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Metadata Record: https://dspace.lboro.ac.uk/2134/9608

Version: Accepted for publication

Publisher: © EURONOISE

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EXCESSIVE GROUND VIBRATIONS ASSOCIATED WITH TRAINS
TRAVELLING AT TRANS-RAYLEIGH SPEEDS

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Key words: Ground vibrations, High-speed railways, Ground vibration boom.

Abstract: A very large increase in ground vibrations generated by high-speed trains (ground vibration boom) has been predicted theoretically in 1994 by the present author. The first experimental observation of this phenomenon was reported in 1997 by C. Madshus who worked with his team on the assessment of the newly opened high-speed railway line from Gothenburg to Malmo in Sweden. As the ground on the site of observation was very soft, the ground vibration boom could be observed for train speeds as low as 160 km/h. It is now well understood that excessive vibrations associated with ground vibration boom represent a serious hazard for the built environment, especially in the cases where high-speed lines are built on very soft soil. The present paper reviews the current status of the theory of ground vibration boom from high-speed trains. Among the problems to be discussed are contributions of different generation mechanisms, effect of track wave resonances on generated ground vibrations, effects of layered geological structure of the ground, waveguide effects of the embankments, and focusing of generated waves due to the track curvature. The results of theoretical calculations are compared with the existing experimental observations.
1. INTRODUCTION

Railway-generated ground vibrations can cause significant disturbance for residents of nearby buildings even when generated by conventional passenger or heavy-freight trains. If train speeds increase, the intensity of railway-generated vibrations generally becomes larger. For modern high-speed trains the increase in ground vibration intensity is especially high when train speeds approach certain critical velocities of waves propagating in a track-ground system. The most important are two such critical velocities: the lowest velocity of Rayleigh surface wave in the ground and the minimal phase velocity of bending waves propagating in a track supported by ballast, the latter velocity being referred to as track critical velocity. Both these velocities can be easily exceeded by modern high-speed trains, especially in the case of very soft soil where both these velocities become very low.

As has been theoretically predicted by the present author [1, 2], if a train speed \( v \) exceeds the Rayleigh wave velocity \( c_R \) in supporting soil, a ground vibration boom occurs which is associated with a very large increase in generated ground vibrations, as compared to the case of conventional trains. This phenomenon is similar to a sonic boom for aircraft crossing the sound barrier, and its existence has been recently confirmed experimentally [3, 4]. The measurements have been carried out on behalf of the Swedish Railway Authorities (Banverket) when their West Coast Line from Gothenburg to Malmö was opened for the X2000 high-speed train. In particular, at the location near Ledsgärd the Rayleigh wave velocity in the ground was only 45 m/s, so the increase in train speed from 140 to 180 km/h lead to about 10 times increase in generated ground vibrations [3].

The above mentioned first observations of ground vibration boom indicate that “supersonic” or (more precisely) “trans-Rayleigh” trains [5-7] have become a reality. The increased attention of railway companies and local authorities to ground vibrations associated with high-speed trains stimulates a growing number of theoretical and experimental investigations in this area (see, e.g. [8-11]).

If train speeds increase further and approach the track critical velocity, then rail deflections due to applied wheel loads become especially large. Possible very large rail deflections at this speed may result even in train derailment, thus representing a serious problem for train and passenger safety [4, 12-14]. From the point of view of generated ground vibrations, these large rail deflections can be responsible for an additional growth of ground vibration amplitudes, as compared to the above mentioned case of ground vibration boom [5, 7,15].

For underground trains, the first critical velocity is the velocity of shear elastic waves, instead of the velocity of Rayleigh waves which are almost not generated in this case. Therefore, one can expect a large increase in the intensity of generated shear waves if train speeds exceed this critical velocity.

In the present paper we review the current status of the theory of ground vibration boom from high-speed trains. Among the problems to be discussed are the quasi-static pressure generation mechanism and the roles of other mechanisms, effects of Rayleigh wave velocity and track wave resonances, effects of layered geological structure of the ground, waveguide
effects of the embankments, and focusing of generated waves due to the track curvature. The results of theoretical calculations for TGV and Eurostar high-speed trains travelling along typical tracks are discussed and compared with the existing experimental observations.

