend highly on what questions the researcher wants to answer. Assumptions made in the model can be justified when the model replicates actual performance well. After the model has been successfully evaluated, it can be used to investigate techniques and to optimise performance.

2.5. Simulation model input

2.5.1. Introduction

A simulation model of the human body requires initial conditions and model parameters as input. Model parameters such as strength and body segmental inertias can be determined directly from experiments or indirectly through an optimisation process when direct measurements are not possible. Body and joint kinematics specifying the initial conditions to the model can be obtained from video recordings of actual performances.

2.5.2. Strength parameters

2.5.2.1. Introduction

In a torque-driven simulation model, simulation outputs are highly dependent on the strength parameters input to the model. In order for the model to predict realistic
human movement, sensible strength parameters within human capacity are needed. This section discusses the current methods in muscle modelling and strength measurement.

2.5.2.2. Muscle modelling

Muscle modelling can be categorised into molecular models and macroscopic models. Molecular models, eg. Huxley's (1957) model, are based on the cross-bridge mechanism of muscle contraction and thus are extremely complex. Macroscopic models, eg. Hill's (1938) model, take a more of a black-box approach based on interpretations of input-output data obtained from controlled experiment.

Classical structures for the Hill model include a contractile element (CE), a series elastic element (SE) and a parallel element (PE) (Figure 2.3). The CE represents the muscle fibres, the SE represents tendons and other elastic tissues in series with the fibres, and the PE represents the passive properties of the fibres and elastic tissue surrounding the muscle fibres. The CE can be described by a hyperbolic force-velocity relationship derived from an experiment on an isolated, tetanically stimulated frog skeletal muscle (Hill, 1938):

\[
(F + a)(v + b) = (F_{\text{max}} + a)b
\]

where \( F \) is muscle tensile force, \( v \) is muscle shortening velocity, \( F_{\text{max}} \) is the maximum isometric force, and \( a \) and \( b \) are constants. Based on basic thermodynamic principles, this model has been used successfully to describe the force-velocity behaviour of muscles. It is also sufficiently simple even if large numbers of such models are incorporated in multi-segment models (Zahalak, 1990).

![Figure 2.3. Classical structures for the Hill model, with contractile element (CE), series elastic element (SE) and parallel elastic element (PE).](image)

Over years, the Hill model has been modified (e.g. Bobbert, Huijing & van Ingen Schenau, 1986; Hatze, 1981) and the activation state of muscles are incorporated (e.g. Meijer, Grootenboer, Koopman, van der Linden & Huijing, 1998; Racz, Beres, Hortobagyi
& Tihanyi, 2002). Attempts have been made to replace muscle force and shortening velocity with torque and joint angular velocity (e.g. Hawkins & Smeulders, 1998). Later experiments show that a double-hyperbolic force-velocity relation describes a single muscle fibre behaviour better than the classical Hill hyperbola (Edman, 1988).

It can be argued that the Hill-type model is too simple and fails to capture certain fundamental features of real muscle. Despite these limitations, the Hill-type model and modifications thereof are widely used in studies of multiple muscle systems due to their simplicity, familiarity and a direct connection with macroscopic experiment (Zahalak, 1990). Application of Hill-type models to sports performance ranges from a single torque generator (Alexander, 1990) to very complex models, for example, Anderson and Pandy’s (1999) model consisting of 10-segment, 23 DOF and 54 muscles.

There is no perfect model to represent muscle behaviour. Some compromise must be made between the complexity of the model and the computational time to simulate movement. For a whole body multi-segment model, a single torque generator for each joint movement is considered adequate though it cannot account for the effect of biarticular muscles.

2.5.2.3. Strength measurement

To obtain muscle parameters for modelling, muscle forces or joint torques of a particular movement have to be determined. Experimental results show that electromyography (EMG) reflect torque production (Cramer et al., 2002) and suggest a linear relationship between EMG and muscle force (see Hof, 1984 for review). Quantitative methods have been proposed to predict muscle force from EMG data (Hof & van den Berg, 1981a, b, c, d), although there have been discrepancies among studies. Within the same muscle group, different EMG / force relationships were found among three surface quadriceps muscles during isometric contraction (Alkner, Tesch & Berg, 2000). Thus, it appears that the relationship between EMG and muscle force remains inconclusive.

For a single torque generator, the net torque about a joint rather than individual muscle force is required. The net joint torque can be measured by using isovelocity dynamometry. Recent electromechanical dynamometers (e.g. Biodex, Cybex, Kin-Com) allow isometric, concentric and eccentric joint movements. Torque data recorded by the dynamometer should be corrected for the effect of gravity (Winter, Wells & Orr, 1981). There are also problems associated with inertial effects and non-rigidity of the machine.
arm and human system (Herzog, 1988). It is recommended that the effects of angular acceleration for the dynamometer and human segment should be accounted for if the initial and final phase of the movement are of interest or if the angular velocity is high (240°/s or higher). Additionally, the joint axis and the axis of rotation of the dynamometer arm must be aligned carefully before each movement and the subject should be strapped firmly to the dynamometer.

The torque / angular velocity relationship over a joint can be obtained by collecting data at several angular velocities. It has been proposed that using peak torque measured at each angular velocity is superior to using angle specific torques (Kawakami, Kubo, Kanehisa & Fukunaga, 2002) and that mean torques fit the Hill equation better than the peak torques (Racz et al., 2002). To include the torque / angle relationship, a 3D surface representation of maximum torque in relation to angle and angular velocity has been used (e.g. Khalaf, Parnianpour & Karakostas, 2000; Wilson, 2003).

It has been demonstrated that knee extension torque is influenced by the hip angle and starting knee angle (Pavol & Grabiner, 2000). Thus, the common practice of characterising joint torque measured over a certain range of motion in a single body position should be accepted with uncertainty in inferring strength capabilities over the range of motion and body positions other than those tested. This could be attributed to role of biarticular muscles which act across two joints. Attempts have been made to examine contribution of biarticular muscle using simulation models (e.g. Bobbert & van Zandwijk, 1994; Jacobs, Bobbert & van Ingen Schenau, 1996). For a single joint torque generator, it is desirable to take into account the influence of adjacent joint movements to the joint torque of interest.

Many muscle-driven simulation models obtain typical muscle parameter values from the literature, and therefore the simulation models are not specific to an individual (Yeadon & Challis, 1994). Hawkins and Smeulders (1999) attempted to develop a generalised model based on average individual torque / velocity behaviour obtained from experiments. They found that inter-subject variations in torque / velocity responses limited the utility of a generic model based on average parameter values.

In order to obtain subject-specific strength parameters, King and Yeadon (2002) measured joint torques from an isovelocity dynamometer. Having simulated the CE and SE characteristics, torque was expressed as an 18-parameter exponential function of joint angle and angular velocity:
where $T$ is torque, $\omega$ is joint angular velocity, $a$, $b$, $c$, $d$, $p$, $q$ are parameters expressed as a quadratic function of joint angle. This method was used to personalise a torque-driven simulation model of tumbling to an individual (Yeadon & King, 2002). Wilson (2003) used a similar method to obtain a 9-parameter function for a torque driven jumping model.

2.5.2.4. Bilateral deficit

Bilateral deficit is a phenomenon where the force produced during a maximal bilateral (BL) action is less than the sum of the forces produced during two maximum unilateral (UL) actions. With a few exceptions, there have been many studies providing evidence of bilateral deficit in both upper and lower limbs (see Jakobi & Chilibeck, 2001 for review). Tasks that have been investigated include: single joint isometric (Kawakami, Sale, MacDougall & Moroz, 1998; Koh, Grabiner & Clough, 1993; Oda & Moritani, 1994; Schantz, Moritani, Karlson, Johansson & Lundh, 1989; Secher, Rube & Elers, 1988) and isovelocity movement (Cresswell & Ovendal, 2002), multi-joint isometric (Behm, Power & Drinkwater, 2003; Secher, 1975) and isovelocity movement (Vandervoort, Sale & Moroz, 1984), and dynamic complex skills such as jumping (Challis, 1998; van Soest, Roebroeck, Bobbert, Huijing & van Ingen Schenau, 1985).

It has been shown that the degree of bilateral deficit depends on joint angular velocity (Vandervoort et al., 1984) and adjacent joint position (Kawakami et al., 1998). On average, the BL force is 10% less than the sum of the UL forces (Jakobi & Chilibeck, 2001). Parallel EMG data providing information on muscle activation during BL and UL actions have been very inconsistent. This inconsistency may be due to the differences in subject background, equipment, testing protocol and data reduction procedures. Overall, there seems to be some agreement on the possible mechanisms that can explain bilateral deficit. Firstly, the decrease in BL force is not due to increased antagonist activity (Behm et al., 2003; Cresswell & Ovendal, 2002; Kawakami et al., 1998). Secondly, it is likely that BL deficit is associated with a reduction of fast motor units recruitment (Kawakami et al., 1998; Koh et al., 1993; Vandervoort et al., 1984). Thirdly, BL deficit is influenced by training (Howard & Enoka, 1991; Secher, 1975; Secher et al., 1988).

Secher observed that there was no BL deficit in experienced rowers. Secher et al. (1988) showed that BL / UL strength ratio increased from $80 \pm 2.5 \%$ to $97 \pm 2.9 \%$ after
five weeks of familiarisation with the experimental apparatus. Howard & Enoka (1991) found that untrained subjects showed BL deficit but trained cyclists did not, and that trained weight-lifters exhibited bilateral facilitation. On the other hand, there are a few contradicting studies which either demonstrate no BL deficit in untrained subjects (e.g. Jakobi & Cafarelli, 1998) or BL deficit in trained subjects (e.g. van Soest et al., 1985). Some of these studies, however, were defective in experimental design and / or methodology. For example, in Jakobi and Cafarelli's (1998) study, visual feedback was provided throughout the testing. The psychological effect of providing feedback has been demonstrated by Secher et al. (1988). There was a problem in the study done by van Soest et al. (1985) that only the left leg was selected without concern about the potential difference in dominant and non-dominant leg. Behm et al. (2003) even made a fundamental mistake in comparing the force differences between leg extension and squat exercises. At the same time, there are also studies in which the results agree with the majority but the methodology is incorrect. For example, Cresswell and Ovendal (2002) employed an inappropriate statistical analysis which calculated group means instead of individual BL / UL ratios.

Overall, most studies agree on the effect of training on BL deficit. The takeoff phase in springboard diving for somersaulting dives is a highly symmetrical movement for good performance. It is therefore speculated that a diver will not demonstrate a high degree of BL deficit.

2.5.2.5. Summary

Regarding the nature of this study, a Hill-based single joint torque generator for each joint action is considered adequate. The maximum joint torque for each movement can be measured using an isovelocity dynamometer to obtain a 3D torque / angle / angular velocity relationship. The torque parameters will be input into the model to limit the strength of the model to a realistic range.

2.5.3. Body segmental inertias

Body segmental inertia parameters include the mass, location of mass centre and principal moment of inertia about the mass centre of each segment. These parameters can be determined directly from the dissection of cadavers (Chandler, 1975; Dempster, 1955). Attempts have been made to estimate such parameters from anthropometric measurement of living subjects by using linear regression equations based on cadaver studies (e.g.
Hinrichs, 1985). In a subject-specific computer simulation model, it is preferable to use personalised segmental inertia parameters. There are several ways to obtain subject specific segment inertia parameters.

Forwood, Neal and Wilson (1985) presented a scaling method and Hinrichs (1985) developed a set of linear regression equations to estimate segmental moments of inertia for individual subjects from cadaver data. Yeadon and Morlock (1989) demonstrated that non-linear equations were superior to linear equations and that non-linear equations could provide reasonable estimates of segmental moments of inertia even when the anthropometric measurements lay outside the sample range of cadaver data.

Another common method is modelling body segments as a series of geometric solids where dimensions can be measured directly on the subject. Inertial properties are then calculated using density data from cadaver studies. Examples of mathematical models are those of Hanavan (1964), Jensen (1976), Hatze (1980) and Yeadon (1990b).

Radiation-based methods provide another means to obtain subject specific segmental inertia parameters. For example, Zatsiorsky and colleagues (1983, 1985, 1990) used a gamma-ray scanning technique to determine the inertia parameters of 100 young men and developed sets of predictive regressive equations. This method was slightly adjusted by de Leva (1996) to facilitate practical application.

Sarfaty and Ladin (1993) developed a video-based system to estimate body segmental inertia parameters for individual subjects. The system consists of an image-processing component which provides anthropometric information and uses body density data from literature. Further investigation and development of this technique is needed. Magnetic resonance imaging (MRI) and other scanning techniques (e.g. Norton, Donaldson & Dekker, 2002) provide accurate measurements but are too costly at the moment.

Kwon (1996; 2000; 2001) investigated the effects of different methods of body segmental inertia parameters estimation on the experimental simulation of a complex airborne movement and assessed the applicability of these methods regarding their accuracy, flexibility and simplicity. He concluded that more individualised methods provided more accurate simulation results.
2.5.4. Kinematic data

2.5.4.1. Image analysis

Recently, 3D image analysis using video cameras has been widely used to investigate sports movements. Performance is recorded using at least two cameras and body landmarks are digitised manually or by using an automatic tracking system. Synchronised data can be obtained by using genlocked video cameras, a timing device, critical events, or a mathematical approach (Pourcelot, Audigie, Degueurce, Geiger & Denoix, 2000; Yeadon & King, 1999). To reconstruct the 3D locations of body landmarks from digitised image coordinates, the direct linear transformation (DLT) (Adbel-Aziz & Karara, 1971) or modifications thereof (e.g. Hatze, 1988) are commonly used. The DLT method requires at least six control points for the calibration of each camera. Once the 11 DLT parameters for each camera are known and the associated image coordinates are obtained, the spatial coordinates can be computed. Control points should be distributed evenly throughout the calibration volume within which the event of interest should occur (Yeadon & Challis, 1994). To overcome problems of analysing events that take place over large areas, panning techniques (Yu, Koh & Hay, 1993) and / or tilting cameras (Yeadon, 1989) have been developed.

2.5.4.2. Data smoothing

Raw kinematic data determined from digitisation are discrete points at a specific time depending on the sampling frequency. Fitting a function through the discrete data points will obtain interpolated values and time derivatives. Due to the nature of numerical differentiation, noise must be reduced to avoid amplified errors in estimated derivatives. When selecting a smoothing technique, consideration must be given to the nature of the technique used and the degree of smoothing required. The most popular functions which have been used include a Butterworth digital filter and finite difference formulae, splines and truncated Fourier series (see Yeadon & Challis, 1994 for review).

The digital filter, splines and Fourier series all produce good fits to displacement data but the digital filter does not provide a smooth analytical function for future computations whilst the Fourier series requires equispaced data (Wood, 1982). Applying to biomechanical data, Wood (1982) identified the spline functions as the most ideal for interpolation of time history data. The quintic spline (Wood & Jennings, 1979) has been shown to be most appropriate for smoothing and obtaining second derivative data (Burkholder & Lieber, 1996; Challis & Kerwin, 1988; Woltring, 1985). It also
demonstrates superiority in producing accurate acceleration data in the endpoint region compared to other popular methods (Vint & Hinrichs, 1996).

2.5.5. Optimisation

Optimisation is often used with simulation models to search for an optimal solution, for example, maximising somersault rotation in tumbling (King & Yeadon, 2004), or to determine unknown model parameters such as the force-sharing among a muscle group (Ait-Haddou, Jinha, Herzog & Binding, 2004). Various optimisation methods have been employed in biomechanical models to optimise sports techniques such as the Simulated Annealing (Corana, Marchesi, Martini & Ridella, 1987), the downhill simplex method (Press, 1997) and the Powell's algorithm (Press, Teukolsky, Vetterling & Flannery, 1992). It has been demonstrated that the Simulated Annealing (Corana et al., 1987) has the advantage over other common conventional algorithms since it is very robust and can find the global solution rather than a local optimum (Goffe, Ferrier & Rogers, 1994). With the advancement in technology, the computation time for optimisation has been reduced enormously over years. This makes optimisation of more complex functions possible, for example, individual muscle modelling in a whole-body model.

2.5.6. Summary

For a torque-driven simulation model, the net torque across individual joint can be measured on an isovelocity dynamometer to obtain subject-specific strength parameters. Body segmental inertia parameters can be calculated from anthropometric measurement by using a mathematical model. Kinematic data of the movement of interest can be obtained from digitisation of video recordings followed by appropriate smoothing.

2.6. Summary of literature review

This chapter has reviewed the literature on diving, springboard characteristics and computer simulation models. The pros and cons of different research techniques are discussed and limitations of previous research are addressed. Based on the information obtained from the literature, this study aims to further investigate the mechanics of springboard diving takeoffs through the use of simulation models.
CHAPTER 3
MODEL DEVELOPMENT

3.1. Introduction

In order to investigate the mechanics of springboard diving takeoffs, a simulation model of a springboard and a diver was developed. For the diver, an angle-driven model and a torque-driven model were developed. This chapter describes the model features in detail.

3.2. The springboard model

During a springboard diving takeoff, the springboard is depressed and then recoils along a curvilinear path, projecting the diver upwards and forwards into the flight. Modelling the springboard alone to reflect its extreme complexity requires extensive computation (Kooi & Kuipers, 1994). In order to incorporate the springboard into a diver / springboard system, a simpler model with reasonable accuracy is preferable. Although a single degree of freedom (DOF) 'bar model' has been proposed (Kooi & Kuipers, 1994), a linear mass-spring model (Sprigings, Stiling & Watson, 1989) is generally accepted. The mass-spring model represents only the vertical behaviour of the springboard and thus the vertical reaction force acting on the diver. It should be noted that the springboard deflects in a curvilinear path, providing also a horizontal reaction force which plays an important role in the generation of angular momentum and board clearance (Miller et al., 1990). In addition, the springboard rotates as it defects which influences the divers' orientation. This chapter presents a new model of the springboard which describes its vertical, horizontal and rotational behaviour.
The springboard was modelled as a 0.3 m rod with three DOF: vertical (z), horizontal (x) and rotational (θ) movement (Figure 3.1).

![Diagram of springboard modelled as a rod with three degrees of freedom. The origin (x = 0, z = 0, θ = 0) is at the board tip when the board is in a resting position.](image)

**3.2.1. Vertical movement**

The vertical behaviour of the springboard was modelled as a linear mass-spring system with no damping (Sprigings et al, 1989):

\[
F_z = -kz \tag{3.1}
\]

where \( F_z \) = vertical reaction force  
\( k \) = vertical spring stiffness  
\( z \) = vertical spring displacement.

The vertical stiffness was allowed to vary depending on foot position such that the further away from the board tip, the stiffer the board. It was expressed as a linear function of foot position:

\[
k = md + c \tag{3.2}
\]

where \( m \) = slope  
\( c \) = constant  
\( d \) = parallel distance between the board tip and the diver’s mass centre along the springboard.
3.2.2. Horizontal movement

The horizontal movement of the board tip was constrained as a function of vertical movement. From experimental data (see Chapter 4), a quadratic function was found to relate the horizontal movement to the vertical movement. This quadratic function can be differentiated twice to obtain horizontal velocity and acceleration:

\[ x = az^2 \]  
\[ \dot{x} = 2azz \]  
\[ \ddot{x} = 2a(zz + z^2) \]

where \( a \) = constant.

3.2.3. Rotational movement

Similarly, the rotational movement was expressed as a function of vertical movement. From experimental data (see Chapter 4), a linear function was found to relate the board rotation angle to the vertical movement. This linear function can be differentiated twice to obtain angular velocity and acceleration:

\[ \theta = bz \]  
\[ \dot{\theta} = b\dot{z} \]  
\[ \ddot{\theta} = b\ddot{z} \]

where \( b \) = constant.

3.3. The diver model

A simulation model of a diver and a single segment of springboard (Figure 3.2) were developed using the software package Autolev 3.4\textsuperscript{TM} based on Kane's method of formulating equations of motion (Kane & Levinson, 1985). The diver was represented by an eight-segment planar model comprising head + neck, upper arm, lower arm + hand, trunk, thigh, shank and a two-segment foot. The foot was modelled as a triangle and a rod connected at the metatarsal-phalangeal joint. The diver's orientation was specified by \( \theta_0 \), the angle between the trunk segment and the horizontal towards the water. The seven internal joint angles were ball (\( \theta_b \)), ankle (\( \theta_a \)), knee (\( \theta_k \)), hip (\( \theta_h \)), shoulder (\( \theta_s \)), elbow (\( \theta_e \)) and head (\( \theta_d \)).
3.3.1. Wobbling mass

Wobbling masses were included within the trunk, thigh and shank segments to represent soft tissue movement (Figure 3.3). Each segment was divided into the fixed and the wobbling component.
The wobbling component was attached to the fixed component through two pairs of identical parallel and perpendicular non-linear damped springs. The springs were nearly critically damped and the spring force was given by the following equation (Pain & Challis, 2001):

\[ F = -kx^3 - cv \]  

where \( F \) = spring force
\( k \) = stiffness
\( c \) = damping
\( x \) = displacement
\( v \) = velocity.

3.3.2. Foot-springboard interface

In a simulation model of diving takeoffs, the springboard was in contact with the diver at the foot. It has been demonstrated that the heel pad deformation plays an important role in energy dissipation during impact (Pain & Challis, 2001) and therefore the elastic properties of a human foot and the springboard should be considered in the model. Gilchrist and Winter (1996) developed a two-segment viscoelastic foot model for gait analysis in which the foot-floor interface was represented by nine pairs of vertical and horizontal spring-dampers. In their study, the horizontal damping was a function of vertical spring force. Besides the magnitude and direction of the spring force, the point of force application relative to the diver’s mass centre is critical in terms of generating angular momentum. In the present study, it was decided to model the foot-springboard interface using three pairs of perpendicular and parallel massless damped springs acting at the toes, ball and heel (Figure 3.4).
Figure 3.4. Modelling the foot-springboard interface with three pairs of visco-elastic spring-dampers acting at the toes, ball and heel.

The perpendicular force $F_z$ in each spring was a function of spring displacement and velocity (Equation (3.10)). The parallel force $F_x$ was dependent on the parallel as well as the perpendicular displacement such that the more pressure on the foot, the more difficult it was for the foot to slide along the springboard surface (Equation (3.11)). When the whole foot was in contact with the springboard, there were forces acting at the heel, ball and toes. During the recoil phase, the foot lost contact from the heel to the toes and there was no force once a point has lost contact.

$$F_z = -k_z z - c_z |z| v_z$$  \hspace{1cm} (3.10)

$$F_x = |z| (-k_x x - c_x |x| v_x)$$ \hspace{1cm} (3.11)

where $k_z =$ perpendicular stiffness
$k_x =$ parallel stiffness
$c_z =$ perpendicular damping
$c_x =$ parallel damping
$z =$ perpendicular displacement
$x =$ parallel displacement
$v_z =$ perpendicular velocity
$v_x =$ parallel velocity.

3.4. The angle-driven model

Since some of the model parameters, for example, the elastic properties of the foot-springboard interface springs, were difficult if not impossible to measure directly from experiment, a subject-specific angle-driven model was developed to determine uncertain
model parameters indirectly. This was achieved by driving the model with known initial conditions and joint angle time histories and optimising the uncertain parameters to minimise the difference between simulation and performance. Subject-specific segmental inertias and springboard parameters determined from experiments were used. The input of the model were initial conditions at touchdown including the foot position, mass centre (CM) horizontal and vertical velocities, trunk angle and angular velocity. Throughout the simulation, the movement of the diver was driven by calculated joint angle time histories obtained from video recordings. The output of the model included time histories of the springboard displacement, the diver’s CM velocities, trunk angle and whole-body angular momentum. Details of optimisation procedures are explained in Chapter 4.

3.5. The torque-driven model

Similar to the angle-driven model, an eight-segment simulation model of a diver with torque generators acting at the metatarsal-phalangeal, ankle, knee, hip and shoulder joint and a single segment springboard model were developed using the software package Autolev 3.4™ (Figure 3.5). The elbow and the head angle were driven by calculated joint angle time histories as in the angle-driven model.

Figure 3.5. An eight-segment torque-driven model of a diver and a springboard.
3.5.1. The torque generators

There were extensor and flexor torque generators at the metatarsal-phalangeal, ankle, knee, hip and shoulder joint. The 10 torque generators were responsible for ball extension (Be), ball flexion (Bf), ankle plantar flexion (Ap), ankle dorsi-flexion (Ad), knee extension (Ke), knee flexion (Kf), hip extension (He), hip flexion (Hf), shoulder extension (Se), and shoulder flexion (Sf). To model the contractile and elastic properties of muscle and tendon, the muscle-tendon complex based on the model of Alexander (1990) was adapted. Each torque generator was modelled as a muscle-tendon complex which included a contractile component (CON) and a series elastic component (SEC). Figure 3.6 shows a graphical representation of the muscle-tendon complex where

- \( \theta \) = joint angle
- \( \theta_{\text{con}} \) = contractile component angle
- \( \theta_{\text{sec}} \) = series elastic component angle.

![Figure 3.6. The muscle-tendon complex consisting of a contractile component and a series elastic component.](image)

The geometric relationships of the joint angle, CON angle and SEC angle were:

- **Flexion:** \( \theta = \theta_{\text{con}} + \theta_{\text{sec}} \)  
- **Extension:** \( \theta = 2\pi - \theta_{\text{con}} - \theta_{\text{sec}} \)
Since the shoulder movement covered a large range, a different definition of shoulder extension angle was used to ensure a positive joint angle. The shoulder flexion and extension angle were:

\[
\text{Shoulder flexion: } \theta = 2\pi - \theta_{\text{con}} - \theta_{\text{sec}} \tag{3.14}
\]

\[
\text{Shoulder extension: } \theta = \theta_{\text{con}} + \theta_{\text{sec}} - \pi \tag{3.15}
\]

The contractile component torque, \( T_{\text{con}} \), was calculated using the 10-parameter function of torque / angle / angular velocity relationship obtained from strength testing on the isokinetic dynamometer (see Chapter 4 for details). The contractile component angle and angular velocity at each time step were calculated by the following procedures.

Initially (time \( t = 0 \)), it was assumed that the contractile component angular velocity \( \dot{\theta}_{\text{con}} \) was the same as the joint angular velocity. \( T_{\text{con}} \), expressed by the 10-parameter function, was set equal to SEC torque, \( T_{\text{sec}} \), which can be given by Equation (3.16)

\[
T_{\text{sec}} = k \theta_{\text{sec}} \tag{3.16}
\]

where \( k \) = SEC stiffness. Substituting for \( \theta_{\text{sec}} \) (Equation (3.12) to (3.15)), the initial \( \theta_{\text{con}} \) was obtained. Using this initial \( \theta_{\text{con}} \) and joint angular velocity, \( T_{\text{con}} \) was then calculated from the 10-parameter function. The \( \theta_{\text{con}} \) was updated for the next time step by integration assuming constant velocity:

\[
\theta_{\text{con}} = \theta_{\text{con}} + \dot{\theta}_{\text{con}} dt \tag{3.17}
\]

where \( dt \) = integration time step.

Afterwards (\( t > 0 \)) in each time step, a new \( \theta_{\text{sec}} \) was calculated from the new joint angle \( \theta \) and the updated \( \theta_{\text{con}} \). The new \( T_{\text{sec}} \) was calculated subsequently. By equating \( T_{\text{con}} \) to \( T_{\text{sec}} \) and substituting for \( \theta_{\text{sec}} \), the new \( \dot{\theta}_{\text{con}} \) could be obtained. Using the new \( \theta_{\text{con}} \) and \( \dot{\theta}_{\text{con}} \), \( T_{\text{con}} \) was calculated using the 10-parameter function. The same procedure as in \( t = 0 \) was used to update \( \theta_{\text{con}} \) for the next time step.
3.5.2. Muscle activation profile

The torque calculated from the 10-parameter function (described in Chapter 4) was the maximum torque that could be produced at a certain CON angle and angular velocity. This torque was then multiplied by an muscle activation level to give the final torque at time \( t \):

\[
TOR(t) = A(t) \cdot T(\theta, \omega) \quad (3.18)
\]

where

- \( TOR(t) \) = torque at time \( t \)
- \( A(t) \) = muscle activation level at time \( t \)
- \( T(\theta, \omega) \) = torque calculated from the torque / angle / angular velocity function.

When the muscle was relaxed, the activation level was 0.0 whereas when the muscle was fully activated, the activation level was 1.0. A quintic function (Yeadon, 1984) was used to ramp up / down the activation level:

\[
q(x) = x^3 (6x^2 - 15x + 10) \quad (3.19)
\]

The function \( q(x) \) increases from 0 to 1 (or decreases from 1 to 0) on interval. The zero end point velocity for its first an second derivatives makes this function favourable for modelling smooth change. The rate of change is slower at the initial and the final phase of the function. This resembles the typical muscle activation pattern recorded by electromyography since muscles are activated and de-activated gradually.

For the extensor torques (and shoulder flexor torque), the activation profile was shown in Figure 3.7. A minimal initial activation level was set to represent pre-landing activation. Six parameters were required to specify the timing and level of activation (Table 3.1).
Figure 3.7. Muscle activation profile for extensor (and shoulder flexor) torque generators.

Table 3.1. Six parameters specifying the extensor muscle activation profile

<table>
<thead>
<tr>
<th>parameters</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$se_1$</td>
<td>starting time of ramping up</td>
</tr>
<tr>
<td>$te_1$</td>
<td>duration of ramping up from zero to maximal activation</td>
</tr>
<tr>
<td>$se_2$</td>
<td>starting time of ramping down</td>
</tr>
<tr>
<td>$te_2$</td>
<td>duration of ramping down from maximal to zero activation</td>
</tr>
<tr>
<td>$le_1$</td>
<td>minimal pre-landing activation level</td>
</tr>
<tr>
<td>$le_2$</td>
<td>maximal activation level</td>
</tr>
</tbody>
</table>

For the flexor torques (and shoulder extensor torque), the activation profile was shown in Figure 3.8. The initial activation was to represent co-contraction at touchdown and the final ramping up activation could contribute to preventing the joint from hyper-extension. Similarly, six parameters were required to specify the timing and level of activation (Table 3.2).
Figure 3.8. Muscle activation profile for flexor (and shoulder extensor) torque generators.

Table 3.2. Six parameters specifying the flexor muscle activation profile

<table>
<thead>
<tr>
<th>parameters</th>
<th>definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s_{f1}$</td>
<td>starting time of ramping down</td>
</tr>
<tr>
<td>$t_{f1}$</td>
<td>duration of ramping down from full to zero activation</td>
</tr>
<tr>
<td>$s_{f2}$</td>
<td>starting time of ramping up</td>
</tr>
<tr>
<td>$t_{f2}$</td>
<td>duration of ramping up from zero to full activation</td>
</tr>
<tr>
<td>$l_{f1}$</td>
<td>maximal pre-activation level</td>
</tr>
<tr>
<td>$l_{f2}$</td>
<td>minimum activation level</td>
</tr>
</tbody>
</table>

3.6. Summary

A model of the springboard with three DOF and a eight-segment linked model of a diver were developed to simulate springboard diving takeoffs. The angle-driven model will be used to determine model parameters which could not be measured directly from experiments. These parameters will then be used in the torque-driven model which, after satisfactory evaluation, will be applied to investigate diving takeoff techniques.
CHAPTER 4
PARAMETER DETERMINATION

4.1. Introduction

Subject-specific parameters were required as inputs to the model. The parameters included 1) springboard, 2) body segmental inertia, 3) strength, 4) wobbling mass, and 5) foot-springboard interface. In addition, kinematic data of diving performance were needed in order to compare simulation outputs with performance for model evaluation. This chapter describes how model parameters and kinematic data were determined directly from experiments and how other parameters were determined indirectly using a subject-specific angle-driven model.

4.2. Kinematic data collection

4.2.1. Camera set-up

A Phantom high speed camera (Vision Research, Inc) was used to record diving performances from a one-meter springboard. The camera was set-up with the field of view covering the whole sequence from the hurdle step to the entry of a dive (see Figure 4.1). The filming rate was 200 Hz with 2502 µs exposure time under normal pool environment with no extra lighting. This camera set-up allowed a maximum 5-second recording time and the video data were stored onto a laptop computer connected to the camera.

4.2.2. Data collection

An elite female diver competing at junior international level (mass = 64.1 kg, height = 1.68 m) participated in the study. Informed consent (Appendix 2a) and a subject profile of training and competition background (Appendix 2b) were obtained. The diver was asked to perform all dives that she could do in the forward and reverse groups from a one-metre springboard. Each dive was performed once if satisfactory otherwise a second attempted was allowed. The diver performed 18 dives in total with no repetitions (see Table 4.1).
Figure 4.1. The camera set-up with a field of view covering the whole sequence from the hurdle step to the entry of a dive.

Table 4.1. Dives performed in the forward and reverse group

<table>
<thead>
<tr>
<th>forward group</th>
<th>reverse group</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive</td>
<td>no.</td>
</tr>
<tr>
<td>forward jump</td>
<td>100A</td>
</tr>
<tr>
<td>forward dive tuck</td>
<td>101C</td>
</tr>
<tr>
<td>forward dive pike</td>
<td>101B</td>
</tr>
<tr>
<td>forward somersault tuck</td>
<td>102C</td>
</tr>
<tr>
<td>forward somersault pike</td>
<td>102B</td>
</tr>
<tr>
<td>forward 1-1/2 somersault tuck</td>
<td>103C</td>
</tr>
<tr>
<td>forward 1-1/2 somersault pike</td>
<td>103B</td>
</tr>
<tr>
<td>forward double somersault tuck</td>
<td>104C</td>
</tr>
<tr>
<td>forward double somersault pike</td>
<td>104B</td>
</tr>
<tr>
<td>forward 2-1/2 somersault tuck</td>
<td>105C</td>
</tr>
<tr>
<td>forward 2-1/2 somersault pike</td>
<td>105B</td>
</tr>
</tbody>
</table>
4.2.3. Dynamic loading of the springboard

The diver used fulcrum number 7.5 for all dives. A dynamic loading of the springboard (Miller & Jones, 1999) at this fulcrum number was performed before and after recording the diving performances. This was done by having the diver standing on the end of the board, setting the board into motion by raising her arms once, and keeping her body as rigid as possible with the arms adducted for about 10 oscillations. The same procedure was used to load the board with a different body weight (mass = 96.8 kg). Five seconds of data were saved to ensure at least five oscillations were recorded.

4.2.4. Camera calibration

After the high speed video recording, a calibration pole with seven balls (ball numbers 0 to 6 from bottom to top) was positioned in six different known locations, three at the far side (P1 - P3) and three at the near side (P4 - P6) of the pool deck. A short period was recorded in each position and the horizontal location of the bottom of the pole was measured. Figure 4.2 shows a two-dimensional (2D) plan of the pool and the measured x-y coordinates of P1 to P6. It was assumed that the RHS of the body (x = 0) was -0.1m away from the middle of the board. It should be noted that the x, y and z directions here were different from those defined in the simulation model.

Figure 4.2. Plan of the pool and x-y coordinates of the calibration pole in six different positions.
Placing the pole horizontally on the pool deck, the distance of each ball to the bottom of the pole was measured. These measured values were comparable to the manufacturing reference values (Table 4.2).

Table 4.2. Distance of balls to the bottom of the calibration pole

<table>
<thead>
<tr>
<th>ball no.</th>
<th>measured distance (mm)</th>
<th>manufacturing reference (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>105</td>
<td>99</td>
</tr>
<tr>
<td>1</td>
<td>1186</td>
<td>1183</td>
</tr>
<tr>
<td>2</td>
<td>2160</td>
<td>2165</td>
</tr>
<tr>
<td>3</td>
<td>3154</td>
<td>3166</td>
</tr>
<tr>
<td>4</td>
<td>4139</td>
<td>4149</td>
</tr>
<tr>
<td>5</td>
<td>5155</td>
<td>5168</td>
</tr>
<tr>
<td>6</td>
<td>6100</td>
<td>6100</td>
</tr>
</tbody>
</table>

Since the pool deck was not level, the pole leaned towards the water (Figure 4.3). The amount of leaning (θ) was calculated by the length of the pole (l) and the estimated horizontal deviation (a) of the top ball (Table 4.3).

Figure 4.3. The x-z coordinates of the control points were adjusted accordingly to the amount of pole leaning.
Table 4.3. Estimated amount of pole leaning

<table>
<thead>
<tr>
<th>pole position</th>
<th>horizontal deviation $a$ (m)</th>
<th>amount of leaning $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.17</td>
<td>1.5970°</td>
</tr>
<tr>
<td>2</td>
<td>0.20</td>
<td>1.8789°</td>
</tr>
<tr>
<td>3</td>
<td>0.23</td>
<td>2.1608°</td>
</tr>
<tr>
<td>4</td>
<td>0.20</td>
<td>1.8789°</td>
</tr>
<tr>
<td>5</td>
<td>0.20</td>
<td>1.8789°</td>
</tr>
<tr>
<td>6</td>
<td>0.20</td>
<td>1.8789°</td>
</tr>
</tbody>
</table>

The x-z coordinates of seven balls of the pole (ball number 0 to 6 from bottom to top) were then calculated from the measured distance of the ball and the degree of pole leaning. As illustrated in Figure 4.3, the height $z$ and horizontal deviation $a$ were calculated as follows:

\[
z_i = l_i \cos \theta \\
a_i = l_i \sin \theta
\]

where $i = 0 - 6$ depending on ball number.

The x-coordinates shown in Figure 4.2 were adjusted with respect to both the positions of the pole and the ball to account for the leaning of the pole.

For position P1 to P3: \[x_i = 21.4 - a_i\]  (4.3)
For position P4 to P6: \[x_i = -6.23 + a_i\]  (4.4)

All seven balls were within the field of view of the camera in position P1, P2 and P3; whereas only ball 2 could be seen in position P4, P5 and P6. The $x,y,z$-coordinates of all calibration points are shown in Table 4.4.
Table 4.4. Space coordinates of the 24 control points for camera calibration

<table>
<thead>
<tr>
<th>point*</th>
<th>x</th>
<th>y</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>P10</td>
<td>21.367</td>
<td>-10.195</td>
<td>0.1050</td>
</tr>
<tr>
<td>P11</td>
<td>21.367</td>
<td>-10.195</td>
<td>1.1795</td>
</tr>
<tr>
<td>P12</td>
<td>21.340</td>
<td>-10.195</td>
<td>2.1592</td>
</tr>
<tr>
<td>P13</td>
<td>21.312</td>
<td>-10.195</td>
<td>3.1528</td>
</tr>
<tr>
<td>P15</td>
<td>21.238</td>
<td>-10.195</td>
<td>5.1530</td>
</tr>
<tr>
<td>P16</td>
<td>21.230</td>
<td>-10.195</td>
<td>6.0976</td>
</tr>
<tr>
<td>P20</td>
<td>21.397</td>
<td>-5.563</td>
<td>0.1049</td>
</tr>
<tr>
<td>P21</td>
<td>21.361</td>
<td>-5.563</td>
<td>1.1794</td>
</tr>
<tr>
<td>P22</td>
<td>21.329</td>
<td>-5.563</td>
<td>2.1588</td>
</tr>
<tr>
<td>P23</td>
<td>21.297</td>
<td>-5.563</td>
<td>3.1523</td>
</tr>
<tr>
<td>P24</td>
<td>21.264</td>
<td>-5.563</td>
<td>4.1368</td>
</tr>
<tr>
<td>P25</td>
<td>21.231</td>
<td>-5.563</td>
<td>5.1522</td>
</tr>
<tr>
<td>P26</td>
<td>21.200</td>
<td>-5.563</td>
<td>6.0967</td>
</tr>
<tr>
<td>P30</td>
<td>21.396</td>
<td>-1.030</td>
<td>0.1049</td>
</tr>
<tr>
<td>P31</td>
<td>21.356</td>
<td>-1.030</td>
<td>1.1792</td>
</tr>
<tr>
<td>P32</td>
<td>21.319</td>
<td>-1.030</td>
<td>2.1585</td>
</tr>
<tr>
<td>P33</td>
<td>21.281</td>
<td>-1.030</td>
<td>3.1517</td>
</tr>
<tr>
<td>P34</td>
<td>21.244</td>
<td>-1.030</td>
<td>4.1361</td>
</tr>
<tr>
<td>P35</td>
<td>21.206</td>
<td>-1.030</td>
<td>5.1513</td>
</tr>
<tr>
<td>P36</td>
<td>21.170</td>
<td>-1.030</td>
<td>6.0957</td>
</tr>
<tr>
<td>P42</td>
<td>-6.159</td>
<td>-2.3700</td>
<td>2.1588</td>
</tr>
<tr>
<td>P52</td>
<td>-6.159</td>
<td>-3.7070</td>
<td>2.1588</td>
</tr>
<tr>
<td>P62</td>
<td>-6.159</td>
<td>-2.9870</td>
<td>2.1588</td>
</tr>
</tbody>
</table>

*Point position P10: 1 = pole position, 0 = ball number, etc.

Note: the x, y and z directions here were different from those defined in the simulation model.
A three-dimensional (3D) direct linear transformation (DLT) method (Adbel-Aziz & Karara, 1971) was used for camera calibration. Each control point in the space gave two equations with respect to the image coordinates:

\[ u = \frac{L_1x + L_2y + L_3z + L_4}{L_9x + L_{10}y + L_{11}z + 1} \]  
\[ v = \frac{L_5x + L_6y + L_7z + L_8}{L_9x + L_{10}y + L_{11}z + 1} \]

where \((u,v)\) = image coordinates
\((y,z)\) = object space coordinates

\(L_1 - L_{11}\) = DLT parameters.

From the 24 control points, 48 equations were obtained to solve for 11 unknowns. These 11 DLT parameters were used in subsequent image reconstruction.

4.2.5. Digitisation

The recorded diving performance was divided into three phases: a) hurdle flight phase; b) takeoff phase; and c) flight phase (Figure 4.4).

The hurdle flight phase was defined as the last frame the supporting leg was in contact with the springboard until the instant of touchdown. The takeoff phase was defined from the instant of touchdown until the last frame that the feet were in contact with the springboard. The flight phase was defined as the last frame the feet were in contact with the springboard until the first frame of entry into the water. Since in some dives the hand /
foot was out of screen before the entry, the last frame in which the whole body could be seen on screen was taken as the last frame of the flight phase.

During the hurdle flight phase, 20 body landmarks were digitised with a sampling frequency of 40 Hz (Table 4.5). Regarding the nature of somersaulting dives, symmetrical movements in the left and right sides of the body were required during the takeoff and the flight phase for good performance. Thus, only the right hand side (RHS) body landmarks together with the middle of the head and the board tip (Point 1-9, 19, 20) were digitised during the takeoff and the flight phase. The takeoff phase was digitised with the full sampling frequency of 200 Hz and the flight phase was sampled at 40 Hz (every fifth frame). To avoid end-point errors in subsequent data processing, digitisation of the takeoff phase started from 15 frames before the touchdown and ended 15 frames after the takeoff.

To minimise digitisation error and variability, the setting of the Phantom software was kept the same for all dives. The image was shown in normal screen view (512 pixel × 512 pixel), zoomed in three times and processed with the same degree of brightness (brightness = 22, contrast = 33, gamma = 1.29).

Table 4.5. Digitisation sequence of body landmarks during hurdle flight phase

<table>
<thead>
<tr>
<th>point</th>
<th>body landmark</th>
<th>point</th>
<th>body landmark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>right wrist</td>
<td>11</td>
<td>left elbow</td>
</tr>
<tr>
<td>2</td>
<td>right elbow</td>
<td>12</td>
<td>left shoulder</td>
</tr>
<tr>
<td>3</td>
<td>right shoulder</td>
<td>13</td>
<td>left hip</td>
</tr>
<tr>
<td>4</td>
<td>right hip</td>
<td>14</td>
<td>left knee</td>
</tr>
<tr>
<td>5</td>
<td>right knee</td>
<td>15</td>
<td>left ankle</td>
</tr>
<tr>
<td>6</td>
<td>right ankle</td>
<td>16</td>
<td>left heel</td>
</tr>
<tr>
<td>7</td>
<td>right heel</td>
<td>17</td>
<td>left ball of foot</td>
</tr>
<tr>
<td>8</td>
<td>right ball of foot</td>
<td>18</td>
<td>left toes</td>
</tr>
<tr>
<td>9</td>
<td>right toes</td>
<td>19</td>
<td>middle of the head</td>
</tr>
<tr>
<td>10</td>
<td>left wrist</td>
<td>20</td>
<td>board tip in line with the right foot</td>
</tr>
</tbody>
</table>

4.2.6. Error estimation

Since the Phantom digitising software does not include sub-pixel resolution, the error arising from the low resolution digitisation was estimated. The field of view in the
camera was about 5 m × 5 m and the resolution was 512 pixel × 512 pixel. One pixel therefore represented approximately 10 mm (Figure 4.5).

![Image of 1 pixel = 10 mm](image)

Figure 4.5. Error estimation due to low resolution digitising system.

Figure 4.5 shows that every point which positions within the 10 mm length can be regarded as the same pixel in the digitising system. A root mean squared (RMS) difference of 3.16 mm error was calculated by Equation (4.7):

\[
RMS = \sqrt{\frac{\sum_{i=-5}^{5} i^2}{11}}
\]

(4.7)

4.2.7. Image Reconstruction

The 11 DLT parameters obtained from camera calibration were used in image reconstruction using a modified 2D DLT method. By setting x to zero, three of the 11 DLT parameters relating to the x-axis were eliminated. The remaining eight DLT parameters were used to reconstruct 2D coordinates. Re-arranging Equations (4.5) and (4.6) and eliminating the x-axis relation parameters:

\[
\begin{bmatrix}
L2 - L10u \\
L6 - L10v
\end{bmatrix}
\begin{bmatrix}
y \\
z
\end{bmatrix}
= \begin{bmatrix}
u - L4 \\
v - L8
\end{bmatrix}
\]

(4.8)

Equation (4.8) was used to reconstruct digitised image coordinates into object space coordinates. After the 2D coordinates were calculated, a set of pseudo-3D space coordinates was then generated by assuming symmetrical movement of the left and right sides of the body. Adjustment of the x-coordinates were made according to the subject-specific anthropometric measurements (see Figure 4.6).
4.2.8. Calculation of kinematic data

The digitised coordinates were used to calculate the diver’s orientation and configuration angles and angular velocities throughout each dive, along with the CM velocities and whole body angular momentum about the mass centre (Yeadon, 1990a, c). The joint angles of interest were: ball, ankle, knee, hip, shoulder, elbow and head. The whole body orientation angle was defined as the angle between the shoulder, the hip and the horizontal pointing towards the water. The CM vertical and horizontal velocities at touchdown and at the last instant of takeoff were determined from the hurdle flight phase and the flight phase respectively. The time history of the orientation and configuration angles were fitted using quintic splines (Wood & Jennings, 1979). The degree of smoothing was initially chosen based on visual inspection of the raw data and the smooth data. Fine adjustment was made by calculating the angular momentum of the forward two and one-half somersault pike from the flight phase kinematic data with different degrees of smoothing close to the initial estimate. The degree of smoothing which produced the least standard error of mean in angular momentum was then used for all other dives.
4.3. Springboard parameters

The springboard parameters included vertical stiffness, effective board mass, the relationships between the vertical board deflection and the horizontal deflection and board rotation angle, and the moment of inertia of the springboard segment.

4.3.1. Vertical stiffness and effective board mass

The physical characteristics of the springboard are governed by the following equation (Miller & Jones, 1999):

\[
m_L = \frac{k}{4\pi^2} T^2 - m_e
\]

where

- \( k \) = vertical stiffness
- \( m_L \) = mass used for loading
- \( m_e \) = effective board mass
- \( T \) = period of oscillation.

From the dynamic loading of the springboard (see Section 4.2.3), the period of oscillation with different loading masses were measured. By plotting \( T^2 \) against \( m_L \), \( k \) and \( m_e \) could be determined from Equation (4.9). The data collected included two trials each of four cycles loaded with 64.1 kg, and one trial of five cycles loaded with 96.8 kg. The average value of \( T \) from all cycles was used to calculate \( T^2 \) for each loaded mass. In addition, the period during free load vibration was obtained from the film data after the diver had taken off from the springboard. Since the board bounced off the fulcrum after takeoff, \( T \) was estimated by assuming the time during which the board was in contact with the fulcrum equalled \( T/2 \). The average value of five cycles from forward jump (100A) and four cycles from reverse double somersault (304C) was used. Table 4.6 shows the values used for plotting \( T^2 \) against \( m_L \) (Figure 4.7). At a fulcrum number of 7.5, the calculated vertical spring stiffness was 5446 N/m and the effective board mass was 8.87 kg. These values are comparable to those reported in the literature for a Maxiflex B springboard (see Chapter 2).
Table 4.6. Loading mass and period of oscillation of the springboard

<table>
<thead>
<tr>
<th>mL (kg)</th>
<th>T(s)</th>
<th>T²(s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2405</td>
<td>0.0578</td>
</tr>
<tr>
<td>64.1</td>
<td>0.7425</td>
<td>0.5513</td>
</tr>
<tr>
<td>96.8</td>
<td>0.8660</td>
<td>0.7500</td>
</tr>
</tbody>
</table>

Figure 4.7. Determination of $k$ and $m_e$ by plotting $m_L$ against $T^2$.

Since the vertical stiffness was allowed to vary depending on foot position, it was assumed that the CM of the diver was 0.15 m from the toes horizontally at touchdown. The vertical stiffness could then be expressed as:

$$k = m(d + 0.15) + c$$

where $m = \text{slope}$

$c = \text{constant}$

$d = \text{parallel distance between the board tip and the toes along the springboard}$

In the study by Sprigings et al. (1990), a nearly linear relation between the stiffness and the position of applied load (PAL) was observed when the PAL was close to the board tip. With the PAL at 0.2 m, the slope with the fulcrum set at the back and at the centre were estimated as follows:
The fulcrum number varies from 1 (front) to 9 (back) and the diver in the present study has chosen 7.5. The slope at fulcrum number 7.5 was calculated as the average between that at the back and the centre:

\[ m = 0.5 \left( \frac{6767 + 7519}{2} \right) = 7143 \]

Taking \( m = 7143 \) and substituting for \( k = 5446 \text{ N/m} \) and \( d = 0 \) in Equation (4.10), the constant \( c \) was calculated as 4375 N/m. The equation for vertical stiffness was therefore:

\[ k = 7143 \left( d + 0.15 \right) + 4375 \]  

(4.11)

It should be noted that the stiffness of the springboard would have a large influence on the diver’s linear and angular momentum. Since the values of \( m \) and \( c \) were not directly measured from the springboard used in this study, these values were allowed to vary slightly later in an optimisation process (see Section 4.7). This would give some flexibility for the model to choose the best stiffness value that matches the simulation with performance.

4.3.2. Vertical and horizontal deflection relationship

The horizontal movement of the springboard was constrained by a geometrical function relating the vertical and the horizontal deflections. From the video recordings, the tip of the springboard during the contact phase of 18 dives in the forward and the reverse groups was digitised. The horizontal deflection \( x \) was regressed against the vertical deflection \( z \). Figure 4.8 shows the regression of \( x \) against \( z \) for a forward one and one-half somersault pike (103B). The regression suggests that a quadratic function fits the data as closely as a cubic function since the cubic term contributes less than 1 mm to the horizontal deflection. The quadratic function was:

\[ x = -0.187z^2 + 0.002z - 0.0026 \]  

(4.12)

It was believed that a simpler function \( x = az^2 \) would be adequate to represent the relationship between \( x \) and \( z \). When \( x \) was plotted against \( z^2 \) using all experimental data of the 18 dives (Figure 4.9), the quadratic function relating the horizontal and vertical deflection was:

\[ x = -0.194z^2 \]  

(4.13)
Figure 4.8. Quadratic and cubic fits to the springboard movement during the contact phase of a forward one and one-half somersault pike (103B).

Figure 4.9. Linear regression of horizontal deflection and vertical deflection squared using experimental data of 18 dives.
4.3.3. Vertical deflection and board rotation relationship

Similarly, the board rotation angle was expressed as a function of the vertical board tip deflection. From the digitised coordinates of the heel, the ball and the board tip, the board angle \( \theta \) was calculated as the angle between the horizontal and a line fitted through the heel, the ball and the board tip. This board angle was regressed against the vertical deflection using all experimental data of the 18 dives. The regression suggests that a linear function is adequate to represent the board angle-vertical deflection relationship (Figure 4.10):

\[
\theta = -28.599z
\]  

\[(4.14)\]

Figure 4.10. Linear regression of the board angle and the vertical deflection using experimental data of 18 dives.

4.3.4. Moment of inertia

The moment of inertia of the springboard \( I_s \) was required as an input to the model to calculate the torque provided by the springboard using an inverse dynamic method. The value of \( I_s \) would not affect the kinematics of the springboard since the rotational movement was constrained by Equation (4.14). \( I_s \) was calculated using the equation for an uniform rod:

\[
I_s = \frac{1}{12} m_s L^2
\]

\[(4.15)\]
\[
\frac{1}{12}(8.87)(0.3)^2 = 0.0665 \text{ kg-m}^2
\]

where \( L \) = length of springboard segment.

4.4. Body segmental inertia parameters

The body segmental inertias of the diver were calculated from 95 anthropometric measurements taken from the diver using a mathematical model (Yeadon, 1990b). Table 4.7 shows the value of mass, length, distance of CM from the proximal joint and the moment of inertia about the transverse axis of each segment. For the limbs, the combined mass and moment of inertia and the average length and CM distance of the left (L) and right (R) side are displayed.

Table 4.7. Body segmental inertias calculated from anthropometric measurements

<table>
<thead>
<tr>
<th>段落</th>
<th>质量 (kg)</th>
<th>长度 (m)</th>
<th>CM from proximal joint (m)</th>
<th>矩心 (kg-m2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>脚 (L + R)</td>
<td>1.37</td>
<td>0.169</td>
<td>0.056</td>
<td>0.003</td>
</tr>
<tr>
<td>(足至脚趾)</td>
<td>0.71</td>
<td>0.035</td>
<td>0.015</td>
<td>0.000013</td>
</tr>
<tr>
<td>(脚踝至足)</td>
<td>1.30</td>
<td>0.134</td>
<td>0.051</td>
<td>0.002</td>
</tr>
<tr>
<td>小腿 (L + R)</td>
<td>7.73</td>
<td>0.403</td>
<td>0.182</td>
<td>0.101</td>
</tr>
<tr>
<td>大腿 (L + R)</td>
<td>17.96</td>
<td>0.411</td>
<td>0.169</td>
<td>0.261</td>
</tr>
<tr>
<td>躯干</td>
<td>25.81</td>
<td>0.541</td>
<td>0.274</td>
<td>1.418</td>
</tr>
<tr>
<td>上臂 (L + R)</td>
<td>3.66</td>
<td>0.263</td>
<td>0.115</td>
<td>0.023</td>
</tr>
<tr>
<td>下臂 (L + R)</td>
<td>2.79</td>
<td>0.425</td>
<td>0.164</td>
<td>0.036</td>
</tr>
<tr>
<td>头 + 颈</td>
<td>4.78</td>
<td>0.265</td>
<td>0.136</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Since the foot was divided into a triangle and a rod segment connected at the ball (Figure 4.11), additional anthropometric measurements (L3 to L9) were required to determine the segmental inertial parameters of the two-segment foot. The dimensions (L4, L7, L8, L9) of the diver’s right foot, which was being digitised, were measured. The mass \( m \) and moment of inertia \( I \) of the whole foot, mass of rod \( m_1 \) and triangle \( m_2 \), moment of inertia of rod \( I_1 \) and triangle \( I_2 \) were taken from the calculated values using Yeadon’s (1990b) model. There was a large difference between the re-measured value of L4 (68 mm) and the previous value (35 mm) used in the mathematical model. It was
believed that the re-measured value was more accurate and therefore this value was used. The value of L3 was calculated from the re-measured L4 value and the length ratio of L3 to L4 obtained from the mathematical model. The remaining parameters to be determined were L5 and L6, which were estimated from the literature using a subject-specific scaling method as shown below.

![Figure 4.11. The foot was modelled as two parts: a triangle and a rod segment.](image)

Horizontally taking moments about the tip of the toes, Equation (4.16) could be used to calculate L5:

\[ mgL_x = m_1gL_3 + m_2g(L_4 + L_5) \]  

(4.16)

where \( g = \) gravitational acceleration

\( L_x = \) horizontal distance of whole foot mass centre to tip of the toes

\[ \frac{L_x}{L_4 + L_7} = 56.3\% \] (Chandler, 1975)

Similarly, height of the triangle mass centre to the ground, L6, could be calculated as:

\[ mgL_z = m_1g(0) + m_2gL_6 \]  

(4.17)

where \( L_z = \) vertical distance of whole foot mass centre to ground
Table 4.8 summarises the measured and calculated dimensions for the two-segment foot.

Table 4.8. Measured and calculated dimensions for the two-segment foot

<table>
<thead>
<tr>
<th>parameters</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3</td>
<td>39 mm</td>
</tr>
<tr>
<td>L4</td>
<td>68 mm</td>
</tr>
<tr>
<td>L5</td>
<td>102 mm</td>
</tr>
<tr>
<td>L6</td>
<td>37 mm</td>
</tr>
<tr>
<td>L7</td>
<td>152 mm</td>
</tr>
<tr>
<td>L8</td>
<td>134 mm</td>
</tr>
<tr>
<td>L9</td>
<td>73 mm</td>
</tr>
</tbody>
</table>

4.5. Strength parameters

4.5.1. Introduction

To ensure that the torque-driven simulation model does not produce movement that exceeds human limit, it is necessary to measure the maximum strength limit of the diver. An isovelocity dynamometer (Cybex Norm) was used to measure isometric and isovelocity joint torques (Figure 4.12).

Figure 4.12. Collecting isometric and isovelocity joint torque using an isovelocity dynamometer.
4.5.2. Experimental set-up

The isovelocity dynamometer was set-up according to the manufacturer’s manual. A spirit level was used to align the crank arm to the vertical position. A laptop computer was connected to the dynamometer via a custom-built box to record the time histories of crank angle and torque at 1000 Hz.

4.5.3. Torque calibration

4.5.3.1. Peak torque on screen

Isometric peak torque with the crank arm in a horizontal position was recorded. Peak torque in the same position loaded with three different body weights were also recorded. The crank arm length (d) was 27 cm. The actual torque (T_{act}) exerted on the crank joint can be calculated as follows:

\[ T_{act} = mgd \]

where \( m \) = mass of the load
\( g \) = gravitational acceleration

\( T_{act} \), after correction for the weight of the crank arm (6 Nm), was compared with the peak torque displayed on the screen (T_{scn}). An average systematic difference of 3.6% was found (see Table 4.9) which could be due to error in measuring the crank arm length.

Table 4.9. Comparison of actual torque and peak torque displayed on screen

<table>
<thead>
<tr>
<th>weight (kg)</th>
<th>( T_{act} ) (Nm)</th>
<th>corrected ( T_{act} ) (Nm)</th>
<th>( T_{scn} ) (Nm)</th>
<th>% difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.2</td>
<td>185</td>
<td>191</td>
<td>198</td>
<td>3.4%</td>
</tr>
<tr>
<td>74.9</td>
<td>198</td>
<td>204</td>
<td>214</td>
<td>4.5%</td>
</tr>
<tr>
<td>89.4</td>
<td>237</td>
<td>243</td>
<td>250</td>
<td>3.0%</td>
</tr>
</tbody>
</table>
4.5.3.2. Voltage output on laptop

In addition to the three trials listed in Table 4.9, an extra trial of one body weight ($T_{scn} = 215$ Nm) was recorded. The peak torque of the four trials loaded with different body weights was linearly regressed against the voltage output ($V_T$) (Figure 4.13). The regression equation obtained (Equation (4.18)) were used later to convert torque voltage output recorded on the laptop to actual torque values for all trials.

$$T_{scn} = 75.204 \ V_T$$  

(4.18)

![Figure 4.13. Linear regression of peak torque against torque voltage output.](image)

4.5.4. Crank angle calibration

Isometric trials of the crank arm at different angles were measured. The known crank angle ($A_c$) was regressed against the recorded angle voltage output ($V_A$) (Figure 4.14). The regression equation obtained (Equation (4.19)) would be used to convert angle voltage output recorded on the laptop to actual crank angle for all trials.

$$A_c = 61.788 \ V_A - 168.42$$  

(4.19)
4.5.5. Testing protocol

The participant provided informed consent (Appendix 2c) and the testing procedures were explained. The four joints tested were the ankle, knee, hip and shoulder. It was assumed that there was no bilateral deficit (see Chapter 2) and therefore only the right limbs were tested. The diver performed both maximal isometric and isovelocity flexion and extension at each joint.

The dynamometer was adjusted so that the segmental joint centre being tested was aligned with the crank joint centre and that the body segment was in the plane of the crank arm during each trial. The diver was then secured to the dynamometer using straps provided by the manufacturer. At each joint, the maximum range of motion (ROM) that the diver felt comfortable with was determined. A safety range was set on the dynamometer to ensure the crank arm would not go beyond the diver’s maximum ROM. In addition, two mechanical stops were placed just beyond the range limits for further safety in case the dynamometer did not stop at the end range. The diver performed a few sub-maximal trials to warm up before actual testing.

At each joint, maximal isometric flexion / extension at six different joint angles distributed evenly within the ROM were recorded for five seconds. A protractor was used to measure the joint angle during each trial. Maximal isovelocity trials at 50°/s, 100°/s and 150°/s (also 200°/s for hip flexion) for two concentric-eccentric contractions then followed.
(King & Yeadon, 2002). Data were recorded for 10 seconds for each isovelocity trial. The diver was asked to exert maximum effort in each trial.

After recording the isometric and isovelocity contractions in one direction (flexion or extension), the diver was allowed to take a break before going on to do the opposite direction at the same joint. During this resting period, an isometric trial was recorded with the crank arm positioned in a horizontal position. This was done for later use to correct for the crank arm weight. The sequence of movements tested was in the order knee extension, knee flexion, hip flexion, hip extension, shoulder flexion, shoulder extension, ankle plantar flexion, and ankle dorsi-flexion. Examples of positioning of the diver and the dynamometer for the different joint movements are shown in Figures 4.15 to 4.18.
4.5.6. Isometric data reduction

4.5.6.1. Calculation of maximum isometric torque

For each isometric trial, the torque time history was obtained from the voltage output file using Equation (4.18). A period was identified over which the isometric torque was stable since movement in the beginning and at the end of the recorded period may produce a bigger torque (Figure 4.19). The maximal value $T_{\text{max}}$ over the isometric period was identified. The value of $T_{\text{max}}$ was systematically greater than the peak torque shown on the screen $T_{\text{scr}}$ and this was probably due to some noise in the signal. To account for this, a new maximum isometric torque, $T_{\text{iso}}$, was calculated by taking the average over
approximately 30 ms near $T_{\text{max}}$. Table 4.10 compares the $T_{\text{scn}}$, $T_{\text{max}}$ and $T_{\text{iso}}$ for isometric knee extension. It can be seen that $T_{\text{iso}}$ is comparable with $T_{\text{scn}}$.

### Torque (Nm)

![Graph showing Torque vs Time](image)

**Figure 4.19. Identification of $T_{\text{max}}$ and the period of stable isometric torque.**

<table>
<thead>
<tr>
<th>Knee angle</th>
<th>$T_{\text{scn}}$ (Nm)</th>
<th>$T_{\text{max}}$ (Nm)</th>
<th>$T_{\text{iso}}$ (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>83°</td>
<td>155</td>
<td>158</td>
<td>157</td>
</tr>
<tr>
<td>105°</td>
<td>197</td>
<td>202</td>
<td>199</td>
</tr>
<tr>
<td>120°</td>
<td>205</td>
<td>211</td>
<td>206</td>
</tr>
<tr>
<td>136°</td>
<td>146</td>
<td>151</td>
<td>148</td>
</tr>
<tr>
<td>157°</td>
<td>122</td>
<td>124</td>
<td>123</td>
</tr>
<tr>
<td>163°</td>
<td>76</td>
<td>79</td>
<td>74</td>
</tr>
</tbody>
</table>
4.5.6.2. Weight Correction

a) Crank arm weight

Depending on the direction of movement, the torque of the crank arm weight can be corrected for using Equation (4.20).

\[ T_{iso} = T_{iso} \pm T_{arm} \cos \theta \]  

where \( T_{arm} \) = isometric torque with the crank arm in a horizontal position
\( \theta \) = angle between the crank arm and the horizontal

![Figure 4.20. Correction for crank arm weight.](image)

b) Segmental weight

Body segmental inertial parameters were obtained from subject-specific anthropometric measurements using Yeadon's (1990b) model. As observed from the video, the diver plantar flexed the ankle during most trials. The shank and the foot are therefore regarded as one rigid segment with the ankle in a fully plantar flexed position. An example of the segmental weight correction for the hip is given in Figure 4.21 where:

- \( H \) = hip joint
- \( K \) = knee joint
- \( \theta_1 \) = angle between the thigh and the horizontal
- \( \theta_2 \) = angle between the shank and the horizontal
- \( m_1 \) = mass centre of the thigh
- \( m_2 \) = mass centre of the combined shank and foot segment
- \( a_1 \) = distance of \( m_1 \) from hip joint
- \( d_1 \) = segmental length of the thigh
- \( a_2 \) = distance of \( m_2 \) from knee joint
Figure 4.21. Correction for segment weight.

Depending on the direction of the movement, segmental weight can be corrected for using Equation (4.21).

\[ T_{iso} = T_{iso} \pm [m_1a_1 \cos \theta_1 + m_2g(d_1 \cos \theta_1 + a_2 \cos \theta_2)] \quad (4.21) \]

4.5.6.3. Results

The corrected isometric torque at corresponding angles are shown in Table 4.11. A linear or quadratic function was fitted to the data to obtain a torque / angle relationship (Figures 4.22 to 4.25).
Table 4.11. Isometric torque at the knee, hip, shoulder and ankle joints

<table>
<thead>
<tr>
<th>joint angle</th>
<th>$T_{iso}$ (Nm)</th>
<th>joint angle</th>
<th>$T_{iso}$ (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee flexion</td>
<td></td>
<td>knee extension</td>
<td></td>
</tr>
<tr>
<td>$87^\circ$</td>
<td>70</td>
<td>$83^\circ$</td>
<td>158</td>
</tr>
<tr>
<td>$99^\circ$</td>
<td>81</td>
<td>$105^\circ$</td>
<td>208</td>
</tr>
<tr>
<td>$107^\circ$</td>
<td>86</td>
<td>$120^\circ$</td>
<td>222</td>
</tr>
<tr>
<td>$129^\circ$</td>
<td>91</td>
<td>$136^\circ$</td>
<td>166</td>
</tr>
<tr>
<td>$140^\circ$</td>
<td>101</td>
<td>$157^\circ$</td>
<td>141</td>
</tr>
<tr>
<td>$156^\circ$</td>
<td>80</td>
<td>$163^\circ$</td>
<td>98</td>
</tr>
<tr>
<td>hip flexion</td>
<td></td>
<td>hip extension</td>
<td></td>
</tr>
<tr>
<td>$68^\circ$</td>
<td>58</td>
<td>$61^\circ$</td>
<td>210</td>
</tr>
<tr>
<td>$87^\circ$</td>
<td>86</td>
<td>$84^\circ$</td>
<td>167</td>
</tr>
<tr>
<td>$113^\circ$</td>
<td>106</td>
<td>$102^\circ$</td>
<td>130</td>
</tr>
<tr>
<td>$123^\circ$</td>
<td>151</td>
<td>$126^\circ$</td>
<td>101</td>
</tr>
<tr>
<td>$143^\circ$</td>
<td>177</td>
<td>$144^\circ$</td>
<td>89</td>
</tr>
<tr>
<td>$171^\circ$</td>
<td>191</td>
<td>$165^\circ$</td>
<td>47</td>
</tr>
<tr>
<td>shoulder flexion</td>
<td></td>
<td>shoulder extension</td>
<td></td>
</tr>
<tr>
<td>$-14^\circ$</td>
<td>47</td>
<td>$12^\circ$</td>
<td>28</td>
</tr>
<tr>
<td>$25^\circ$</td>
<td>40</td>
<td>$32^\circ$</td>
<td>33</td>
</tr>
<tr>
<td>$49^\circ$</td>
<td>33</td>
<td>$64^\circ$</td>
<td>34</td>
</tr>
<tr>
<td>$88^\circ$</td>
<td>30</td>
<td>$83^\circ$</td>
<td>38</td>
</tr>
<tr>
<td>$132^\circ$</td>
<td>18</td>
<td>$108^\circ$</td>
<td>43</td>
</tr>
<tr>
<td>$162^\circ$</td>
<td>11</td>
<td>$138^\circ$</td>
<td>44</td>
</tr>
<tr>
<td>ankle plantar flexion</td>
<td></td>
<td>ankle dorsi-flexion</td>
<td></td>
</tr>
<tr>
<td>$86^\circ$</td>
<td>94</td>
<td>$78^\circ$</td>
<td>33</td>
</tr>
<tr>
<td>$93^\circ$</td>
<td>87</td>
<td>$78^\circ$</td>
<td>38</td>
</tr>
<tr>
<td>$95^\circ$</td>
<td>105</td>
<td>$83^\circ$</td>
<td>38</td>
</tr>
<tr>
<td>$109^\circ$</td>
<td>100</td>
<td>$87^\circ$</td>
<td>42</td>
</tr>
<tr>
<td>$120^\circ$</td>
<td>59</td>
<td>$96^\circ$</td>
<td>42</td>
</tr>
<tr>
<td>$128^\circ$</td>
<td>56</td>
<td>$98^\circ$</td>
<td>46</td>
</tr>
</tbody>
</table>
Figure 4.22. Isometric torque data for knee flexion and extension.

Figure 4.23. Isometric torque data for hip flexion and extension.
Figure 4.24. Isometric torque data for shoulder flexion and extension.

Figure 4.25. Isometric torque data for ankle plantar flexion and dorsi-flexion.
4.5.7. Isovelocity data reduction

4.5.7.1. Weight correction

The torque and crank angle voltage output of isovelocity trials were converted into actual torque and crank angle values using Equations (4.18) and (4.19). The same procedures used in the isometric trials were used to correct for crank arm and segmental weights throughout the whole torque time history for the isovelocity trials.

4.5.7.2. Crank / joint angular velocity

The angle obtained from the voltage output was the crank angle. This was different from the joint angle since adjacent segments moved from their resting positions during muscle contraction. This implied that the crank angular velocity \( (\omega_c) \) would be different from the joint angular velocity \( (\omega_j) \) which was of interest. In order to obtain joint angular velocity, the crank angular velocity was converted into joint angular velocity using the ratios reported in Wilson (2003) for the knee and the hip joint (Table 4.12). Wilson (2003) obtained these ratios by comparing the crank angle time history with the joint angle time history measured using goniometers. For the shoulder joint, \( \omega_j \) was assumed to be the same as \( \omega_c \) since the diver was strapped tightly onto the machine at both the shoulder and the hip joints.
Table 4.12. Ratio of crank to joint angular velocity obtained from Wilson (2003)

<table>
<thead>
<tr>
<th>Joint</th>
<th>( \omega_c (°/s) )</th>
<th>( \omega_j (°/s) )</th>
<th>ratio (( \omega_j/\omega_c ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Knee extension</td>
<td>49.2</td>
<td>34.6</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>-49.8</td>
<td>-36.8</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>98.3</td>
<td>68.7</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>-99.0</td>
<td>-70.3</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>147.0</td>
<td>102.9</td>
<td>0.70</td>
</tr>
<tr>
<td></td>
<td>-148.6</td>
<td>-108.3</td>
<td>0.73</td>
</tr>
<tr>
<td>Knee flexion</td>
<td>49.8</td>
<td>31.9</td>
<td>0.64</td>
</tr>
<tr>
<td></td>
<td>-50.2</td>
<td>-29.9</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>99.4</td>
<td>62.6</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>-99.9</td>
<td>-58.7</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>149.0</td>
<td>94.2</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>-149.4</td>
<td>-88.4</td>
<td>0.59</td>
</tr>
<tr>
<td>Hip extension</td>
<td>49.6</td>
<td>20.6</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>-49.8</td>
<td>-28.4</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>99.5</td>
<td>48.3</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>-99.5</td>
<td>-52.6</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>148.9</td>
<td>66.2</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>-149.0</td>
<td>-71.5</td>
<td>0.48</td>
</tr>
<tr>
<td>Hip flexion</td>
<td>50.0</td>
<td>26.1</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>-49.6</td>
<td>-26.6</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>99.6</td>
<td>53.2</td>
<td>0.53</td>
</tr>
<tr>
<td></td>
<td>-99.1</td>
<td>-56.1</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>149.3</td>
<td>74.2</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>-148.5</td>
<td>-83.6</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>199.1</td>
<td>97.5</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>-198.3</td>
<td>-109.0</td>
<td>0.55</td>
</tr>
</tbody>
</table>
4.5.7.3. Calculation of maximum concentric and eccentric torque

From the angle data, a period of constant velocity was identified over which the peak concentric or eccentric torque occurred (Figure 4.26). A quintic spline (Wood & Jennings, 1979) was fitted to smooth the torque over this period and the average torque was calculated. The maximum and minimum torque and the corresponding angle were also identified.

Since the maximum torque from the raw data occurred at different angles, a new maximum torque ($T_{\text{max}}$) was calculated by multiplying the average torque by a percentage (%) increase level (except knee extension). The % increase level was determined from the ratio of maximum torque to average torque during eccentric contraction (see Figure 4.27). For knee extension, the maximum values from raw data were used because the average values appeared to vary inconsistently.
Figure 4.27. A new maximum torque ($T_{\text{max}}$) was calculated based on the average torque and the % increase level during eccentric contraction (knee flexion).

4.5.7.4. Calculation of isometric torque

Based on the new torque / angular velocity relationship, the torque value at zero angular velocity ($T_0$) was determined by taking the average of $T_{\text{max}}$ at the lowest angular velocity in both concentric and eccentric contraction (Figure 4.28). Table 4.13 compares this $T_0$ with the maximum $T_{\text{iso}}$ obtained from isometric trials and the corresponding joint angle.

Table 4.13. Isometric torques obtained from isovelocity and isometric data

<table>
<thead>
<tr>
<th>joint / movement</th>
<th>$T_0$ (Nm)</th>
<th>$T_{\text{iso}}$ (Nm)</th>
<th>$T_{\text{iso}}$ joint angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee flexion</td>
<td>113</td>
<td>101</td>
<td>140°</td>
</tr>
<tr>
<td>knee extension</td>
<td>190</td>
<td>222</td>
<td>120°</td>
</tr>
<tr>
<td>hip flexion</td>
<td>148</td>
<td>190</td>
<td>171°</td>
</tr>
<tr>
<td>hip extension</td>
<td>213</td>
<td>210</td>
<td>61°</td>
</tr>
<tr>
<td>shoulder flexion</td>
<td>34</td>
<td>47</td>
<td>-14°</td>
</tr>
<tr>
<td>shoulder extension</td>
<td>38</td>
<td>44</td>
<td>138°</td>
</tr>
</tbody>
</table>
Figure 4.28. $T_o$ was determined by taking the average of $T_{\text{max}}$ at the lowest concentric and eccentric contraction velocity (knee flexion).

4.5.7.5. Calculation of maximum angular velocity

Joint angular velocity time histories for each diving performance were calculated from digitised video recordings (see Section 4.2). The maximum angular velocity ($\omega_{\text{max}}$) at the ankle, knee, hip and shoulder joint were identified from the takeoff and the flight phases. This value would be used to fit a 7-parameter function to the torque / angular velocity data (details follow). Table 4.14 shows the value $\omega_{\text{max}}$ for each movement and the corresponding dive that it occurs in.
Table 4.14. Maximum angular velocity obtained from video analysis of the diving performances

<table>
<thead>
<tr>
<th>joint movement</th>
<th>$\omega_{\text{max}}$ (rad/s)</th>
<th>dive</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee extension</td>
<td>20.5</td>
<td>304C</td>
</tr>
<tr>
<td>knee flexion</td>
<td>18.6</td>
<td>304C</td>
</tr>
<tr>
<td>hip extension</td>
<td>16.1</td>
<td>104B</td>
</tr>
<tr>
<td>hip flexion</td>
<td>12.7</td>
<td>105C</td>
</tr>
<tr>
<td>shoulder extension</td>
<td>11.4</td>
<td>105B</td>
</tr>
<tr>
<td>shoulder flexion</td>
<td>38.9</td>
<td>304C</td>
</tr>
<tr>
<td>ankle plantar flexion</td>
<td>15.9</td>
<td>101B</td>
</tr>
<tr>
<td>ankle dorsi-flexion</td>
<td>17.2</td>
<td>100A</td>
</tr>
</tbody>
</table>

4.5.7.6. Fitting a 7-parameter function

From the isovelocity trials, torque data are only available at certain angular velocities. To obtain a complete torque / angular velocity relationship, a 7-parameter function was fit to the raw data using the optimisation algorithm Simulated Annealing (Corana, Marchesi, Martini, & Ridella, 1987). The seven parameters included four parameters defining two hyperbolic functions and three parameters defining differential activation (Wilson, 2003). In the concentric phase, a rotational equivalent of the classic Hill’s (1938) hyperbola was used to represent the maximum torque / angular velocity relationship whereas in the eccentric phase an inverted rectangular hyperbola was used. Figure 4.29 describes the relationship between torque (T) and angular velocity ($\omega$) where

- $T_0$ = maximum isometric torque
- $T_{\text{max}}$ = maximum torque in eccentric phase
- $\omega_{\text{max}}$ = maximum angular velocity at which torque equals zero
- $T_c / T_e$ = asymptote of torque in concentric / eccentric hyperbola
- $\omega_c / \omega_e$ = asymptote of angular velocity in concentric / eccentric hyperbola
In the concentric phase, the torque-angular velocity relationship is governed by Hill’s (1938) hyperbola. This relationship can be described in Equation (4.22) which has asymptotes at $T = -T_c$ and $\omega = -\omega_c$:

$$(T + T_c)(\omega + \omega_c) = C$$

where

$$T_c = \frac{T_o \omega_c}{\omega_{\text{max}}}$$

$$C = T_c (\omega_{\text{max}} + \omega_c)$$

Similarly in the eccentric phase, the torque-angular velocity relationship is given by Equation (4.23) which has asymptotes at $T = T_e$ and $\omega = \omega_e$:

$$(T_e - T)(\omega_e - \omega) = -E$$

where

$$\omega_e = \frac{(T_{\text{max}} - T_o)}{kT_o} \cdot \frac{\omega_{\text{max}} \omega_c}{(\omega_{\text{max}} + \omega_c)}$$

$$E = -\omega_e (T_{\text{max}} - T_o)$$

$k =$ ratio of slopes between concentric and eccentric phase.
The tetanic torque function requires four parameters: \( T_0 \), \( T_{\text{max}} \), \( \omega_c \) and \( \omega_{\text{max}} \). A differential activation function (Wilson, 2003) can be used to adjust the theoretical torque predicted by the tetanic torque function \( T_4 \) to better fit the experimental data (Figure 4.30). This function, given by Equation (4.24), calculates an activation level \( a \) to re-define the maximum torque \( T_\omega \) by using Equation (4.25). The three additional parameters are: \( a_{\text{min}} \), \( m \) and \( \omega_1 \).

\[
\omega - \omega_1 = \frac{m[a - 0.5(a_{\text{min}} + a_{\text{max}})]}{(a_{\text{max}} - a)(a - a_{\text{min}})}
\]  
(4.24)

\[
T_\omega = a \cdot T_4
\]
(4.25)

where \( a_{\text{min}} \) = minimum activation level in eccentric phase

\( a_{\text{max}} \) = maximum activation level in concentric phase

\( m \) = parameter that governs the rate at which the activation increases with angular velocity (1/\( m \) is proportional to the slope at the point of inflection)

\( \omega_1 \) = angular velocity at the mid-point of the slope.

Figure 4.30. A 3-parameter function representing differential activation.

The 7-parameter function described above was used to fit the experimental data for each movement. The value of \( k \) was set at 4.3, the theoretical value predicted by Huxley's (1957) original model. The value of \( a_{\text{max}} \) was assumed to be 1.0. The value of \( T_{\text{max}} \) was
assumed to be 1.5 times \( T_o \) (Harry, Ward, Heglund, Morgan, & McMahon, 1990). The lower and upper limits of the seven parameters are shown in Table 4.15.

Table 4.15. Upper and lower limits of the seven parameters

<table>
<thead>
<tr>
<th>parameter</th>
<th>lower bound (LB)</th>
<th>upper bound (UB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_o )</td>
<td>( T_o ) from isovelocity trials</td>
<td>20% larger than LB</td>
</tr>
<tr>
<td>( T_{\text{max}} )</td>
<td>1.5 times ( T_o )</td>
<td>20% larger than LB</td>
</tr>
<tr>
<td>( \omega_{\text{max}} )</td>
<td>from video analysis (see Table 4.14)</td>
<td>20% larger than LB</td>
</tr>
<tr>
<td>( \omega_c )</td>
<td>0.0</td>
<td>50.0</td>
</tr>
<tr>
<td>( a_{\text{min}} )</td>
<td>0.0</td>
<td>0.995</td>
</tr>
<tr>
<td>( m )</td>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>( \omega_1 )</td>
<td>-6.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

The seven parameters were optimised by using Simulated Annealing (Corana et al., 1987) to minimise the root mean square (RMS) between the known raw torques (\( T_{\text{max}} \)) and the calculated torques (\( T \)) at each angular velocity:

\[
\text{RMS} = \sqrt{\frac{\sum_{i=1}^{n}(T - T_{\text{max}})^2}{n}} \tag{4.26}
\]

where \( n \) = number of angular velocities at which the torque was measured.

Optimisation results are shown in Table 4.16. For hip flexion and extension, there was little effect of including the differential activation parameters. It was therefore decided that a 4-parameter function was sufficient to fit those data. Examples of the resulting functions are given in Figure 4.31 and Figure 4.32.
Table 4.16. Optimisation results for the 7-parameter torque / angular velocity relationship

<table>
<thead>
<tr>
<th>parameter</th>
<th>knee</th>
<th>hip</th>
<th>shoulder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>flexion</td>
<td>extension</td>
<td>flexion</td>
</tr>
<tr>
<td>$T_0$ (Nm)</td>
<td>135</td>
<td>208</td>
<td>148</td>
</tr>
<tr>
<td>$T_{max}$ (Nm)</td>
<td>204</td>
<td>312</td>
<td>222</td>
</tr>
<tr>
<td>$\omega_{max}$ (rad/s)</td>
<td>22.3</td>
<td>23.6</td>
<td>15.3</td>
</tr>
<tr>
<td>$\omega_c$ (rad/s)</td>
<td>4.83</td>
<td>3.08</td>
<td>2.89</td>
</tr>
<tr>
<td>$\alpha_{min}$</td>
<td>0.77</td>
<td>0.63</td>
<td>-</td>
</tr>
<tr>
<td>m</td>
<td>0.10</td>
<td>0.13</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
<td>1.19</td>
<td>0.00</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4.31. A 7-parameter fit to the isovelocity knee flexion data.
Figure 4.32. A 4-parameter fit to the isovelocity hip extension data.

4.5.7.7. Fitting a 12-parameter function

The 7-parameter function provides a continuous torque / angular velocity relationship. In actual human movement, the torque is also dependent on the joint angle. It is therefore necessary to obtain a function which defines how torque changes over both angle and angular velocity. A quadratic function can be introduced to serve this purpose:

\[ T_{\theta,\omega} = T_\omega \times p \left[ 1 - q (\theta - \theta_{\text{opt}})^2 \right] \]  

(4.27)

where \( T_{\theta,\omega} \) = angle and angular velocity dependent torque

\( T_\omega \) = angular velocity dependent torque calculated from the 7-parameter function

\( p \) = constant to adjust maximal voluntary torque level

\( q \) = rate at which torque drops off from the optimum angle

\( \theta_{\text{opt}} \) = optimum angle at which maximum torque occurs

To allow the rate of drop off and optimum angle to be angular velocity dependent, \( q \) and \( \theta_{\text{opt}} \) can further be described as:

\[ q = r \omega + s \]  

(4.28)
\[ \theta_{\text{opt}} = u \omega + v \]  

(4.29)

where \( r, s, u, v \) are constants.

This function includes the seven parameters used in the 7-parameter fit and five additional parameters \( (p, r, s, u, v) \). To determine the new function parameters, raw torque values at ten angles equally spaced over the isovelocity period were calculated using splines at each angular velocity. The five parameters were determined by using Simulated Annealing to minimise the RMS between the known raw torques and the torques calculated using Equation (4.27). A penalty score was added to constrain \( r \) and \( s \) so that \( q \) was always positive.

The seven parameters obtained from the previous 7-parameter optimisation were used as fixed input to the 12-parameter fit. The lower and upper bounds of the additional five parameters were set as follows:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound (LB)</th>
<th>Upper Bound (UB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p )</td>
<td>0.5</td>
<td>2.0</td>
</tr>
<tr>
<td>( r )</td>
<td>-0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>( s )</td>
<td>0.0</td>
<td>1.5</td>
</tr>
<tr>
<td>( u )</td>
<td>-0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>( v )</td>
<td>1.0</td>
<td>6.0</td>
</tr>
</tbody>
</table>

Optimisation results of the 12 parameters are shown in Table 4.18. An example of a surface plot of the 12-parameter function with raw data is given in Figure 4.33. Within the range of angle and angular velocity obtained from the isovelocity trials, the surface plot appears to fit the raw data points very well. At each angular velocity, there is an optimum angle \( (\theta_{\text{opt}}) \) at which maximum torque occurs; and that \( \theta_{\text{opt}} \) varies with angular velocity. At each angle, the torque follows the 7-parameter function with a rate of drop off \( q \) and that \( q \) depends on angular velocity.
Figure 4.33. A 12-parameter fit to the knee extension data (dot = raw data).
Table 4.18. Optimisation results for the 12-parameter torque / angle / angular velocity relationship

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Knee flexion</th>
<th>Knee extension</th>
<th>Hip flexion</th>
<th>Hip extension</th>
<th>Shoulder flexion</th>
<th>Shoulder extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (Nm)</td>
<td>135</td>
<td>208</td>
<td>148</td>
<td>213</td>
<td>40</td>
<td>42</td>
</tr>
<tr>
<td>$T_{\text{max}}$ (Nm)</td>
<td>204</td>
<td>312</td>
<td>222</td>
<td>320</td>
<td>60</td>
<td>63</td>
</tr>
<tr>
<td>$\omega_{\text{max}}$ (rad/s)</td>
<td>22.3</td>
<td>23.6</td>
<td>15.3</td>
<td>16.1</td>
<td>43.2</td>
<td>11.7</td>
</tr>
<tr>
<td>$\omega_c$ (rad/s)</td>
<td>4.83</td>
<td>3.08</td>
<td>2.89</td>
<td>1.71</td>
<td>4.40</td>
<td>2.17</td>
</tr>
<tr>
<td>$a_{\text{min}}$</td>
<td>0.77</td>
<td>0.63</td>
<td>-</td>
<td>-</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>$m$</td>
<td>0.10</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
<td>0.10</td>
<td>0.37</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
<td>1.19</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>2.01</td>
<td>1.28</td>
</tr>
<tr>
<td>$p$</td>
<td>1.21</td>
<td>0.87</td>
<td>1.49</td>
<td>0.76</td>
<td>0.99</td>
<td>0.90</td>
</tr>
<tr>
<td>$r$</td>
<td>0.40</td>
<td>-0.11</td>
<td>0.05</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.01</td>
</tr>
<tr>
<td>$s$</td>
<td>0.23</td>
<td>1.15</td>
<td>0.03</td>
<td>0.36</td>
<td>0.30</td>
<td>0.31</td>
</tr>
<tr>
<td>$u$</td>
<td>-0.16</td>
<td>0.03</td>
<td>-0.42</td>
<td>0.06</td>
<td>-0.10</td>
<td>0.14</td>
</tr>
<tr>
<td>$v$</td>
<td>3.18</td>
<td>2.20</td>
<td>6.00</td>
<td>1.56</td>
<td>1.10</td>
<td>1.90</td>
</tr>
</tbody>
</table>

In an actual diving takeoff, the range of angle and angular velocity is greater than the measured range on the isovelocity dynamometer (Table 4.19). The 12-parameter function was therefore extrapolated to the range based on actual diving takeoffs. The value of $q$ was forced between zero and the maximum value within the range of experimental data. Figure 4.34 shows an example of the extrapolated 12-parameter function surface plot for knee extension.