2. OUTLINE OF THE THEORY

2.1 Mechanisms of generating ground vibrations

One can distinguish several mechanisms of railway-generated ground vibrations that may contribute to the total ground vibration level in different frequency bands. Among these mechanisms are the wheel-axle pressure, the effects of joints in unwelded rails, the unevenness of wheels or rails, and the dynamically induced forces of carriage- and wheel-axle vibrations excited mainly by unevenness of wheels and rails. In this paper we consider only the most common generation mechanism which is present even for ideally flat rails and wheels - the quasi-static pressure of wheel axles onto the track. As will be demonstrated below, this mechanism is also responsible for railway-generated ground vibration boom.

2.2 Track dynamic properties

An essential aspect of analysing the above mentioned wheel-axle pressure generation mechanism is calculation of the track deflection curve as function of the applied axle load and train speed. One can treat each rail as an Euler-Bernoulli elastic beam of uniform mass $m_0$ lying on a visco-elastic half space $z > 0$ and use the following dynamic equation to describe its vertical deflections (see, e.g. [16]):

$$EI \frac{\partial^4 w}{\partial x^4} + m_0 \frac{\partial^2 w}{\partial t^2} + 2m_0 \omega_b \frac{\partial w}{\partial t} + \alpha w = T \delta(x-vt).$$

(1)

Here $w$ is the beam deflection magnitude, $E$ and $I$ are Young's modulus and the cross-sectional momentum of the beam, $\omega_b$ is a circular frequency of damping, $\alpha$ is the proportionality coefficient of the equivalent Winkler elastic foundation, $x$ is the distance along the beam, $T$ is a wheel-axle load considered as a vertical point force, $v$ is its speed, and $\delta(x)$ is the Dirac’s delta-function. It is useful first to discuss free wave propagation in the supported beam without damping, i.e., to analyse eqn (1) with $T = 0$ and $\omega_b = 0$. In this case, the substitution of the solution in the form of harmonic bending waves

$$w = A \exp(ikx - i\omega t)$$

(2)

into (1) gives the following dispersion equation for track waves propagating in the system:

$$\omega = (\alpha + EIk^4)^{1/2}/m_0^{1/2}.$$

(3)
Here $k$ is the wavenumber of track waves, and $\omega$ is circular frequency. In the quasi-static (long-wave) approximation ($k = 0$) the dispersion equation (3) reduces to the well-known expression for the so-called track on ballast resonance frequency: $\omega_{tb} = \alpha^{1/2}/m_0^{1/2}$. For example, for typical parameters $\alpha = 52.6 \text{ MN/m}^2$ [17], and $m_0 = 300 \text{ kg/m}$ this gives $F_{tb} = \omega_{tb}/2\pi = 67 \text{ Hz}$. The frequency $F_{tb}$ represents the minimal frequency of propagating track waves. It also follows from eqn (3) that the frequency-dependent velocity of track wave propagation $c = \omega/k$ is determined by the expression

$$c = (\alpha/k^2 + EI k^2/m_0^{1/2})^{1/2},$$

(4)

which shows that at $k = (\alpha/EI)^{1/4}$ the velocity $c$ has a minimum

$$c_{\min} = (4 \alpha EI/m_0^{3})^{1/4}.$$  

(5)

The value $c_{\min}$ is often referred to as track critical velocity. For the above mentioned typical track and ballast parameters and for the value of $EI$ equal to $4.85 \text{ MNm}^2$ it follows from (5) that $c_{\min} = 326 \text{ m/s (1174 km/h)}$ which is much larger than the speeds of the fastest modern trains. However, for very soft soils, e.g. alluvial soils, characterised by very low $\alpha$, values of $c_{\min}$ can be as low as $60-70 \text{ m/s}$ and can be easily exceeded by even relatively moderate high-speed trains. In practice, the value of $c_{\min}$ for a particular location can be estimated using eqn (5) in which the stiffness of equivalent Winkler foundation $\alpha$ is expressed in terms of real elastic moduli of the ground. There are different theoretical models that give such expressions (see, e.g. [14]). Generally, it follows from these models that track critical velocity is normally larger by 10-30% than Rayleigh wave velocity for the same ground. The solution of eqn (1) with the right-hand side different from zero has different forms for small and large values of time $t$. For the problem under consideration we are interested in the “established” solution for large values of $t$ which describes the track deflections being at rest relative to the coordinate system moving at train speed $v$ - the so-called stationary solution. Obviously, this solution must depend only on the combination $x - vt$. Using the notation $p = \beta(x - vt)$, where $\beta = (\alpha/EI)^{1/4}$, it is easy to obtain the stationary solution of (1) in the Fourier domain, $W(p)$, (see [16]), where $W(p) = \int_{-\infty}^{\infty} w(z) e^{-ipz} dz$. Taking the inverse Fourier transform of $W(p)$ allows the calculation of the function $w(x-vt)$ which has different forms depending on whether $v < c_{\min}$, $v = c_{\min}$ or $v > c_{\min}$. The behaviour of $w(x-vt)$ in all these cases is well known and will not be discussed here. It is worth mentioning, however, that if a train speed $v$ approaches and exceeds the minimal phase velocity $c_{\min}$, the rail deflection amplitudes $w$ experience a large resonance increase limited by track damping. Note that possible large rail deflections at train speeds approaching track critical velocity may result even in train derailment, thus representing a serious problem also from the point of view of...
train and passenger safety. Different aspects of this problem are now widely investigated (see, e.g. [4,12-14]).