It can be seen that the function does not predict reasonable torque values once it is extrapolated outside the range of experimental data. The rate of drop off in the eccentric phase is so high that torque drops to zero very quickly. In reality, a higher torque value will be expected even at extreme angle range. The 12-parameter function is, therefore, not considered good enough to represent the torque / angle / angular velocity relationship during diving takeoffs.
Table 4.19. Range of angle and angular velocity during the takeoff phase

<table>
<thead>
<tr>
<th>joint movement</th>
<th>angle</th>
<th>angular velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee extension</td>
<td>114°-180°</td>
<td>-782°/s to 570°/s</td>
</tr>
<tr>
<td>knee flexion</td>
<td>130°-114°</td>
<td>-570°/s to 782°/s</td>
</tr>
<tr>
<td>hip extension</td>
<td>105°-196°</td>
<td>-648°/s to 543°/s</td>
</tr>
<tr>
<td>hip flexion</td>
<td>164°-120°</td>
<td>-543°/s to 648°/s</td>
</tr>
<tr>
<td>shoulder extension</td>
<td>191°-72°</td>
<td>-2060°/s to 652°/s</td>
</tr>
<tr>
<td>shoulder flexion</td>
<td>-32°-192°</td>
<td>-652°/s to 2060°/s</td>
</tr>
<tr>
<td>ankle plantar flexion</td>
<td>86°-180°</td>
<td>-987°/s to 910°/s</td>
</tr>
<tr>
<td>ankle dorsi-flexion</td>
<td>112°-86°</td>
<td>-910°/s to 987°/s</td>
</tr>
</tbody>
</table>

Figure 4.34. The 12-parameter function for knee extension extrapolated to the actual range of angle and angular velocity used in actual diving takeoffs (dot = raw data).
4.5.7.8. Fitting a 10-parameter function

To solve the problem that the rate of drop off was too high, the 12-parameter function (Equation 4.27) was modified into a 10-parameter function where q and $\theta_{opt}$ were constants instead of angular velocity dependent. The same procedures used in fitting the 12-parameter function were employed to search for optimal p, q and $\theta_{opt}$. The lower and upper limits for q were 0.0 to 1.0 and for $\theta_{opt}$ were 0.0 to 6.0. Optimisation results are tabulated in Table 4.20. Graphical representation of the 10-parameter function for knee extension is displayed in Figure 4.35. The figure was plotted using the range of angle and angular velocity in actual diving takeoffs. Raw data collected from isovelocity measurement are included for comparison. It can be seen from Figure 4.35 that the 10-parameter function predicts sensible torques over the whole range of interest.

Table 4.20. Optimisation results for the 10-parameter torque / angle / angular velocity relationship

<table>
<thead>
<tr>
<th>parameter</th>
<th>knee flexion</th>
<th>extension</th>
<th>hip flexion</th>
<th>extension</th>
<th>shoulder flexion</th>
<th>extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_c$ (Nm)</td>
<td>135</td>
<td>208</td>
<td>148</td>
<td>213</td>
<td>40</td>
<td>42</td>
</tr>
<tr>
<td>$T_{max}$ (Nm)</td>
<td>204</td>
<td>312</td>
<td>222</td>
<td>320</td>
<td>60</td>
<td>63</td>
</tr>
<tr>
<td>$\omega_{max}$ (rad/s)</td>
<td>22.3</td>
<td>23.6</td>
<td>15.3</td>
<td>16.1</td>
<td>43.2</td>
<td>11.7</td>
</tr>
<tr>
<td>$\omega_{c}$ (rad/s)</td>
<td>4.83</td>
<td>3.08</td>
<td>2.89</td>
<td>1.71</td>
<td>4.40</td>
<td>2.17</td>
</tr>
<tr>
<td>$a_{min}$</td>
<td>0.77</td>
<td>0.63</td>
<td>-</td>
<td>-</td>
<td>0.84</td>
<td>0.72</td>
</tr>
<tr>
<td>m</td>
<td>0.10</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
<td>0.10</td>
<td>0.37</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
<td>1.19</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>2.01</td>
<td>1.28</td>
</tr>
<tr>
<td>p</td>
<td>1.04</td>
<td>0.86</td>
<td>1.07</td>
<td>0.75</td>
<td>0.98</td>
<td>0.97</td>
</tr>
<tr>
<td>q</td>
<td>0.40</td>
<td>1.23</td>
<td>0.81</td>
<td>0.40</td>
<td>0.31</td>
<td>0.11</td>
</tr>
<tr>
<td>$\theta_{opt}$ (rad)</td>
<td>2.67</td>
<td>2.18</td>
<td>4.13</td>
<td>1.51</td>
<td>1.19</td>
<td>2.64</td>
</tr>
</tbody>
</table>
4.5.8. The ankle joint

Since only isometric torque data were collected for the ankle joint, the torque-angle-angular velocity relationship had to be determined in a different way. The maximum $T_{iso}$ from isometric trials was used as $T_0$ and $T_{max}$ was set to be 1.5 times $T_0$. The value of $\omega_{max}$ was set to be 10% bigger than $\omega_{max}$ determined from video data (see Table 4.14). The values of $\omega_c$, $a_{min}$, $m$ and $\omega_1$ were estimated by taking the average value from all the other joints, after the highest and the lowest values had been removed. The values of $p$, $q$ and $\theta_{opt}$ were determined by minimising the RMS difference between the calculated torque and the isometric data at which angular velocity was zero using Simulated Annealing. Optimised results of the 10-parameter function are listed in Table 4.21. Figure 4.36 and Figure 4.37 show the extrapolated surface plot of the 10-parameter function for ankle plantar flexion and dorsi-flexion.
Table 4.21. Optimisation results of the 10 parameters for the ankle joint

<table>
<thead>
<tr>
<th>parameter</th>
<th>plantar flexion</th>
<th>dorsi-flexion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (Nm)</td>
<td>105</td>
<td>46.0</td>
</tr>
<tr>
<td>$T_{\text{max}}$ (Nm)</td>
<td>157.5</td>
<td>69.0</td>
</tr>
<tr>
<td>$\omega_{\text{max}}$ (rad/s)</td>
<td>17.47</td>
<td>18.94</td>
</tr>
<tr>
<td>$\omega_c$ (rad/s)</td>
<td>3.14</td>
<td>3.14</td>
</tr>
<tr>
<td>$a_{\text{min}}$</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>$m$</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>$p$</td>
<td>0.94</td>
<td>1.03</td>
</tr>
<tr>
<td>$q$</td>
<td>1.66</td>
<td>0.61</td>
</tr>
<tr>
<td>$\theta_{\text{opt}}$ (rad)</td>
<td>1.69</td>
<td>2.00</td>
</tr>
</tbody>
</table>

In Figure 4.31, the torque values drop to zero at high angular velocity (near 1000°/s). For the ankle joint, $\omega_{\text{max}}$ was set to be 10% larger than the video data whereas in other joints $\omega_{\text{max}}$ was allowed to vary up to 20% larger than the video data. It can be argued that $\omega_{\text{max}}$ can reach further than this 10% limit and that at the 10% limit a higher torque value will be expected. However, since the torque is very small at high angular velocities, it is believed that allowing a larger variation of $\omega_{\text{max}}$ would not make a large difference.

Figure 4.31 also shows that the torque values drop to zero before the angle reaches its maximal range of 180°. It should be noted that near the end of takeoff at which plantar flexion is approaching its maximum range, the ankle dorsi-flexors should be activated to facilitate the slow-down of plantar flexion. This muscle co-contraction serves as a protective mechanism to ensure that the joint will not go beyond its limit. Although a non-zero torque value may be expected near the end of actual diving takeoffs, the value will be too small to have a large influence.
Figure 4.36. A 10-parameter fit to the ankle plantar flexion data extrapolated to the range of angle and angular velocity used in actual diving takeoffs (dot = raw data).

Figure 4.37. A 10-parameter fit to the isometric ankle dorsi-flexion data extrapolated to the range of angle and angular velocity used in actual diving takeoffs (dot = raw data).
4.5.9. Transforming joint angle to contractile component angle

The torque / angle / angular velocity relationship established above refers to joint angle and joint angular velocity. For subsequent use in the muscle-tendon complex (see Section 3.5.1), it is necessary to convert the joint angle and joint angular velocity into a contractile component angle and angular velocity.

It has been shown that the joint angular velocity is approximately equal (or opposite and equal) to the contractile component angular velocity for isovelocity data (King & Yeadon, 2002). The joint angle can be transformed into a contractile component (CON) angle $\theta_{\text{con}}$ and a series elastic component (SEC) angle $\theta_{\text{sec}}$ based on a geometric relationship (Equations (3.12) to (3.15)). The torque measured from the isovelocity dynamometer is generated by the contractile component. By setting SEC torque equal to the CON torque, the value of $\theta_{\text{sec}}$ can be determined once the SEC stiffness $k$ is known (Equation (3.16)). The value of $\theta_{\text{con}}$ can be calculated subsequently using Equations (3.12) to (3.15). To obtain the value of $k$ for each joint movement, two additional pieces of information are required: 1) SEC length; and 2) moment arm of major muscle groups.

4.5.9.1 Calculation of SEC length

![Figure 4.38. Geometry of pennate and parallel fibred muscles (adapted from Pierrynowski, 1995).](image-url)

Figure 4.38. Geometry of pennate and parallel fibred muscles (adapted from Pierrynowski, 1995).
From Figure 4.38, the SEC length can be calculated by geometry:

\[ \text{SEC length} = L_b + L_t - L_f \cos \alpha \]  \hspace{1cm} (4.30)

where

\( L_b = \) muscle belly length
\( L_t = \) tendon length
\( L_f = \) muscle fibre length
\( \alpha = \) pennation angle

Using data from the literature (Pierrynowski, 1995), the SEC length was scaled to the diver in the present study using Equation (4.31):

\[ \text{scaled SEC length} = \text{SEC length} \times \frac{H_{\text{sub}}}{H_{\text{lit}}} \]  \hspace{1cm} (4.31)

where

\( H_{\text{sub}} = \) height of subject in the present study
\( H_{\text{lit}} = \) height of subject in the literature

Selected muscle groups with major contribution to movements in the sagittal plane were chosen: soleus (SO), gastrocnemius (lateral / medial) (GA(l) / (m)), plantaris (PLT), tibialis anterior (TA), extensor digitorum longus (EDL), extensor hallucis longus (EHL), peroneus tertius (PT), rectus femoris (RF), vastus lateralis (VL), vastus intermedius (VI), vastus medialis (VM), gracilis (GR), semimembranosus (SM), semitendinosus (ST), biceps femoris (long / short) ((BF(l)) / (s)), adductor magnus (middle / posterior) (AM(m) / (p)), psoasmajor (PM), iliacus (IL), gluteus maximus (deep / superficial) (GM(d) / (s)). Table 4.22 shows the values reported in the literature and the new values scaled to the diver of the present study.
### Table 4.22. SEC length obtained from the literature and scaled to the diver

<table>
<thead>
<tr>
<th>muscle group</th>
<th>$\alpha$ (°)</th>
<th>$L_f$ (mm)</th>
<th>$L_t$ (mm)</th>
<th>$L_b$ (mm)</th>
<th>SEC length* (mm)</th>
<th>literature</th>
<th>scaled</th>
</tr>
</thead>
<tbody>
<tr>
<td>soleus</td>
<td>SO</td>
<td>26</td>
<td>49</td>
<td>227</td>
<td>129</td>
<td>312.0</td>
<td>294.4</td>
</tr>
<tr>
<td>gastrocnemius (lateral)</td>
<td>GA(l)</td>
<td>11</td>
<td>88</td>
<td>226</td>
<td>225</td>
<td>364.6</td>
<td>344.1</td>
</tr>
<tr>
<td>gastrocnemius (medial)</td>
<td>GA(m)</td>
<td>14</td>
<td>68</td>
<td>207</td>
<td>248</td>
<td>389.0</td>
<td>367.2</td>
</tr>
<tr>
<td>plantaris</td>
<td>PLT</td>
<td>4</td>
<td>73</td>
<td>359</td>
<td>90</td>
<td>376.2</td>
<td>355.0</td>
</tr>
<tr>
<td>tiliabas anterior</td>
<td>TA</td>
<td>9</td>
<td>99</td>
<td>217</td>
<td>117</td>
<td>236.2</td>
<td>222.9</td>
</tr>
<tr>
<td>extensor digitorum</td>
<td>EDL</td>
<td>11</td>
<td>101</td>
<td>344</td>
<td>124</td>
<td>368.9</td>
<td>348.1</td>
</tr>
<tr>
<td>extensor hallucis</td>
<td>EHL</td>
<td>7</td>
<td>92</td>
<td>248</td>
<td>111</td>
<td>267.7</td>
<td>252.6</td>
</tr>
<tr>
<td>peroneus tertius</td>
<td>PT</td>
<td>12</td>
<td>75</td>
<td>112</td>
<td>85</td>
<td>123.6</td>
<td>116.7</td>
</tr>
<tr>
<td>rectus femoris</td>
<td>RF</td>
<td>10</td>
<td>88</td>
<td>186</td>
<td>302</td>
<td>401.3</td>
<td>378.8</td>
</tr>
<tr>
<td>vastus lateralis</td>
<td>VL</td>
<td>11</td>
<td>110</td>
<td>138</td>
<td>273</td>
<td>303.0</td>
<td>286.0</td>
</tr>
<tr>
<td>vastus intermedius</td>
<td>VI</td>
<td>6</td>
<td>106</td>
<td>87</td>
<td>320</td>
<td>301.6</td>
<td>284.6</td>
</tr>
<tr>
<td>vastus medialis</td>
<td>VM</td>
<td>10</td>
<td>112</td>
<td>49</td>
<td>360</td>
<td>298.7</td>
<td>284.6</td>
</tr>
<tr>
<td>gracilis</td>
<td>GR</td>
<td>2</td>
<td>310</td>
<td>148</td>
<td>322</td>
<td>160.2</td>
<td>151.2</td>
</tr>
<tr>
<td>semimembranosus</td>
<td>SM</td>
<td>15</td>
<td>79</td>
<td>116</td>
<td>304</td>
<td>343.7</td>
<td>324.4</td>
</tr>
<tr>
<td>semitendinosus</td>
<td>ST</td>
<td>4</td>
<td>175</td>
<td>196</td>
<td>288</td>
<td>309.4</td>
<td>292.0</td>
</tr>
<tr>
<td>biceps femoris long</td>
<td>BF(l)</td>
<td>7</td>
<td>101</td>
<td>158</td>
<td>274</td>
<td>331.8</td>
<td>313.1</td>
</tr>
<tr>
<td>biceps femoris short</td>
<td>BF(s)</td>
<td>15</td>
<td>146</td>
<td>96</td>
<td>152</td>
<td>107.0</td>
<td>101.0</td>
</tr>
<tr>
<td>adductor magnus</td>
<td>AM(m)</td>
<td>3</td>
<td>163</td>
<td>0</td>
<td>196</td>
<td>33.2</td>
<td>31.4</td>
</tr>
<tr>
<td>middle</td>
<td>AM(p)</td>
<td>3</td>
<td>194</td>
<td>81</td>
<td>242</td>
<td>129.3</td>
<td>122.0</td>
</tr>
<tr>
<td>posterior</td>
<td>PM</td>
<td>5</td>
<td>190</td>
<td>54</td>
<td>238</td>
<td>102.7</td>
<td>97.0</td>
</tr>
<tr>
<td>iliacus</td>
<td>IL</td>
<td>5</td>
<td>164</td>
<td>0</td>
<td>220</td>
<td>56.6</td>
<td>53.4</td>
</tr>
<tr>
<td>gluteus maximus (deep)</td>
<td>GM(d)</td>
<td>3</td>
<td>154</td>
<td>39</td>
<td>174</td>
<td>59.2</td>
<td>55.9</td>
</tr>
<tr>
<td>gluteus maximus (superficial)</td>
<td>GM(s)</td>
<td>0</td>
<td>171</td>
<td>409</td>
<td>171</td>
<td>409.0</td>
<td>386.0</td>
</tr>
</tbody>
</table>

*$H_{lit} = 1.78$ m (Pierrynowski, 1995); $H_{sub} = 1.68$ m
4.5.9.2. Calculation of moment arm

The moment arms of the corresponding muscle groups were obtained from literature. Since there was no complete data set reported in one study, moment arm values were taken from different sources with appropriate scaling. Due to the distinct gender difference in muscle architecture (Chow et al., 2000), only data of female subjects were used for the knee and the hip joint. Since female subject data were not available for the ankle joint, male subject data were used instead. The moment arm data were scaled to the diver of the present study by using Equation (4.32).

\[ m = r^2 \cdot L \quad \text{(since mass } \alpha \text{ volume)} \]
\[ r = \sqrt{\frac{m}{L}} \]

Scaled \[ d = d \times \frac{r_{\text{sub}}}{r_{\text{lit}}} \]  \hspace{1cm} (4.32)

where

- \( m \) = mass
- \( L \) = length
- \( d \) = moment arm
- \( r_{\text{sub}} \) = ratio of diver in the present study
- \( r_{\text{lit}} \) = ratio of subject in literature

When the mass of subjects used in the literature was not reported, a linear scaling of length measurement was adopted. In the case where only femur length was given, the femur length was first scaled to thigh length defined as the distance between hip joint centre and knee joint centre. This was done by obtaining a ratio of femur length to thigh length from a MRI scan of an adult male subject. When more than one moment arm value were available, average value was taken after discarding the smallest and the largest values. Table 4.23 lists the moment arm values before and after scaling, and the corresponding literature.
Table 4.23. Moment arm obtained from the literature and scaled to the diver

<table>
<thead>
<tr>
<th>muscle / tendon</th>
<th>reference</th>
<th>d (mm)</th>
<th>scaled d (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ankle dorsi-flexion</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TA</td>
<td>Magnaris &amp; Paul (1999)</td>
<td>36</td>
<td>33.3 (discarded)</td>
</tr>
<tr>
<td></td>
<td>Maganaris (2000) MRI</td>
<td>38</td>
<td>36.8</td>
</tr>
<tr>
<td></td>
<td>Fukunaga, Roy et al. (1996b)</td>
<td>40.4</td>
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<tr>
<td></td>
<td>Rugg et al. (1990)</td>
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</tr>
<tr>
<td></td>
<td>Fukunaga, Ito et al. (1996a)</td>
<td>46.5</td>
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</tr>
<tr>
<td></td>
<td>Magnanis (2000) ultrasonic</td>
<td>50</td>
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</tr>
<tr>
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<td>Ito et al. (2000)</td>
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<tr>
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<tr>
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<td>average</td>
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<td>43.0</td>
</tr>
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<td></td>
</tr>
<tr>
<td>Achillis tendon</td>
<td>Magnanis &amp; Paul (2002)</td>
<td>63</td>
<td>60.8</td>
</tr>
<tr>
<td></td>
<td>Rugg et al. (1990)</td>
<td>56.3</td>
<td>54.3</td>
</tr>
<tr>
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<td>average</td>
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</tr>
<tr>
<td>knee flexion</td>
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</tr>
<tr>
<td>GA(l)</td>
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<td>37.6</td>
</tr>
<tr>
<td></td>
<td>Duda et al. (1996)</td>
<td>32</td>
<td>36.1</td>
</tr>
<tr>
<td></td>
<td>average</td>
<td></td>
<td>36.8</td>
</tr>
<tr>
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<td>36.4</td>
<td>38.2</td>
</tr>
<tr>
<td></td>
<td>Duda et al. (1996)</td>
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<td>34.4</td>
</tr>
<tr>
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<td>average</td>
<td></td>
<td>36.3</td>
</tr>
<tr>
<td>BF(s)</td>
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<tr>
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<td>31</td>
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<tr>
<td></td>
<td>Duda et al. (1996)</td>
<td>25</td>
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<td>average</td>
<td></td>
<td>30.4</td>
</tr>
<tr>
<td>SM</td>
<td>Wretenberg et al. (1996)</td>
<td>24.1</td>
<td>25.3</td>
</tr>
<tr>
<td></td>
<td>Duda et al. (1996)</td>
<td>16.5</td>
<td>18.6</td>
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<td>average</td>
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</tr>
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<td>Wretenberg et al. (1996)</td>
<td>30</td>
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</tr>
<tr>
<td></td>
<td>Duda et al. (1996)</td>
<td>26.5</td>
<td>29.9</td>
</tr>
</tbody>
</table>

96
4.5.9.3. Calculation of SEC stiffness

It has been demonstrated that the tendon and aponeurosis in the human gastrocnemius muscle stretch homogenously during muscle contraction (average 5.1% and 5.9% respectively) and therefore the SEC can be operated as a single unit (Muramatsu et al., 2001). In order to calculate SEC stiffness, it was assumed that the SEC length was stretched up to 5% during maximal isometric contraction. The corresponding change in $\theta_{sec}$ was calculated as follows:

$$\Delta \theta_{sec} = \frac{\Delta \text{SEC length}}{d}$$  \hspace{1cm} (4.33)
The SEC stiffness for an individual muscle group, $k_m$, was calculated by dividing the maximum torque by $\Delta \theta_{sec}$:

$$k_m = \frac{T_m}{\Delta \theta_{sec}} \quad (4.34)$$

The maximum torque ($T_m$) provided by an individual muscle group was estimated by multiplying the maximum isometric torque ($T_{iso}$) measured on the isovelocity dynamometer by a ratio of muscle moment arm and physiological cross-sectional area (pCSA):

$$T_m = T_{iso} \frac{d_i \times pCSA_i}{\sum_i (d_i \times pCSA_i)} \quad (4.35)$$

where $n = \text{number of muscle groups}$.

The SEC stiffness $k$ of a joint movement was calculated by summing up the stiffness of muscle groups $k_m$ that contributed to the movement. Due to lack of data for the shoulder joint, the stiffness of shoulder flexion and extension was set to be 1500 Nm/rad based on the literature (King, 1998). Values of pCSA were taken from Pierrynowski (1995) and the resulting stiffness of each joint movement were tabulated in Table 4.24. Once $k$ was known for each joint movement, the joint angle could be transformed to contractile component angle using Equation (3.12) to Equation (3.15).

### Table 4.24. SEC stiffness calculated from major muscle groups at each joint

<table>
<thead>
<tr>
<th>muscle</th>
<th>pCSA (mm$^2$)</th>
<th>$d \times pCSA$ (mm$^3$)</th>
<th>$T_m$ (Nm)</th>
<th>$k_m$ (Nm/rad)</th>
<th>$k$ (Nm/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ankle plantar flexion ($T_{iso} = 105$ Nm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>384.7</td>
</tr>
<tr>
<td>SO</td>
<td>11868</td>
<td>683158</td>
<td>68.3</td>
<td>267.1</td>
<td></td>
</tr>
<tr>
<td>GA(l)</td>
<td>1990</td>
<td>114550</td>
<td>11.5</td>
<td>38.3</td>
<td></td>
</tr>
<tr>
<td>GA(m)</td>
<td>4177</td>
<td>240441</td>
<td>24.0</td>
<td>75.4</td>
<td></td>
</tr>
<tr>
<td>PLT</td>
<td>209</td>
<td>12031</td>
<td>1.2</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td>ankle dorsi-flexion ($T_{iso} = 46$ Nm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>171.9</td>
</tr>
<tr>
<td>TA</td>
<td>2040</td>
<td>87723</td>
<td>24.0</td>
<td>92.4</td>
<td></td>
</tr>
<tr>
<td>EDL</td>
<td>1050</td>
<td>45152</td>
<td>12.3</td>
<td>30.5</td>
<td></td>
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<tr>
<td>EHL</td>
<td>485</td>
<td>20856</td>
<td>5.7</td>
<td>19.4</td>
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<td>PT</td>
<td>342</td>
<td>14707</td>
<td>4.0</td>
<td>29.6</td>
<td></td>
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<tr>
<td>Joint</td>
<td>Flexion</td>
<td>Extension</td>
<td>Flexion</td>
<td>Extension</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td>-----------</td>
<td>---------</td>
<td>-----------</td>
<td></td>
</tr>
<tr>
<td>Knee</td>
<td>(T_{iso} = 101 Nm)</td>
<td>(T_{iso} = 222 Nm)</td>
<td>(T_{iso} = 190 Nm)</td>
<td>(T_{iso} = 210 Nm)</td>
<td></td>
</tr>
<tr>
<td>GA(l)</td>
<td>1990</td>
<td>114550</td>
<td>323.2</td>
<td>323.2</td>
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<tr>
<td>GA(m)</td>
<td>4177</td>
<td>240441</td>
<td>36.4</td>
<td>36.4</td>
<td></td>
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<tr>
<td>PLT</td>
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<td>12031</td>
<td>1.8</td>
<td>1.8</td>
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</tr>
<tr>
<td>GR</td>
<td>340</td>
<td>10438</td>
<td>1.6</td>
<td>1.6</td>
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<td>87586</td>
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<td>938</td>
<td>28496</td>
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<tr>
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<tr>
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<td>171160</td>
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<td>350783</td>
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<td>VI</td>
<td>5368</td>
<td>273696</td>
<td>58.8</td>
<td>58.8</td>
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<td>238308</td>
<td>51.2</td>
<td>51.2</td>
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<tr>
<td>RF</td>
<td>3357</td>
<td>206365</td>
<td>179.0</td>
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<td></td>
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<tr>
<td>PM</td>
<td>1383</td>
<td>5460</td>
<td>4.7</td>
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</tr>
<tr>
<td>IL</td>
<td>1817</td>
<td>7173</td>
<td>6.2</td>
<td>6.2</td>
<td></td>
</tr>
<tr>
<td>GM (d)</td>
<td>1986</td>
<td>95204</td>
<td>23.2</td>
<td>23.2</td>
<td></td>
</tr>
<tr>
<td>GM (s)</td>
<td>2185</td>
<td>150338</td>
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<td>36.6</td>
<td></td>
</tr>
<tr>
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<td>3988</td>
<td>197923</td>
<td>48.2</td>
<td>48.2</td>
<td></td>
</tr>
<tr>
<td>ST</td>
<td>938</td>
<td>50785</td>
<td>12.4</td>
<td>12.4</td>
<td></td>
</tr>
<tr>
<td>BF (l)</td>
<td>2881</td>
<td>154356</td>
<td>37.6</td>
<td>37.6</td>
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</tr>
<tr>
<td>AM (m)</td>
<td>1362</td>
<td>104466</td>
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<td>25.4</td>
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</tr>
<tr>
<td>AM (p)</td>
<td>1674</td>
<td>109514</td>
<td>26.7</td>
<td>26.7</td>
<td></td>
</tr>
</tbody>
</table>

Shoulder flexion
Shoulder extension

---

dez and Voigt (2001; 2002) used a quick release method to measure the SEC stiffness in ankle dorsiflexors and plantar flexors. The average values at 34 Nm of three female subjects for dorsiflexors were 174 Nm/rad (voluntary) and 116 Nm/rad (electrical stimulation). The SEC stiffness calculated in this study was 179 Nm/rad, which was
comparable to de Zee and Voigt's study. For plantar flexors, the reported values at 100 Nm were 484 Nm/rad (voluntary) and 515 Nm/rad (electrical stimulation). Using similar calculation procedures for the male subjects in Maganaris and Paul (2002)'s study, the average SEC stiffness of gastrocnemius tendon was 312 Nm/rad ($T_{iso} = 46$Nm, stretch = 11.1mm, $d = 63$mm). Stefanyshyn and Nigg (1998) calculated the dynamic angular stiffness of ankle plantar flexion of male distance runners and sprinters from kinematic data. The average torsional spring stiffness was 327 Nm/rad for runners and 423 Nm/rad for sprinters. It is promising that the calculated value in this study for a female subject (385 Nm/rad) is comparable to values reported in literature.

There is minimal research in this area, thus discrepancies among studies are not unexpected. These discrepancies could be due to differences in methodology and individual subject characteristics, and/or assumptions made in the calculation procedures. Data of muscle architecture and moment arm taken from the literature, though after scaling, might not accurately represent that of the subject. Muscle groups which make minor contributions to the movement of interest (e.g. adductor longus in hip flexion/extension depending of hip angle) are not included. Regardless of these undetermined parameters, it is believed the calculated SEC stiffness values are reasonable estimates for subsequent use in the muscle-tendon model.

4.5.10. Re-optimisation of the 10-parameter function

After transforming the joint angle into $\theta_{con}$, the 10-parameter function was re-optimised using the new $\theta_{con}$.

$$T(\theta_{con}) = T_0 \times p[1 - q(\theta_{con} - \theta_{opt})^2]$$  \hspace{1cm} (4.36)

The seven angular velocity dependent parameters were kept unchanged whilst three angle dependent parameters ($p$, $q$ and $\theta_{opt}$) were re-optimised. The re-optimisation results of the three parameters are shown in Table 4.25. Surface plots of the 10-parameter function using $\theta_{con}$ and the corresponding joint angle are displayed in Figure 4.39 to Figure 4.46.
Table 4.25. Re-optimised results of the 10-parameter function using $\theta_{\text{con}}$

<table>
<thead>
<tr>
<th>parameter</th>
<th>knee</th>
<th></th>
<th>hip</th>
<th></th>
<th>shoulder</th>
<th></th>
<th>ankle</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fle</td>
<td>ext</td>
<td>fle</td>
<td>ext</td>
<td>fle</td>
<td>ext</td>
<td>dorsi</td>
<td>plantar</td>
</tr>
<tr>
<td>$T_0$ (Nm)</td>
<td>135</td>
<td>208</td>
<td>148</td>
<td>213</td>
<td>40</td>
<td>42</td>
<td>46</td>
<td>105</td>
</tr>
<tr>
<td>$T_{\text{max}}$ (Nm)</td>
<td>204</td>
<td>312</td>
<td>222</td>
<td>320</td>
<td>60</td>
<td>63</td>
<td>69</td>
<td>158</td>
</tr>
<tr>
<td>$\omega_{\text{max}}$ (rad/s)</td>
<td>22.3</td>
<td>23.6</td>
<td>15.3</td>
<td>16.1</td>
<td>43.2</td>
<td>11.7</td>
<td>19.0</td>
<td>17.5</td>
</tr>
<tr>
<td>$\omega_c$ (rad/s)</td>
<td>4.83</td>
<td>3.08</td>
<td>2.89</td>
<td>1.71</td>
<td>4.40</td>
<td>2.17</td>
<td>3.14</td>
<td>3.14</td>
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<tr>
<td>$\alpha_{\text{min}}$</td>
<td>0.77</td>
<td>0.63</td>
<td>-</td>
<td>-</td>
<td>0.84</td>
<td>0.72</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>m</td>
<td>0.10</td>
<td>0.13</td>
<td>-</td>
<td>-</td>
<td>0.10</td>
<td>0.37</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>$\omega_1$ (rad/s)</td>
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<td>0.00</td>
<td>-</td>
<td>-</td>
<td>2.01</td>
<td>1.28</td>
<td>1.26</td>
<td>1.26</td>
</tr>
<tr>
<td>p</td>
<td>1.24</td>
<td>0.87</td>
<td>1.45</td>
<td>0.76</td>
<td>0.98</td>
<td>0.99</td>
<td>1.03</td>
<td>0.94</td>
</tr>
<tr>
<td>q</td>
<td>0.16</td>
<td>1.26</td>
<td>0.03</td>
<td>0.38</td>
<td>0.30</td>
<td>0.10</td>
<td>0.61</td>
<td>1.66</td>
</tr>
<tr>
<td>$\theta_{\text{opt}}$ (rad)</td>
<td>3.00</td>
<td>3.88</td>
<td>6.00</td>
<td>4.68</td>
<td>5.07</td>
<td>2.73</td>
<td>1.73</td>
<td>4.32</td>
</tr>
</tbody>
</table>

* fle = flexion; ext = extension; dorsi = dorsi-flexion; plantar = plantar flexion.

Figure 4.39. Knee extension 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).
Figure 4.40. Knee flexion 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).

Figure 4.41. Hip extension 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).
Figure 4.42. Hip flexion 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).

Figure 4.43. Shoulder flexion 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).
Figure 4.44. Shoulder extension 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).

Figure 4.45. Ankle plantar flexion 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).
Figure 4.46. Ankle dorsi-flexion 10-parameter fit to contractile element angle and the corresponding joint angle (dot = raw data).

4.5.11. The metatarsal-phalangeal joint

Since no data for the metatarsal-phalangeal joint were collected, the halluc extensor torque was estimated as follows. Consider a person is standing still with the ankle (point A) plantar flexed and the ball (point B) maximally extended so that the point of pressure (point O) is near the tip of the toes and there is a large reaction force R acting at point O (Figure 4.47).

In order to maintain a fixed configuration, there must be a torque acting at the ankle ($T_A$) and the ball ($T_B$). Taking moment about the ankle and the ball respectively,
\[ T_A = Rd_A \]  \hspace{1cm} (4.37)
\[ T_B = Rd_B \]  \hspace{1cm} (4.38)

where \( d_A \) = moment arm of \( R \) to point \( A \)
\( d_B \) = moment arm of \( R \) to point \( B \)

It can be assumed that the moment arm \( d_B \) is about half of the moment arm \( d_A \). Substituting for \( d_B \) in Equation (4.38), the ball torque can be estimated as half of the ankle torque:
\[ T_B = R(0.5d_A) \]
\[ T_B = 0.5T_A \]  \hspace{1cm} (4.39)

Since the ball extensor was comparable to be half as strong as the ankle plantar flexor, it was assumed that the ball flexor was half as strong as the ankle dorsi-flexor. In order to avoid extrapolating the ankle angle to the range of motion of the ball, only a 7-parameter torque / angular velocity function was used for the ball.

4.5.12. Summary

From the isometric and isovelocity data collected, a torque / angle / angular velocity relationship was established by fitting a 10-parameter function. The joint angle was transformed to the contractile component angle to be used in the muscle-tendon complex in the torque-driven model. This 10-parameter function would be used to calculate the maximum torque at a specific contractile element angle and angular velocity in the simulation. However, it was found later that the torque given by this function was not strong enough to reproduce actual diving performance and therefore the function was further adjusted (details explained in Chapter 5).

4.6. Wobbling mass parameters

The wobbling mass parameters required include the spring stiffness and damping, the mass distribution, CM location and the moment of inertia of the fixed and wobbling components.

4.6.1. Spring stiffness and damping

Each wobbling mass was attached to the fixed component through two pairs of identical spring-dampers (see Section 3.3.1). The stiffness was chosen based on estimated maximum spring displacement (Table 4.26). A near-critical damping coefficient was selected based on visual inspection of vibration frequency (Pain & Challis, 2001). Figure
4.48 shows a typical displacement-time relationship and its corresponding spring force time history.

Table 4.26. Stiffness and damping values of wobbling mass springs

<table>
<thead>
<tr>
<th>wobbling mass</th>
<th>stiffness (10^8) N/m³</th>
<th>damping (Ns/m)</th>
<th>maximum displacement (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>trunk</td>
<td>1</td>
<td>80</td>
<td>16</td>
</tr>
<tr>
<td>thigh</td>
<td>7</td>
<td>120</td>
<td>8</td>
</tr>
<tr>
<td>shank</td>
<td>15</td>
<td>150</td>
<td>6.5</td>
</tr>
</tbody>
</table>

Figure 4.48. Typical displacement and force time history of a wobbling mass spring.

4.6.2. Mass distribution

The mass of the whole segment was determined using Yeadon's (1990b) model. The mass of the fixed and wobbling components were estimated from body composition data. This was done by re-calculating the bone to non-bone ratio reported in the literature using the percentage (%) body fat of the diver.
4.6.2.1. Measuring body composition

A 3-site skinfold formula for women (Jackson & Pollock, 1985) was used to determine the body density of the diver. The skinfold sites measured were 1) triceps; 2) suprailiac; and 3) thigh. Duplicated measures were taken if duplicated measurements were not within 1 to 2 mm (Table 4.27). Once the body density was determined, percent body fat was calculated using a conversion equation (Siri, 1956). Calculation procedures and results are shown below:

Figure 4.49. A 3-site skinfold test was carried out to determine body composition.

Table 4.27. Skinfold measurements of triceps, suprailiac and thigh

<table>
<thead>
<tr>
<th>Sites</th>
<th>Measure 1 (mm)</th>
<th>Measure 2 (mm)</th>
<th>Measure 3 (mm)</th>
<th>Average of 2 measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triceps</td>
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<td>13.4</td>
<td></td>
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<tr>
<td>Suprailiac</td>
<td>9.6 (discarded)</td>
<td>8.4</td>
<td>8.7</td>
<td>8.55</td>
</tr>
<tr>
<td>Thigh</td>
<td>20.0</td>
<td>19.4</td>
<td></td>
<td>19.7</td>
</tr>
</tbody>
</table>

| Sum of 3 skinfolds | 41.45 |

Age = 17

Body Density = 1.099421 - 0.0009929 (sum of 3 skinfolds) + 0.0000023 (sum of 3 skinfolds)^2 - 0.0001392 (Age) (Jackson & Pollock, 1985)

= 1.05985

% body fat = (495/body density) - 450 (Siri, 1956)

= 17.0%
The average body mass and % body fat of the cadavers from the literature are 64.3 kg and 34.6% respectively (Clarys, Martin, & Drinkwater, 1984). Taking the shank as an example, the scaling procedures are shown below:

- **mass diver** = 64.1 kg
- **% fat diver** = 17%
- **shank mass** = 2.136 kg (Clarys & Marfell-Jones, 1986)
- **fat mass** = 0.615 kg (Clarys & Marfell-Jones, 1986)
- **fat-free mass** = 2.136 - 0.615 = 1.521 kg
  
  \[
  \text{fat in shank} = \frac{0.615}{2.136} = 28.78\%
  \]

  \[
  \text{fat ratio} = \frac{\text{% fat in shank}}{\text{whole body % fat}} = \frac{28.78}{34.6} = 0.8318
  \]

  \[
  \text{new % fat in shank} = 0.8318 \times 17\% = 14.14\% \quad \text{(subject-specific)}
  \]

  \[
  \text{new fat mass} = 2.136 \times 14.14\% = 0.302 \text{ kg}
  \]

  \[
  \text{new shank mass} = \text{fat-free mass} + \text{new fat mass}
  \]

  \[
  = 1.521 + 0.302
  \]

  \[
  = 1.823 \text{ kg}
  \]

- **bone mass** = 0.463 kg (Clarys & Marfell-Jones, 1986)

  \[
  \text{fixed mass ratio} = \frac{\text{bone mass}}{\text{new shank mass}} = \frac{0.463}{1.823} = 25.42\%
  \]

  \[
  \text{wobbling mass ratio} = (100-25.42)\% = 74.58\%
  \]

The same procedures were applied to the thigh. For the trunk, the mass ratio was obtained in a slightly different way:

- **limb mass** = 26.854 kg (Clarys & Marfell-Jones, 1986)
- **trunk mass** = 64.3 - 26.854 = 37.445 kg
- **total fat mass** = 64.3 \times 34.6\% = 22.248 kg
- **limb fat mass** = 10.051 kg (Clarys & Marfell-Jones, 1986)
- **trunk fat mass** = 22.248 - 10.051 = 12.197 kg
- **fat-free mass** = 37.455 - 12.197 = 25.248 kg

  \[
  \text{% fat in trunk} = \frac{12.197}{37.445} = 32.57\%
  \]
fat ratio = \frac{\% \text{ fat in trunk}}{\text{whole body } \% \text{ fat}} = \frac{32.57}{34.6} = 0.9414

new \% \text{ fat in trunk} = 0.9414 \times 17\% = 16\% \text{ (subject-specific)}

new fat mass = 37.445 \times 16\% = 5.993 \text{ kg}

new trunk mass = \text{fat-free mass + new fat mass}
= 25.248 + 5.992
= 31.241 \text{ kg}

total \% \text{ bone} = 13.4\% \text{ (Clarys et al., 1984)}

total bone mass = 64.3 \times 13.4\% = 8.616 \text{ kg}

limb bone mass = 3.736 \text{ kg (Clarys & Marfell-Jones, 1986)}

trunk bone mass = 8.616 - 3.736 = 4.88 \text{ kg}

fixed mass ratio = \frac{\text{bone mass}}{\text{new trunk mass}} = \frac{4.88}{31.241} = 15.62\%

wobbling mass ratio = (100-15.62)\% = 84.38\%

Using the above fixed to wobbling ratio and the segmental mass obtained from Yeadon's (1990b) model, the mass of the fixed (m_f) and wobbling (m_w) components were calculated (Table 4.28).

<table>
<thead>
<tr>
<th>Table 4.28. Masses of fixed and wobbling components</th>
</tr>
</thead>
<tbody>
<tr>
<td>shank</td>
</tr>
<tr>
<td>m_f (kg)</td>
</tr>
<tr>
<td>m_w (kg)</td>
</tr>
</tbody>
</table>

4.6.3. CM location and moment of inertia

The moment of inertia of the whole segment (I_g) determined from Yeadon's (1990b) model was the summation of the moment of inertia of the fixed (I_f) and wobbling component (I_w) using the Parallel Axis Theorem:

\[ I_g = I_f + m_f(z_f - z)^2 + I_w + m_w(z_w - z)^2 \]  

(4.37)

where \( z = \) distance of the whole segment CM to proximal joint
\( z_f = \) distance of the fixed component CM to proximal joint
\( z_w = \) distance of the wobbling component CM to proximal joint.
Figure 4.50. Modelling a segment as a fixed and a wobbling component.

It was assumed that the segmental length of the fixed and the wobbling component are the same as the length (L) of whole segment. Modelling the fixed component as an uniform cylinder, the volume of the fixed component can be given by:

$$\text{volume} = \frac{\text{mass} (m_f)}{\text{density} (d)}$$

Taking the density values from the literature (Table 4.29), the radius (r) of the cylinder can be calculated using Equation (4.39):

$$\pi r^2 L = \frac{m_f}{d} \quad (4.39)$$

Table 4.29. Density values used to calculate the radius of the fixed component

<table>
<thead>
<tr>
<th></th>
<th>shank</th>
<th>thigh</th>
<th>trunk</th>
</tr>
</thead>
<tbody>
<tr>
<td>density (gm/cc$^3$)</td>
<td>1.2075</td>
<td>1.218</td>
<td>1.1</td>
</tr>
<tr>
<td>density (kg/m$^3$)</td>
<td>1207.5</td>
<td>1218</td>
<td>1100</td>
</tr>
<tr>
<td>reference</td>
<td>(Clarys &amp; Marfell-Jones, 1986)</td>
<td>(Dempster, 1955)</td>
<td></td>
</tr>
</tbody>
</table>

After the radius has been determined, $I_f$ can be determined using moment of inertia equation for an uniform cylinder:

$$I_f = \frac{1}{12} m_f L^2 + \frac{1}{4} m_f r^2 \quad (4.40)$$

For the fixed component with uniform density, $z_f$ can be taken as:

$$z_f = 0.5L \quad (4.41)$$

For the wobbling component, $z_w$ can be calculated by Equation (4.42):

$$m_f z_f + m_w z_w = (m_f + m_w) z \quad (4.42)$$
Finally after all parameters are determined, $I_w$ can be calculated using Equation (4.37). Table 4.30 summarises the results of segmental mass, length, distance of CM to proximal joint, and moments of inertia for the shank, thigh and trunk.

Table 4.30. Segmental inertia parameters for the fixed and wobbling components

<table>
<thead>
<tr>
<th></th>
<th>shank</th>
<th>thigh</th>
<th>trunk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_f$ (kg)</td>
<td>1.033</td>
<td>1.104</td>
<td>4.033</td>
</tr>
<tr>
<td>$m_w$ (kg)</td>
<td>3.030</td>
<td>8.335</td>
<td>21.786</td>
</tr>
<tr>
<td>$I_f$ (kg·m²)</td>
<td>0.0141</td>
<td>0.0157</td>
<td>0.1031</td>
</tr>
<tr>
<td>$I_w$ (kg·m²)</td>
<td>0.0378</td>
<td>0.1197</td>
<td>0.6119</td>
</tr>
<tr>
<td>$z_f$ (m)</td>
<td>0.202</td>
<td>0.205</td>
<td>0.274</td>
</tr>
<tr>
<td>$z_w$ (m)</td>
<td>0.175</td>
<td>0.164</td>
<td>0.274</td>
</tr>
<tr>
<td>$L$ (m)</td>
<td>0.403</td>
<td>0.411</td>
<td>0.548</td>
</tr>
<tr>
<td>$r$ (m)</td>
<td>0.026</td>
<td>0.027</td>
<td>0.046</td>
</tr>
</tbody>
</table>

4.7. Foot-springboard interface parameters

4.7.1. Introduction

The stiffness and damping of the visco-elastic elements in the foot-springboard interface, along with the refined estimation of wobbling mass and springboard parameters, were determined using an angle-driven model. This was done by driving the model with joint angle time histories obtained from video and optimising the values of the unknown visco-elastic parameters to minimise the difference between simulation and performance.

4.7.2. Model input

The input to the angle-driven model included initial conditions at touchdown and joint angle time histories throughout the simulation. The initial conditions comprised CM horizontal $v_x$ and vertical $v_z$ velocities, orientation angle $\theta_i$ and angular velocity $\omega_i$, and the foot position $d$ (horizontal distance from the toes to the tip of the board). Four dives which required minimal and maximal angular momentum in each dive group were selected. These dives were forward dive pike (101B), forward two and one-half somersault pike (105B), reverse dive tuck (301C) and reverse one and one-half somersault pike (303B). Since these four dives require very different rotational requirements, the parameters determined from these dives should be suitable for use with other dives from these two
groups with some confidence. The initial conditions for the four dives are shown in Table 4.31.

Table 4.31. Initial conditions at touchdown for the four dives as input into the model

<table>
<thead>
<tr>
<th>initial conditions</th>
<th>101B</th>
<th>105B</th>
<th>301C</th>
<th>303B</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_x ) (m/s)</td>
<td>0.38</td>
<td>0.47</td>
<td>0.54</td>
<td>0.48</td>
</tr>
<tr>
<td>( v_z ) (m/s)</td>
<td>-3.56</td>
<td>-3.85</td>
<td>-3.53</td>
<td>-3.40</td>
</tr>
<tr>
<td>( \theta_t )</td>
<td>71°</td>
<td>57°</td>
<td>68°</td>
<td>64°</td>
</tr>
<tr>
<td>( \omega_t ) (rad/s)</td>
<td>-2.05</td>
<td>-1.97</td>
<td>-1.22</td>
<td>-1.72</td>
</tr>
<tr>
<td>( d ) (m)</td>
<td>0.000</td>
<td>0.007</td>
<td>0.013</td>
<td>0.001</td>
</tr>
</tbody>
</table>

4.7.3. Model parameters

The parameters for the angle-driven model include body segmental inertias, springboard parameters, and the visco-elastic parameters of the wobbling mass and the foot-springboard interface. The inertia parameters were determined from anthropometric measurement (see Section 4.3) and the springboard parameters were calculated from experiment (see Section 4.4). The visco-elastic parameters were determined in an individual and a combined matching process which minimised the difference between simulation and performance.

4.7.3.1. Individual matching

The spring stiffness and damping of the foot-springboard interface were initially estimated based on predicted spring deformation and reaction force. From a preliminary matching simulation, the peak vertical springboard reaction force was about 3600 N near maximum springboard depression with the heel, ball and toes in contact with the springboard. It was assumed that the foot-spring displacement was 5 mm and the perpendicular stiffness of the foot-springs was therefore estimated as:

\[
F_z = -k_z z
\]

\[
3600 = -k_z (-0.005)
\]

\[
k_z = 0.72 \times 10^6 \text{ N/m}
\]

Assuming \( F_z \) was shared evenly among the three points of contact, \( k_z \) for each foot-spring would be \( 0.24 \times 10^6 \text{ N/m} \). The stiffness of the parallel foot-springs \( k_x \) was assumed to be
similar to $k_z$. Since there was a $z$ term in the parallel foot-spring equation (Equation (3.11)), $k_x$ was calculated as follows:

$$k_z = |z| k_x$$

$$0.24 \times 10^6 = (0.005) k_x$$

$$k_x = 48 \times 10^6 \text{ N/m}^2$$

Based on these values of $k_z$ and $k_x$, the lower (LB) and upper bound (UB) in the optimisation process were set to be one-third and three times of their initial estimates. Damping values were varied within the same bounds as the stiffness values.

In addition to the foot-springboard spring parameters, variables with uncertainties in initial estimation were also allowed to be optimised. These variables included: springboard constant $c$ in Equation (4.11) estimated from Sprigings et al. (1990); ball correction angle; triangular foot angle (the angle between the ankle, ball and heel); initial trunk angle and angular velocity. The correction ball angle was a fixed angle added to the calculated ball angle and this correction. Such correction was needed because the coordinate data were projected onto a plane for the planar model and the calculated ball angle was less than the actual angle. The triangular foot angle should be a fixed value defining the shape of the foot but due to digitisation error it might vary from the expected value. Slight flexibility of $\pm 1^\circ$ for initial trunk angle and $\pm 1 \text{ rad/s}$ for initial trunk angular velocity was allowed to compensate for digitisation error. Table 4.32 summarises the lower and upper bounds of the parameters that were allowed to vary in the individual matching.

<table>
<thead>
<tr>
<th>parameters</th>
<th>LB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallel foot-springs stiffness $k_x (x10^6 \text{ N/m}^2)$</td>
<td>16</td>
<td>144</td>
</tr>
<tr>
<td>parallel foot-springs damping $c_x (x10^6 \text{ Ns/m})$</td>
<td>16</td>
<td>144</td>
</tr>
<tr>
<td>perpendicular foot-springs stiffness $k_z (x10^6 \text{ Ns/m})$</td>
<td>0.08</td>
<td>0.72</td>
</tr>
<tr>
<td>perpendicular foot-springs damping $c_z (x10^6 \text{ Ns/m}^2)$</td>
<td>0.08</td>
<td>0.72</td>
</tr>
<tr>
<td>springboard constant $c (\text{N/m})$</td>
<td>2000</td>
<td>6000</td>
</tr>
<tr>
<td>ball correction angle</td>
<td>+12$^\circ$</td>
<td>+16$^\circ$</td>
</tr>
<tr>
<td>triangular foot angle</td>
<td>22$^\circ$</td>
<td>26$^\circ$</td>
</tr>
<tr>
<td>initial trunk angle $\theta_t$</td>
<td>-1$^\circ$</td>
<td>+1$^\circ$</td>
</tr>
<tr>
<td>initial trunk angular velocity $\omega_t$</td>
<td>-1 rad/s</td>
<td>+1 rad/s</td>
</tr>
</tbody>
</table>
4.7.3.2. Combined matching

The four chosen dives were used together in a combined matching to obtain a common set of parameters that would work for all four dives. In the combined matching, the foot-springboard interface parameters, springboard constant \( c \), triangular foot angle and ball correction angle were varied around the values found in the individual matchings. In addition, the springboard parameter \( m \) in Equation (4.11) estimated from Sprigings et al. (1990) and wobbling mass parameters (stiffness, damping and mass of the wobbling component) and the were varied ±20% from their initial estimates. A flexibility of ± 1° for initial trunk angle and ± 1 rad/s for initial trunk angular velocity was again allowed to compensate for digitisation error. Table 4.33 summarises the lower and upper bounds of the parameters that were allowed to vary in the combined matching.

Table 4.33. Lower and upper bounds of the 25 parameters in combined matching

<table>
<thead>
<tr>
<th>parameters</th>
<th>LB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallel foot-spring stiffness ( k_x ) (×10^6 N/m^2)</td>
<td>59</td>
<td>90</td>
</tr>
<tr>
<td>parallel foot-spring damping ( c_x ) (×10^6 Ns/m^3)</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>perpendicular foot-spring stiffness ( k_z ) (×10^6 Ns/m)</td>
<td>0.2</td>
<td>0.28</td>
</tr>
<tr>
<td>perpendicular foot-spring damping ( c_z ) (×10^6 Ns/m^2)</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>springboard constant ( c ) (N/m)</td>
<td>2800</td>
<td>3700</td>
</tr>
<tr>
<td>springboard constant ( m ) (N/m)</td>
<td>-20%</td>
<td>+20%</td>
</tr>
<tr>
<td>ball angle correction</td>
<td>+12°</td>
<td>+13°</td>
</tr>
<tr>
<td>triangular foot angle</td>
<td>22°</td>
<td>23°</td>
</tr>
<tr>
<td>initial trunk angle ( \theta_t ) (× 4 dives)</td>
<td>-1°</td>
<td>+1°</td>
</tr>
<tr>
<td>initial trunk angular velocity ( \omega_t ) (× 4 dives)</td>
<td>-1 rad/s</td>
<td>+1 rad/s</td>
</tr>
<tr>
<td>wobbling mass stiffness (× 3 segments)</td>
<td>-20%</td>
<td>+20%</td>
</tr>
<tr>
<td>wobbling mass damping (× 3 segments)</td>
<td>-20%</td>
<td>+20%</td>
</tr>
<tr>
<td>mass of wobbling component (× 3 segments)</td>
<td>-20%</td>
<td>+20%</td>
</tr>
</tbody>
</table>

4.7.4. Model output

The output variables chosen for comparison were takeoff time \( t \), maximum board depression \( z_{max} \), and conditions at takeoff including the CM horizontal \( v_x \) and vertical \( v_z \) velocities, angular momentum \( H \) and orientation angle \( \theta_t \). An objective score \( S_{ANG} \) was calculated as the average percentage difference between the simulation and the
performance for the above variables. To weight each of the six variables comparably, a 0.001 s difference in takeoff time and 1° difference in orientation were counted equivalent to 1% difference in other variables. Additional penalty scores were used to constrain the foot spring displacements to be within 10 mm in the perpendicular and 15 mm in the parallel directions respectively. If the spring deformation exceed these limits, 1 mm difference would introduce one point of penalty. Figure 4.51 summarises the components of the score $S_{ANG}$.

![Diagram](image)

Figure 4.51. Summary of the components of the objective score $S_{ANG}$.

4.7.5. Optimisation

For matching, nine parameters each of the four dives individually, nine parameters (see Table 4.32) were varied until the best match between simulation and performance was found. This matching was achieved by minimising the score $S_{ANG}$ using the Simulated Annealing optimisation algorithm (Corana et al., 1987). To obtain a common set of parameters that could be applied to any dive, the four dives were combined in a single optimisation program with a total of 25 parameters (see Table 4.33). A mean score was calculated as the average of the four $S_{ANG}$ obtained from the individual matchings. This mean score was minimised to match the four simulations to the performances with the same set of parameters. A maximum number of 200,000 simulations took 12 days to run.

4.7.6. Results

When each dive was matched individually, the simulation matched the performance very well with scores of 4.4% (101B), 2.2% (105B), 3.7% (301C) and 7.3% (303B). No penalty occurred in the foot-springboard interface for all four dives. Table 4.34 shows the parameters determined from each individual matching. These results, after
discarding values which were very different from the others, formed the upper and lower parameter bounds for the combined matching. Since the parallel damping $c_x$ was chosen to be the lower bound of $16 \times 10^6$ Ns/m$^3$ in two dives, the lower bound of $c_x$ was set as $8 \times 10^6$ Ns/m$^3$.

Table 4.34. Parameters determined from individual matching of four different dives*

<table>
<thead>
<tr>
<th>parameters</th>
<th>101B</th>
<th>105B</th>
<th>301C</th>
<th>303B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_x$ ($\times 10^6$ N/m$^2$)</td>
<td>89.60</td>
<td>58.85</td>
<td>74.89</td>
<td>(27.34)</td>
</tr>
<tr>
<td>$c_x$ ($\times 10^6$ Ns/m$^3$)</td>
<td>16.00</td>
<td>23.50</td>
<td>16.00</td>
<td>16.32</td>
</tr>
<tr>
<td>$k_z$ (Ns/m)</td>
<td>(0.72)</td>
<td>0.20</td>
<td>0.28</td>
<td>0.22</td>
</tr>
<tr>
<td>$c_z$ (Ns/m$^2$)</td>
<td>(0.08)</td>
<td>0.71</td>
<td>0.72</td>
<td>0.66</td>
</tr>
<tr>
<td>$c$ (N/m)</td>
<td>2830</td>
<td>2904</td>
<td>3348</td>
<td>3703</td>
</tr>
<tr>
<td>ball angle correction</td>
<td>$+13.43^\circ$</td>
<td>$+13.03^\circ$</td>
<td>(+16.00$^\circ$)</td>
<td>$+13.91^\circ$</td>
</tr>
<tr>
<td>triangular foot angle</td>
<td>$22.66^\circ$</td>
<td>$23.03^\circ$</td>
<td>$22.00^\circ$</td>
<td>$22.94^\circ$</td>
</tr>
</tbody>
</table>

* numbers in brackets were discarded in setting bounds for combined matching

In the combined matching, reasonably good agreement between the simulation and the performance was found with a mean score of 11.3%. The score $S_{ANG}$ for individual dives were 13.0% (101B), 11.1% (105B), 10.0% (301C), and 9.5% (303B). There were no penalties in general except for 301C with a 1.6 mm penalty in the heel spring perpendicular displacement. The largest discrepancy was that the simulation could not produce sufficient angular momentum. The triangular foot angle was $22.6^\circ$ and the ball angle was corrected by $+13.1^\circ$. The springboard parameters $m$ and $c$ were 7158 N/m$^2$ and 3663 N/m respectively. The stiffness and damping of the foot-springboard interface and the wobbling masses are shown in Table 4.35.

Table 4.35. Stiffness and damping parameters determined from combined matching

<table>
<thead>
<tr>
<th>parameter</th>
<th>stiffness</th>
<th>damping</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallel foot-spring</td>
<td>$80.9 \times 10^6$ N/m$^2$</td>
<td>$8.58 \times 10^6$ Ns/m$^3$</td>
</tr>
<tr>
<td>perpendicular foot-spring</td>
<td>$0.23 \times 10^6$ N/m</td>
<td>$0.76 \times 10^6$ Ns/m$^2$</td>
</tr>
<tr>
<td>shank wobbling mass</td>
<td>$1536 \times 10^6$ N/m$^3$</td>
<td>124 Ns/m</td>
</tr>
<tr>
<td>thigh wobbling mass</td>
<td>$817 \times 10^6$ N/m$^3$</td>
<td>102 Ns/m</td>
</tr>
<tr>
<td>trunk wobbling mass</td>
<td>$115 \times 10^6$ N/m$^3$</td>
<td>70 Ns/m</td>
</tr>
</tbody>
</table>
Personalised segmental inertias of the fixed and wobbling components are provided in Table 4.36. Detailed comparison of the matching simulation (individual and combined) of the four selected dives with their corresponding performance are listed in Table 4.37 to Table 4.40.

### Table 4.36. Personalised segmental inertias of the fixed and wobbling components

<table>
<thead>
<tr>
<th>segment*</th>
<th>mass (kg)</th>
<th>length (m)</th>
<th>CM from proximal joint (m)</th>
<th>moment of inertia (kg·m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>shank</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(fixed)</td>
<td>2.10</td>
<td>0.202</td>
<td>0.202</td>
<td>0.029</td>
</tr>
<tr>
<td>(wobbling)</td>
<td>6.03</td>
<td>0.214</td>
<td></td>
<td>0.053</td>
</tr>
<tr>
<td>thigh</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(fixed)</td>
<td>2.14</td>
<td>0.205</td>
<td></td>
<td>0.030</td>
</tr>
<tr>
<td>(wobbling)</td>
<td>16.74</td>
<td>0.180</td>
<td></td>
<td>0.226</td>
</tr>
<tr>
<td>trunk</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(fixed)</td>
<td>3.96</td>
<td>0.271</td>
<td></td>
<td>0.099</td>
</tr>
<tr>
<td>(wobbling)</td>
<td>21.86</td>
<td>0.299</td>
<td></td>
<td>0.529</td>
</tr>
</tbody>
</table>

*values for mass and moment of inertia of the left and right limbs are combined.

### Table 4.37. Comparison of the forward dive pike (101B) matching simulation and performance

<table>
<thead>
<tr>
<th>variable</th>
<th>performance</th>
<th>matching simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>individual</td>
</tr>
<tr>
<td>t</td>
<td>0.485 s</td>
<td>0.485 s</td>
</tr>
<tr>
<td>z_{max}</td>
<td>-0.75 m</td>
<td>-0.75 m</td>
</tr>
<tr>
<td>v_x</td>
<td>0.87 m/s</td>
<td>0.87 m/s</td>
</tr>
<tr>
<td>v_z</td>
<td>4.75 m/s</td>
<td>4.94 m/s</td>
</tr>
<tr>
<td>H</td>
<td>17.05 kg·m²</td>
<td>20.67 kg·m²</td>
</tr>
<tr>
<td>\theta_t</td>
<td>69°</td>
<td>70°</td>
</tr>
<tr>
<td>average score S_{ANG}</td>
<td>4.4%</td>
<td>13.0%</td>
</tr>
</tbody>
</table>
Table 4.38. Comparison of the forward two and one-half somersault pike (105B) matching simulation and performance

<table>
<thead>
<tr>
<th>variable</th>
<th>performance</th>
<th>matching simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>individual</td>
</tr>
<tr>
<td>t</td>
<td>0.435 s</td>
<td>0.435 s</td>
</tr>
<tr>
<td>z\text{max}</td>
<td>-0.73 m</td>
<td>-0.73 m</td>
</tr>
<tr>
<td>v_x</td>
<td>1.33 m/s</td>
<td>1.33 m/s</td>
</tr>
<tr>
<td>v_z</td>
<td>4.39 m/s</td>
<td>4.63 m/s</td>
</tr>
<tr>
<td>H</td>
<td>58.91 kg\cdot m^2</td>
<td>58.90 kg\cdot m^2</td>
</tr>
<tr>
<td>\theta_t</td>
<td>14°</td>
<td>21°</td>
</tr>
<tr>
<td>average score S\text{ANG}</td>
<td>2.2%</td>
<td>11.1%</td>
</tr>
</tbody>
</table>

Table 4.39. Comparison of the reverse dive tuck (301C) matching simulation and performance

<table>
<thead>
<tr>
<th>variable</th>
<th>performance</th>
<th>matching simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>individual</td>
</tr>
<tr>
<td>t</td>
<td>0.480 s</td>
<td>0.470 s</td>
</tr>
<tr>
<td>z\text{max}</td>
<td>-0.73 m</td>
<td>-0.74 m</td>
</tr>
<tr>
<td>v_x</td>
<td>1.55 m/s</td>
<td>1.55 m/s</td>
</tr>
<tr>
<td>v_z</td>
<td>4.55 m/s</td>
<td>4.97 m/s</td>
</tr>
<tr>
<td>H</td>
<td>26.80 kg\cdot m^2</td>
<td>26.80 kg\cdot m^2</td>
</tr>
<tr>
<td>\theta_t</td>
<td>92°</td>
<td>95°</td>
</tr>
<tr>
<td>average score S\text{ANG}</td>
<td>3.7%</td>
<td>10.0%</td>
</tr>
</tbody>
</table>
Table 4.40. Comparison of the reverse one and one-half somersault pike (303B) matching simulation and performance

<table>
<thead>
<tr>
<th>variable</th>
<th>performance</th>
<th>matching simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>individual</td>
</tr>
<tr>
<td>t</td>
<td>0.495 s</td>
<td>0.469 s</td>
</tr>
<tr>
<td>z_{max}</td>
<td>-0.74 m</td>
<td>-0.70 m</td>
</tr>
<tr>
<td>v_x</td>
<td>1.25 m/s</td>
<td>1.25 m/s</td>
</tr>
<tr>
<td>v_z</td>
<td>4.40 m/s</td>
<td>4.94 m/s</td>
</tr>
<tr>
<td>H</td>
<td>53.94 kg·m^2</td>
<td>53.94 kg·m^2</td>
</tr>
<tr>
<td>θ_l</td>
<td>119°</td>
<td>119°</td>
</tr>
<tr>
<td>average score S_{ANG}</td>
<td>7.3%</td>
<td>9.5%</td>
</tr>
</tbody>
</table>

Graphical comparison of the takeoff phase in combined matching and performance for 105B and 303B are displayed in Figures 4.52 and 4.53. It can be seen that both simulations takeoff earlier than in the performance. The body configuration at the last instant of the takeoff will therefore be different.

Figure 4.52. Graphical comparison of 105B takeoff: performance (upper) and combined matching simulation (lower).
4.7.7. Sensitivity of parameters

The sensitivity of the parameters determined from individual matching (see Table 4.34) was assessed by using the parameter set determined from one dive in the simulations of other three dives. The matching scores resulting from using a different parameter set ranged from 6.6% to 34.2% excluding the penalties incurred. There were penalties in most simulations with the poorest matching for 301C when parameters obtained from 101B were used. Table 4.41 compares the scores obtained from individual matching, combined matching and using the parameter set of other dives.
Table 4.41. Comparison of scores obtained from individual matching, combined matching and using the parameter set of other dives.

<table>
<thead>
<tr>
<th>dive</th>
<th>101B</th>
<th>105B</th>
<th>301C</th>
<th>303B</th>
</tr>
</thead>
<tbody>
<tr>
<td>101B</td>
<td>4.4% (13.0%)*</td>
<td>9.4%</td>
<td>22.1%</td>
<td>19.1%</td>
</tr>
<tr>
<td>105B</td>
<td>6.6%</td>
<td>2.2% (11.1%)*</td>
<td>18.6%</td>
<td>20.1%</td>
</tr>
<tr>
<td>301C</td>
<td>9.0%</td>
<td>8.9%</td>
<td>3.7% (10.0%)*</td>
<td>9.6%</td>
</tr>
<tr>
<td>303B</td>
<td>33.6%</td>
<td>34.2%</td>
<td>23.0%</td>
<td>7.3% (9.5%)*</td>
</tr>
</tbody>
</table>

*individual (combined) matching score for each dive

Using the parameters of 101B in 105B and vice versa appears to work quite well with less than 10% in matching score but this is not the case for other dives. Even with a similar set of parameters as for 105B and 301C, there is a substantial difference in the matching score when the parameters are inter-changed. 301C seems to be less sensitive to the parameters in that all sets of parameters produce matching scores of less than 10%. On the other hand, 303B is the most sensitive resulting in a poor match if any other parameter set is used. These results suggest that parameters determined from individual matching may not work for another dive. It is therefore necessary to use more than one dive in a combined matching to determine a parameter set that can be used with all dives.

4.7.8. Discussion

In individual matching simulations, there is good agreement in the takeoff time and maximum board depression for all dives in general. This suggests that the springboard model successfully captures the physical characteristics of the springboard. This model, which allows horizontal board tip movement and rotation in addition to vertical movement, is an improved representation of the springboard beyond a simple mass-spring system. The CM horizontal velocity and angular momentum (except 101B) match very well for all four dives, though the CM vertical velocity is systematically higher in the simulation. The general good agreement demonstrates the potential of modelling the foot-springboard interface using spring-dampers. It also shows that the springboard model can be incorporated with the diver model to reproduce diving takeoff movement accurately.

In the combined optimisation from which a common set of spring parameters were obtained, there was a large discrepancy in matching angular momentum. This discrepancy
could be associated with errors in manual digitisation especially in locating the heel, ball and toes on the same plane. In the angle-driven model, digitisation error could lead to a fluctuation in joint kinematics, resulting in unrealistic joint torques at the ankle and the ball. These joint torques would affect the foot-springboard interface kinematics and thus the reaction force acting on the diver. An example of the torque time history of 105B calculated in the angle-driven model is shown in Figure 4.54.

Figure 4.54. Torque time history of 105B calculated in the angle-driven model.

Moreover, the CM position relative to the point of force application has a large influence in terms of generating angular momentum. During the recoil phase of the takeoff, the heel lost contact with the springboard gradually such that there were only reaction forces acting on the ball and the toes. If the CM was behind the ball, reverse angular momentum would be generated whereas if the CM was close to the toes, forward angular momentum was favoured. Since the distance between the ball and the toes was only 68 mm, a slight fluctuation in CM position due to digitisation error might cause problems in matching both linear and angular momentum.
There could also be a problem of using a straight segment to represent the trunk in the model. The natural curvature of a human spine can arch (hyper-extend) and dish (protract) and this may facilitate the generation of angular momentum. In the later torque-driven model, the hip angle was allowed to hyper-extend slightly more in order to compensate for this model deficiency.

Using the parameter set determined from an individual matching with another dive could result in a very poor match between simulation and performance. This indicates that parameters determined from individual matching are sensitive to an individual performance and cannot be applied to other performances. Using the combined parameter set on the other hand demonstrated a close correspondence for all four dives although the matches between simulation and performance were less good than the individual matching particularly for the angular momentum generated. This problem might be assessed in the future using a torque-driven model. If joint torque time histories can be found which result in a close matching of both joint angles and angular momentum between simulation and performance then the problem will have little implication for use of the combined parameters in a torque-driven model. Since four dives with different angular momentum requirements from different dive groups were used in the determination of the combined set of visco-elastic parameters, it may be expected that this parameter set could be used for simulation of other dives. In the torque-driven model, joint kinematics will be calculated from a torque / angle / angular velocity function determined from experimental data. Realistic joint torques within the diver's strength capacity would be expected. It is therefore speculated that a better simulation result will be obtained.

4.8. Summary

This chapter demonstrates how model parameters were measured directly from experiments or indirectly determined using a subject-specific angle-driven model. Kinematic data were obtained from video recordings of diving performances. Springboard parameters were determined from experimental board loading data. Body segmental inertias were calculated from anthropometric measurements taken on the diver. Maximum isometric and isovelocity torques were measured on an isovelocity dynamometer. Wobbling mass segmental inertias were estimated from body composition and information in the literature. Visco-elastic parameters of the foot-springboard interface and wobbling mass were determined in a combined matching of the angle-driven model using four different dives. The parameters determined will be used in the torque-driven model which,
after satisfactory evaluation, will be used to investigate takeoff techniques and optimise diving performance.
CHAPTER 5
MODEL EVALUATION

5.1. Introduction

Before simulation models can be used with any confidence, a successful evaluation must be demonstrated to ensure that the model produces realistic human movements. This can be done by comparing the simulation with an actual performance. Only if good agreement is found between the simulation and the performance should the model be used for further analysis and optimisation. This chapter describes how the torque-driven model is evaluated based on an objective measurement of the difference between simulation and performance. In addition, kinetic and kinematic analysis of the wobbling mass, springboard and foot-springboard interface are also included.

5.2. Evaluation

5.2.1. Model input

The torque-driven model was evaluated by driving the model with known initial conditions and optimising muscle activation parameters until the best match between the simulation and the performance was found. Each of the four dives used in the angle-driven model was evaluated individually. The input to the torque-driven model includes initial conditions at touchdown (see Chapter 4) and the muscle activation time histories of the 10 torque generators. Subject-specific model parameters including segmental inertia, strength, wobbling mass, foot-springboard interface and springboard parameters previously determined from experiments or the combined matching of the angle-driven model were used (see Chapter 4). The torque calculated from the 10-parameter torque / angle / angular velocity function and muscle activation level was doubled to represent the torque generated by two limbs assuming that there was no bilateral deficit due to the symmetrical nature of diving takeoff (see Chapter 2).

5.2.2. Model variables

The 60 muscle activation parameters (see Section 3.5.2), plus the initial trunk angle (±1°) and angular velocity (±1 rad/s) were varied to search for the best match between the simulation and the performance. The lower bound (LB) and upper bound (UB) of each of
the 12 muscle activation parameters at each joint were set based upon information from the literature.

It has been well demonstrated that muscles are activated before landing. This pre-landing activation time has been recorded between 66 ms to 139 ms depending on landing height (Arampatzis, Morey-Klapsing & Bruggemann, 2003; Santello & McDonagh, 1998) and 44 ms to 48 ms in tumbling takeoffs (McNeal, Sands & Shultz, 2003). Thus, it was decided to set the initial starting times \( s_1 \) and \( s_f \) (see Chapter 3.5.2) to be between \(-150\) ms and 0 where touchdown occurred at \( t = 0 \). There was more flexibility in the final starting time \( s_{e2} \) and \( s_{f2} \) which were allowed to vary between 0.1 s to 0.5 s. If \( s_{e2} \) or \( s_{f2} \) was larger than the total takeoff time (less than 0.5 s), there would be no final ramp up / down. For the reverse group, the knee flexor activation was not allowed to ramp up again at the end. This was because the knee was flexing instead of extending at the end and therefore no need to prevent hyper-extension.

It has been observed that the time required for zero to maximal activation is about 100 ms in landing from 0.45 m (Duncan & McDonagh, 2000) and between 100 ms to 200 ms in jumping activities (Finni, Komi & Lepola, 2000; Jacobs, Bobbert & van Ingen Schenau, 1996; Komi & Gollhofer, 1997), maximum voluntary contraction on a dynamometer (Grabiner & Owings, 2002), fast elbow movement (Gottlieb, 1998; Shapiro, Gottlieb, Moore & Corcos, 2002), and landing from different heights (Arampatzis et al., 2003; Santello & McDonagh, 1998). The lower bound for ramping duration \( t_{e1}, t_{e2}, t_{f1} \) and \( t_{f2} \) was therefore set as 100 ms. There should not be a upper bound theoretically but it was set at 0.5 s within which the whole diving takeoff would have completed.

The level of extensor and flexor activation at touchdown in a drop jump from 0.5 m is about 50% of maximal activation (Horita, Komi, Nicol & Kyrolainen, 2002). Although some higher pre-activation levels have been reported (Arampatzis et al., 2003; Santello & McDonagh, 1998), it was decided to set the upper bound for pre-activation level \( l_e \) at 0.5. For the extensors, the maximum activation level \( l_e \) was set between 0.6 and 1.0. Since the maximum torque that can be produced by the ball extensor (flexor) was assumed to be half of that of the ankle plantar (dorsi) flexor (see Chapter 4.5.11), the strength parameters for the ankle were used for the ball except that the activation level \( l_e \) for the ball extensor was set between 0.0 and 0.1 and \( l_e \) between 0.1 and 0.5. For the flexors, a minimal activation level of 1% to 5% was set so that the SEC angle did not fall to zero. Table 5.1 shows the lower and upper bounds for the 12 parameters at each joint.
Table 5.1. Lower and upper bounds for muscle activation parameters

<table>
<thead>
<tr>
<th>parameters</th>
<th>symbol</th>
<th>LB</th>
<th>UB</th>
</tr>
</thead>
<tbody>
<tr>
<td>extensors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>starting time for ramping up</td>
<td>se₁ (s)</td>
<td>-0.15</td>
<td>0.0</td>
</tr>
<tr>
<td>duration for ramping up</td>
<td>te₁ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>starting time for ramping down</td>
<td>se₂ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>duration for ramping down</td>
<td>te₂ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>minimal pre-landing activation level</td>
<td>le₁</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>maximum activation level</td>
<td>le₂</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>flexors</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>starting time for ramping down</td>
<td>tf₁ (s)</td>
<td>-0.15</td>
<td>0.0</td>
</tr>
<tr>
<td>duration of ramping down</td>
<td>tf₂ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>starting time of ramping up</td>
<td>sf₂ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>duration of ramping up</td>
<td>te₂ (s)</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>maximal pre-activation level</td>
<td>lf₁</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>minimum activation level</td>
<td>lf₂</td>
<td>0.01</td>
<td>0.05</td>
</tr>
</tbody>
</table>

5.2.3. Objective score

In order to evaluate the model, an objective score $S_{TOR}$ was used as a measure of the average percentage (%) difference between a simulation and the performance. This objective score was the average of five equally weighted parts. The five parts were S1: joint angles, S2: orientation angle, S3: linear momentum, S4: angular momentum, and S5: springboard.

The joint angles score S1 was the average root mean squared (RMS) difference between simulation and performance in degrees of the five joint angles: ball ($\theta_b$), ankle ($\theta_a$), knee ($\theta_k$), hip ($\theta_h$), and shoulder ($\theta_s$). The orientation angle score S2 was calculated as the RMS difference in the trunk angle ($\theta_t$) time history. The linear momentum score S3 was the average % difference in CM horizontal $v_x$ and vertical velocities $v_z$ at takeoff. The angular momentum score S4 was the % difference of the angular momentum $H$ at takeoff. The springboard score S5 was the average of the takeoff time difference and the % difference in maximum vertical board depression $z_{max}$. For the takeoff time, 0.001 s was counted as 1%. For the scores S1 and S2 which were calculated in degrees, 1° was considered to be comparable to 1% for other scores and therefore 1° was counted as 1%.

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The effect of these weightings was to weight each component of $S_{TOR}$ comparably. Penalty scores were used to limit the joint angles during takeoff and 0.1 s after takeoff within the range observed in video recordings to prevent hyper-extension in flight using Equation (5.4). Table 5.2 summarised the range of motion that constrained each joint angle.

$$\theta_2 = \theta + \omega (0.1)$$  \hspace{2cm} (5.4)

where $\theta_2$ = predicted angle at 0.1 s after takeoff

$\theta$ = joint angle at takeoff

$\omega$ = angular velocity at takeoff.

<table>
<thead>
<tr>
<th>Joint Angle</th>
<th>During Takeoff</th>
<th>0.1 s After Takeoff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>Ball</td>
<td>85°</td>
<td>190°</td>
</tr>
<tr>
<td>Ankle</td>
<td>85°</td>
<td>180°</td>
</tr>
<tr>
<td>Knee</td>
<td>180°</td>
<td>110°</td>
</tr>
<tr>
<td>Hip</td>
<td>200°</td>
<td>195°</td>
</tr>
<tr>
<td>Shoulder</td>
<td>195°</td>
<td>195°</td>
</tr>
</tbody>
</table>

These constraints ensured that the joint angular velocity at takeoff was sensible and would not lead to injury in the flight phase. The hip angle in flight was allowed to hyper-extend to 220° (instead of the observed 200°) in order not to over-constrain and to compensate for using a straight spine in the model. It was believed that the diver should be able to hyper-extend more than 220° but being too arched at the takeoff would inhibit pulling into shape in the flight phase and also affect the aesthetic aspect of the dive. The minimum knee angle in flight was set at 110° based on the observed takeoff angle (133°) and angular velocity (3.6 rad/s) even though this was not a physical limit. This was to prevent too much knee flexion at the takeoff in the reverse group. Equation (5.5) shows how the objective score $S_{TOR}$ was calculated and Figure 5.1 summarises the components of $S_{TOR}$.

$$S_{TOR} = \frac{S_1 + S_2 + S_3 + S_4 + S_5}{5} + \text{penalties}$$  \hspace{2cm} (5.5)
5.2.4. Optimisation

With known inputs and initial conditions, the 60 muscle activation parameters along with the initial trunk angle (±1°) and angular velocity (±1 rad/s) in the model were varied until the best match between the simulation and the performance was found. This was achieved by minimising \( S_{TOR} \) using the Simulated Annealing optimisation algorithm (Corana et al., 1987). The starting simulation was chosen manually by trial and error until the model performed a reasonable dive. An integration time step of 0.0001s was used. The maximum number of evaluations was set at 20,000 and it took about 12 days to complete.

5.2.5. Strength adjustment

Using the 10-parameter torque / angle / angular velocity relationships determined on the isovelocity dynamometer could not produce a good match between the simulation and the performance. At many joints, the joint could not extend in the same manner as the performance even though it was fully activated. The torques calculated from the angle-driven model suggested that the subject had produced a larger torques in performing dives than on the dynamometer. It was believed that the subject did not produce maximum torque on the dynamometer due to unfamiliarity. Having closely examined the torque / angular velocity data and the maximum isometric torque, it was concluded that the diver
should be capable of producing higher concentric torque at high angular velocities. The eccentric torque and isometric torque values obtained from the dynamometer were thought to be more reasonable.

In order to obtain a function which could predict maximum torque at high angular velocities, it was decided to scale a 7-parameter torque / angular velocity function of an elite male gymnast (Mills, 2004, personal communication) to the subject in this study based on her isometric torque data. Since diving and gymnastics share similar techniques of somersault takeoffs, it was speculated that the muscle properties of divers and gymnasts would be similar. The seven parameters for the torque / angular velocity relationship were the same as those described earlier (see Section 4.5.7.6). The highest isometric torque measured was taken to be $T_o$ and $T_{\text{max}}$ was set as 1.5 times $T_o$. The other five parameters were taken from those of the male gymnast.

An additional two parameters $q$ and $\theta_{opt}$ introducing angle dependency were obtained from fitting a quadratic function to the isometric data using Equation (5.5). The 9-parameter function representing the torque / angle / angular velocity relationship was therefore expressed using Equation (5.6). The values for this scaled 9-parameter function are shown in Table 5.3.

$$T_\theta = T_o \times [1 - q (\theta - \theta_{\text{opt}})^2]$$  \hspace{1cm} (5.5)

$$T(\theta, \omega) = T_o \times [1 - q (\theta - \theta_{\text{opt}})^2]$$  \hspace{1cm} (5.6)

where

$T_\theta = \text{angle dependent torque}$

$T_o = \text{torque calculated from a torque / angular velocity relationship}$

$T(\theta, \omega) = \text{torque calculated from a torque / angle / angular velocity relationship}$
Table 5.3. Nine torque parameters scaled from an elite male gymnast

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Knee flex*</th>
<th>Ext*</th>
<th>Hip flex</th>
<th>Ext</th>
<th>Shoulder flex</th>
<th>Ext</th>
<th>Dorsi-flexion</th>
<th>Plantar flexion</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (Nm)</td>
<td>126</td>
<td>222</td>
<td>191</td>
<td>231</td>
<td>47</td>
<td>44</td>
<td>46</td>
<td>105</td>
</tr>
<tr>
<td>$T_{\text{max}}$ (Nm)</td>
<td>189</td>
<td>333</td>
<td>289</td>
<td>347</td>
<td>71</td>
<td>66</td>
<td>69</td>
<td>158</td>
</tr>
<tr>
<td>$\omega_{\text{max}}$ ($^\circ$/s)</td>
<td>2000</td>
<td>2000</td>
<td>1600</td>
<td>1600</td>
<td>2000</td>
<td>2000</td>
<td>1600</td>
<td>1600</td>
</tr>
<tr>
<td>$\omega_c$ ($^\circ$/s)</td>
<td>382</td>
<td>665</td>
<td>203</td>
<td>523</td>
<td>364</td>
<td>578</td>
<td>316</td>
<td>589</td>
</tr>
<tr>
<td>$a_{\text{min}}$</td>
<td>0.80</td>
<td>0.90</td>
<td>0.80</td>
<td>0.80</td>
<td>0.90</td>
<td>0.80</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>$m$</td>
<td>3.6</td>
<td>15.5</td>
<td>5.1</td>
<td>15.8</td>
<td>7.3</td>
<td>18.4</td>
<td>5.3</td>
<td>16.6</td>
</tr>
<tr>
<td>$\omega_1$ ($^\circ$/s)</td>
<td>-26</td>
<td>-131</td>
<td>-17</td>
<td>-44</td>
<td>-114</td>
<td>-72</td>
<td>-52</td>
<td>-83</td>
</tr>
<tr>
<td>$q$ ($x 10^4$)</td>
<td>2.2</td>
<td>2.5</td>
<td>0.43</td>
<td>0.29</td>
<td>0.12</td>
<td>0.13</td>
<td>2.4</td>
<td>6.6</td>
</tr>
<tr>
<td>$\theta_{\text{opt}}$ ($^\circ$)</td>
<td>127</td>
<td>244</td>
<td>199</td>
<td>359</td>
<td>449</td>
<td>363</td>
<td>109</td>
<td>261</td>
</tr>
</tbody>
</table>

*flex = flexion; ext = extension

After scaling the torque parameters, the simulation matched the performance better but some optimised activation was fully activated throughout the simulation. This again suggested that the torque was not large enough to re-produce the diving performance. In some movements, the torque calculated from the 9-parameter function was still smaller than that of the angle-driven model. It was then decided to further increase the value of $T_0$ until a good match was found provided that the adjusted torque did not exceed the torque calculated from the angle-driven model. The final adjustment of $T_0$ in all movements are listed in Table 5.4.

There were approximately two times increase in the $T_0$ value four movements (hip extension, shoulder extension / flexion, and ankle plantar flexion). No increase was required for the knee extension, hip flexion and ankle dorsi-flexion. The small increase in knee flexion was due to the higher isometric torque measured on the dynamometer compared to the $T_0$ value estimated from the isovelocity movements since a springboard diving takeoff would not require a large knee flexion torque.
### Table 5.4. Adjustment of isometric torque to scale strength parameters

<table>
<thead>
<tr>
<th>movement</th>
<th>isometric $T_0$ (Nm)</th>
<th>adjusted $T_0$ (Nm)</th>
<th>adjustment factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee flexion</td>
<td>101</td>
<td>126</td>
<td>1.25</td>
</tr>
<tr>
<td>knee extension</td>
<td>222</td>
<td>222</td>
<td>1.00</td>
</tr>
<tr>
<td>hip flexion</td>
<td>191</td>
<td>191</td>
<td>1.00</td>
</tr>
<tr>
<td>hip extension</td>
<td>210</td>
<td>397</td>
<td>1.89</td>
</tr>
<tr>
<td>shoulder flexion</td>
<td>47</td>
<td>91</td>
<td>1.93</td>
</tr>
<tr>
<td>shoulder extension</td>
<td>44</td>
<td>88</td>
<td>2.00</td>
</tr>
<tr>
<td>ankle plantar flexion</td>
<td>105</td>
<td>215</td>
<td>2.04</td>
</tr>
<tr>
<td>ankle dorsi-flexion</td>
<td>46</td>
<td>46</td>
<td>1.00</td>
</tr>
</tbody>
</table>

### 5.2.6. Results

The average $S_{TOR}$ of the four dives was 6.3% and $S_{TOR}$ for individual dives were 4.9% (101B), 6.5% (105B), 7.2% (301C), and 6.6% (303B). Details of each component of the score and muscle activation parameters for each dive are shown below.