2.2 Forces applied from sleepers to the ground

To describe generation of ground vibrations by moving trains one needs to know the frequency spectra of dynamic forces applied from each sleeper to the ground, rather than the time dependence of these forces. Since the dynamic forces associated with any particular sleepers are related to each other via the distance between them and the train speed, it is sufficient to consider only the force $P(t)$ associated with the sleeper located at $x = 0$. Note that, whereas a time-domain solution $P(t)$ has different forms for $v < c_{min}$, $v = c_{min}$ and $v > c_{min}$, its Fourier representation $P(\omega)$ has the same form for all these cases. It can be shown that the corresponding expression for $P(\omega)$ can be written as follows [15]:

$$P(\omega) = \frac{-12.8Td}{\omega^2 - \sqrt{\frac{\omega^2}{\beta^4 v^4} - \frac{\omega^2}{c_{min}^2 \beta^2} - 8i \frac{g\omega}{c_{min} \beta} + 4}},$$

(6)

where $d$ is a sleeper periodicity, and $g = (m_0/\alpha)^{1/2} \omega_b$ is a non-dimensional damping parameter. For relatively low train speeds, i.e., for $v < c_R$, the dynamic solution (6) for the force spectrum $P(\omega)$ goes over to the quasi-static one, as expected. When train speeds increase and approach or exceed the minimal track wave velocity the spectra $P(\omega)$ become broader and larger in amplitudes, and a second peak appears at higher frequencies [15].

The values of $P(\omega)$ in the model under consideration are limited by track wave damping described by the non-dimensional damping parameter $g$. As one can see, the effect of track damping on the spectra $P$ is more pronounced for high train speeds, $v > c_{min}$. For low train speeds, $v < c_R$, the effect of track damping is negligibly small.

2.3 Calculation of generated ground vibrations

To calculate ground vibrations generated by trains one needs superposition of waves generated by each sleeper activated by wheel axles of all carriages, with the time and space differences between sources (activated sleepers) being taken into account. To derive this in a precise way one can use the Green’s function approach [2, 5-7]. The corresponding analytical formula relating the frequency spectra of vertical component of surface ground vibration velocity $v_z(0, y_0, \omega)$ at the point of observation $x = 0$ and $y = y_0$ with the sleeper force spectra $P(\omega)$ and geometrical parameters of track and train has the following form [2, 5-7]:
\[ v_z (0, y_0, \omega) = P(\omega)D(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} \left[ \exp(-\gamma \rho_m/\rho_m) \right] \left[ 1 + \exp(iM\omega/v) \right] \exp(i(\omega/v)(md + nL) + i(\omega/c_R)\rho_m), \]  

(7)