#### 5.2.6.1. Forward dive pike

The score of forward dive pike (101B) was 4.9% with excellent match (0% difference) in angular momentum. There were also very good agreements in orientation ($4^\circ$ difference), CM horizontal ($0.7\%$ difference) and vertical ($2.2\%$ difference) velocities. There were considerable difference in joint angle time histories, maximum board depression and takeoff time and the largest discrepancy lay in the ball angle time history. Details of the score and comparison of simulation outputs and performance are displayed in Table 5.5.
Table 5.5. Evaluation score of forward dive pike (101B)

<table>
<thead>
<tr>
<th>component / variables</th>
<th>performance</th>
<th>simulation</th>
<th>sub-score</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: joint angles</td>
<td></td>
<td></td>
<td></td>
<td>9.8°</td>
</tr>
<tr>
<td>( \theta_b )</td>
<td>14.5°</td>
<td>14.5°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_a )</td>
<td>11.5°</td>
<td>11.5°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_k )</td>
<td>12.4°</td>
<td>12.4°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_h )</td>
<td>4.0°</td>
<td>4.0°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_s )</td>
<td>6.8°</td>
<td>6.8°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2: orientation angle</td>
<td>( \theta_t )</td>
<td>4.2°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3: linear momentum</td>
<td></td>
<td>1.5%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v_x )</td>
<td>0.87 m/s</td>
<td>0.86 m/s</td>
<td>0.7%</td>
<td></td>
</tr>
<tr>
<td>( v_z )</td>
<td>4.75 m/s</td>
<td>4.65 m/s</td>
<td>2.2%</td>
<td></td>
</tr>
<tr>
<td>S4: angular momentum H</td>
<td>17.05 kg·m²</td>
<td>17.05 kg·m²</td>
<td>0%</td>
<td></td>
</tr>
<tr>
<td>S5: springboard</td>
<td></td>
<td>9.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z_{max} )</td>
<td>-0.75 m</td>
<td>-0.66 m</td>
<td>11.0%</td>
<td></td>
</tr>
<tr>
<td>( t )</td>
<td>0.485 s</td>
<td>0.478 s</td>
<td>7.2%</td>
<td></td>
</tr>
<tr>
<td>average score ( S_{TOR} )</td>
<td></td>
<td>4.9%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.2 compares the joint angle and orientation angle time histories in the simulation with those calculated from the performance. There is good agreement in general for the hip and shoulder angles and trunk orientation throughout the whole simulation. The knee and ankle angles in the simulation were similar to the performance initially and the difference increased in the later half of the simulation. This suggests that the board depression phase in the simulation closely matches the performance and the difference mainly lies in the recoil phase during which angular momentum is generated. The ball angle calculated from the video recordings is not considered to be very accurate and therefore a match in pattern rather than the exact value is acceptable. The ball angle in the simulation was quite stable for the first 0.3 s as in the performance but failed to replicate the flexion / extension pattern towards the end of the simulation. Since the contribution of the ball flexor / extensor is relatively small compared to other joints, the difference in ball angle should have little influence on the overall simulation output.
Figure 5.2. Comparison of joint angle and orientation time histories for 101B (solid line = simulation, dotted line = performance).
The muscle activation profiles of the 10 torque generators are shown in Figure 5.3 and their corresponding values are included in Table 5.6. There was co-contraction at touchdown in each joint since both extensors and flexors were activated initially. This is in agreement with the well established finding that muscles are activated prior to landing.

Figure 5.3. Muscle activation time histories for the matching 101B simulation (kemact = knee extension muscle activation, etc).

The net torque time histories obtained from the torque-driven and angle-driven models are compared in Figure 5.4. It can be seen that both time histories display a similar pattern and that the torque in the torque-driven model is a lot smoother than in the angle-driven model. The only obvious difference in the net torque pattern was that the decrease in knee torque during the recoil phase in the torque-driven model was later than in the angle-driven model. Despite the difference in the knee torque pattern, the torque values in the torque-driven model did not exceed any of the peak values in the angle-driven model.
Table 5.6. Muscle activation parameters for the matching 101B simulation

<table>
<thead>
<tr>
<th></th>
<th>torque generator</th>
<th>muscle activation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>se1/sf1(s)</td>
<td>te1/tf1(s)</td>
</tr>
<tr>
<td>knee extensor</td>
<td>-0.047</td>
<td>0.253</td>
</tr>
<tr>
<td>knee flexor</td>
<td>-0.041</td>
<td>0.203</td>
</tr>
<tr>
<td>hip extensor</td>
<td>-0.049</td>
<td>0.291</td>
</tr>
<tr>
<td>hip flexor</td>
<td>-0.076</td>
<td>0.218</td>
</tr>
<tr>
<td>shoulder extensor</td>
<td>-0.064</td>
<td>0.281</td>
</tr>
<tr>
<td>shoulder flexor</td>
<td>-0.045</td>
<td>0.205</td>
</tr>
<tr>
<td>ankle plantar flexor</td>
<td>-0.013</td>
<td>0.314</td>
</tr>
<tr>
<td>ankle dorsi-flexor</td>
<td>-0.091</td>
<td>0.316</td>
</tr>
<tr>
<td>ball extensor</td>
<td>-0.059</td>
<td>0.430</td>
</tr>
<tr>
<td>ball flexor</td>
<td>-0.083</td>
<td>0.483</td>
</tr>
</tbody>
</table>

Figure 5.4. Torque time histories for 101B obtained from the torque-driven (solid line) and angle-driven (dashed line) models.
Graphics sequences of the hurdle, takeoff and flight phase of the performance and the simulation are displayed in Figure 5.5. The hurdle phase configuration was obtained from video digitisation and therefore was identical for the performance and simulation. The flight phase was simulated based on takeoff conditions of the simulation and body configuration of the performance using an 11-segment simulation model of aerial movement (Yeadon, Atha & Hales, 1990). The takeoff configuration was merged into the configuration of the performance in the first 100 ms using a quintic function (Yeadon & Hiley, 2000). The orientation and position of the diver at the entry reflect the takeoff conditions of the dive. In this particular dive, there is a good match in angular momentum and CM horizontal velocity. Thus, the diver travels similar distances in the two dives and shows similar orientations at the entry with slightly more rotation in the simulation since the CM vertical velocity is 2.2% higher at takeoff.

Figure 5.5. Graphs comparison of the performance (upper) and matching simulation (lower) for 101B (with an addition of 0.6 m horizontal spacing between figures).
5.2.6.2. Forward two and one-half somersault pike

The score of the forward two and one-half somersault pike (105B) was 6.5% with excellent match (0% difference) in angular momentum and CM horizontal velocity. There was also very good agreements in orientation angle (less than 3°) and linear momentum (less than 2% difference) at takeoff. There are considerable differences in joint angle time histories (12°) and maximum springboard depression (11%) but the largest difference was in the takeoff time (21%). Details of the score and comparison of simulation outputs and performance are displayed in Table 5.7.

Table 5.7. Evaluation score of forward two and one-half somersault pike (105B)

<table>
<thead>
<tr>
<th>component / variables</th>
<th>performance</th>
<th>simulation</th>
<th>sub-score</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: joint angles</td>
<td></td>
<td></td>
<td></td>
<td>12.1°</td>
</tr>
<tr>
<td>( \theta_b )</td>
<td>17.6°</td>
<td>17.2°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_a )</td>
<td>9.7°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_k )</td>
<td>9.1°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_h )</td>
<td>6.7°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_s )</td>
<td>17.2°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2: orientation angle</td>
<td>( \theta_t )</td>
<td>2.8°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3: linear momentum</td>
<td></td>
<td>1.3%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( v_x )</td>
<td>1.33 m/s</td>
<td>1.34 m/s</td>
<td>0.7%</td>
<td></td>
</tr>
<tr>
<td>( v_z )</td>
<td>4.39 m/s</td>
<td>4.30 m/s</td>
<td>1.9%</td>
<td></td>
</tr>
<tr>
<td>S4: angular momentum H</td>
<td>58.91 kg⋅m²</td>
<td>58.88 kg⋅m²</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>S5: springboard</td>
<td></td>
<td>16.1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( z_{\text{max}} )</td>
<td>-0.73 m</td>
<td>-0.65 m</td>
<td>10.9%</td>
<td></td>
</tr>
<tr>
<td>( t )</td>
<td>0.435 s</td>
<td>0.414 s</td>
<td>21.3%</td>
<td></td>
</tr>
<tr>
<td>average score ( S_{\text{TOR}} )</td>
<td>6.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.6 compares the joint angle and orientation time histories in the simulation with those calculated from the performance. Similar to 101B, there was a close match in the beginning of the simulation and the differences were mainly in the recoil phase. The orientation angle matched well throughout the simulation and the shoulder angle matched extremely well up to 0.2 s. There was some initial vibration at the ball angle just after the touchdown and the angle became stable thereafter although it did not extend to the same extent towards the end of the simulation as in the performance.

Figure 5.6. Comparison of joint angle and orientation time histories for 105B (solid line = simulation, dotted line = performance).

The muscle activation profiles of the 10 torque generators are shown in Figure 5.7 and their corresponding values are given in Table 5.8. There is co-contraction at each joint at touchdown, as in the case of 101B. The shoulder flexor and hip extensor activation ramp down quickly during the recoil phase while the shoulder extensor and hip flexor were fully activated to generate forward angular momentum. Other extensors were gradually ramping down while the flexors were ramping up toward the end of the takeoff to prevent hyper-extension of the joint.
Figure 5.7. Muscle activation time histories for the matching 105B simulation (kemact = knee extension muscle activation, etc).

Table 5.8. Muscle activation parameters for the matching 105B simulation

<table>
<thead>
<tr>
<th>torque generator</th>
<th>se₁/sf₁ (s)</th>
<th>te₁/tf₁ (s)</th>
<th>se₂/sf₂ (s)</th>
<th>te₂/tf₂ (s)</th>
<th>le₁/lf₁</th>
<th>le₂/lf₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee extensor</td>
<td>-0.042</td>
<td>0.309</td>
<td>0.231</td>
<td>0.417</td>
<td>0.110</td>
<td>0.808</td>
</tr>
<tr>
<td>knee flexor</td>
<td>-0.091</td>
<td>0.166</td>
<td>0.171</td>
<td>0.189</td>
<td>0.433</td>
<td>0.017</td>
</tr>
<tr>
<td>hip extensor</td>
<td>-0.088</td>
<td>0.362</td>
<td>0.236</td>
<td>0.104</td>
<td>0.297</td>
<td>0.951</td>
</tr>
<tr>
<td>hip flexor</td>
<td>-0.113</td>
<td>0.170</td>
<td>0.175</td>
<td>0.170</td>
<td>0.451</td>
<td>0.032</td>
</tr>
<tr>
<td>shoulder extensor</td>
<td>-0.029</td>
<td>0.194</td>
<td>0.124</td>
<td>0.105</td>
<td>0.052</td>
<td>0.031</td>
</tr>
<tr>
<td>shoulder flexor</td>
<td>-0.061</td>
<td>0.223</td>
<td>0.227</td>
<td>0.130</td>
<td>0.316</td>
<td>0.770</td>
</tr>
<tr>
<td>ankle plantar flexor</td>
<td>-0.050</td>
<td>0.208</td>
<td>0.243</td>
<td>0.305</td>
<td>0.250</td>
<td>0.818</td>
</tr>
<tr>
<td>ankle dorsi-flexor</td>
<td>-0.081</td>
<td>0.228</td>
<td>0.142</td>
<td>0.296</td>
<td>0.352</td>
<td>0.049</td>
</tr>
<tr>
<td>ball extensor</td>
<td>-0.112</td>
<td>0.203</td>
<td>0.118</td>
<td>0.419</td>
<td>0.077</td>
<td>0.244</td>
</tr>
<tr>
<td>ball flexor</td>
<td>-0.003</td>
<td>0.324</td>
<td>0.353</td>
<td>0.286</td>
<td>0.232</td>
<td>0.019</td>
</tr>
</tbody>
</table>
The net torque time histories obtained from the torque-driven and angle-driven models are compared in Figure 5.8. The two torque time histories of the ankle displayed a very similar pattern throughout the simulation and there was good agreement for all other joints during the board depression phase. The major difference occurred in the recoil phase where angular momentum was generated.

Figure 5.8. Torque time histories for 105B obtained from the torque-driven (solid line) and angle-driven (dashed line) models.
Graphics sequences of the hurdle, takeoff and flight phase of the performance and the simulation are displayed in Figure 5.9. Since the simulation produces a good match in both linear and angular momentum at takeoff as in the performance, the orientation and position of the diver at the entry in the two dives are very similar.

Figure 5.9. Graphics comparison of the performance (upper) and matching simulation (lower) for 105B (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).
5.2.6.3. Reverse dive tuck

The score of the reverse dive tuck (301C) was 7.2% with excellent match (0% difference) in angular momentum, CM horizontal velocity and takeoff time. There was also good agreements in the orientation and CM vertical velocity. The maximum board depression in the simulation was 13% less than in the performance. There were however large differences in the joint angle time histories especially for the knee angle with an average difference of 30.2°. Details of the score and comparison of simulation outputs and performance are displayed in Table 5.9.

Table 5.9. Evaluation score of reverse dive tuck (301C)

<table>
<thead>
<tr>
<th>component / variables</th>
<th>performance</th>
<th>simulation</th>
<th>sub-score</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: joint angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_0 )</td>
<td></td>
<td>16.7°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_a )</td>
<td></td>
<td>27.4°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_k )</td>
<td></td>
<td>30.2°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_h )</td>
<td></td>
<td>22.1°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_s )</td>
<td></td>
<td>18.3°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2: orientation angle</td>
<td>( \theta_t )</td>
<td>3.5°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S3: linear momentum</td>
<td>( v_x )</td>
<td>1.55 m/s</td>
<td>1.55 m/s</td>
<td>0.2%</td>
</tr>
<tr>
<td></td>
<td>( v_z )</td>
<td>4.55 m/s</td>
<td>4.33 m/s</td>
<td>4.9%</td>
</tr>
<tr>
<td>S4: angular momentum</td>
<td>H</td>
<td>26.80 kg·m²</td>
<td>26.81 kg·m²</td>
<td>0.0%</td>
</tr>
<tr>
<td>S5: springboard</td>
<td>( z_{max} )</td>
<td>-0.73 m</td>
<td>-0.64 m</td>
<td>13.0%</td>
</tr>
<tr>
<td></td>
<td>( t )</td>
<td>0.480 s</td>
<td>0.480 s</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

average score \( S_{TOR} \) = 7.2%

Figure 5.10 compares the joint angle and orientation time histories of the simulation with the actual value calculated from the performance. It can be seen that the hip, knee and ankle do not match very well throughout the simulation. Despite the large difference in joint angle time histories, there is very good agreement in both linear and
angular momentum generated. This implies that a different technique is used in the simulation as in the performance to generate similar takeoff conditions.

Figure 5.10. Comparison of joint angle and orientation time histories for 301C (solid line = simulation, dotted line = performance).

The muscle activation profiles of the 10 torque generators are shown in Figure 5.11 and their corresponding values are included in Table 5.10. Similar to the forward rotating dives, there was co-contraction at touchdown and the flexors were activated at the end of the simulation to protect the joint except for the knee flexor which was not allowed to come on again once it ramped down. Another simulation was run allowing the knee flexor activation to ramp up at the end and similar matching results were obtained. This confirms that it is not the knee flexor activation which inhibits the matching of joint angles.
Figure 5.11. Muscle activation time histories for the matching 301C simulation (kemact = knee extension muscle activation, etc).

Table 5.10. Muscle activation parameters for the matching 301C simulation

<table>
<thead>
<tr>
<th>torque generator</th>
<th>se1/sf1(s)</th>
<th>te1/tf1(s)</th>
<th>se2/sf2 (s)</th>
<th>te2/tf2 (s)</th>
<th>le1/lf1</th>
<th>le2/lf2</th>
</tr>
</thead>
<tbody>
<tr>
<td>knee extensor</td>
<td>-0.019</td>
<td>0.256</td>
<td>0.127</td>
<td>0.838</td>
<td>0.215</td>
<td>0.200</td>
</tr>
<tr>
<td>knee flexor</td>
<td>-0.061</td>
<td>0.282</td>
<td>/</td>
<td>/</td>
<td>0.203</td>
<td>0.017</td>
</tr>
<tr>
<td>hip extensor</td>
<td>-0.010</td>
<td>0.284</td>
<td>0.252</td>
<td>0.389</td>
<td>0.118</td>
<td>0.588</td>
</tr>
<tr>
<td>hip flexor</td>
<td>-0.088</td>
<td>0.195</td>
<td>0.434</td>
<td>0.126</td>
<td>0.214</td>
<td>0.033</td>
</tr>
<tr>
<td>shoulder extensor</td>
<td>-0.104</td>
<td>0.109</td>
<td>0.390</td>
<td>0.278</td>
<td>0.200</td>
<td>0.042</td>
</tr>
<tr>
<td>shoulder flexor</td>
<td>-0.138</td>
<td>0.293</td>
<td>0.481</td>
<td>0.342</td>
<td>0.064</td>
<td>0.420</td>
</tr>
<tr>
<td>ankle plantar flexor</td>
<td>-0.033</td>
<td>0.356</td>
<td>0.285</td>
<td>0.312</td>
<td>0.146</td>
<td>0.587</td>
</tr>
<tr>
<td>ankle dorsi-flexor</td>
<td>-0.050</td>
<td>0.202</td>
<td>0.388</td>
<td>0.184</td>
<td>0.198</td>
<td>0.016</td>
</tr>
<tr>
<td>ball extensor</td>
<td>-0.079</td>
<td>0.491</td>
<td>0.226</td>
<td>0.446</td>
<td>0.082</td>
<td>0.306</td>
</tr>
<tr>
<td>ball flexor</td>
<td>-0.027</td>
<td>0.477</td>
<td>0.325</td>
<td>0.437</td>
<td>0.078</td>
<td>0.024</td>
</tr>
</tbody>
</table>
The torque time histories obtained from the torque-driven and angle-driven models are compared in Figure 5.12. There were increasing positive torques at the ball and ankle joint during the board depression phase in the torque-driven model while the torque was near zero in the angle-driven model for the first half of the simulation. In order to maintain zero net torque, there must be an equal but opposite torque acting on each side of the joint. In the muscle activation profile used, the initial ramping time of the torque generator was set between -0.15 s to 0.00 s to ensure pre-landing activation. Once the flexor / extensor is activated, the activation either ramps up or down following the quintic function. It might not be possible to find the activation parameter set that can produce constant zero net torque for a long period of time. There is a good match in the hip torque throughout the simulation and also the knee torque during the board depression phase. The shoulder torque calculated in the angle-driven model was very noisy whereas the torque-driven model produced a much smoother torque to a similar peak value.

Figure 5.12. Torque time histories for 301C obtained from the torque-driven (solid line) and angle-driven (dashed line) models.
Graphics sequences of the hurdle, takeoff and flight phase of the performance and the simulation are displayed in Figure 5.13. Although the simulation generates the same amount of angular momentum as in the performance, the model is slightly over-rotated at the entry since it takes off with a higher vertical velocity and thus has more time in the air for rotation. The distance travelled in the two dives are similar since there is a good match in CM horizontal velocity at takeoff.

Figure 5.13. Graphics comparison of the performance (upper) and matching simulation (lower) for 301C (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).
5.2.6.4. Reverse one and one-half somersault pike

The score for the reverse one and one-half somersault pike (303B) was 6.6% with an excellent match (0% difference) in linear and angular momentum. There is also good agreement in orientation, springboard depression and takeoff time. The largest difference was again the joint angle time histories as in the reverse dive tuck. Details of the score and comparison of simulation outputs and performance are displayed in Table 5.11.

Table 5.11. Evaluation score of reverse one and one-half somersault pike (303B)

<table>
<thead>
<tr>
<th>component / variables</th>
<th>performance</th>
<th>simulation</th>
<th>sub-score</th>
<th>score</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1: joint angles</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_b)</td>
<td></td>
<td></td>
<td></td>
<td>19.6°</td>
</tr>
<tr>
<td>(\theta_a)</td>
<td>29.5°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_k)</td>
<td>20.7°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_h)</td>
<td>21.6°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\theta_s)</td>
<td>21.8°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>S2: orientation angle</td>
<td>(\theta_t)</td>
<td></td>
<td></td>
<td>5.7°</td>
</tr>
<tr>
<td>S3: linear momentum</td>
<td></td>
<td></td>
<td></td>
<td>0.1%</td>
</tr>
<tr>
<td>(v_x)</td>
<td>1.25 m/s</td>
<td>1.249 m/s</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>(v_z)</td>
<td>4.40 m/s</td>
<td>4.395 m/s</td>
<td>0.1%</td>
<td></td>
</tr>
<tr>
<td>S4: angular momentum (H)</td>
<td>53.94 kg\cdot m^2</td>
<td>54.07 kg\cdot m^2</td>
<td>0.2%</td>
<td></td>
</tr>
<tr>
<td>S5: springboard</td>
<td></td>
<td></td>
<td></td>
<td>7.2%</td>
</tr>
<tr>
<td>(z_{\text{max}})</td>
<td>-0.72 m</td>
<td>-0.67 m</td>
<td>8.5%</td>
<td></td>
</tr>
<tr>
<td>(t)</td>
<td>0.495 s</td>
<td>0.489 s</td>
<td>6%</td>
<td></td>
</tr>
<tr>
<td>average score (S_{\text{TOR}})</td>
<td></td>
<td></td>
<td></td>
<td>6.6%</td>
</tr>
</tbody>
</table>

Figure 5.14 compares the joint angle and orientation time histories of the simulation with the actual values calculated from the performance. There is good agreement in the shoulder and orientation time histories but the hip, knee, ankle and ball do not match very well throughout the simulation. Similar to the reverse dive tuck, these results suggest that somewhat different movement patterns are used in the simulation to generate similar takeoff conditions as in the performance since there is excellent match in both linear and angular momentum.
Figure 5.14. Comparison of joint angle and orientation time histories for 303B (solid line = simulation, dotted line = performance).

The muscle activation profiles of the 10 torque generators are shown in Figure 5.16 and the corresponding values are given in Table 5.12. The joint angle time histories shows that there is insufficient extension in the hip, knee and ankle and these torque generators are not fully activated as shown in Figure 5.15. This implies that it is not the strength parameter that limit the extension of these joints. Rather, the model tends to choose the activation parameters that will minimise the overall evaluation score.
Figure 5.15. Muscle activation time histories for the matching 303B simulation (kemact = knee extension muscle activation, etc).

Table 5.12. Muscle activation parameters for the matching 303B simulation

<table>
<thead>
<tr>
<th>torque generator</th>
<th>muscle activation parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>se_1/sf_1(s)</td>
</tr>
<tr>
<td>knee extensor</td>
<td>-0.030</td>
</tr>
<tr>
<td>knee flexor</td>
<td>-0.138</td>
</tr>
<tr>
<td>hip extensor</td>
<td>-0.011</td>
</tr>
<tr>
<td>hip flexor</td>
<td>-0.150</td>
</tr>
<tr>
<td>shoulder extensor</td>
<td>-0.112</td>
</tr>
<tr>
<td>shoulder flexor</td>
<td>-0.147</td>
</tr>
<tr>
<td>ankle plantar flexor</td>
<td>-0.083</td>
</tr>
<tr>
<td>ankle dorsi-flexor</td>
<td>-0.062</td>
</tr>
<tr>
<td>ball extensor</td>
<td>-0.051</td>
</tr>
<tr>
<td>ball flexor</td>
<td>-0.097</td>
</tr>
</tbody>
</table>
The torque time histories obtained from the torque-driven and angle-driven models are compared in Figure 5.16. Similar patterns can be observed in the hip, shoulder and initial knee torque time histories. The net torque at the ball and ankle increases earlier in the torque-driven model as in the reverse dive tuck. The peak torque at each joint produced in the torque-driven model were comparable to that of the angle-driven model.

Figure 5.16. Torque time histories for 303B obtained from the torque-driven (solid line) and angle-driven (dashed line) models.
Graphics sequences of the hurdle, takeoff and flight phase of the performance and the simulation are displayed in Figure 5.17. The diver rotates more in the simulation due to the difference in orientation at takeoff and the 0.2% higher angular momentum generated. The orientation angle at takeoff in the simulation (125°) is larger than in the performance (119°) so the model has already rotated more at the start of the flight phase.

Figure 5.17. Graphics comparison of the performance (upper) and matching (lower) simulation of 303B (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).

5.2.7. Discussion

The torque-driven model has produced good agreement between the simulation and the performance in all four dives. This indicates that the model parameters determined from the combined matching of the four dives can be used successfully in the torque-driven model to reproduce the takeoff in springboard diving. In the combined matching,
the simulation did not produce sufficient angular momentum but this problem does not exist in the torque-driven model which produces excellent matches in angular momentum for all four dives. The torques calculated from the 9-parameter function were much smoother than those calculated from the angle-driven model as expected and this may explain the better match in angular momentum. Moreover, the hip was allowed to hyper-extend 20° more than the observed range. The good agreement in the reverse angular momentum generated in the torque-driven model suggests that the limitation of using a straight spine to represent the trunk can be compensated by allowing extra hip hyper-extension.

Co-contraction is observed at all joints in the four simulations in that both the flexor and extensor are activated at touchdown. The initial activation level \( se / sf \) does not reach the upper bound of 0.5 in any of the torque generators. This suggests that the upper bound of 50% activation is appropriate although higher pre-activation levels have been reported (Arampatsis et al., 2003; Santello & McDonagh, 1998). The activation levels in many studies are reported as the percentage of the recorded maximum torque, most of the time the peak isometric torque. If the peak isometric torque is not measured at the optimum joint angle, it is likely that the recorded maximum torque is lower than the actual maximum torque that can be produced at the optimum angle.

There were relatively large differences in the joint angle time histories between simulation and performance in both of the reverse somersaulting dives. While the results imply that the simulation has chosen different techniques to produce similar takeoff conditions, the reason for being unable to find a solution that matches both joint angle and takeoff conditions is unclear. It is likely that multiple solutions are able to produce the same takeoff conditions and a very different local minimum is found in the optimisation process. The weightings of the score may have favoured matching takeoff conditions while sacrificing the matching of joint angles. The assumptions made in developing the model might limit the model's ability to match the takeoff conditions using the same techniques as in the performance. The problem in matching the joint angles could also be due to the limitation in controlling the muscle activation pattern using two quintic functions which cannot replicate the actual joint torque pattern produced in a reverse dive. Despite the problem of matching joint angle time histories, the good match in takeoff conditions in terms of linear and angular momenta suggests that the model is of producing appropriate takeoff conditions for forward and reverse somersaulting dives.
5.2.8. Conclusion

Using model parameters determined from the combined matching, the torque-driven model produces good agreement between simulation and performance for all four dives. It can be concluded that this torque-driven model is a realistic representation of springboard diving takeoff movements. The model can now be used for further analysis and application.

5.3. Kinetic and kinematics analysis

5.3.1. Wobbling mass

The wobbling mass spring parameters and fixed to wobbling mass ratios determined from the angle-driven model were used in the torque-driven model. Similar spring displacements were expected in both models for the same dive. The wobbling mass rotation angle can influence the evaluation of joint angles. During digitisation, the joint centre was identified from whole segment (fixed + wobbling). In the simulation model, the joint angle was defined by the fixed segment positions. If the wobbling mass moves away from the fixed segment by a large extent, the joint angle based on fixed segment position will be expected to be different from the whole segment joint angle even for the same body configuration. Figure 5.18 shows the wobbling mass rotation angle, parallel (x) and perpendicular (z) displacement of the shank, thigh and trunk of 105B. It can be seen from the figure that the spring displacements lie within the predicted range (see Section 4.6). The rotation angle is less than 2° and therefore will have little influence on the joint angles.
Figure 5.18. Wobbling mass rotation and displacement time histories for 105B.

5.3.2. Springboard reaction forces

The vertical and horizontal springboard reaction forces were calculated in the simulations (Figure 5.19). The peak vertical ($R_z$) and horizontal ($R_x$) forces in each dive were identified (Table 5.13). A positive horizontal force projects the diver forward into the pool whereas a negative force retards forward translation. The value of $R_z$ ranged between 5.0 BW (301C) and 5.8 BW (105B) and within each dive group $R_z$ is higher in the dive with more rotational requirement. This suggests that the diver presses the board with more effort when performing a more difficult dive within the same dive group. $R_z$ peaks earlier in the forward group near to the point of maximum board depression and slightly later in the reverse group during the recoil phase. This could be associated with the different rotational requirements of the two dive groups in that the diver needs to generate angular momentum in opposite directions.

The horizontal force time histories were noisy in the beginning of the takeoff phase. This could be due to the initial vibration at the ball and ankle after the touchdown from the hurdle. Once all three points (heel, ball and toes) of the foot-springboard interface were in
contact with the board, a general trend could be observed though the horizontal force was less smooth than the vertical force. The positive peak value $R_x$ ranged between 0.7 BW (301C) to 1.0 BW (105B) and the negative $R_x$ was 0.9 BW (105B). $R_x$ peaks earlier in the forward group than the reverse group as in the case of $R_z$. There is a double positive peak in both 301C and 303B during the recoil phase. In the reverse group, hip hyperextension associated with knee flexion are required to produce reverse rotation and move the diver's mass centre forward. This body configuration results in a positive reaction force which favours reverse rotation and projects the diver forward into the water. Unlike the reverse group in which the horizontal force remains highly positive during the recoil phase, the force drops to negative in the forward group and there is a sustained negative force period for 105B just before takeoff. This force pattern is consistent with the techniques used in generating forward angular momentum. As the diver flexes the hip during the recoil phase to produce forward rotation, the foot will be pushed forward resulting in an opposite backward (negative) reaction force.

Table 5.13. Peak vertical and horizontal springboard reaction forces

<table>
<thead>
<tr>
<th>dive</th>
<th>peak vertical force</th>
<th>peak horizontal force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_z$ (N)</td>
<td>$R_z$ (BW)</td>
</tr>
<tr>
<td>101B</td>
<td>3551</td>
<td>5.65</td>
</tr>
<tr>
<td>105B</td>
<td>3669</td>
<td>5.84</td>
</tr>
<tr>
<td>301C</td>
<td>3197</td>
<td>5.08</td>
</tr>
<tr>
<td>303B</td>
<td>3420</td>
<td>5.44</td>
</tr>
</tbody>
</table>

* this $R_x$ value was due to noise resulting from initial foot vibration
5.3.3. Foot-springboard interface

The reaction forces acting on the diver are applied through the heel, the ball and the toes. Figure 5.20 displays the resultant and individual parallel and perpendicular spring force time histories for 303B. There is an initial vibration just after the touchdown and afterwards both the parallel and perpendicular force follow a smooth pattern. The parallel force peaks slightly earlier than the perpendicular force near the point of maximum board depression. It is clear that the foot loses contact gradually from heel to toes as indicated by zero reaction force. The force distribution to the ball increases as the heel loses contact. It can be concluded that the three pairs of parallel and perpendicular spring dampers successfully model the elasticity of the foot-springboard interface.
5.4. Summary

It has been demonstrated in the evaluation that the torque-driven model successfully simulates realistic springboard diving takeoffs. There is good agreement between the simulation and the performance in both forward and reverse dive groups with an average difference of 6.3% among the four dives evaluated. The validity of the model is further supported by reasonable kinetic and kinematic outputs. This model is ready for use in investigating takeoff techniques and optimising performance.
CHAPTER 6

OPTIMISATION AND APPLICATIONS

6.1. Introduction

This study aims to understand the mechanics of springboard diving takeoffs in terms of generating both linear and angular momentum. There are two major research questions that this study is trying to answer. The first question concerns maximising dive height and the second question concerns maximising somersault rotation.

Q1. For a specific dive with fixed rotational requirement, what is the optimal takeoff technique to obtain maximum height while generating sufficient angular momentum and travelling safely away from the springboard?

Dive height attained has both a direct and indirect influence on the dive score (McCormick, 1982). Height itself is a factor that judges take into account when judging a dive. Moreover, greater height allows more time for rotation, for adopting a good shape and for preparing for the entry in an unrushed manner (Sanders, 2001). According to the laws of mechanics, the mass centre of the diver travels in a parabolic path in the air assuming that air resistance is negligible. To increase dive height, there will be a decrease in the distance travelled. During the springboard depression, energy is stored in the springboard and returned to the diver in the recoil phase during which the diver flexes or extends the joints to generate rotation. The more the diver flexes or extends the joints, the more energy is lost in gaining height (Sanders & Wilson, 1988). Thus, it is clear that gaining dive height will lead to compromises in distance and rotation. In competitive diving, a dive is specified by its takeoff and somersaulting direction, rotational requirement and body position. To perform a specific dive, the correct amount of angular momentum must be generated during the takeoff. An optimal takeoff technique will allow the diver to gain maximum dive height while generating the required angular momentum and keeping a safe distance from the springboard. In the sections below, the four dives with minimum and maximum angular momentum in the forward and reverse groups will be optimised for maximum dive height.
Q2. What is the maximum rotation the diver can perform in the forward and reverse groups without an increase in strength?

The limiting dive for the diver in this study was a two and one-half somersault pike in the forward group and a one and one-half somersault pike in the reverse group. Can she perform a more difficult dive without an increase in strength? It has been shown that as the rotational requirement increases, there is an increase in forward lean and hip flexion for the forward group and hip hyper-extension for the reverse group. The increase in flexion or hyper-extension generates more angular momentum but at the same time reduces dive height. The loss in height reduces the time for somersault rotation and therefore the increase in angular momentum may not produce maximum rotation. In terms of distance travelled, forward lean facilitates rotation in the forward group but decreases height. For the reverse group, hyper-extension facilitates reverse rotation but may bring the diver's mass centre too close to the springboard. In this chapter, an optimisation will be carried out to search for maximal rotational potential with reasonable dive height and board clearance distance in the forward and reverse groups based on the diver's strength capability.

6.2. Calculations of kinematic variables

6.2.1. Introduction

The procedures for calculating flight time t, board clearance distance d, dive height H (Figure 6.1) and rotational potential \( \phi \) are explained in the following sections.

![Figure 6.1. Dive height and board clearance distance in the flight phase.](image)
6.2.2. Flight time

The flight time $t$ was defined as the time from the last instant of takeoff until the diver's CM was 1.0 m above the water, i.e., level with the springboard:

$$t = \frac{v_z + \sqrt{v_z^2 - 2gh_1}}{g}$$  \hspace{1cm} (6.1)

where $v_z =$ diver's CM vertical velocity at takeoff

$h_1 =$ the height of the diver's CM above a resting springboard (1.0 m)

$g =$ gravitational acceleration (9.81 m/s$^2$)

6.2.3. Board clearance distance

The board clearance distance $d$ was defined as the distance between the diver's CM and the free end of the springboard when the CM was level with the springboard height at 1.0 m. It was the summation of the initial CM horizontal position at takeoff and the distance travelled in flight. The distance travelled in flight could be calculated as the product of the diver's CM horizontal velocity $v_x$ at takeoff and the flight time.

$$d = d_1 + v_x t$$  \hspace{1cm} (6.2)

where $d_1 =$ diver's CM horizontal position at takeoff

6.2.4. Dive height

The dive height $H$ was defined as the maximum height that the diver's CM reached in the air (Figure 6.1). It could be divided into the pre-flight height $h_1$ and the flight height $h_2$. The flight height was the height that the CM reached from the last instant of takeoff to the maximum height and it could be calculated by Equation (6.3). After $h_2$ was determined, $H$ could be calculated using Equation (6.4).

$$h_2 = \frac{v_x^2}{2g}$$  \hspace{1cm} (6.3)

$$H = 1.0 + h_1 + h_2$$  \hspace{1cm} (6.4)

6.2.5. Rotational potential

The rotational potential was a measure of how much rotation the model had completed when it's CM was 1.0 m above the water. It took into account the initial body orientation and angular momentum at takeoff as well as the flight time available to
complete the rotation. In order to compare among different divers, the rotation was normalised to the number of somersaults achieved in a standard straight position. The initial somersault angle $\phi_i$ was calculated based on takeoff configuration (Figure 6.2) using Equation (6.5). The weightings of the upper (two-third) and lower (one-third) body angles were comparable to their mass ratio.

$$\phi_i = \frac{1}{3} \phi_1 + \frac{2}{3} \phi_2$$  \hfill (6.5)

where $\phi_1$ = angle between the vertical and a line joining the hip to the ankle

$$\phi_2 = \text{angle between the vertical and a line joining the shoulder to the hip.}$$

![Figure 6.2. The initial somersault angle was calculated based on takeoff configuration.](image)

The moment of inertia $I_{ss}$ of the diver in a standard straight position with adducted arms was calculated as 10.6 kg·m² using Yeadon’s (1990b) model. The rotation during flight $\phi_f$ was calculated as the product of the angular velocity $\omega$ and the flight time:

$$\phi_f = \omega t$$  \hfill (6.6)

where

$$\omega = \frac{H}{I_{ss}}$$

The total rotation $\phi$ was the summation of initial somersault angle and flight phase rotation and was then normalised to the number of straight somersaults:

$$\phi = \frac{\phi_i + \phi_f}{2\pi}$$  \hfill (6.7)
6.3. Optimisation

6.3.1. Optimisation for maximum height

The four dives evaluated previously were optimised for maximum dive height while the board clearance distance and rotational potential were kept unchanged. The 60 muscle activation parameters and initial trunk angle ($\pm 1^\circ$) and angular velocity ($\pm 1 \text{ rad/s}$) were varied to search for an optimal activation profile. An objective optimisation score $S_{H}$ was calculated to maximise dive height using the Simulated Annealing optimisation algorithm. This score was based on the dive height $H$ and penalty scores constraining rotational potential, board clearance distance and joint angles. A 1 cm difference in board clearance distance $d$ and 1% difference in rotation $\phi$ between the matching and the optimised simulation were each counted as one penalty point. The joint angles during takeoff and 0.1 s after takeoff were limited to sensible ranges of motion using the same procedures as in the evaluation (see Chapter 5.2.3). One additional penalty was used to constrain the minimum knee angle at takeoff to be no less than $130^\circ$ ($133^\circ$ observed from video recordings) in order to prevent excessive knee flexion for the reverse somersaulting dives. Once the angle exceeded the limited range, one point of penalty was introduced for one-degree difference. The penalty score for each joint angle was the average value of all penalties introduced at that joint. The penalty score for each joint angle was weighted equivalent to the score for distance and rotational potential. The optimisation score $S_{H}$ was then calculated as:

$$S_{H} = 10H - \text{penalty scores}$$

(6.8)

This score was maximised in the optimisation to search for an optimal muscle activation profile that would achieve maximum dive height. The muscle activation parameters for the matching simulation were used as the starting point of the optimisation. If there were penalties in the matching simulation, the particular constraint that introduced the penalties was adjusted until there was no penalty incurred initially.

6.3.2. Optimisation for maximum rotation

The dives with maximum angular momentum in the forward and reverse groups were optimised for maximum rotational potential. For the forward group, the maximum board clearance distance was set as the distance travelled in the forward two and one-half somersault pike matching simulation. If the distance exceed this maximum limit, 1 cm difference was counted as one penalty point. Similarly for the reverse group, the minimum
board clearance distance was set as the distance travelled in the reverse one and one-half somersault pike matching simulation. If the distance was less than this minimum limit, 1 cm difference was counted as one penalty point. The joint angle limits were set the same as in optimisation for height (Section 6.3.1). The objective optimisation score $S_R$ to be maximised was calculated as:

$$S_R = 100 \phi - \text{penalty scores} \quad (6.9)$$

### 6.4. Results

#### 6.4.1. Optimisation for maximum height

##### 6.4.1.1. Forward dive pike

The optimisation of the forward dive pike (101B) for maximum dive height could not achieve any higher than it was in the matching simulation. Table 6.1 shows the dive height, flight time, board clearance distance, maximum depression, takeoff time and rotational potential of the matching simulation.

<table>
<thead>
<tr>
<th>variables</th>
<th>matching simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>3.207</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.146</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.009</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.745</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.476</td>
</tr>
<tr>
<td>angular momentum (kg m²)</td>
<td>18.10</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>0.357</td>
</tr>
</tbody>
</table>

##### 6.4.1.2. Forward two and one-half somersault pike

The optimised forward two and one-half somersault pike (105B) had a 1.7 cm increase in dive height, 0.005 s increase in flight time and 0.9 cm decrease in board clearance distance than the matching simulation. There was no difference in the takeoff time, maximum board depression and rotational potential between the two simulations. Comparison of kinematic variables of the optimised and the matching simulations of 105B are tabulated in Table 6.2. Figure 6.3 shows the graphics comparison of matching and the optimised simulations. The joint angle time histories (Figure 6.4), muscle activation profile...
(Figure 6.5) and torque time histories (Figure 6.6) of the optimised simulation are comparable to those of the matching simulation.

Table 6.2. Kinematic variables of the matching and optimised for maximum height simulations for the forward two and one-half somersault pike (105B)

<table>
<thead>
<tr>
<th>Variables</th>
<th>matching</th>
<th>optimised</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>2.881</td>
<td>2.898</td>
<td>+0.6%</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.059</td>
<td>1.064</td>
<td>+0.5%</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.601</td>
<td>1.592</td>
<td>-0.6%</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.730</td>
<td>-0.730</td>
<td>0.0%</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.414</td>
<td>0.414</td>
<td>0.0%</td>
</tr>
<tr>
<td>angular momentum (kg·m²)</td>
<td>59.24</td>
<td>58.97</td>
<td>-0.5%</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>1.073</td>
<td>1.073</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Figure 6.3. Graphics comparison of the matching (upper) and optimised for maximum height (lower) simulation for 105B (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).
Figure 6.4. Joint angle and orientation time histories of the matching (dashed) and the optimised (solid) 105B simulations.
Figure 6.5. Muscle activation time histories of the matching (dashed) and optimised (solid) 105B simulations (kemact = knee extension muscle activation, etc).
6.4.1.3. Reverse dive tuck

The optimised reverse dive tuck (301C) had a 1.2 cm increase in dive height and 0.003 s increase in flight time compared with the matching simulation. There was similar board clearance distance and takeoff time in the two simulations and no change in maximum board depression or rotational potential. Comparison of kinematic variables of the optimised and the matching simulations of 301C are tabulated in Table 6.3. The joint angle time histories (Figure 6.7), muscle activation profile (Figure 6.8) and torque time histories (Figure 6.9) of the optimised simulation were very similar to those of the matching simulation. Graphics comparison is omitted due to the similarity of the two simulations.
Table 6.3. Kinematic variables of the matching and optimised for maximum height simulations for reverse dive tuck (301C)

<table>
<thead>
<tr>
<th>variables</th>
<th>matching</th>
<th>optimised</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>2.874</td>
<td>2.886</td>
<td>+0.4%</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.060</td>
<td>1.063</td>
<td>+0.3%</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.773</td>
<td>1.772</td>
<td>-0.1%</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.732</td>
<td>-0.732</td>
<td>0.0%</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.481</td>
<td>0.482</td>
<td>+0.2%</td>
</tr>
<tr>
<td>angular momentum (kg-m$^2$)</td>
<td>26.60</td>
<td>26.59</td>
<td>0.0%</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>0.440</td>
<td>0.440</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Figure 6.7. Joint angle and orientation time histories of the matching (dashed) and the optimised (solid) 301C simulations.
Figure 6.8. Muscle activation time histories of the matching (dashed) and optimised (solid) 301C simulations (kemact = knee extension muscle activation, etc).
Figure 6.9. Torque time histories of matching (dashed) and the optimised (solid) 301C simulations.

6.4.1.4. Reverse one and one-half somersault pike

The optimised reverse one and one-half somersault pike (303B) had a 2.0 cm increase in dive height and 0.008 s increase in flight time compared to the matching simulation. The increase in dive height resulted in a 2.7 cm decrease in board clearance distance 1.3% decrease in rotational potential in the optimised dive. There was no difference in maximum board depression and takeoff time for the two simulations. Other comparisons are not shown due to the similarity of the two simulations.
Table 6.4. Kinematic variables of the matching and optimised for maximum height simulations for reverse one and one-half somersault pike (303B)

<table>
<thead>
<tr>
<th>variables</th>
<th>matching</th>
<th>optimised</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>2.915</td>
<td>2.935</td>
<td>+0.7%</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.073</td>
<td>1.081</td>
<td>+0.8%</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.362</td>
<td>1.335</td>
<td>-2.0%</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.737</td>
<td>-0.737</td>
<td>0.0%</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.488</td>
<td>0.488</td>
<td>0.0%</td>
</tr>
<tr>
<td>angular momentum (kg-m²)</td>
<td>54.21</td>
<td>53.19</td>
<td>-1.9%</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>0.839</td>
<td>0.828</td>
<td>-1.3%</td>
</tr>
</tbody>
</table>

6.4.2. Optimisation for maximum rotation

6.4.2.1. Maximum forward rotation

In the optimisation for maximum forward rotation, there was a 28% increase in rotational potential compared with the matching simulation of 105B. Comparison of kinematic variables of the optimised and the matching simulations are tabulated in Table 6.5. The 44% increase in angular momentum resulted in a compromise in dive height (8 cm lower) and thus flight time (0.08 s shorter) than in the matching simulation. This implies that although the model can rotate faster, there is less time in the air to complete the rotation. There was similar board clearance distance and no change in the maximum board depression suggesting that the diver presses the board with similar effort in both dives. The 5% decrease in takeoff time would be due to the different techniques used during the recoil phase.
Table 6.5. Kinematic variables of the matching 105B and the maximised forward rotating dive simulations

<table>
<thead>
<tr>
<th>variables</th>
<th>matching 105B</th>
<th>maximised rotation</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>2.881</td>
<td>2.623</td>
<td>-9.0%</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.059</td>
<td>0.983</td>
<td>-7.2%</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.601</td>
<td>1.571</td>
<td>-1.2%</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.730</td>
<td>-0.730</td>
<td>0.0%</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.414</td>
<td>0.382</td>
<td>-7.7%</td>
</tr>
<tr>
<td>angular momentum (kg·m²)</td>
<td>59.24</td>
<td>85.54</td>
<td>+44.4%</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>1.073</td>
<td>1.378</td>
<td>+28.4%</td>
</tr>
</tbody>
</table>

The joint angle and orientation time histories of the maximised forward rotating dive and the matching simulation of 105B are displayed in Figure 6.10. It can be seen that the hip extends faster and more before flexing in the optimised dive than in the matching simulation. The rate of change of hip flexion during the recoil phase is similar in both simulations. There is also an obvious increase in knee angle throughout the optimised simulation whereas not much difference is observed at the ball, ankle and shoulder. The orientation time history suggests that the body is more upright during the board depression phase and there is increasing forward lean during the recoil phase.
Figure 6.10. Joint angle and orientation time histories of the matching 105B (dashed) and the maximum forward rotating dive (solid) simulations.

The torque time histories of the optimised dive and the matching simulation 105B are compared in Figure 6.11. The knee torque remains high between 0.2 s to 0.3 s in the optimised simulation whereas it decreases and then increases again in the matching 105B simulation during the same period. The higher knee torque in the optimised simulation is consistent with the increased knee extension shown in the joint angle time history. The hip torque drops from positive to negative earlier in the optimised simulation, suggesting that the hip flexor torque generator is activated earlier. A more negative peak torque at the hip and shoulder are observed in the optimised simulation and this implies that more hip flexion torque and shoulder extension torque are required to generate forward rotation. The two ankle torque time histories follow a similar pattern and the difference in ball torque is too small to have any large influence on the simulation results. It can be concluded that the increase in forward angular momentum is generated by increased knee extension, hip flexion and shoulder extension torque.
Figure 6.11. Joint torque time histories of the matching 105B (dashed) and the maximum forward rotating dive (solid) simulations.

Figure 6.12 displays the muscle activation time history for the optimised forward rotating dive and the matching 105B simulation. There is an obvious increase in hip extensor activation during the board depression phase followed by an earlier hip flexor activation in the optimised simulation. This activation pattern is in agreement with the increased hip extension followed by a stronger hip flexion as shown in the joint angle and torque time histories. The two knee extensor activation histories were very similar up to 0.3 s after which the activation ramped down faster in the optimised simulation. The increase in net knee extension torque can be explained by the delayed final ramp up of the knee flexor which results in a reduced knee flexor torque. Similarly, the increase in net shoulder extension torque in the optimised simulation is due to a lower shoulder flexor activation since there is a delayed shoulder extensor activation.
Figure 6.12. Muscle activation time histories of the matching 105B (dashed) and the maximum forward rotating dive (solid) simulations (kemact = knee extension muscle activation, etc).

Figure 6.13 compares the reaction force time histories of the maximum forward rotating dive and the matching 105B simulation. Both vertical and horizontal forces for the two simulations follow a similar pattern except that the total takeoff time is shorter in the optimised dive. There seems to be a more positive horizontal reaction force during the depression phase in the optimised dive and this force changes to negative earlier and quicker. Such changes in horizontal reaction force could be associated with the increase in hip flexor and shoulder extensor torque during the recoil phase to generate forward angular momentum. In the forward group a negative horizontal reaction force will promote forward rotation, though the contribution of the horizontal force is smaller than that of the vertical force.
Based on the takeoff conditions of the maximised forward rotating dive, the flight phase performance was predicted based on the joint configurations of the 105B performance using a simulation model of aerial movement (Yeadon et al., 1990). Figure 6.14 compares the performance of 105B and the maximised forward rotation dive. In addition, the flight phase joint configurations of the forward one and one-half somersault pike (103B) and the forward two and one-half somersault tuck (105C) were also used to demonstrate how adopting different techniques in flight could influence the orientation at the entry based on the same set of takeoff conditions (Figure 6.15). The aerial phase of all simulations were run until the vertical position of the pelvis was at about 1.0 m (level with a resting springboard).
Figure 6.14. Graphics comparison of the 105B performance (upper) and the maximised forward rotating dive simulation (lower) using joint configuration of 105B in flight (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).

Figure 6.14 shows that with increased angular momentum, the diver over-rotates and performs a forward triple somersault pike (106B) if the same techniques as in the 105B performance is used in the flight phase. It is speculated she will have time to come out and perform a 106B with an extended body at the entry. Although a feet-first entry is seldom used in high level competitions since it is unlikely to gain a high score from the judges, it is often practised as a lead-up for the head-first entry dive with an additional half somersault rotation from a three-metre springboard. If the diver can perform a 106B from a one-metre springboard, he or she should be able to perform a three and one-half somersault pike (107B) from a three-metre springboard using the same takeoff techniques.

If the diver use the optimised takeoff technique to perform a 105B, she can come out a lot earlier and have plenty of time to prepare for entry in a straight body position.
This can be proved by using the flight phase joint configuration of 103B as displayed in Figure 6.15 that the diver can be fully extended before the entry. This can be an improved takeoff technique for 105B compared to the performance in which the diver enters the water in a pike position and extends the body during the entry. If a tuck instead of a pike position is used, the diver can perform a three and one-half somersault tuck (107C) (Figure 6.15).

Figure 6.15. Graphical comparison of the maximised forward rotating dive simulation using joint configuration of 103B (upper) and 105C (lower) in flight (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).
6.4.2.2. Maximum reverse rotation

In the optimisation for maximum reverse rotation, there was about an 11% increase in rotational potential compared with the matching 303B simulation. Comparison of kinematic variables of the optimised and the matching 303B simulations are tabulated in Table 6.6. The 11% increase in angular momentum results in a compromise in dive height which is 11 cm lower than in the matching 303B. The board clearance distance in the two simulations were very similar. There was no change in the maximum board depression but a small decrease in takeoff time. This suggests that the diver presses the board with similar effort in both dives and it is during the recoil phase that takeoff time differs.

Table 6.6. Kinematic variables of the matching 303B and the maximised reverse rotating dive simulations

<table>
<thead>
<tr>
<th>variables</th>
<th>matching 303B</th>
<th>maximised rotation</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>dive height (m)</td>
<td>2.915</td>
<td>2.803</td>
<td>-3.8%</td>
</tr>
<tr>
<td>flight time (s)</td>
<td>1.073</td>
<td>1.054</td>
<td>-1.8%</td>
</tr>
<tr>
<td>board clearance distance (m)</td>
<td>1.362</td>
<td>1.383</td>
<td>+1.5%</td>
</tr>
<tr>
<td>maximum board depression (m)</td>
<td>-0.737</td>
<td>-0.737</td>
<td>0.0%</td>
</tr>
<tr>
<td>takeoff time (s)</td>
<td>0.488</td>
<td>0.470</td>
<td>-3.7%</td>
</tr>
<tr>
<td>angular momentum (kg-m²)</td>
<td>54.21</td>
<td>60.17</td>
<td>+11.0%</td>
</tr>
<tr>
<td>rotation (SS)</td>
<td>0.839</td>
<td>0.930</td>
<td>+10.8%</td>
</tr>
</tbody>
</table>

The joint angle and orientation time histories of the optimised and the matching simulations of 303B are displayed in Figure 6.16. The major difference lays in the recoil phase with increased hip hyper-extension and reduced knee and shoulder angles. The ball and angle time histories in the optimised dive were similar to that of the matching simulation. The two orientation time histories were similar during the board depression phase and there was increased backward lean during the recoil phase in the optimised dive. These results are in agreement with other studies (Sanders et al., 2002; Sprigings & Miller, 2004) where as reverse rotational requirement increases, there is increased hip hyper-extension and knee flexion at takeoff.
Figure 6.16. Joint angle and orientation time histories of the matching 303B (dashed) and the maximum reverse rotating dive (solid) simulations.

The net torque time histories of the optimised dive were compared with that of the matching simulation of 303B (Figure 6.20). Since increasing reverse rotation is characterized by increased hip hyper-extension at takeoff, it is surprising that the optimised dive does not require a larger net hip extension torque to produce more rotation. There is even a reduction in hip torque during the mid recoil phase (between 0.3 s to 0.4 s). On the other hand, there is substantial increase in the net knee extension torque through most of the recoil phase (from 0.2 s to 0.4 s). This suggests that although a reduced knee angle at the instant of takeoff is observed, there is no reduction in knee extension torque during the recoil phase. In contrast, it appears that an increased knee extension torque is required to generate more reverse angular momentum. This can be explained by the joint configurations with increased hip hyper-extension during the recoil phase. The more arched body position will require a higher knee extension torque to maintain the joint configurations since the mass centre is further away from the knee horizontally than it is in a less arched body position. There is no obvious difference in the torque time histories of
the ankle and shoulder for the two simulations. The net ball extension torque is generally lower in the optimised dive but since the ball torque is relatively low compared to the mass of the diver, it should not have large influence on the overall results.

![Joint torque time histories of the matching 303B (dashed) and the maximum reverse rotating dive (solid) simulations.](image)

Figure 6.17. Joint torque time histories of the matching 303B (dashed) and the maximum reverse rotating dive (solid) simulations.

Figure 6.18 displays the muscle activation time history for the optimised reverse rotation dive and the matching 303B. The slower rate of ramping down for the knee extensor results in a higher activation level during the recoil phase. There is also a delay in ramping down for the hip extensor and this increased activation can explain the increase in hip hyper-extension towards the end of the recoil phase. The lower activation level of the shoulder flexor may be balanced by the reduced shoulder extensor activation so that the overall net shoulder torque does not change very much. The small difference in the ball and ankle activation profiles will not have a large influence on the simulation results.
Figure 6.18. Muscle activation time histories of the matching 303B (dashed) and the maximum reverse rotating dive (solid) simulations (kemact = knee extension muscle activation, etc).

Figure 6.19 compares the reaction force time histories of the maximum reverse rotating dive and the matching 303B simulation. It can been seen that the vertical reaction force time history of the two simulations are more or less the same whereas the horizontal reaction force during the recoil phase differs. In the optimised simulation, there is a higher but later peak horizontal force just before the takeoff. This change in horizontal force pattern could be associated with the change in body configuration and joint torque. The increase in hip hyper-extension will push the legs backward, resulting in a larger positive horizontal reaction force. This positive horizontal will project the diver forward to travel away from the board and generate a torque about the diver’s CM which facilitates reverse rotation.
Based on the takeoff conditions of the maximised reverse rotation dive, the flight phase performance was again predicted using a simulation model of aerial movement (Yeadon et al., 1990). The flight phase configuration was based on the performance of 303B and also the reverse double somersault tuck (304C). Figure 6.20 compares the performance of 303B and the maximised reverse rotating dive using the two flight phase configurations.
Figure 6.20. Graphic comparison of 1) top: 303B performance, 2) middle: maximised reverse rotation using joint configuration of 303B in flight, and 3) bottom: maximised reverse rotation using joint configuration of 304C in flight (with an addition of 0.6 m horizontal spacing during contact and 0.4 m in flight between figures).
It can be seen from Figure 6.20 that the diver over-rotates a lot if the same technique as 303B is used in flight. If the come-out at the end of the flight phase is delayed and/or the arms are kept close to the trunk without shoulder flexion, she can probably make a reverse double somersault pike (304B). When the flight configuration of 304C is used, the diver can nearly perform a reverse two and one-half somersault tuck (305C). If the diver comes out and straighten the body at the entry, the rotation will be less than a two and one-half somersault. This means that that diver can still perform the dive 305C but will not gain a high score from the judges due to being short of rotation at entry. The score in diving comprises two parts: the judges' score and the degree of difficulty of the dive performed. Whilst the degree of difficulty of 305C on a one-metre springboard is 3.0 which is much higher than that of 303B (2.4) and 304B (2.6), the advantage of adopting a more difficult dive may be outweighed by a low score from the judges if the dive is executed with poor form. If the diver can perform a more difficult dive with reasonable quality, the benefit of the higher degree of difficulty will increase the total dive score markedly.

6.5. Discussion

The optimisations for maximum height result in only small increases in dive height for three dives and no increase for 101B. The reason why the simulation cannot achieve much higher may be associated with the maximum strength limit. In the evaluation of the torque-driven model, the strength parameters were adjusted so that the simulation could match the performance well as long as the maximum torque did not exceed the torque calculated from the angle-driven model. The maximum strength limit used in the optimisation is therefore the strength required to produce the matching simulation. The diver might be able to produce a higher torque in reality and if so a greater dive height might be achieved. The 101B matching simulation achieved a dive height of 3.2 m which was higher than all other matching or optimised simulations. It may be that the maximum dive height, or very close to that, has already been reached in the matching simulation so there is no further improvement based on the same strength limits.

The second reason for the small increase in dive height could be due to the limited number (5000 to 10,000) of simulations run in the optimisation process. Since the optimisation uses the matching simulation results as a starting point, it is likely that a local minimum close to the matching simulation is found. If this is the case, allowing more
simulations and starting at different points may allow Simulated Annealing to search for a global optimum.

Penalty scores were used to control the board clearance distance and rotational potential in the optimisation. In the optimisation for maximum height, a penalty score was introduced if the distance or rotational potential was not exactly the same as that of the matching simulation. If a slight change in the distance and rotational potential introduces severe penalties that outweigh the increase in height, such penalties may be too strict to allow the search for maximum height. In the optimisation for maximum rotation, a penalty for distance was only introduced once the distance went beyond the lower or upper limit. This allows more freedom in the optimisation and may partly explain why solutions with larger increase in rotational potential can be found. If the penalty score for distance is set the same as in the optimisation for maximum height, less increase in rotational potential will be obtained. On the other hand if more flexibility in distance, ±10% for example, is allowed in the optimisation for maximum height, greater dive height may be achieved for each dive. Future research can further investigate the effect of penalty scores on the optimisation results.

There are 60 muscle activation parameters to be varied in the optimisation and this number is perhaps too larger for Simulated Annealing to find the global optimum. If the number of parameters is reduced, it may be easier and faster to find a solution. This can be done by optimising muscle activation parameters in two phases separately: 40 parameters for ramping up and 20 for ramping down. The optimisation results in this chapter suggest that it is mainly during the recoil phase that muscle activation differs. Future research may optimise only the muscle activation parameters for the recoil phase while keeping those for the board depression phase unchanged.

In the 2004 Olympic Games, most female divers performed a forward three and one-half somersault either in a pike or a tuck position in the three-metre springboard competition final. Using the takeoff techniques in performing a three and one-half somersault from a three-metre springboard, the diver should be able to perform a triple somersault in the same body position (pike or tuck) from a one-metre springboard. The optimisation for maximum forward rotation dive predicts that the diver in this study can perform a triple somersault pike from a one-metre springboard. Since the diver can already perform a two and one-half somersault pike, it seems reasonable to perform a triple somersault pike based on the same strength characteristics. If a tuck position is used, the
diver can perform a three and one-half somersault tuck (107C). Although the degree of difficulty of a 106B (3.2) is higher than that of a 107C (3.0) and a 105B (2.6), a feet-first entry is seldom used in competition since it is unlikely to obtain a high score from the judges. On the other hand, there will be increased muscular torque requirements to hold a tight tuck shape while performing a 107C since the angular velocity of a 107C is faster than that of a 105B (Miller & Sprigings, 2001).

During the data collection, the diver performed a reverse double somersault tuck (304C) with less perceived effort than a reverse one and one-half somersault pike (303B). This is consistent with the fact that the angular momentum required to perform a 303B in a pike position is actually higher than for a 304C in a tuck position though less rotation is completed. The maximised reverse rotating dive possessed 11% more angular momentum than 303B and therefore performing a 305C is within reach. In a recent diving competition (2004 Speedo USA National Diving Championships, results available online at http://www.usadiving.org/USD_03redesign/events/results/04sr_nats_detail.pdf), the winner of the women one-metre springboard used a 305C in her list. She performed this dive with poor form in the semi-final (average judges' score of 4.5 out of 10) and quarter-final (score = 2.5) but managed to execute this difficult dive with high quality in the final with an average score of 8.5. This example is consistent with the optimisation results of this study that it is possible for women divers to perform a 305C from a one-metre springboard. However, the quality and consistency of the dive should also be considered when selecting the dive list.

It should be noted that only the muscle activation parameters were varied in the optimisation to search for the best techniques. Even if the strength and inertia properties remain unchanged, other factors including initial mass centre velocity, whole-body orientation and joint configurations at the touchdown will also influence the takeoff techniques. For example, a higher hurdle step might allow the diver to takeoff with greater vertical velocity (Sanders and Wilson, 1988) and therefore have more time in the air to complete the somersault rotation. If the initial conditions at touchdown are allowed to vary as well as the muscle activation parameters, it is expected that the optimal solution will achieve more dive height or produce more rotation compared to the present optimisation results.

Figure 6.21 shows the angular momentum time histories of the four matching simulations and the two maximised rotating dives. It can be seen that there is hardly any angular momentum at touchdown and during the board depression phase, and that angular
momentum is mainly built up during the recoil phase as found in experimental studies (Miller, 1981; Miller & Munro, 1985).

![Angular momentum time histories for the forward and reverse somersaulting dives.](image)

It is interesting to note that the angular momentum levels off toward the end of the takeoff in the reverse group but not as obvious in the forward group. This is probably associated with the need to gain sufficient horizontal velocity in the reverse group and that shifting the mass centre forward will inhibit reverse rotation. Miller (2000b) calculated the angular momentum time histories of five somersaulting dives performed by a male diver in the 1996 Olympics. In these reported data, the angular momentum does not level off in any dive groups and there is a two-stage increase in angular momentum in the reverse somersaulting dives. The difference between the angular momentum generation pattern in this study and that of Miller (2000b) might be due to the different techniques of investigation. In Miller’s (2000b) study, the angular momentum is calculated based on kinematic data using an inverse dynamics method and therefore joint kinematics are dependent on sampling rate and the degree of smoothing. The present study uses a forward
dynamics approach in which angular momentum is calculated directly from the forces acting on the diver.

6.6. Conclusion

The optimisations for maximum height result in only small increases in dive height for three of the four dives and no increase for 101B. The reason why a greater dive height cannot be achieved may be associated with the maximum strength limit, the large number of parameters and the limited number of simulations run in the optimisation process, and the weightings of the penalty scores used to control the board clearance distance and rotational potential.

The optimisations for maximum rotation show a 28% increase in forward rotational potential compared to a forward two and one-half somersault pike and a 11% increase in reverse rotational potential compared to a reverse one and one-half somersault pike. These results suggest that the diver can perform a forward three and one-half somersault tuck and almost a reverse two and one-half somersault tuck by only changing takeoff techniques. With the increase in angular momentum, the diver can also improve the forward two and one-half somersault pike by completing the rotation earlier to allow sufficient time to prepare for the entry. The increased net knee extension, hip flexion and shoulder extension torque contribute to the increase in forward angular momentum. For the reverse group, the increased reverse angular momentum is characterised by increased hip hyper-extension and a reduced knee angle at takeoff which, in turns, require a larger knee extension torque to maintain the more arched body configuration.
CHAPTER 7
SUMMARY AND DISCUSSION

7.1. Introduction
This chapter summarises the main findings of the present study. The limitations of the methodology will be discussed with suggestions for improvement in the future. Finally, the potential use of the model developed in this study for further investigation of springboard diving takeoff techniques is addressed.

7.2. Summary of main findings
A computer simulation model of a diver and a springboard was developed to investigate the takeoff techniques in springboard diving. This model was evaluated to successfully reproduce realistic takeoff movements with a mean difference of 6.3% between simulation and performance for four dives in the forward and reverse groups. It was then used to maximise dive height for the four dives and also maximise rotational potential in these two dive groups based on the same strength characteristics.

In the optimisations for maximum dive height, the optimised dive achieved 1.7 cm (105B), 1.2 cm (301C) and 2.0 cm (303B) higher than in the corresponding matching simulations and there was no increase for 101B. These results suggest that by modifying the takeoff techniques a higher dive height can be achieved while travelling the same distance and producing the same amount of rotation at the entry. The reason for the small or no increase in dive height may be associated with the maximum strength limit, the large number of parameters and limited number of simulations run in the optimisation process, and the weightings of the penalty scores used to control the board clearance distance and rotational potential.

The limiting dives of the diver in this study at the time of data collection appear to be the forward two and one-half somersault pike (105B) and the reverse one and one-half somersault pike (303B) from a one-metre springboard. In the optimisation for maximised rotation, the simulation has generated a 28% increase in forward rotational potential and a 11 % increase in reverse rotational potential than the corresponding limiting dives. Based on the optimised takeoff conditions, it is predicted that the diver can perform a forward three and one-half somersault tuck (107C) and nearly a reverse two and one-half
somersault tuck (305C). The only difference between the matching and optimised simulations is the muscle activation parameters. This suggests that the diver can increase the rotational potential by changing technique alone without any increase in strength. In the forward group, the increased rotational potential was a result of an increase in knee extension torque, hip flexion torque and shoulder extension torque. On the other hand, the increased reverse angular momentum is characterised by increased hip hyper-extension and knee flexion at the end of the takeoff phase which, in turns, require a larger knee extension torque to maintain the more arched body configuration.

7.3. Discussion

7.3.1. Kinematic data collection

Diving performances were recorded using a high speed camera at 200 Hz and were digitised manually to obtain kinematic data. The field of view of the camera was set too small such that the point of entry into the water was out of view for some dives when the diver jumped slightly off to the right side of the springboard. During camera set-up, only the dive height and the horizontal distance were considered since side travel was not expected. In the future, the camera view should be set larger to ensure that the whole diving sequence can be seen even when the diver jumps off to the side.

There was noise in the kinematic data due to the poor resolution of the digitising system and difficulties in locating joint centres during digitisation especially for the shoulder and ball and toes of the foot. Such errors will have been reduced by data smoothing using splines and allowing corrections in the triangular foot angle and the ball angle in the angle-driven model. An alternate method to manual digitisation is to use an automatic tracking system to further reduce digitisation error and the time of data processing. Marker-based automatic tracking system is becoming more common and many commercial products are available (eg. VICON, Motion Analysis, Peak). In a pool environment, markers can be used to measure springboard movements but may not be suitable for the diver who is diving into the water. If the diver takes off from a dryland board and lands on crash mats or in a foam pit, markers can be used to obtain joint kinematics. With a dryland board, muscle activation time histories of the diver can also be measured using electromyography.
7.3.2. Springboard parameters

The vertical stiffness and effective mass of the springboard were measured using the dynamic loading method described in Miller and Jones (1999). In this study, only two different weights were used to load the springboard at the diver’s preferred fulcrum number of 7.5. The free oscillation frequency of the springboard with no load was estimated from the springboard movement after the diver had taken off from the springboard but the board bounced off the fulcrum. This free oscillation frequency could be measured more accurately in the future by applying a light force to set the board oscillating while in contact with the fulcrum.

The vertical stiffness of the springboard was allowed to vary depending on the foot position on the board using a linear function. Since data were only collected at one point of load application with the toes at the tip of the springboard, the slope and constant of this linear equation were initially estimated from the reported values in the literature (Sprigings et al., 1990) and were then optimised in the angle-driven model. Moreover, the effective board mass determined from the dynamic loading method is fixed at 8.87 kg in the present study whilst it should increase as the point of load application moves further away from the tip of the springboard. It can be argued that this mass is relatively small compared with the mass of the diver and therefore will have little effect on the simulations.

It is desirable to obtain a complete set of springboard parameters that can be applied to any fulcrum number and foot position. Future research could perform the dynamic loading test at more fulcrum numbers (1 to 9), more points of load application, and with more loading weights including a trial of free board oscillation. With more information, the vertical stiffness and the effective mass of the springboard can both be expressed as a function of the fulcrum number and the foot position.

7.3.3. Strength parameters

It was believed that the maximum torques produced by the diver on the isovelocity dynamometer was less than the maximum torques that she was capable of. This hypothesis was supported by the joint torques calculated from the angle-driven model in the four diving takeoffs. The main reason for not being able to produce maximum torque on the isovelocity dynamometer is likely to have been that she had not been sufficiently familiarised with the machine especially for high velocity trials. Future research should allow more time for the participant to practise on the isovelocity dynamometer before data are collected.
In this study, isovelocity data were collected only up to 150°/s and 200°/s for hip flexion. This was due to time limitations and the expectation that the diver could not produce maximum torque at high velocities without sufficient practice. In the future, if time permits, collecting more data at higher velocities will reduce the extrapolation of data in the torque / angle / angular velocity relationship.

The crank angular velocity was converted into joint angular velocity using the ratios reported by Wilson (2003) for the knee and hip joints. Joint angular velocity can be measured directly using goniometers as in Wilson (2003). However, it is difficult to position the goniometer in conjunction with the straps used to stabilise the participant. Moreover, the equipment and constraints placed on the participant may prevent the participant from exerting maximum force in a natural manner. Another way of obtaining joint angular velocity can be achieved by using an automatic tracking system in which only a few markers are needed to be placed on the participant.

The segmental weight correction was done by taking inertia data calculated from Yeadon’s (1990b) model and estimating the segment orientation from video recordings. An alternate method of measuring segmental and crank arm weights is to measure the passive isometric torque with the segment relaxed on the crank arm. This passive isometric torque can be measured over the range of crank angle used in isometric and isovelocity trials to obtain a regression equation that calculates the weight correction required at a specific crank angle. This method takes direct measurements of the segment on the isovelocity dynamometer and avoids errors arising from the assumptions made in the inertia model and the estimation of segment orientation from video recordings. There is a potential limitation of the alternate method in that the position of the segmental orientation may be different when the muscles are contracting and in a relaxed state. The resulting difference in torque, however, will be small compared with the maximum joint torque and therefore should not have a large influence on the overall results.

A single joint torque generator for flexion or extension was used to produce movement at each joint. In strength measurement, adjacent joint angles were either fixed (eg. the hip when measuring the knee) or free to move with no constraints (eg. the ankle when measuring the knee). This method does not take into account the effect of bi-articular muscles that produce movement across two joints. Future work using single joint torque generators could measure a torque / angle / angular velocity relationship that is also dependent on adjacent joint angles and joint angular velocities. The concept of modelling
individual muscles instead of the resultant joint torque is appealing but may be too complex for whole-body modelling. Moreover, individual modelling of muscles results in an over-determined system in which many model parameters cannot be measured directly.

In the present study, muscle parameters such as muscle moment arms and cross-sectional areas were scaled from the reported values in the literature in order to determine the SEC stiffness. Subject-specific muscle parameters can be measured directly using MRI scans, for example, though there will be cost implications. There is one research group (de Zee and Voigt, 2001; 2002) using the quick release method to measure SEC stiffness of the ankle joint directly. In the future, there may be well-established experimental protocols to determine SEC stiffness for all joints.

7.3.4. Body segmental inertia parameters

The body segmental inertias calculated from the model of Yeadon (1990b) have been used in various rigid body simulation models and shown to reproduce realistic human movement (e.g. Yeadon et al., 1990; Yeadon & Hiley, 2000; Yeadon & King, 2002). However, little is known about the inertial properties of the soft tissues within a segment. In the present study, the segmental inertias of the fixed and wobbling components were distributed based on body composition and bone to non-bone mass ratios reported in the literature. These initial estimates were then varied ±20% in the combined matching of the angle-driven model until the best match between the simulation and the performance was found. In the future, more accurate subject-specific inertias could be obtained using dual energy X-ray absorptiometry (DEXA), for example.

7.3.5. Visco-elastic parameters

Subject-specific visco-elastic parameters were determined using an angle-driven model which minimised the difference between simulation and performance. Using the parameter set determined from an individual matching with another dive could result in a very poor match between simulation and performance. This indicates that parameters determined from an individual matching are sensitive to the individual performance and cannot be applied to other performances. Using the combined parameter set on the other hand demonstrated a close correspondence for all four dives although the matches between simulation and performance were less good than in the individual matching particularly for the angular momentum generated. This discrepancy might be associated with errors in joint
kinematics obtained from manual digitisation. In an angle-driven model, digitisation errors can result in unrealistic joint torques and thus inaccurate reaction forces acting on the diver. This problem is particularly acute for the foot-springboard interface parameters due to the difficulties in locating the heel, ball and toes positions in the same plane during digitisation. The wobbling mass parameters, on the other hand, are considered to be less influential on the kinematics of the model (Pain & Challis, 2004).

The visco-elastic properties at the ball, toes and heel were assumed to be identical and so were the parallel and perpendicular wobbling mass spring-damper properties in order to reduce the number of parameters to be determined in the angle-driven model. Since it was difficult to measure accurately the deformation of the foot-springboard interface and the movement of the wobbling masses, penalty scores were used to prevent excessive spring displacements. Assumptions made in a simulation model could be justified if the model could successfully reproduce realistic takeoff movements. Since the torque-driven model has shown good agreement with diving performances, it can be concluded that the visco-elastic properties of the diver can be represented by equal parallel and perpendicular spring-damper parameters.

Using the combined parameter set determined from four different dives in the torque-driven model of the four dives resulted in good matches between simulation and performance. Since these four dives require different angular momenta and are from different dive groups, it may be expected that this parameter set can be used for simulations of other dives from these two groups with some confidence.

7.3.6. Model limitations

Assumptions made in developing the model and determining parameters may have limited its potential to match the recorded performances. The major limitation of the two models used in the present study is the use of a single segment to represent the trunk without taking into account the curvature of the spine which might play an important role in generating angular momentum. When producing forward somersault rotation, the trunk curves forward with protraction instead of remaining straight as in the model. When producing reverse rotation, the trunk arches backward with retraction. Such movements within the trunk segment are not included in the model. The advantage of modelling the trunk as a single straight segment rather than multiple segments to represent curvature in the spine is that the resultant joint torque across each joint can be measured directly on an isovelocity dynamometer. In the torque-driven model, the hip was allowed to hyper-extend
slightly more to facilitate the generation of angular momentum. The good agreement in the reverse angular momentum generated in the torque-driven model suggests that the limitation of using a straight segment to represent the trunk can be compensated by allowing extra hip hyper-extension.

In addition, the model was planar based on the assumption that there should be symmetrical movements for a good somersaulting dive takeoff. Such an assumption can be justified for the trunk and the legs which more or less move in the same plane throughout the takeoff. However, the armswing during takeoff and the lateral arm movements in flight, while symmetrical, were clearly 3D movements. Projecting 3D coordinates onto a plane for the 2D model would have resulted in inaccurate joint angles and the derived torque time histories may have influenced the orientation and angular momentum of the diver.

7.3.7. Optimisation algorithm

The Simulated Annealing optimisation algorithm (Corana et al., 1987) was used in this study since it has been shown to be very robust and able to find a global optimum rather than a local optimum (Goffe et al., 1994). However, the function to be optimise in the evaluation and the optimisation process is a very complex function with 62 parameters and takes a relatively long time to run a single simulation (between 30 seconds to one minute for the torque-driven model). If a high initial temperature is used to ensure the searching for the global minimum, a large number of simulations will be required and it will take too long. In order to save on running time, the starting point was chosen manually by trial and error until the simulation produced a reasonable dive. Since a low initial temperature was used and only a limited number of simulations were run, it is likely that the solution found was a local minimum that was close to the initial starting point rather than the global minimum. Future research can start the simulation at different points to see if there is any difference in the optimised solution. If time is sufficient or a computer with a faster processor is available, more simulations can be run with a higher initial temperature. There might also be multiple solutions that a different combination of muscle activation parameters can produce the same takeoff conditions.

For the individual matching of the angle-driven model with nine parameters, the optimisation parameters were set as: initial temperature $T = 5.0$, temperature reduction factor $RT = 0.85$, number of cycles $NS = 20$, number of iterations before temperature reduction $NT = 10$. Other optimisation parameters were set according to the recommendations by Goffe et al.(1994). A maximum number of 200,000 simulations took
about 3 days to complete. The combined matching took four times as long as the individual matching to complete the same number of simulations. For the torque-driven model with 62 parameters, the optimisation parameters were set as: \( T = 5.0 \), \( RT = 0.85 \), \( NS = 4 \), \( NT = 4 \). A maximum number of 20,000 simulations in the evaluation process took about 12 days to complete. Due to the limited time, only 5,000 to 10,000 simulations were run in the optimisations for maximum height and rotational potential. Since the optimisation used the matching simulation results as a starting point, it is likely that a local minimum close to the matching simulation was found. If time permits, more simulations could be run with an increased value for \( NS \) and \( NT \) to encourage a wider search before any temperature reduction.

### 7.3.8. Optimisation score

The objective function or score that is to be maximised or minimised is the most important part of any optimisations. The weightings of each component of the score and the penalties may have a large influence on the solution found. In the matching simulation, there is large discrepancy in the joint angle time histories compared with the other four components of the matching score. If the root mean square instead of the average value of the five components is used, it may be expected that the score will be distributed more evenly among the components.

The optimisation score was calculated as 10 times of the dive height achieved and 100 times the number of straight somersaults produced in the optimisation for maximum height and rotational potential respectively. The weighting between the optimisation score and the penalty scores may have a considerable effect on the solution found and therefore should be set with careful thought.

### 7.3.9. Penalty scores

As mentioned before, the weighting of the penalty scores may be very important for the optimisation result. If the penalties are too small compared to the optimisation score, violation of the constraints may be ignored as long as the overall optimisation score is improving. The matching simulation of 303B is an example that despite the good match in linear and angular momentum generation, there are penalty scores for the joint angle constraints. Increasing the weightings of the penalties might encourage the optimisation to find a solution with no penalties although the matching results would not be as good. On the other hand if the penalty scores are set too strictly, it may limit the optimisation to
search for the global solution. This may be the case for the optimisation for maximum dive height in which penalty scores are introduced if the board clearance distance and the rotational potential are not exactly the same as in the matching simulation.

Since the penalty scores may have a large influence on the optimisation result, they should be carefully set within sensible ranges. In this study, the joint angle constraints were limited to the range of motion observed from video recordings of diving performances and the physical limit of a joint, 180° for a straight knee for example. These constraints are used to ensure that the simulation results will not correspond to an injury in reality. While the actual maximum range may exceed the observed range, it is better to under-estimate than over-estimate the joint limits for this purpose. In some pilot work of this study, simulations were run without the joint angle constraints in flight (0.1 s after takeoff) and therefore the joint angular velocity at takeoff was not constrained. The matching simulation results appeared promising initially with an average of 4.1% difference between the simulation and performance but there was a 90% increase in rotational potential in the optimisation for maximum reverse rotation with excessive knee flexion. These results highlight the need to constrain both maximum and minimum joint angle and joint angular velocity to be within a sensible range for the optimisation to search for a reasonable solution.

7.4. Future research

Once a simulation model has been developed and successfully evaluated, it can be a useful tool to answer research questions. The torque-driven model developed in this study can be used to investigate further springboard diving takeoff techniques. Following are some examples that future research can address.

7.4.1. Robustness

A diver can perform the same dive with slightly different techniques and make adjustments to compensate for any mistakes during the takeoff, flight, and/or entry. The sensitivity of the simulation to perturbations can be assessed by varying the model input such as the initial foot position on the board and the muscle activation time histories. For the optimisation for maximum rotation, it is particularly important to ensure that a solution is robust to minor perturbations and can still produce appropriate takeoff conditions. For example, if the muscles are activated 10 ms earlier (as the diver ‘rushed’ into the dive) and
there is a significant reduction in board clearance distance, it is unlikely that the diver will use this optimum takeoff technique in reality for the reason of safety.

7.4.2. Initial conditions

It has been shown that a higher hurdle step is associated with more dive height (Sanders & Wilson, 1988). The advantage of using the new hurdle pre-flight techniques to increase dive height as opposed to the traditional walking or running approach is not yet fully understood (Miller et al., 2002). Divers with longer and faster hurdles tended to reduce their horizontal velocity during takeoff whilst those with shorter and slower hurdles tended to increase their horizontal velocity during takeoff (Miller, 1984, 2000). There can also be variations in the joint configuration at touchdown due to the different timing of the armswing for example. By varying the initial CM velocity and joint configurations at touchdown, the model can be applied to investigate the optimal takeoff techniques with different initial conditions.

7.4.3. Inertial parameters

Modifying technique according to changing body size plays an important role for growing junior divers. The optimal takeoff techniques for divers with different segmental inertias can be investigated by inputting a different set of inertia parameters to the model. The optimal takeoff technique may be expected to be different even to perform the same dive for different inertial parameter sets. The extent to which these techniques differ will have a strong implication for coaching since there is a wide range of body height and size among divers.

7.4.4. Strength parameters

Similar to inertia parameters, the strength of an individual diver will influence the optimal takeoff technique. Besides the different strength characteristics in male / female and senior / junior divers, the physical condition of a diver may be different throughout the year especially after an injury. The influence of strength on diving performance is also important in designing a strength and conditioning program. By using a different set of strength parameters in the model, the effect of strength on diving performance can be assessed.
7.4.5. Board clearance distance

In the optimisation for height, a penalty score was used to keep the board clearance distance the same as that travelled in the matching simulation. This distance, however, may not be the optimal distance for maximum height and safety. For example, the reverse dive tuck travelled 1.77 m when the mass centre was level with the board and this distance is considered far too long for a good performance. Setting the board clearance distance within a sensible range will allow more freedom for the optimisation algorithm to search for the optimal solution within the range. It is expected that some simulations will push to the lower limit of the range and therefore the minimum board clearance distance for safety should be set with caution. Alternately, optimisations with different fixed board clearance distances could be used to investigate the maximum height and rotation that can be generated at a particular distance.

7.4.6. Head position

There has been a long standing argument of whether the head should be thrown back vigorously (Hobden, 1936; Billingsley, 1965) or kept in line with the body (Barone, 1973; Batterman, 1968; O'Brien, 1992; Zhang, 1996) to generate backward and reverse angular momentum. The recent trend tends to favour the latter argument though the reason behind is not fully understood. Whilst hyper-extending the head at takeoff may slow down the pulling into shape in the flight phase, the effect of the head movement in generating angular momentum is unclear. Since the trunk leans backward during a reverse somersaulting dive takeoff, head hyper-extension may encourage greater hip-hyperextension which facilitates the generation of reverse angular momentum.

7.4.7. Timing of the armswing

Sprigings and Watson (1985) suggested a late armswing in that the upward acceleration of the arms with respect to the shoulders should commence at the moment of board contact. The timing of the relative force patterns of the arms, torso and legs during takeoff has also been investigated (Sprigings et al., 1986). The effect of the timing and the armswing movement on achieving dive height and generating angular momentum may be studied by varying the muscle activation parameters of the shoulder torque generators.
7.4.8. Application to other dive groups

The model developed in this study was based on running dives including the forward and reverse groups. It can be further extended to investigate techniques in standing dives: the inward and backward groups. The major difference between modelling a running dive and a standing dive takeoff is that the heel is not in contact with the springboard in a standing dive. Thus, the model developed in this study should also be suitable to simulate a standing somersaulting dive takeoff but it still needs to be evaluated before being applied to investigate optimum takeoff techniques in the inward and backward groups.

7.4.9. Sensitivity to model parameters

The sensitivity of the model to the model parameters can be assessed by varying the individual parameter values to see how these parameters affect the overall results. The wobbling mass parameters, for example, have found to have little influence on the kinematics of a simulation model of landing (Pain & Challis, 2004). Whether wobbling mass is necessary in a springboard diving takeoff model has yet to be established. If increasing the stiffness in the wobbling mass springs has no obvious effect on the simulation results, rigid-body modelling may be sufficient for the purpose of investigating optimal takeoff techniques in the future. The sensitivity of the model to other parameters such as the SEC stiffness, muscle moment arm and foot-springboard interface stiffness and damping could also be examined.

7.5. Conclusion

The torque-driven model developed in this study has been successfully evaluated and applied to optimise takeoff techniques for forward and reverse somersaulting dives. The limitations in data collection, model parameter determination and the optimisation process along with the assumptions made in developing the model have been discussed with suggestions for improvement in the future. The model can be used for further investigation of springboard diving takeoff techniques and to answer the 'what if' questions that cannot be addressed by experimental studies.
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APPENDIX 1

DEVELOPMENT OF SIMULATION MODELS

USING AUTOLEV 3.4™

Appendix 1a. Autolev commands used for the angle-driven model of a diver and a springboard

Appendix 1b. Autolev commands used for the torque-driven model of a diver and a springboard
Appendix 1a

%DV8SEG.AL

% ANGLE DRIVEN 8-SEGMENT MODEL FOR SPRINGBOARD DIVING TAKEOFF
% - END OF BOARD HORIZONTAL MOVEMENT CONSTRAINED BY BOARD DEPRESSION
% - USE AUXILIARY TO CALCULATE HORIZONTAL SPRINGBOARD FORCE RX
% - BOARD ANGLE CONSTRAINED BY BOARD DEPRESSION
% - TRIANGULAR 2-SEGMENT FOOT
% - BOARD STIFFNESS CHANGES WITH PLACEMENT OF FOOT SD
% - 3 WOBBLING MASSES (TRUNK, THIGH, SHANK)
% - NON-LINEAR SPRING EQUATIONS FOR WOBBLING MASS SPRINGS
% - ELIMINATE SPRINGBOARD FROM BODY IN P_O_CM> AND ANGMOM

% ---------------------------------------------------------------
% PHYSICAL DECLARATION

NEWTONIAN N
FRAMES T % TRIANGULAR FOOT
BODIES S,A,B,C,D,E,F,G,H,J,K,L
POINTS O,P1,P2,P3,P4,P5,P6,P7,P8,P9,P10,P11,P12,P13,P14,P15,CW1,CW2, &
DW1,DW2,EW1,EW2,CM

% ---------------------------------------------------------------
% MATHEMATICAL DECLARATION

INERTIA S, 0,0, IS
INERTIA A, 0,0, IA
INERTIA B, 0,0, IB
INERTIA C, 0,0, IC
INERTIA D, 0,0, ID
INERTIA E, 0,0, IE
INERTIA F, 0,0, IF
INERTIA G, 0,0, IG
INERTIA H, 0,0, IH
INERTIA J, 0,0, IJ
INERTIA K, 0,0, IK
INERTIA L, 0,0, IL
SPECIFIED X", Theta" % SPRINGBOARD CONSTRAINTS
SPECIFIED QBA", QA", QK", QH", QS", QEL", QHE" % JOINT ANGLES
SPECIFIED RX, RZ, & % SPRINGBOARD REACTION FORCES
R3, R4, R5, R6, R7, R8, & % FOOT SPRING FORCE
FC(4), FD(4), FE(4), & % WOBBLING MASS FORCES
TBAL, TANK, THIP, TSHD, TELB, THEA, TBRD % TORQUE
CONSTANTS HORCON, ANGCON, SD, SM, SC, & % SPRINGBOARD CONSTANTS
THETA2, & % TRIANGULAR FOOT ANGLE
G,L(20), & % GRAVITY, LENGTH
K, K3, K4, K5, K6, K7, K8, KC(4), KD(4), KE(4), & % STIFFNESS
KK3, KK4, KK5, KK6, KK7, KK8, KK(4), KD(4), KE(4) % DAMPING
VARIABLES Q"(14), U"(22) % DEGREE OF FREEDOM
VARIABLES POSOX, POSOZ, POAOX, POAOZ, POBOX, POBOX, POCOX, POCOX, PODOX, PODOZ, &
POEOX, POEOX, POFOX, POFOX, POFOZ, POFOZ, POHDX, POHDX, POHOZ, POHOZ, POJOZ, &
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VOP4X, VOP4Z, VOP5X, VOP5Z, VOP6X, VOP6Z, VOP7X, VOP7Z, VOP8X, VOP8Z, &
VOP9X, VOP9Z, VOP10X, VOP10Z, VOP11X, VOP11Z, VOP12X, VOP12Z, &
VP5P8X, VP5P8Z, VCW2X, VCW2Z, VDW2X, VDW2Z, VEW2X, VEW2Z

VARIABLES POCMX, POCMZ, VOCMX, VOCMZ, AOCMX, AOCMZ, &
KET, KECM, KES, KEA, KEB, KEC, KED, KEF, KEG, KEH, KEJ, KEL, &
PET, PECM, PES, PEA, PEC, PED, PEE, PEF, PEG, PEH, PEJ, PEK, PEL, &
PEQ2, PEQ3, PEQ4, PEBX, PEBZ, PEHX, PEHZ, PE1C, PE2C, PE3C, PE4C, &
PED1, PED2, PED3, PED4, PEEL1, PEE2, PEE3, PEE4

VARIABLES ANGMOM, HORMOM, VERMOM, & % MOMENTUM
QCM, M, MT % CM ANGLE, MASSES

AUTOZ ON
ZEE_NOT = [RX, RZ, R3, R4, R5, R6, TBAL, TANK, TKNE, THIP, TSHD, TELB, THEA, TBRA, &
THEA', X, X', X'']

M = MA + MB + MC + MD + ME + MF + MG + MH + MJ + MK + ML
MT = MS + MA + MB + MC + MD + ME + MF + MG + MH + MJ + MK + ML

K = SM*(SD + 0.15) + SC % SPRINGBOARD STIFFNESS
X = HORNCON*Q2^2 % HORIZONTAL BOARD MOVEMENT
THETA = ANGCON*Q2 % BOARD ANGLE IN RADIANS

QBAL = T^3 % CALL VALQ3 IN .F FILE FOR ANGLE AND W
QANK = T^3 % TO OVER-WRITE THE Q VALUESE HERE
QKNE = T^3
QHIP = T^3
QSHD = T^3
QELB = T^3
QHEA = T^3

%-----------------------------------------------
% GEOMETRICAL RELATION
%-----------------------------------------------

SIMPROT(N, S, 3, THETA) % BOARD/HORIZONTAL
SIMPROT(N, E, 3, Q14) % TRUNK/HORIZONTAL
SIMPROT(D, E, 3, PI+QHIP) % TRUNK/THIGH
SIMPROT(D, C, 3, PI+QKNE) % THIGH/KNEE
SIMPROT(B, C, 3, QANK) % KNEE/FOOT
SIMPROT(A, B, 3, QBAL) % FOOT/TOES *CORRECTED AS PI+QBAL LATER
SIMPROT(B, T, 3, QANK) % FIXED TRIANGULAR FOOT
SIMPROT(E, F, 3, PI+QSHD) % TRUNK/UPPER ARM
SIMPROT(G, F, 3, PI+QELB) % UPPER/LOWER ARM
SIMPROT(E, H, 3, PI+QHEA) % TRUNK/HEAD

%-----------------------------------------------
% POSITION
%-----------------------------------------------

P_O_P1> = (X+Q1)*N1> + Q2*N2>
P_P1_S0> = -L1*S1>
P_P1_P2> = -L2*S1>
P_P1_P3> = -SD*S1>
P_P3_P4> = -L4*S1>
P_P4_P5> = -L7*S1>
P_P3_P6> = Q3*S1> + Q4*S2>
P_P6_A0> = -(L4-L3)*A1>
P_P6_P7> = -L4*A1>
P_P7_B0> = -L5*T1> + L6*T2>
P_P7_P8> = -L7*T1>
P_P7_P9> = -L8*B1>
P_P4_P7> = -P_P3_P4> + P_P3_P6> + P_P6_P7> % BALL SPRING
Appendix 1a

\[ \text{P}_P5_P8> = \text{-P}_P4_P5> + \text{P}_P4_P7> + \text{P}_P7_P8> \]  
\[ \text{P}_P9_CO> = (\text{L}10-\text{L}9)*\text{C}1> \]  
\[ \text{P}_P9_P10> = \text{L}10*\text{C}1> \]  
\[ \text{P}_P10_D0> = (\text{L}12-\text{L}11)*\text{D}1> \]  
\[ \text{P}_P10_P11> = \text{L}12*\text{D}1> \]  
\[ \text{P}_P11_E0> = \text{L}13*\text{E}1> \]  
\[ \text{P}_P11_P12> = \text{L}14*\text{E}1> \]  
\[ \text{P}_P12_F0> = \text{L}15*\text{F}1> \]  
\[ \text{P}_P12_P13> = \text{L}16*\text{F}1> \]  
\[ \text{P}_P13_G0> = \text{L}17*\text{G}1> \]  
\[ \text{P}_P13_P14> = \text{L}18*\text{G}1> \]  
\[ \text{P}_P12_H0> = \text{L}19*\text{H}1> \]  
\[ \text{P}_P12_P15> = \text{L}20*\text{H}1> \]  
\[ \text{P}_P0_S0> = \text{P}_O_P1> + \text{P}_P1_S0> \]  
\[ \text{P}_P0_P2> = \text{P}_O_P1> + \text{P}_P1_P2> \]  
\[ \text{P}_P0_P3> = \text{P}_O_P1> + \text{P}_P1_P3> \]  
\[ \text{P}_P0_P4> = \text{P}_O_P3> + \text{P}_P3_P4> \]  
\[ \text{P}_P0_P5> = \text{P}_O_P4> + \text{P}_P4_P5> \]  
\[ \text{P}_P0_P6> = \text{P}_O_P5> + \text{P}_P5_P6> \]  
\[ \text{P}_P0_A0> = \text{P}_O_P6> + \text{P}_P6_A0> \]  
\[ \text{P}_P0_P7> = \text{P}_O_P6> + \text{P}_P6_P7> \]  
\[ \text{P}_P0_B0> = \text{P}_O_P7> + \text{P}_P7_B0> \]  
\[ \text{P}_P0_P9> = \text{P}_O_P7> + \text{P}_P7_P9> \]  
\[ \text{P}_P0_CO> = \text{P}_O_P9> + \text{P}_P9_CO> \]  
\[ \text{P}_P0_P10> = \text{P}_O_P9> + \text{P}_P9_P10> \]  
\[ \text{P}_P0_D0> = \text{P}_O_P10> + \text{P}_P10_D0> \]  
\[ \text{P}_P0_P11> = \text{P}_O_P10> + \text{P}_P10_P11> \]  
\[ \text{P}_P0_E0> = \text{P}_O_P11> + \text{P}_P11_E0> \]  
\[ \text{P}_P0_P12> = \text{P}_O_P11> + \text{P}_P11_P12> \]  
\[ \text{P}_P0_F0> = \text{P}_O_P12> + \text{P}_P12_F0> \]  
\[ \text{P}_P0_P13> = \text{P}_O_P12> + \text{P}_P12_P13> \]  
\[ \text{P}_P0_G0> = \text{P}_O_P13> + \text{P}_P13_G0> \]  
\[ \text{P}_P0_P14> = \text{P}_O_P13> + \text{P}_P13_P14> \]  
\[ \text{P}_P0_H0> = \text{P}_O_P12> + \text{P}_P12_H0> \]  
\[ \text{P}_P0_P15> = \text{P}_O_P12> + \text{P}_P12_P15> \]  

SIMPROT(C, J, 3, Q7)

\[ \text{P}_P9_CW1> = \text{Q}5*\text{C}1> + \text{Q}6*\text{C}2> \]  
\[ \text{P}_CW1JO> = (\text{L}10-\text{L}9)*\text{J}1> \]  
\[ \text{P}_CW1CW2> = \text{L}10*\text{J}1> \]  
\[ \text{P}_P10_CW2> = \text{-P}_P9_P10> + \text{P}_P9_CW1> + \text{P}_CW1CW2> \]  
\[ \text{P}_O_CW1> = \text{P}_O_P9> + \text{P}_P9_CW1> \]  
\[ \text{P}_O_CW1JO> = \text{P}_O_CW1> + \text{P}_CW1JO> \]  
\[ \text{P}_O_CW2> = \text{P}_O_CW1> + \text{P}_CW1CW2> \]  
\[ \text{PCW2X} = \text{DOT}(\text{P}_P10_CW2>, \text{C}1>) \]  
\[ \text{PCW2Z} = \text{DOT}(\text{P}_P10_CW2>, \text{C}2>) \]  
\[ \text{PCW1X} = \text{DOT}(\text{P}_O_CW1>, \text{N}1>) \]  
\[ \text{PCW1Z} = \text{DOT}(\text{P}_O_CW1>, \text{N}2>) \]  
\[ \text{POJOX} = \text{DOT}(\text{P}_OJO>, \text{N}1>) \]  
\[ \text{POJOZ} = \text{DOT}(\text{P}_OJO>, \text{N}2>) \]  
\[ \text{POCW2X} = \text{DOT}(\text{P}_O_CW2>, \text{N}1>) \]  
\[ \text{POCW2Z} = \text{DOT}(\text{P}_O_CW2>, \text{N}2>) \]  

SIMPROT(D, K, 3, Q10)

\[ \text{P}_P10_DW1> = \text{Q}8*\text{D}1> + \text{Q}9*\text{D}2> \]  
\[ \text{P}_DW1KO> = (\text{L}12-\text{L}11)*\text{K}1> \]  
\[ \text{P}_DW1DW2> = \text{L}12*\text{K}1> \]  
\[ \text{P}_P11_DW2> = \text{-P}_P10_P11> + \text{P}_P10_DW1> + \text{P}_DW1_DW2> \]  
\[ \text{P}_O_DW1> = \text{P}_O_P10> + \text{P}_P10_DW1> \]  
\[ \text{P}_O_KO> = \text{P}_O_DW1> + \text{P}_DW1_KO> \]  

% HEEL SPRING

% WOBBLING MASS J IN C

% WOBBLING MASS K IN D
Appendix 1a

\[ P_{0\_DW2} = P_{0\_DW1} + P_{DW1\_DW2} \]

\[ PDW2X = DOT(P_{P\_11\_DW2}, D1) \]

\[ PDW2Z = DOT(P_{P\_11\_DW2}, D2) \]

\[ PODW1X = DOT(P_{0\_DW1}, N1) \]

\[ PODW1Z = DOT(P_{0\_DW1}, N2) \]

\[ POKOX = DOT(P_{0\_KO}, N1) \]

\[ POKOZ = DOT(P_{0\_KO}, N2) \]

\[ PODW2X = DOT(P_{0\_DW2}, N1) \]

\[ PODW2Z = DOT(P_{0\_DW2}, N2) \]

\[ SIMPROT(E, L, 3, Q13) \% \]

\[ PP11_EW1 = Q11*E1 + Q12*E2 \]

\[ P_{EW1\_LO} = L13*L1 \]

\[ P_{EW1\_EW2} = L14*L1 \]

\[ P_{P\_11\_P12} = -P_{P\_11\_P12} + P_{P\_11\_EW1} + P_{EW1\_EW2} \]

\[ P_{0\_LO} = P_{0\_P11} + P_{P\_11\_P12} \]

\[ P_{0\_EW1} = P_{0\_EW1} + P_{EW1\_LO} \]

\[ P_{0\_EW2} = P_{0\_EW2} + P_{EW2\_EW2} \]

\[ PEW2X = DOT(P_{P\_12\_EW2}, E1) \]

\[ PEW2Z = DOT(P_{P\_12\_EW2}, E2) \]

\[ POE1WX = DOT(P_{0\_EW1}, N1) \]

\[ POE1WZ = DOT(P_{0\_EW1}, N2) \]

\[ POE2WZ = DOT(P_{0\_EW2}, N1) \]

\[ POE2WZ = DOT(P_{0\_EW2}, N2) \]

\[ P_{0\_CM} = CM(O, A, B, C, D, E, F, G, H, J, K, L) \% NOT INCLUDING SPRINGBOARD S

\[ F0P1X = DOT(P_{0\_P1}, N1) \]

\[ F0P1Z = DOT(P_{0\_P1}, N2) \]

\[ F0P2X = DOT(P_{0\_P2}, N1) \]

\[ F0P2Z = DOT(P_{0\_P2}, N2) \]

\[ F0P3X = DOT(P_{0\_P3}, N1) \]

\[ F0P3Z = DOT(P_{0\_P3}, N2) \]

\[ F0P4X = DOT(P_{0\_P4}, N1) \]

\[ F0P4Z = DOT(P_{0\_P4}, N2) \]

\[ F0P5X = DOT(P_{0\_P5}, N1) \]

\[ F0P5Z = DOT(P_{0\_P5}, N2) \]

\[ F0P6X = DOT(P_{0\_P6}, N1) \]

\[ F0P6Z = DOT(P_{0\_P6}, N2) \]

\[ F0P7X = DOT(P_{0\_P7}, N1) \]

\[ F0P7Z = DOT(P_{0\_P7}, N2) \]

\[ F0P8X = DOT(P_{0\_P8}, N1) \]

\[ F0P8Z = DOT(P_{0\_P8}, N2) \]

\[ F0P9X = DOT(P_{0\_P9}, N1) \]

\[ F0P9Z = DOT(P_{0\_P9}, N2) \]

\[ F0P10X = DOT(P_{0\_P10}, N1) \]

\[ F0P10Z = DOT(P_{0\_P10}, N2) \]

\[ F0P11X = DOT(P_{0\_P11}, N1) \]

\[ F0P11Z = DOT(P_{0\_P11}, N2) \]

\[ F0P12X = DOT(P_{0\_P12}, N1) \]

\[ F0P12Z = DOT(P_{0\_P12}, N2) \]

\[ F0P13X = DOT(P_{0\_P13}, N1) \]

\[ F0P13Z = DOT(P_{0\_P13}, N2) \]

\[ F0P14X = DOT(P_{0\_P14}, N1) \]

\[ F0P14Z = DOT(P_{0\_P14}, N2) \]

\[ F0P15X = DOT(P_{0\_P15}, N1) \]

\[ F0P15Z = DOT(P_{0\_P15}, N2) \]

\[ F0POSX = DOT(P_{0\_P10}, N1) \]

\[ F0POSZ = DOT(P_{0\_P10}, N2) \]

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Appendix 1a

POAOX = DOT(P_0_AO>, N1>)
POAOZ = DOT(P_0_AO>, N2>)
POBOX = DOT(P_0_BO>, N1>)
POBOZ = DOT(P_0_BO>, N2>)
POCOX = DOT(P_0_CO>, N1>)
POCOZ = DOT(P_0_CO>, N2>)
PODOX = DOT(P_0_DO>, N1>)
PODOZ = DOT(P_0_DO>, N2>)
POEOX = DOT(P_0_EO>, N1>)
POEOZ = DOT(P_0_EO>, N2>)
POFOX = DOT(P_0_FO>, N1>)
POFOZ = DOT(P_0_FO>, N2>)
POGOX = DOT(P_0_GO>, N1>)
POGOZ = DOT(P_0_GO>, N2>)
POHOX = DOT(P_0_HO>, N1>)
POHOZ = DOT(P_0_HO>, N2>)
PP4P7X = DOT(P_P4_P7>, S1>)
PP4P7Z = DOT(P_P4_P7>, S2>)
PP5P8X = DOT(P_P5_P8>, S1>)
PP5P8Z = DOT(P_P5_P8>, S2>)
POCMX = DOT(P_0_CM>, N1>)
POCMZ = DOT(P_0_CM>, N2>)
QCM = ATAN(POCMZ/POCMX) % BODY MASS CENTRE ANGLE

% KINEMATICAL DIFFERENTIAL EQUATIONS

Q1' = U1
Q2' = U2
Q3' = U3
Q4' = U4
Q5' = U5
Q6' = U6
Q7' = U7
Q8' = U8
Q9' = U9
Q10' = U10
Q11' = U11
Q12' = U12
Q13' = U13
Q14' = U14
X' = DT(X)
X'' = DT(X')
THETA'' = DT(THETA')
THETA' = DT(THETA)

% ANGULAR VELOCITY AND ACCELERATION

W_S_N> = THETA''*S3> + U22*S3> % USE GENERALISED SPEED TO CALCULATE TORQUE
W_E_N> = U14*E3>
W_E_D> = QHIP''*E3> + U15*E3>
W_C_D> = QKNE''*C3> + U16*C3>
W_C_B> = QANK''*C3> + U17*C3>
W_B_A> = QBAL''*B3> + U18*B3>
W_F_E> = QSHD''*F3> + U19*F3>
W_F_C> = QELB''*F3> + U20*F3>
W_H_E> = QHEA''*H3> + U21*H3>
W_T_B> = 0>

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\[ W_{\text{J}_C} = U7*J3 > \]
\[ W_{\text{K}_D} = U10*K3 > \]
\[ W_{\text{L}_E} = U13*L3 > \]

\[ \text{ALF}_S_{\text{N}} = \text{THETA}'^*S3 > \]
\[ \text{ALF}_E_{\text{N}} = U14'^*E3 > \]
\[ \text{ALF}_E_{\text{D}} = QHIP'^*E3 > \]
\[ \text{ALF}_C_{\text{D}} = QKNE'^*C3 > \]
\[ \text{ALF}_C_{\text{B}} = QANK'^*C3 > \]
\[ \text{ALF}_B_{\text{A}} = QBAL'^*B3 > \]
\[ \text{ALF}_F_{\text{E}} = QSHE'^*F3 > \]
\[ \text{ALF}_F_{\text{G}} = QELB'^*F3 > \]
\[ \text{ALF}_H_{\text{E}} = QHEA'^*H3 > \]
\[ \text{ALF}_T_{\text{B}} = 0 > \]
\[ \text{ALF}_J_{\text{C}} = U7'^*J3 > \]
\[ \text{ALF}_K_{\text{D}} = U10'^*K3 > \]
\[ \text{ALF}_L_{\text{E}} = U13'^*L3 > \]

%----------------------------------------------------------
% LINEAR VELOCITY
%----------------------------------------------------------

\[ V_{\text{O}_N} = 0 > \]
\[ V_{\text{P}1_{N}} = DT(P_{\text{O}_P1}_{N}, N) \]
\[ V2PTS(N,S,P1,S0) \]
\[ V2PTS(N,S,P1,P2) \]
\[ V2PTS(N,S,P1,P3) \]
\[ V2PTS(N,S,P1,P4) \]
\[ V2PTS(N,S,P1,P5) \]
\[ V_{\text{P}6_{N}} = DT(P_{\text{P}_P6}_{N}, N) \]
\[ V2PTS(N,A,P6,A0) \]
\[ V2PTS(N,A,P6,P7) \]
\[ V2PTS(N,T,P7,P8) \]
\[ V2PTS(N,T,P7,B0) \]
\[ V2PTS(N,B,P7,P9) \]
\[ V2PTS(N,C,P9,C0) \]
\[ V2PTS(N,C,P9,F0) \]
\[ V2PTS(N,D,P10,D0) \]
\[ V2PTS(N,D,P10,P11) \]
\[ V2PTS(N,E,P11,E0) \]
\[ V2PTS(N,E,P11,P12) \]
\[ V2PTS(N,F,P12,F0) \]
\[ V2PTS(N,F,P12,P13) \]
\[ V2PTS(N,G,P13,G0) \]
\[ V2PTS(N,G,P13,P14) \]
\[ V2PTS(N,H,P12,H0) \]
\[ V2PTS(N,H,P12,P15) \]
\[ V_{P7_S} = DT(P_{P4_P7}, S) \]
\[ V_{P8_S} = DT(P_{P5_P8}, S) \]

\[ V_{\text{CW}1_{C}} = DT(P_{P9_{CW1}}, C) \]
\[ V2PTS(C,J,CW1,J0) \]
\[ V2PTS(C,J,CW1,CW2) \]
\[ V_{\text{CW}1_{N}} = DT(P_{P_O_{CW1}}, N) \]
\[ V2PTS(N,J,CW1,J0) \]
\[ V2PTS(N,J,CW1,CW2) \]
\[ VCW2X = DOT(V_{CW2_C}, C1) \]
\[ VCW2Z = DOT(V_{CW2_C}, C2) \]

\[ V_{\text{DW}1_{D}} = DT(P_{P10_{DW1}}, D) \]
\[ V2PTS(D,K,DW1,K0) \]
\[ V2PTS(D,K,DW1,DW2) \]

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VDW1N> = DT(P_O_DW1>, N)
V2PTS(N, K, DW1, KO)
VDW2X = DOT(V_DW2_D>, D1>)
VDW2Z = DOT(V_DW2_D>, D2>)

V_EW1_E> = DT(P_P11_EW1>, E)
V2PTS(E, L, EW1, LO)
V_EW1_N> = DT(P_O_EW1>, N)
V2PTS(N, L, EW1, LO)
V_EW1_N> = DT(P_O_EW1>, N)
V2PTS(N, L, EW1, EW2)
VEW2X = DOT(V_EW2_E>, E1>)
VEW2Z = DOT(V_EW2_E>, E2>)

V_CM_N> = DT(P_O_CM>, N)
VOP1X = DOT(V_P1_N>, N1>)
VOP1Z = DOT(V_P1_N>, N2>)
VOP2X = DOT(V_P2_N>, N1>)
VOP2Z = DOT(V_P2_N>, N2>)
VOP3X = DOT(V_P3_N>, N1>)
VOP3Z = DOT(V_P3_N>, N2>)
VOP4X = DOT(V_P4_N>, N1>)
VOP4Z = DOT(V_P4_N>, N2>)
VOP5X = DOT(V_P5_N>, N1>)
VOP5Z = DOT(V_P5_N>, N2>)
VOP6X = DOT(V_P6_N>, N1>)
VOP6Z = DOT(V_P6_N>, N2>)
VOP7X = DOT(V_P7_N>, N1>)
VOP7Z = DOT(V_P7_N>, N2>)
VOP8X = DOT(V_P8_N>, N1>)
VOP8Z = DOT(V_P8_N>, N2>)
VOP9X = DOT(V_P9_N>, N1>)
VOP9Z = DOT(V_P9_N>, N2>)
VOP10X = DOT(V_P10_N>, N1>)
VOP10Z = DOT(V_P10_N>, N2>)
VOP11X = DOT(V_P11_N>, N1>)
VOP11Z = DOT(V_P11_N>, N2>)
VOP12X = DOT(V_P12_N>, N1>)
VOP12Z = DOT(V_P12_N>, N2>)
VOP13X = DOT(V_P13_N>, N1>)
VOP13Z = DOT(V_P13_N>, N2>)
VOP14X = DOT(V_P14_N>, N1>)
VOP14Z = DOT(V_P14_N>, N2>)
VOP15X = DOT(V_P15_N>, N1>)
VOP15Z = DOT(V_P15_N>, N2>)
VOSOX = DOT(V_SO_N>, N1>)
VOSOZ = DOT(V_SO_N>, N2>)
VOAGX = DOT(V_AO_N>, N1>)
VOAOZ = DOT(V_AO_N>, N2>)
VOBOX = DOT(V_BO_N>, N1>)
VOBOZ = DOT(V_BO_N>, N2>)
VOCOX = DOT(V_CO_N>, N1>)
VOCOZ = DOT(V_CO_N>, N2>)
VODOX = DOT(V_DO_N>, N1>)
VODOZ = DOT(V_DO_N>, N2>)
VOEOX = DOT(V_EO_N>, N1>)
VOEOZ = DOT(V_EO_N>, N2>)
VOFox = DOT(V_FO_N>, N1>)
VFOOZ = DOT(V_FO_N>, N2>
VOGOX = DOT(V_GO_N>, N1>)
VOGOZ = DOT(V_GO_N>, N2>)
VOHOX = DOT(V_HO_N>, N1>)
VOH0Z = DOT(V_HO_N>, N2>)
VOJOX = DOT(V_J0_N>, N1>)
VOJOZ = DOT(V_J0_N>, N2>)
VOKOX = DOT(V_KO_N>, N1>)
VOKOZ = DOT(V_KO_N>, N2>)
VOLOX = DOT(V_LO_N>, N1>)
VOLOZ = DOT(V_LO_N>, N2>)
VP4P7X = DOT(V_P7_S>, S1>)
VP4P7Z = DOT(V_P7_S>, S2>)
VP5P8X = DOT(V_P8_S>, S1>)
VP5P8Z = DOT(V_P8_S>, S2>)

VOCMX = DOT(V_CM_N>, N1>)
VOCMZ = DOT(V_CM_N>, N2>)

% LINEAR ACCELERATION

% LINEAR ACCELERATION

A0_N> = 0>
A_P1_N> = DT(V_P1_N>, N)
A_P2_N> = DT(V_P2_N>, N)
A_P3_N> = DT(V_P3_N>, N)
A_P4_N> = DT(V_P4_N>, N)
A_P5_N> = DT(V_P5_N>, N)
A_P6_N> = DT(V_P6_N>, N)
A_P7_N> = DT(V_P7_N>, N)
A_P8_N> = DT(V_P8_N>, N)
A_P9_N> = DT(V_P9_N>, N)
A_P10_N> = DT(V_P10_N>, N)
A_P11_N> = DT(V_P11_N>, N)
A_P12_N> = DT(V_P12_N>, N)
A_P13_N> = DT(V_P13_N>, N)
A_P14_N> = DT(V_P14_N>, N)
A_P15_N> = DT(V_P15_N>, N)
A_S0_N> = DT(V_S0_N>, N)
A_A0_N> = DT(V_A0_N>, N)
A_BO_N> = DT(V_BO_N>, N)
A_CO_N> = DT(V_CO_N>, N)
A_DO_N> = DT(V_DO_N>, N)
A_E0_N> = DT(V_E0_N>, N)
A_F0_N> = DT(V_F0_N>, N)
A_G0_N> = DT(V_G0_N>, N)
A_H0_N> = DT(V_H0_N>, N)
A_CM_N> = DT(V_CM_N>, N)
A_CW1_C> = DT(V_CW1_C>, C)
A_CW2_C> = DT(V_CW2_C>, C)
A_DW1_D> = DT(V_DW1_D>, D)
A_DW2_D> = DT(V_DW2_D>, D)
A_EW1_E> = DT(V_EW1_E>, E)
A_EW2_E> = DT(V_EW2_E>, E)
A_JO_C> = DT(V_JO_C>, C)
A_K0_D> = DT(V_K0_D>, D)
A_LO_E> = DT(V_LO_E>, E)
A_JO_N> = DT(V_JO_N>, N)
A_KO_N> = DT(V_KO_N>, N)
A_LO_N> = DT(V_LO_N>, N)

% WOBBLING MASS WRT SEGMENT

AOCMX = DOT(A_CM_N>, N1>)
Appendix 1a

\[ \text{AOCMZ} = \text{DOT}(A_{CM\ N}, N2) \]

%---------------------------------------------------------------------
% AUXILIARY CONSTRAIN

AUXILIARY[1] = U1
AUXILIARY[2] = U15
AUXILIARY[3] = U16
AUXILIARY[4] = U17
AUXILIARY[5] = U18
AUXILIARY[6] = U19
AUXILIARY[7] = U20
AUXILIARY[8] = U21
AUXILIARY[9] = U22
CONSTRAIN (AUXILIARY[U1,U15,U16,U17,U18,U19,U20,U21,U22])

%---------------------------------------------------------------------
% ENERGY

KES = KE(S) \hspace{0.5cm} % KINETIC ENERGY
KEA = KE(A)
KEB = KE(B)
KEC = KE(C)
KED = KE(D)
KEE = KE(E)
KEF = KE(F)
KEG = KE(G)
KEH = KE(H)
KEJ = KE(J)
KEK = KE(K)
KEL = KE(L)
KET = KE(S,A,B,C,D,E,F,G,H,J,K,L)

PES = -MS*G*POSOZ
PEA = -MA*G*POA0Z
PEB = -MB*G*POBOZ
PEC = -MC*G*POCOZ
PED = -MD*G*PODOZ
PEE = -ME*G*POE0Z
PEF = -MF*G*POFOZ
PEG = -MG*G*POGOZ
PEH = -MH*G*POHOZ
PEJ = -MJ*G*POJOZ
PEK = -MK*G*POKOZ
PEL = -ML*G*POLOZ
PECM = -M*G*POCMZ

PEQ2 = 0.5*K*Q2^2 \hspace{0.5cm} % SPRING POTENTIAL ENERGY
PEQ3 = 0.5*K3*Q3^2
PEQ4 = 0.5*K4*Q4^2
PEBX = 0.5*K5*PP4P7X^2
PEBZ = 0.5*K6*PP4P7Z^2
PEHX = 0.5*K7*PP5P8X^2
PEHZ = 0.5*K8*PP5P8Z^2
PEC1 = 0.5*KC1*Q5^2
PEC2 = 0.5*KC2*Q6^2
PEC3 = 0.5*KC3*PCW2X^2
PEC4 = 0.5*KC4*PCW2Z^2
PED1 = 0.5*KD1*Q8^2
PED2 = 0.5*KD2*Q9^2
Appendix 1a

PED3 = 0.5*KD3*PDW2X^2
PED4 = 0.5*KD4*PDW2Z^2
PEE1 = 0.5*KE1*Q11^2
PEE2 = 0.5*KE2*Q12^2
PEE3 = 0.5*KE3*PEW2X^2
PEE4 = 0.5*KE4*PEW2Z^2
PESPR = PEQ2 + PEQ3 + PEQ4 + PEBX + FEBZ + PEHX + PEHZ + &
PCE1 + PCE2 + PCE3 + PCE4 + PED1 + PED2 + PED3 + PED4 + &
PEE1 + PEE2 + PEE3 + PEE4
PET = PES + PECM + PESPR

% ANGULAR AND LINEAR MOMENTUM

ANOM> = MOMENTUM(ANGULAR, CM) - MOMENTUM(ANGULAR, CM, S)
ANGMOM = DOT(ANOM>, N3>)
LMOM> = MOMENTUM(LINEAR) - MOMENTUM(LINEAR, S)
HORMOM = DOT(LMOM>, N1>)
VERMOM = DOT(LMOM>, N2>)

% FORCES

GRAVITY (G*N2>)

RZ = -K*Q2
FORCE(SO, RX*N1> + RZ*N2>)

R3 = -K3*Q3 - KK3*U3*ABS(Q3)
R4 = -K4*Q4 - KK4*U4*ABS(Q4)
FORCE(P3/P6, R3*S1> + R4*S2>)

R5 = -K5*PP4P7X - KK5*VP4P7X*ABS(PP4P7X)
R6 = -K6*PP4P7Z - KK6*VP4P7Z*ABS(PP4P7Z)
FORCE(P4/P7, R5*S1> + R6*S2>)

R7 = -K7*PP5P8X - KK7*VP5P8X*ABS(PP5P8X)
R8 = -K8*PP5P8Z - KK8*VP5P8Z*ABS(PP5P8Z)
FORCE(P5/P8, R7*S1> + R8*S2>)

FC1 = -KC1*Q5^2*(ABS(Q5)/Q5) - KKC1*U5*ABS(Q5) % WOBBLING MASS J
FC2 = -KC2*Q6^2*(ABS(Q6)/Q6) - KKC2*U6*ABS(Q6)
FORCE(P9/CW1, FC1*C1> + FC2*C2>)

FC3 = -KC3*PCW2X^2*(ABS(PCW2X)/PCW2X) - KKC3*VCW2X*ABS(PCW2X)
FC4 = -KC4*PCW2Z^2*(ABS(PCW2Z)/PCW2Z) - KKC4*VCW2Z*ABS(PCW2Z)
FORCE(P10/CW2, FC3*C1> + FC4*C2>)

FD1 = -KD1*Q8^2*(ABS(Q8)/Q8) - KKD1*U8*ABS(Q8) % WOBBLING MASS K
FD2 = -KD2*Q9^2*(ABS(Q9)/Q9) - KKD2*U9*ABS(Q9)
FORCE(P10/DW1, FD1*D1> + FD2*D2>)

FE1 = -KE1*Q11^2*(ABS(Q11)/Q11) - KKE1*U11*ABS(Q11) % WOBBLING MASS L
FE2 = -KE2*Q12^2*(ABS(Q12)/Q12) - KKE2*U12*ABS(Q12)
FORCE(P11/EW1, FE1*E1> + FE2*E2>)

FE3 = -KE3*PEW2X^2*(ABS(PEW2X)/PEW2X) - KKE3*VEW2X*ABS(PEW2X)
FE4 = -KE4*PEW2Z^2*(ABS(PEW2Z)/PEW2Z) - KKE4*VEW2Z*ABS(PEW2Z)
FORCE(P12/EW2, FE3*E1> + FE4*E2>)
Appendix 1a

TORQUE(B/A, TBAL*N3>)
TORQUE(C/B, TANK*N3>)
TORQUE(C/D, TKNE*N3>)
TORQUE(E/D, THIP*N3>)
TORQUE(F/E, TSHD*N3>)
TORQUE(F/G, TELB*N3>)
TORQUE(H/E, THEA*N3>)
TORQUE(N/S, TBRD*N3>)

% SET +VE AS EXTENSION

% EQUATIONS OF MOTION

ZERO = FR() + FRSTAR()

KANE (RX, TBAL, TANK, TKNE, THIP, TSHD, TELB, THEA, TBRD)

% INPUTS

INPUT TINITIAL=0.0, TFINAL=0.55, INTEGRSTP=0.001, PRINTINT=100
INPUT ABSERR=1.0E-08, RELERR=1.0E-07
INPUT G=-9.806, HORCON=-0.194, ANGCON=-28.599
INPUT THETA2=27, SD=0.00, SM=10000, SC=3946
INPUT M3=8.87, MA=5.0, MB=10.0, MD=10, ME=25, MF=5, MG=4, &
MH=2, MJ=2, MK=3, ML=5
INPUT Q2=0, U2=0, Q3=0, U3=0, Q4=0, U4=0, Q5=0, U5=0, Q6=0, U6=0, Q7=0, U7=0, &
Q8=0, U8=0, Q9=0, U9=0, Q10=0, U10=0, Q11=0, U11=0, Q12=0, U12=0, &
Q13=0, U13=0, Q14=70, U14=0
INPUT K3=400000, KK3=2000, K4=200000, KK4=2000, &
K5=200000, K6=100000, K7=20000, K8=100000, K9=2000
INPUT KC1=100000, KK3=10000, KC2=100000, KCC2=100000, KC3=100000, KC4=100000, &
KC5=100000, KC6=100000, KD1=100000, KKD1=100000, KD2=100000, KKD2=100000, &
KD3=100000, KD4=100000, KD5=100000, KE1=100000, KE2=100000, KE3=100000, KE4=100000, &
INPUT L1=0.1, L2=0.3, L3=0.2, L4=0.5, L5=0.1, L6=0.1, L7=0.1, L8=0.1, L9=0.2, &
L10=1, L11=1, L12=1, L13=1, L14=1, L15=2, L16=2, L17=1, L18=2, L19=1, L20=2
INPUT IS=0.5, IA=1, IB=1, IC=1, ID=1, IF=1, IG=1, IH=1, IJ=1, IK=1, IL=1

% OUTPUTS

OUTPUT T, POP1X, POP1Z, POP2X, POP2Z, POP3X, POP3Z, POP6X, POP6Z, POP7X, POP7Z, &
POP13X, POP13Z, POP14X, POP14Z, POCMX, POCMZ
OUTPUT T, VOP1X, VOP1Z, VOP2X, VOP2Z, VOP6X, VOP6Z, VOP7X, VOP7Z, VOP8X, VOP8Z, &
VOP9X, VOP9Z, VOP10X, VOP10Z, VOP11X, VOP11Z, VOP12X, VOP12Z, &
VOP13X, VOP13Z, VOP14X, VOP14Z, VOP15X, VOP15Z
OUTPUT T, POCLX, POCLZ, VOCM, VOPCMZ, AOCLM, AOCLMZ
OUTPUT T, TBAL, TANK, TKNE, THIP, TSHD, TELB, THEA
OUTPUT T, QBAL, QANK, QKNE, QHIP, QSHD, QELB, QHEA
OUTPUT T, QBAL', QANK', QKNE', QHIP', QSHD', QELB', QHEA'
OUTPUT T, QBAL'', QANK'', QKNE'', QHIP'', QSHD'', QELB'', QHEA''
OUTPUT T, QC, QCM, Q14, U14, U14'
OUTPUT T, X, X', X'', RX, TBRD
OUTPUT T, Q2, U2, U2', R2, K
OUTPUT T, Q3, U3, R3, Q4, U4, R4
OUTPUT T, PP47X, VP47X, R5, PP47Z, VP47Z, R6
OUTPUT T, PP58X, VP58X, R7, PP58Z, VP58Z, R8
OUTPUT T, Q5, U6, Q6, U7, PCW2X, VCW2X, PCW2Z, VCW2Z, FC1, FC2, FC3, FC4
OUTPUT T, Q8, U9, Q9, U10, PDW2X, VDW2X, PDW2Z, VDW2Z, FD1, FD2, FD3, FD4
OUTPUT T, Q11, U12, Q12, U13, PEW2X, VEW2X, PEW2Z, VEW2Z, FE1, FE2, FE3, FE4
OUTPUT T, KET, KECM, KES, KEA, KEB, KEC, KED, KEE, KEG, KEH, KEJ, KEL
OUTPUT T, PET, PECM, PES, PEA, PEB, PEC, PED, PEE, PEG, PEH, PEJ, PEK, PEL
% --------------------------------------------------------------------
% UNITS
UNITS [Q7, Q10, Q13, Q14] = DEG, [U7, U10, U13, U14] = RAD/S
UNITS [QBAL, QANK, QKNE, QHIP, QSHD, QELB, QHEA, THETA, THETA2, QCM] = DEG
UNITS [QBAL', QANK', QKNE', QHIP', QSHD', QELB', QHEA', THETA'] = RAD/S
UNITS [U7', U10', U13', U14', QBAL'', QANK'', QKNE'', QHIP''] = RAD/S^2
UNITS [QSHD'', QELB'', QHEA'', THETA''] = RAD/S^2
UNITS [IS, IA, IB, IC, ID, IE, IF, IG, IH, IJ, IK, IL] = KGM^2
UNITS [SC, K3, K4, K5, K6, K7, K8, KC1, KC2, KC3, KC4] = N/M
UNITS [KD1, KD2, KD3, KD4, KE1, KE2, KE3, KE4] = N/M
UNITS [KK3, KK4, KK5, KK6, KK7, KK8, KC1K, KC2K, KC3K, KC4K] = N/M
UNITS [KKD1, KKDK2, KKDK3, KKDK4, KKE1, KKE2, KKE3, KKE4K] = NS/M
UNITS [Q1, Q2, Q3, Q4, Q5, Q6, Q8, Q9, Q11, Q12, X, SD] = M
UNITS [POP1X, POP12, POP2X, POP2Z, POP3X, POP3Z, POP4X, POP4Z] = M
UNITS [PCW2X, PCW2Z, POCWX, POCWZX, PCW2WX, PCW2WZX] = M
UNITS [PDW2X, PDW2Z, PODWX, PODWZX, PODW2WX, PODW2WZX] = M
UNITS [PEW2X, PEW2Z, POEWX, POEWZX, POEW2WX, POEW2WZX] = M
UNITS [U1, U2, U3, U4, U5, U6, U7, U8, U9, U11, U12, *] = M/S
UNITS [VP4P7X, VP4P7Z, VP5P8X, VP5P8Z, VOCMX, VOCMZ] = M/S
UNITS [VCW2X, VCW2Z, VDW2X, VDW2Z, VEW2X, VEW2Z] = M/S
UNITS [RX, RZ, R3, R4, R5, R6, R7, R8] = N
UNITS [FC1, FC2, FC3, FC4, FD1, FD2, FD3, FD4, FE1, FE2, FE3, FE4] = N
UNITS [TBAL, TANK, TKNE, THIP, TSHD, TELB, THEA, TBDR] = NM
UNITS ANGMOM = KGM^2/S
UNITS [HORMOM, VERMOM] = KGM/S
UNITS [KET, KES, KECM, KEA, KEB, KEC, KED, KEE, KEJ, KELK] = J
UNITS [PET, PECM, PES, PEA, PEB, PEC, PED, PEE, PEG, PEH, PEJ, PEK] = J
UNITS [PESPR, PEQ2, PEQ3, PEQ4, PEBX, PEBZ, PEHX, PEBZ] = J
UNITS [PEC1, PEC2, PEC3, PEC4, PED1, PED2, PED3, PED4, PEE1, PEE2, PEE3, PEE4] = J
% --------------------------------------------------------------------
SAVE C:\\AL\\VENI\\DV8SEG. ALL

CODE DYNAMICS() C:\ALVENI\DV8SEG.FOR, SUBS
% --------------------------------------------------------------------
% END END END END END END END END END END END END END
Appendix lb

TQ8SEG. AL

% TORQUE DRIVEN 8-SEGMENT MODEL FOR SPRINGBOARD DIVING TAKEOFF
% - ELBOW AND HEAD ANGLE DRIVEN BY ANGLE
% - END OF BOARD HORIZONTAL MOVEMENT CONSTRAINED BY BOARD DEPRESSION
% - USE AUXILIARY TO CALCULATE HORIZONTAL SPRINGBOARD FORCE RX
% - BOARD ANGLE CONSTRAINED BY BOARD DEPRESSION
% - TRIANGULAR 2-SEGMENT FOOT
% - BOARD STIFFNESS CHANGES WITH PLACEMENT OF FOOT SD
% - THREE SPRING-DAMPERS ACTING ON THE FOOT
% - 3 WOBBLING MASSES (TRUNK, THIGH, SHANK)
% - NON-LINEAR SPRING EQUATIONS FOR WOBBLING MASS SPRINGS
% - ELIMINATE SPRINGBOARD FROM BODY IN P_O_CM> AND ANGMOM

% PHYSICAL DECLARATION

NEWTONIAN N
FRAMES T
BODIES S, A, B, C, D, E, F, G, H, J, K, L
POINTS O, P1, P2, P3, P4, P5, P6, P7, P8, P9, P10, P11, P12, P13, P14, P15, CW1, CW2, &
   DW1, DW2, EW1, EW2, CM

% MATHEMATICAL DECLARATION

INERTIA S, 0,0,IS
INERTIA A, 0,0,IA
INERTIA B, 0,0,IB
INERTIA C, 0,0,IC
INERTIA D, 0,0,ID
INERTIA E, 0,0,IE
INERTIA F, 0,0,IF
INERTIA G, 0,0,IG
INERTIA H, 0,0,IH
INERTIA J, 0,0,IJ
INERTIA K, 0,0,IK
INERTIA L, 0,0,IL

SPECIFIED X'', THETA'', TORBRD, & % SPRINGBOARD CONSTRAINTS
   RX, RZ, & % SPRINGBOARD REACTION FORCES
   R3, R4, R5, R6, R7, R8, & % FOOT SPRING FORCE
   FC(4), FD(4), FE(4), & % WOBBLING MASS FORCES
   QELB'', QHEA'', & % ANGLE DRIVEN ELBOW AND HEAD
   TORBAL, TORANK, TORSHE, TORSHIP, TORSHD, TORELB, TORHEA % TORQUE

CONSTANTS HORCON, ANGCON, SD, SM, SC, & % SPRINGBOARD CONSTANTS
   THETA2, & % TRIANGULAR FOOT ANGLE
   G, L{20},, & % GRAVITY, LENGTH
   K, K3, K4, K5, K6, K7, K8, KC{4}, KD{4}, KE{4}, & % STIFFNESS
   KK3, KK4, KK5, KK6, KK7, KK8, KKC{4}, KKD{4}, KKE{4} % DAMPING

VARIABLES Q{19}', U{22}' % DEGREE OF FREEDOM

VARIABLES POSOX, POSOZ, POAOX, POAOZ, POBOX, POBOZ, POCOX, POCOZ, PODOX, PODOZ, &
   POEOX, POEOZ, POFOX, POFOZ, POFOX, POFOZ, POHOX, POHOZ, POJOX, POJOZ, &
   POFOK, POFOZ, POLOX, POLOZ, POP1X, POP1Z, POP12, POP12X, POP12Z, &
   PP58X, PP58Z, PCN2X, PCN2Z, PCW1X, PCW1Z, PCW2X, PCW2Z, &
   PDW2X, PDW2Z, PDW1X, PDW1Z, PDW2X, PDW2Z, PEW2X, PEW2Z, &
   POEW1X, POEW1Z, POEW2X, POEW2Z

VARIABLES VOSOX, VOSOZ, VOAOX, VOAOZ, VOBOX, VOBOZ, VOCOX, VOCOZ, VODOX, VODOZ, &
Appendix 1b

VOEOX, VOEOZ, VOFOX, VOFOZ, VOFOX, VOFOZ, VOHOX, VOHOZ, VOJOX, VOJOZ, &
VOOKX, VOOKZ, VOOLF, VOOLF, VOP1X, VOP1Z, VOP3X, VOP3Z, &
VOP4X, VOP4Z, VOP5X, VOP5Z, VOP6X, VOP6Z, VOP7X, VOP7Z, VOP8X, VOP8Z, &
VOP9X, VOP9Z, VOP10X, VOP10Z, VOP11X, VOP11Z, VOP12X, VOP12Z, &
VOP5P8X, VOP5P8Z, VCW2X, VCW2Z, VDW2X, VDW2Z, VEW2X, VEW2Z

VARIABLES
POCMX, POCMZ, VOCMX, VOCMZ, AOCMX, AOCMZ, &
KET, KECM, KES, KEA, KEB, KEC, KED, KEE, KEF, KEH, KEJ, KEK, KEL, &
PET, PECM, PES, PEA, PEB, PEC, PED, PEE, PEF, PEG, PEH, PEJ, PEK, PEL, &
PEQ2, PEQ3, PEQ4, PEBX, PEHZ, PEC1, PEC2, PEC3, PEC4, &
PED1, PED2, PED3, PED4, PEE1, PEE2, PEE3, PEE4

VARIABLES ANGMOM, HORMOM, VERMOM, &
% MOMENTUM
M, MT
% CM MASSES

AUTOZ ON
ZEE_NOT = [RX, RZ, R3, R4, R5, R6, TORELB, TORHEA, TORBRD, THETA '', X, X', X'']
M = MA + MB + MC + MD + ME + MF + MG + MH + MJ + MK + ML
MT = MS + MA + MB + MC + MD + ME + MF + MG + MH + MJ + MK + ML
K = SM*(SD + 0.15) + SC
X = HORCON*Q2^2
% HORIZONTAL BOARD MOVEMENT
THETA = ANGCON*Q2
% BOARD ANGLE IN RADIANS
QELB = T^3
% OVER-WRITE IN FORTRAN
QHEA = T^3

% GEOMETRICAL RELATION

% BOARD/HORIZONTAL
SIMPROT(N, S, 3, THETA)

% TRUNK/HORIZONTAL
SIMPROT(N, E, 3, Q14)

% THIGH/KNEE
SIMPROT(D, E, 3, PI+Q18)

% KNEE/FOOT
SIMPROT(B, C, 3, Q16)

% FOOT/TOES
SIMPROT(A, B, 3, Q15)

% FIXED TRIANGULAR FOOT
SIMPROT(B, T, 3, Q17)

% TRUNK/UPPER ARM
SIMPROT(E, F, 3, Q19)

% UPPER/LOWER ARM
SIMPROT(G, F, 3, QELB)

% TRUNK/HEAD
SIMPROT(E, H, 3, QHEA)

% BALL SPRING
P_O_P1> = (X+Q1)*N1> + Q2*N2>
P_P1_SO> = -L1*S1>
P_P1_P2> = -L2*S1>
P_P1_P3> = -SD*S1>
P_P3_P4> = -(L4-L3)*A1>
P_P4_P5> = -L7*S1>
P_P3_P6> = Q3*S1> + Q4*S2>
P_P6_AO> = -(L4-L3)*A1>
P_P6_P7> = -L4*A1>
P_P7_BO> = -L5*T1> + L6*T2>
P_P7_P8> = -L7*T1>
P_P7_P9> = -L8*B1>
P_P4_P7> = -P_P3_P4> + P_P3_P6> + P_P6_P7>
P_P5_P8> = -P_P4_P5> + P_P4_P7> + P_P7_P8>
P_P9_CO> = (L10-L9)*C1>
P_P9_P10> = L10*C1>
P_P10_DO> = (L12-L11)*D1>
P_P10_P11> = L12*D1>
P_P11_EO> = L13*E1>

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\[ P_{P11\_P12} = L14*E1 > \]
\[ P_{P12\_F0} = L15*F1 > \]
\[ P_{P12\_P13} = L16*F1 > \]
\[ P_{P13\_G0} = L17*G1 > \]
\[ P_{P13\_P14} = L18*G1 > \]
\[ P_{P12\_H0} = L19*H1 > \]
\[ P_{P12\_P15} = L20*H1 > \]
\[ P_{O\_S0} = P_{O\_P1} > + P_{P1\_S0} > \]
\[ P_{O\_P2} = P_{O\_P1} > + P_{P1\_P2} > \]
\[ P_{O\_P3} = P_{O\_P1} > + P_{P1\_P3} > \]
\[ P_{O\_P4} = P_{O\_P3} > + P_{P3\_P4} > \]
\[ P_{O\_P5} = P_{O\_P4} > + P_{P4\_P5} > \]
\[ P_{O\_P6} = P_{O\_P3} > + P_{P3\_P6} > \]
\[ P_{O\_A0} = P_{O\_P6} > + P_{P6\_A0} > \]
\[ P_{O\_P7} = P_{O\_P6} > + P_{P6\_P7} > \]
\[ P_{O\_P8} = P_{O\_P7} > + P_{P7\_P8} > \]
\[ P_{O\_B0} = P_{O\_P7} > + P_{P7\_B0} > \]
\[ P_{O\_P9} = P_{O\_P7} > + P_{P7\_P9} > \]
\[ P_{O\_C0} = P_{O\_P9} > + P_{P9\_C0} > \]
\[ P_{O\_P10} = P_{O\_P9} > + P_{P9\_P10} > \]
\[ P_{O\_D0} = P_{O\_P10} > + P_{P10\_D0} > \]
\[ P_{O\_P11} = P_{O\_P10} > + P_{P10\_P11} > \]
\[ P_{O\_E0} = P_{O\_P11} > + P_{P11\_E0} > \]
\[ P_{O\_P12} = P_{O\_P11} > + P_{P11\_P12} > \]
\[ P_{O\_F0} = P_{O\_P12} > + P_{P12\_F0} > \]
\[ P_{O\_P13} = P_{O\_P12} > + P_{P12\_P13} > \]
\[ P_{O\_G0} = P_{O\_P13} > + P_{P13\_G0} > \]
\[ P_{O\_P14} = P_{O\_P13} > + P_{P13\_P14} > \]
\[ P_{O\_H0} = P_{O\_P12} > + P_{P12\_H0} > \]
\[ P_{O\_P15} = P_{O\_P12} > + P_{P12\_P15} > \]

SIMPROT(C,J,3,Q7)
\[ P_{P9\_CW1} = Q5*C1 > + Q6*C2 > \]
\[ P_{CW1\_JO} = (L10-L9)*J1 > \]
\[ P_{CW1\_CW2} = L10*J1 > \]
\[ P_{P10\_CW2} = -P_{P9\_P10} > + P_{P9\_CW1} > + P_{CW1\_CW2} > \]
\[ P_{O\_CW1} = P_{O\_P9} > + P_{P9\_CW1} > \]
\[ P_{O\_JO} = P_{O\_CW1} > + P_{CW1\_JO} > \]
\[ P_{O\_CW2} = P_{O\_CW1} > + P_{CW1\_CW2} > \]
\[ PCW2X = DOT(P_{P10\_CW2},C1) \]
\[ PCW2Z = DOT(P_{P10\_CW2},C2) \]
\[ POCW1X = DOT(P_{O\_CW1},N1) \]
\[ POCW1Z = DOT(P_{O\_CW1},N2) \]
\[ POJOX = DOT(P_{O\_JO},N1) \]
\[ POJOZ = DOT(P_{O\_JO},N2) \]
\[ POCW2X = DOT(P_{O\_CW2},N1) \]
\[ POCW2Z = DOT(P_{O\_CW2},N2) \]

SIMPROT(D,K,3,Q10)
\[ P_{P10\_DW1} = Q8*D1 > + Q9*D2 > \]
\[ P_{DW1\_KO} = (L12-L11)*K1 > \]
\[ P_{DW1\_DW2} = L12*K1 > \]
\[ P_{P11\_DW2} = -P_{P10\_P11} > + P_{P10\_DW1} > + P_{DW1\_DW2} > \]
\[ P_{O\_DW1} = P_{O\_P10} > + P_{P10\_DW1} > \]
\[ P_{O\_KO} = P_{O\_DW1} > + P_{DW1\_KO} > \]
\[ P_{O\_DW2} = P_{O\_DW1} > + P_{DW1\_DW2} > \]
\[ PDW2X = DOT(P_{P11\_DW2},D1) \]
\[ PDW2Z = DOT(P_{P11\_DW2},D2) \]
\[ PODW1X = DOT(P_{O\_DW1},N1) \]
\[ PODW1Z = DOT(P_{O\_DW1},N2) \]
\[ POKOX = DOT(P_{O\_KO},N1) \]

$WOBBLING MASS J IN C$

$WOBBLING MASS K IN D$

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POKOZ = DOT(P\_O\_KO>, N2>)
PODW2X = DOT(P\_O\_DW2>, N1>)
PODW2Z = DOT(P\_O\_DW2>, N2>)

SIMPROT(E, L, 3, Q13) % WOBBLING MASS L IN E
P\_P11\_EW1> = Q11*E1> + Q12*E2>
P\_EW1\_LO> = L13*L1>
P\_EW1\_EW2> = L14*L1>
P\_P12\_EW2> = -P\_P11\_P12> + P\_P11\_EW1> + P\_EW1\_EW2>
P\_O\_EW1> = P\_O\_P11> + P\_P11\_EW1>
P\_O\_LO> = P\_O\_EW1> + P\_EW1\_LO>
P\_O\_EW2> = P\_O\_EW1> + P\_EW1\_EW2>
PEW2X = DOT(P\_P12\_EW2>, E1>)
PEW2Z = DOT(P\_P12\_EW2>, E2>)
P\_O\_CM> = CM(O, A, B, C, D, E, F, G, H, J, K, L) % NOT INCLUDING SPRINGBOARD S

POP1X = DOT(P\_O\_P1>, N1>)
POP1Z = DOT(P\_O\_P1>, N2>)
POP2X = DOT(P\_O\_P2>, N1>)
POP2Z = DOT(P\_O\_P2>, N2>)
POP3X = DOT(P\_O\_P3>, N1>)
POP3Z = DOT(P\_O\_P3>, N2>)
POP4X = DOT(P\_O\_P4>, N1>)
POP4Z = DOT(P\_O\_P4>, N2>)
POP5X = DOT(P\_O\_P5>, N1>)
POP5Z = DOT(P\_O\_P5>, N2>)
POP6X = DOT(P\_O\_P6>, N1>)
POP6Z = DOT(P\_O\_P6>, N2>)
POP7X = DOT(P\_O\_P7>, N1>)
POP7Z = DOT(P\_O\_P7>, N2>)
POP8X = DOT(P\_O\_P8>, N1>)
POP8Z = DOT(P\_O\_P8>, N2>)
POP9X = DOT(P\_O\_P9>, N1>)
POP9Z = DOT(P\_O\_P9>, N2>)
POP10X = DOT(P\_O\_P10>, N1>)
POP10Z = DOT(P\_O\_P10>, N2>)
POP11X = DOT(P\_O\_P11>, N1>)
POP11Z = DOT(P\_O\_P11>, N2>)
POP12X = DOT(P\_O\_P12>, N1>)
POP12Z = DOT(P\_O\_P12>, N2>)
POP13X = DOT(P\_O\_P13>, N1>)
POP13Z = DOT(P\_O\_P13>, N2>)
POP14X = DOT(P\_O\_P14>, N1>)
POP14Z = DOT(P\_O\_P14>, N2>)
POP15X = DOT(P\_O\_P15>, N1>)
POP15Z = DOT(P\_O\_P15>, N2>)
POSOX = DOT(P\_O\_SO>, N1>)
POS0Z = DOT(P\_O\_SO>, N2>)
POAOX = DOT(P\_O\_AO>, N1>)
POAOZ = DOT(P\_O\_AO>, N2>)
POBOX = DOT(P\_O\_BO>, N1>)
POBO2 = DOT(P\_O\_BO>, N2>)
POCOX = DOT(P\_O\_CO>, N1>)
POCOZ = DOT(P\_O\_CO>, N2>)
PODOX = DOT(P_O_DO>, N1>)
PODOZ = DOT(P_O_DO>, N2>)
POEOX = DOT(P_O_EO>, N1>)
POEOZ = DOT(P_O_EO>, N2>)
POFOX = DOT(P_O_FO>, N1>)
POFOZ = DOT(P_O_FO>, N2>)
POGOX = DOT(P_O_GO>, N1>)
POGOZ = DOT(P_O_GO>, N2>)
POHOX = DOT(P_O_HO>, N1>)
POHOZ = DOT(P_O_HO>, N2>)
PP4P7X = DOT(P_P4_P7>, S1>)
PP4P7Z = DOT(P_P4_P7>, S2>)
PP5P8X = DOT(P_P5_P8>, S1>)
PP5P8Z = DOT(P_P5_P8>, S2>)
POCMX = DOT(P_O_CM>, N1>)
POCMZ = DOT(P_O_CM>, N2>)

% KINEMATICAL DIFFERENTIAL EQUATIONS

Q1' = U1
Q2' = U2
Q3' = U3
Q4' = U4
Q5' = U5
Q6' = U6
Q7' = U7
Q8' = U8
Q9' = U9
Q10' = U10
Q11' = U11
Q12' = U12
Q13' = U13
Q14' = U14
Q15' = U15
Q16' = U16
Q17' = U17
Q18' = U18
Q19' = U19
X' = DT(X)
X'' = DT(X')
THETA'' = DT(THETA')
THETA' = DT(THETA)
QELB' = DT(QELB)
QELB'' = DT(QELB')
QHEA' = DT(QHEA)
QHEA'' = DT(QHEA')

% ANGULAR VELOCITY AND ACCELERATION

W_S_N> = THETA'*S3> + U22*S3>  % USE GENERALISED SPEED TO CALCULATE TORQUE
W_E_N> = U14*E3>
W_E_D> = U18*E3>
W_C_D> = U17*C3>
W_C_B> = U16*C3>
W_B_A> = U15*B3>
W_F_E> = U19*F3>
W_F_G> = QELB'*F3> + U20*F3>  % GENERALISED SPEED U20

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\[ W_{H,E} = QHEA'*H3> + U21*H3> \quad \% \text{GENERALISED SPEED U21} \]
\[ W_{T,B} = 0 > \]
\[ W_{J,C} = U7*J3> \]
\[ W_{L,E} = U13*L3> \]
\[ ALF_{S,N} = \Theta ''*S3> \]
\[ ALF_{E,N} = U14'*E3> \]
\[ ALF_{E,D} = U18'*E3> \]
\[ ALF_{C,D} = U17'*C3> \]
\[ ALF_{C,B} = U16'*C3> \]
\[ ALF_{B,A} = U15'*B3> \]
\[ ALF_{F,E} = U19'*F3> \]
\[ ALF_{F,G} = QELB''*F3> \]
\[ ALF_{H,E} = QHEA''*H3> \]
\[ ALF_{T,B} = 0 > \]
\[ ALF_{J,C} = U7'*J3> \]
\[ ALF_{K,D} = U10'*K3> \]
\[ ALF_{L,E} = U13'*L3> \]

\%--------------------------------------------------------------------
\% LINEAR VELOCITY
\[ V_{O,N} = 0 > \]
\[ V_P1_N = DT(P_O_P1>, N) \]
\[ V2PTS(N,S,P1,S0) \]
\[ V2PTS(N,S,P1,P2) \]
\[ V2PTS(N,S,P1,P3) \]
\[ V2PTS(N,S,P1,P4) \]
\[ V2PTS(N,S,P1,P5) \]
\[ V_P6_N = DT(P_O_P6>, N) \]
\[ V2PTS(N,A,P6,AO) \]
\[ V2PTS(N,A,P6,P7) \]
\[ V2PTS(N,T,P7,P8) \]
\[ V2PTS(N,T,P7,BO) \]
\[ V2PTS(N,B,P7,P9) \]
\[ V2PTS(N,C,P9,CO) \]
\[ V2PTS(N,C,P9,P10) \]
\[ V2PTS(N,D,P10,DO) \]
\[ V2PTS(N,D,P10,P11) \]
\[ V2PTS(N,E,P11,EO) \]
\[ V2PTS(N,E,P11,P12) \]
\[ V2PTS(N,F,P12,FO) \]
\[ V2PTS(N,F,P12,F13) \]
\[ V2PTS(N,G,P13,GO) \]
\[ V2PTS(N,G,P13,F14) \]
\[ V2PTS(N,H,P12,HO) \]
\[ V2PTS(N,H,P12,P15) \]
\[ V_P7_S = DT(P_P4_P7>, S) \]
\[ V_P8_S = DT(P_P5_P8>, S) \]
\% FOOT SPRING
\[ V_CW1_C = DT(P_P9_CW1>, C) \]
\[ V2PTS(C,J,CW1,J0) \]
\[ V2PTS(C,J,CW1,CW2) \]
\[ V_CW1_N = DT(P_O_CW1>, N) \]
\[ V2PTS(N,J,CW1,J0) \]
\[ V2PTS(N,J,CW1,CW2) \]
\[ VCW2X = DOT(V_CW2_C>, C1>) \]
\[ VCW2Z = DOT(V_CW2_C>, C2>) \]
V_DW1_D> = DT(P_P10_DW1>, D)
V2PTS(D, K, DW1, KO)
V2PTS(D, K, DW1, DW2)
V_DW1_N> = DT(P_O_DW1>, N)
V2PTS(N, K, DW1, KO)
V2PTS(N, K, DW1, DW2)
VDW2X = DOT(V_DW2_D>, D1>)
VDW2Z = DOT(V_DW2_D>, D2>)

V_EW1_E> = DT(P_P11_EW1>, E)
V2PTS(E, L, EW1, LO)
V2PTS(E, L, EW1, EW2)
V_EW1_N> = DT(P_O_EW1>, N)
V2PTS(N, L, EW1, LO)
V2PTS(N, L, EW1, EW2)
VEW2X = DOT(V_EW2_E>, E1>)
VEW2Z = DOT(V_EW2_E>, E2>)

V_CM_N> = DT(P_O_CM>, N)

VOP1X = DOT(V_P1_N>, N1>)
VOP1Z = DOT(V_P1_N>, N2>)
VOP2X = DOT(V_P2_N>, N1>)
VOP2Z = DOT(V_P2_N>, N2>)
VOP3X = DOT(V_P3_N>, N1>)
VOP3Z = DOT(V_P3_N>, N2>)
VOP4X = DOT(V_P4_N>, N1>)
VOP4Z = DOT(V_P4_N>, N2>)
VOP5X = DOT(V_P5_N>, N1>)
VOP5Z = DOT(V_P5_N>, N2>)
VOP6X = DOT(V_P6_N>, N1>)
VOP6Z = DOT(V_P6_N>, N2>)
VOP7X = DOT(V_P7_N>, N1>)
VOP7Z = DOT(V_P7_N>, N2>)
VOP8X = DOT(V_P8_N>, N1>)
VOP8Z = DOT(V_P8_N>, N2>)
VOP9X = DOT(V_P9_N>, N1>)
VOP9Z = DOT(V_P9_N>, N2>)
VOP10X = DOT(V_P10_N>, N1>)
VOP10Z = DOT(V_P10_N>, N2>)
VOP11X = DOT(V_P11_N>, N1>)
VOP11Z = DOT(V_P11_N>, N2>)
VOP12X = DOT(V_P12_N>, N1>)
VOP12Z = DOT(V_P12_N>, N2>)
VOP13X = DOT(V_P13_N>, N1>)
VOP13Z = DOT(V_P13_N>, N2>)
VOP14X = DOT(V_P14_N>, N1>)
VOP14Z = DOT(V_P14_N>, N2>)
VOP15X = DOT(V_P15_N>, N1>)
VOP15Z = DOT(V_P15_N>, N2>)
VOSOX = DOT(V_SO_N>, N1>)
VOSOZ = DOT(V_SO_N>, N2>)
VQA0X = DOT(V_A0_N>, N1>)
VQA0Z = DOT(V_A0_N>, N2>)
VBOX = DOT(V_BO_N>, N1>)
VBOZ = DOT(V_BO_N>, N2>)
VOCOX = DOT(V_CO_N>, N1>)
VOCOZ = DOT(V_CO_N>, N2>)
VODOX = DOT(V_DO_N>, N1>)
VODOZ = DOT(V_DO_N>, N2>)
VOEOX = DOT(V_EO_N>, N1>)
Appendix 1b

:\section*{Appendix 1b}

\input{appendix1b}

\noindent% LINEAR ACCELERATION
\begin{verbatim}
A_0_N> = 0>
A_P1_N> = DT(V_P1_N>, N)
A_P2_N> = DT(V_P2_N>, N)
A_P3_N> = DT(V_P3_N>, N)
A_P4_N> = DT(V_P4_N>, N)
A_P5_N> = DT(V_P5_N>, N)
A_P6_N> = DT(V_P6_N>, N)
A_P7_N> = DT(V_P7_N>, N)
A_P8_N> = DT(V_P8_N>, N)
A_P9_N> = DT(V_P9_N>, N)
A_P10_N> = DT(V_P10_N>, N)
A_P11_N> = DT(V_P11_N>, N)
A_P12_N> = DT(V_P12_N>, N)
A_P13_N> = DT(V_P13_N>, N)
A_P14_N> = DT(V_P14_N>, N)
A_P15_N> = DT(V_P15_N>, N)
A_SO_N> = DT(V_SO_N>, N)
A_AO_N> = DT(V_AO_N>, N)
A_BO_N> = DT(V_BO_N>, N)
A_CO_N> = DT(V_CO_N>, N)
A_DO_N> = DT(V_DO_N>, N)
A_EO_N> = DT(V_EO_N>, N)
A FO_N> = DT(V_FO_N>, N)
A_GO_N> = DT(V_GO_N>, N)
A_HO_N> = DT(V_HO_N>, N)
A_CM_N> = DT(V_CM_N>, N)
A_CW1_C> = DT(V_CW1_C>, C)
A_CW2_C> = DT(V_CW2_C>, C)
A_DM1_D> = DT(V_DM1_D>, D)
A_DM2_D> = DT(V_DM2_D>, D)
A_EW1_E> = DT(V_EW1_E>, E)
A_EW2_E> = DT(V_EW2_E>, E)
A_JO_C> = DT(V_JO_C>, C)
A_KO_D> = DT(V_KO_D>, D)
A_LO_E> = DT(V_LO_E>, E)
A_JO_N> = DT(V_JO_N>, N)
A_KO_N> = DT(V_KO_N>, N)
\end{verbatim}

\noindent% WOBBLING MASS WRT SEGMENT
A_LO_N> = DT(V_LO_N>, N)
AOCMX = DOT(A_CM_N>, N1>)
AOCMZ = DOT(A_CM_N>, N2>)

% --------------------------------------------------------------------
% AUXILIARY CONSTRAIN

AUXILIARY[1] = U1
AUXILIARY[2] = U20
AUXILIARY[3] = U21
AUXILIARY[4] = U22
CONSTRAIN (AUXILIARY[U1, U20, U21, U22])

% --------------------------------------------------------------------
% ENERGY

KES = KE(S) % KINETIC ENERGY
KEA = KE(A)
KEB = KE(B)
KEC = KE(C)
KED = KE(D)
KEE = KE(E)
KEF = KE(F)
KEG = KE(G)
KEH = KE(H)
KEJ = KE(J)
KEK = KE(K)
KEL = KE(L)
KET = KE(S, A, B, C, D, E, F, G, H, J, K, L)

PES = -MS*G*POSOZ
PEA = -MA*G*POAOZ
PEB = -MB*G*POBOZ
PEC = -MC*G*POCOZ
PED = -MD*G*PODOZ
PEE = -ME*G*POEOZ
PEF = -MF*G*POFOZ
PEG = -MG*G*POGOZ
PEH = -MH*G*POHOZ
PEJ = -MJ*G*POJOZ
PEK = -MK*G*POKOZ
PEL = -ML*G*POLOZ
PECM = -M*G*POCMZ

PEQ2 = 0.5*K*Q2^2
PEQ3 = 0.5*K3*Q3^2
PEQ4 = 0.5*K4*Q4^2
PEBX = 0.5*K5*PP4P7X^2
PEBZ = 0.5*K6*PP4P7Z^2
PEHX = 0.5*K7*PP5P8X^2
PEHZ = 0.5*K8*PP5P8Z^2
PEC1 = 0.5*KC1*Q5^2
PEC2 = 0.5*KC2*Q6^2
PEC3 = 0.5*KC3*PCW2X^2
PEC4 = 0.5*KC4*PCW2Z^2
PED1 = 0.5*KD1*Q8^2
PED2 = 0.5*KD2*Q9^2
PED3 = 0.5*KD3*PDW2X^2
PED4 = 0.5*KD4*PDW2Z^2
PEE1 = 0.5*KE1*Q11^2
PEE2 = 0.5*KE2*Q12^2
PEE3 = 0.5*KE3*PEW2X^2
PEE4 = 0.5*KE4*PEW2Z^2
PESPR = PEQ2 + PEQ3 + PEQ4 + PEBX + PEBZ + PEHX + PEHZ + &
PEC1 + PEC2 + PEC3 + PEC4 + PED1 + PED2 + PED3 + PED4 + &
PEE1 + PEE2 + PEE3 + PEE4
PET = PES + PECM + PESPR

% -------------------------------------------------------------
% ANGULAR AND LINEAR MOMENTUM

AMOM> = MOMENTUM(ANGULAR, CM) - MOMENTUM(ANGULAR, CM, S)
ANGMOM = DOT(AMOM>, N3>)

LMOM> = MOMENTUM(LINEAR) - MOMENTUM(LINEAR, S)
HORMOM = DOT(LMOM>, N1>)
VERMOM = DOT(LMOM>, N2>)

% -------------------------------------------------------------
% FORCES

GRAVITY (G*N2>)
RZ = -K*Q2
FORCE(SO, RX*N1> + RZ*N2>) % SPRINGBOARD REACTION FORCE

R3 = ABS(Q4)*(-K3*Q3 - KK3*U3*ABS(Q3))
DAMPERS
R4 = -K4*Q4 - KK4*U4*ABS(Q4)
FORCE(P3/P6, R3*S1> + R4*S2>)

R5 = ABS(PP4P7Z)*(-K5*PP4P7X - KK5*VP4P7X*ABS(PP4P7X))
% BALL SPRING DAMPERS
R6 = -K6*PP4P7Z - KK6*VP4P7Z*ABS(PP4P7Z)
FORCE(P4/P7, R5*S1> + R6*S2>)

R7 = ABS(PP5P8Z)*(-K7*PP5P8X - KK7*VP5P8X*ABS(PP5P8X))
% ANKLE SPRING DAMPERS
R8 = -K8*PP5P8Z - KK8*VP5P8Z*ABS(PP5P8Z)
FORCE(P5/P8, R7*S1> + R8*S2>)

FC1 = -KC1*Q5^3 - KKC1*U5
FC2 = -KC2*Q6^3 - KKC2*U6
FORCE(P9/CW1, FC1*C1> + FC2*C2>)

FC3 = -KC3*PCW2X^3 - KKC3*VCW2X
FC4 = -KC4*PCW2Z^3 - KKC4*VCW2Z
FORCE(P10/CW2, FC3*C1> + FC4*C2>)

FD1 = -KD1*Q8^3 - KKD1*U8
FD2 = -KD2*Q9^3 - KKD2*U9
FORCE(P10/DW1, FD1*D1> + FD2*D2>)

FD3 = -KD3*PDW2X^3 - KKD3*VDW2X
FD4 = -KD4*PDW2Z^3 - KKD4*VDW2Z
FORCE(P11/DW2, FD3*D1> + FD4*D2>)

FE1 = -KE1*Q11^3 - KKE1*U11
FE2 = -KE2*Q12^3 - KKE2*U12
FORCE(P11/EW1, FE1*E1> + FE2*E2>)

FE3 = -KE3*PEW2X^3 - KKE3*VEW2X
FE4 = -KE4*PEW2Z^3 - KKE4*VEW2Z

% WOBBLING MASS J
% WOBBLING MASS K
% WOBBLING MASS L
FORCE\((P12/EW2, \ FE3*E1> + FE4*E2>)\)

\[
\begin{align*}
\text{TORBAL} & = T^3 & \% \text{ CALL TORQUE SUBROUTINE TO OVER-WRITE IN FORTRAN} \\
\text{TORANK} & = T^3 \\
\text{TORKNE} & = T^3 \\
\text{TORHIP} & = T^3 \\
\text{TORSHD} & = T^3 \\
\text{TORQUE(A/B, TORBAL*N3>)} & \% \ -VE \ AS \ EXTENSION \ HERE \\
\text{TORQUE(B/C, TORANK*N3>)} & \% \ +VE \ EXTENSION \ IN \ TORQUE \ MODEL \\
\text{TORQUE(D/C, TORKNE*N3>)} \\
\text{TORQUE(D/E, TORHIP*N3>)} \\
\text{TORQUE(E/F, TORSHD*N3>)} \\
\text{TORQUE(G/F, TORELB*N3>)} \\
\text{TORQUE(S/N, TORELD*N3>)}
\end{align*}
\]

% --------------------------------------------------------------------
% EQUATIONS OF MOTION

\[
\begin{align*}
\text{ZERO} & = FR() + FRSTAR() \\
\text{KANE} & (RX, TORELB, TORHEA, TORELD)
\end{align*}
\]

% --------------------------------------------------------------------
% INPUTS

\[
\begin{align*}
\text{INPUT TINITIAL} & = 0.0, \ TFINAL = 0.55, \ \text{INTEGRSTP} = 0.001, \ \text{PRINTINT} = 100 \\
\text{INPUT ABSERR} & = 1.0E-08, \ RELERR = 1.0E-07 \\
\text{INPUT G} & = -9.806, \ \text{HORCON} = -0.194, \ \text{ANGCON} = -28.599 \\
\text{INPUT theta2} & = 22.17, \ \text{SD} = 0.00, \ \text{SM} = 8526, \ \text{SC} = 3130 \\
\text{INPUT MS} & = 8.87, \ \text{MA} = 5.0, \ \text{MB} = 10.0, \ \text{MC} = 10, \ \text{MD} = 10, \ \text{MF} = 5, \ \text{MG} = 4, \\
\text{MJ} & = 2, \ \text{MK} = 3, \ \text{ML} = 5 \\
\text{INPUT Q2} & = 0, \ \text{U2} = 0, \ \text{Q3} = 0, \ \text{U3} = 0, \ \text{Q4} = 0, \ \text{U4} = 0, \ \text{Q5} = 0, \ \text{U5} = 0, \ \text{Q6} = 0, \ \text{U6} = 0, \ \text{Q7} = 0, \ \text{U7} = 0, \\
\text{Q8} & = 0, \ \text{U8} = 0, \ \text{Q9} = 0, \ \text{U9} = 0, \ \text{Q10} = 0, \ \text{U10} = 0, \ \text{Q11} = 0, \ \text{U11} = 0, \ \text{Q12} = 0, \ \text{U12} = 0, \\
\text{Q13} & = 0, \ \text{U13} = 0, \ \text{Q14} = 70, \ \text{U14} = 0 \\
\text{INPUT K3} & = 138333332, \ \text{KK3} = 9049775, \ \text{K4} = 954615, \ \text{KK4} = 91075, \\
\text{K5} & = 11272129, \ \text{KK5} = 9109247, \ \text{K6} = 963527, \ \text{KK6} = 93112, \\
\text{K7} & = 12020908, \ \text{KK7} = 10028846, \ \text{K8} = 798581, \ \text{KK8} = 100551 \\
\text{INPUT KCl} & = 1461677206, \ \text{KC2} = 1461677206, \ \text{KC3} = 1461677206, \ \text{KC4} = 1461677206, \\
\text{KKC1} & = 136, \ \text{KKC2} = 136, \ \text{KKC3} = 136, \ \text{KKC4} = 136, \\
\text{KDD1} & = 753104877, \ \text{KDD2} = 753104877, \ \text{KDD3} = 753104877, \ \text{KDD4} = 753104877, \\
\text{KKD1} & = 119, \ \text{KKD2} = 119, \ \text{KKD3} = 119, \ \text{KKD4} = 119, \\
\text{KE1} & = 103073974, \ \text{KE2} = 103073974, \ \text{KE3} = 103073974, \ \text{KE4} = 103073974, \\
\text{KKE1} & = 73, \ \text{KKE2} = 73, \ \text{KKE3} = 73, \ \text{KKE4} = 73 \\
\text{INPUT L1} & = 0.1, \ \text{L2} = 0.3, \ \text{L3} = 0.2, \ \text{L4} = 0.5, \ \text{L5} = 0.1, \ \text{L6} = 0.1, \ \text{L7} = 0.1, \ \text{L8} = 0.1, \ \text{L9} = 0.2, \\
\text{L10} & = 1, \ \text{L11} = 1, \ \text{L12} = 1, \ \text{L13} = 1, \ \text{L14} = 1, \ \text{L15} = 2, \ \text{L16} = 2, \ \text{L17} = 1, \ \text{L18} = 2, \ \text{L19} = 1, \ \text{L20} = 2 \\
\text{INPUT IS} & = 0.5, \ \text{IA} = 1, \ \text{IB} = 1, \ \text{IC} = 1, \ \text{ID} = 1, \ \text{IF} = 1, \ \text{IG} = 1, \ \text{IH} = 1, \ \text{IJ} = 1, \ \text{IK} = 1, \ \text{IL} = 1
\end{align*}
\]

% --------------------------------------------------------------------
% OUTPUTS

\[
\begin{align*}
\text{OUTPUT T, \ POP1X, \ POP1Z, \ POP2X, \ POP2Z, \ POP3X, \ POP3Z, \ POP6X, \ POP6Z, \ POP7X, \ POP7Z, \\
& \ POP8X, \ POP8Z, \ POP9X, \ POP9Z, \ POP7X, \ POP7Z, \ POP9X, \ POP9Z, \ POP10X, \ POP10Z, \\
& \ POP11X, \ POP11Z, \ POP12X, \ POP12Z, \ POP15X, \ POP15Z, \ POP12X, \ POP12Z, \\
& \ POP13X, \ POP13Z, \ POP14X, \ POP14Z, \ FOCMX, \ FOCMZ \}
\end{align*}
\]

\[
\begin{align*}
\text{OUTPUT T, \ VOP1X, \ VOP1Z, \ VOP2X, \ VOP2Z, \ VOP6X, \ VOP6Z, \ VOP7X, \ VOP7Z, \ VOP8X, \ VOP8Z, \\
& \ VOP9X, \ VOP9Z, \ VOP10X, \ VOP10Z, \ VOP11X, \ VOP11Z, \ VOP12X, \ VOP12Z, \\
& \ VOP13X, \ VOP13Z, \ VOP14X, \ VOP14Z, \ VOP15X, \ VOP15Z \\
\end{align*}
\]

\[
\begin{align*}
\text{OUTPUT T, \ FOCMX, \ FOCMZ, \ VOCMX, \ VCMZ, \ AOCMX, \ AOCMZ} \\
\text{OUTPUT T, \ TORBAL, \ TORANK, \ TORKNE, \ TORHIP, \ TORSHD, \ TORELB, \ TORHEA}
\end{align*}
\]
OUTPUT T, Q15, Q16, Q17, Q18, Q19, QELB, QHEA
OUTPUT T, U15, U16, U17, U18, U19, QELB', QHEA'
OUTPUT T, U15', U16', U17', U18', U19', QELB '', QHEA ''
OUTPUT T, Q14, U14, U14'
OUTPUT T, X, X', X '', RX, TORBRD
OUTPUT T, Q2, U2, U2', RZ, K
OUTPUT T, Q3, U3, R3, Q4, U4, R4
OUTPUT T, PP5P8X, VP5P8X, R7, PP5P8Z, VP5P8Z, R8
OUTPUT T, Q5, Q6, Q7, U7, PCW2X, VCW2X, PVCM2, FC1, FC2, FC3, FC4
OUTPUT T, Q8, U8, Q9, U9, Q10, U10, DOW2X, DOW2X, DOW2Z, D1, D2, D3, D4
OUTPUT T, Q11, U11, U12, U12, Q13, U13, PEW2X, PEW2X, PEW2Z, PEW2Z, FE1, FE2, FE3, FE4

% --------------------------------------------------------------------
% UNITS

UNITS [Q7, Q10, Q13, Q14, Q15, Q16, Q17, Q18, Q19, QELB, QHEA]=DEG
UNITS [U7, U10, U13, U14, U15, U16, U17, U18, U19, QELB', QHEA ']=RAD/S
UNITS

% --------------------------------------------------------------------
SAVE C:\\AL\\VENI\\TQ8SEG. ALL
CODE DYNAMICS() C:\AL\VENI\TQ8SEG.FOR, SUBS

$-----------------------------------------------$
$ END END END END END END END END END END END END$
$-----------------------------------------------$
APPENDIX 2

INFORMED CONSENT AND SUBJECT PROFILE

Appendix 2a. Informed consent for kinematic data collection

Appendix 2b. Subject profile of training and competition background

Appendix 2c. Informed consent for strength measurement
INFORMED CONSENT

School of Sport and Exercise Science, Loughborough University

RESEARCH TOPIC  Mechanics of the takeoff in springboard diving

PURPOSE  To obtain video data of a diver performing dives in the forward and reverse group from a 1m springboard.

PROCEDURES  Performance of dives from the 1m springboard will be recorded by a video camera and the recordings will form the basis for further analysis.

Anthropometric measurements of the diver will be taken.

QUESTIONS  The researcher will be pleased to answer any questions which you may wish to ask.

WITHDRAWAL  You are free to withdraw from the study at any time for whatever reason without prejudice.

CONFIDENTIALITY  Your identity will remain confidential in any material resulting from this work.

I have read and understood the information on this form and agree to participate in this study. As far as I am aware I do not have any injury nor infirmity which would be affected by the procedures outlined.

Name: ............................. Signed: ........................ Date: .....................

Parent’s name (for under 18 years old): ....................................................

Signed: .............................  Date: .................................
SUBJECT PROFILE

PERSONAL INFORMATION

Name:                                                    Coach:
Date of Birth:                                          Sex:         M / F
Contact numbers (Hm):
(Mobile):
Email address:

DIVING EXPERIENCE

How many years have you been diving?

What level of competition are you currently at?

Diving Achievements:
(eg. Major competitions participated, ranking, scores, …… etc.)

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
What fulcrum number do you normally use for dives in the *forward* group?

(If you are using a different fulcrum number for any dive in this group, please specify: ..........................................................)

What fulcrum number do you normally use for dives in the *reverse* group?

(If you are using a different fulcrum number for any dive in this group, please specify: ..........................................................)

What factors would you recognise as the key of a good springboard takeoff?

How would you describe your springboard takeoff technique?

Is there any particular aspect of technique regarding the springboard takeoff that you would like to improve on?
INFORMED CONSENT

School of Sport and Exercise Science, Loughborough University

RESEARCH TOPIC  Mechanics of the takeoff in springboard diving

PURPOSE  To obtain subject specific strength parameters

PROCEDURES  Perform maximum isometric contractions at the shoulder, hip, knee and ankle joints on an isokinetic dynamometer (Cybex) and a custom built rig attached to the force plate in the biomechanics laboratory.

QUESTIONS  The researcher will be pleased to answer any questions which you may wish to ask.

WITHDRAWAL  You are free to withdraw from the study at any time for whatever reason without prejudice.

CONFIDENTIALITY  Your identity will remain confidential in any material resulting from this work.

I have read and understood the information on this form and agree to participate in this study. As far as I am aware I do not have any injury nor infirmity which would be affected by the procedures outlined.

Name: .......................... Signed: ........................ Date: .....................

Parent’s name (for under 18 years old): ..................................................

Signed: ............................ Date: ...........................