where \( D(\omega) = (1/2\pi)^{1/2}(-i\omega)qk_R^{1/2}k_t^2 \exp(-i3\pi/4)/\mu F'(k_R) \) is a function describing properties of Rayleigh surface waves generated by a single sleeper (for the problem under consideration, only the Rayleigh surface wave contribution need be considered since Rayleigh waves transfer most of the vibration energy to remote locations). Here \( k_R = \omega/c_R \) is the wavenumber of a Rayleigh wave propagating through the ground with the velocity \( c_R \); terms \( k_l = \omega/c_l \) and \( k_t = \omega/c_t \) are wavenumbers of longitudinal and shear bulk elastic waves in the ground, \( c_l, c_t \) and \( \mu \) are the corresponding wave velocities and shear modulus; \( \gamma = 0.001 - 0.1 \) is an empirical constant describing the "strength" of dissipation of Rayleigh waves in soil. The term \( q \) is defined as \( q = \sqrt{(k_R^2 - (k_l^2)^2)} \) and the factor \( F'(k_R) \) is the derivative \( d/dk \) of the so called Rayleigh determinant \( F(k) = (2k^2-k_t^2)^2-4k^2(k_t^2-k_t^2)^{1/2}(k_t^2-k_l^2)^{1/2} \) taken at \( k = k_R \). \( \rho_m = \left[ y_0^2 + (md)^2 \right]^{1/2} \) is the distance between a current radiating sleeper characterised by the number \( m \) and the point of observation \( (x=0, y=y_0) \); \( N \) is the number of carriages, \( M \) is the distance between the centres of bogies in each carriage and \( L \) is the total carriage length. Dimensionless quantity \( A_n \) is an amplitude weight-factor to account for different carriage masses (for simplicity we assume all carriage masses to be equal, i.e., \( A_n = 1 \)). Note that eqn (7) is applicable for trains travelling at arbitrary speeds.

2.4 Special case of trans-Rayleigh trains

For "trans-Rayleigh trains", i.e., trains travelling at speeds higher than Rayleigh wave velocity in the ground, it follows from eqn (7) that maximum radiation of ground vibrations (a ground vibration boom) takes place if the train speed \( v \) and Rayleigh wave velocity \( c_R \) satisfy the relation [1,2]

\[ \cos \Theta = c_R/v, \]  

(8)

where \( \Theta \) is the observation angle. Since the observation angle \( \Theta \) must be real \( (\cos \Theta \leq 1) \), the value of \( v/c_R \) should be larger than 1, i.e., the train speed \( v \) should be larger than Rayleigh wave velocity \( c_R \). Under this condition, a ground vibration boom takes place, i.e., ground vibrations are generated as quasi-plane Rayleigh surface waves symmetrically propagating at angles \( \Theta \) with respect to the track, and with amplitudes much larger than in the case of conventional trains. Note that for trans-Rayleigh trains these Rayleigh surface waves are generated equally well on tracks with and without railway sleepers, whereas for conventional trains the presence of sleepers is paramount [5,7]. Without them no propagating waves are generated in the framework of quasi-static pressure generation mechanism [18].
3. CALCULATION OF GROUND VIBRATIONS GENERATED BY SURFACE HIGH-SPEED TRAINS

3.1 Ground vibrations generated at very low and very high train speeds.

Numerical calculations of ground vibrations generated by high-speed trains can be carried out according to eqn (7) for different values of train speed, different parameters characterising Rayleigh wave dispersion in layered ground, and for different geometrical and physical parameters of both track and train.

The results for ground vibration spectra (in dB, relative to the reference level of $10^{-9}$ m/s) generated by complete TGV or Eurostar trains travelling on homogeneous ground with Rayleigh wave velocity $c_R = 125$ m/s at sub-Rayleigh and trans-Rayleigh train speeds ($v = 13.8$ m/s (50 km/h) and $v = 138.8$ m/s (500 km/h) respectively) show [6] that the averaged ground vibration level from a train moving at trans-Rayleigh speed ($138.8$ m/s) is by approximately 70 dB higher than from a train travelling at speed of ($13.8$ m/s).

3.2 Effects of layered structure of the ground

Taking into account the effect of a stiffer lower layer, causing the Rayleigh wave dispersion and increase in its velocity for lower frequencies, results in decrease of ground vibration amplitudes generated by a trans-Rayleigh train at low frequencies [6]. If a three- or four-layer ground model is used, with the softest layer being in the depth of the ground, then the Rayleigh wave velocity has a minimum at a certain frequency (typically around 4-8 Hz). In these case a decrease in levels of ground vibrations generated by a trans-Rayleigh train may take place for both low and high frequencies. For the range of frequencies corresponding to the minimum of Rayleigh wave velocity (around 4-8 Hz) the levels of generated ground vibrations remain very high.

3.3 Effect of track dynamics

Calculations of ground vibration frequency spectra generated by complete TGV or Eurostar trains travelling on a very soft homogeneous ground (with $c_R = 45$ m/s, $c_{min} = 65$ m/s, and $g = 0.1$) for three values of train speed ($v = 20, 50$ and $70$ m/s) demonstrate [15] that for the trans-Rayleigh train speed of 50 m/s corresponding to the case $c_R < v < c_{min}$ the overall level of generated ground vibrations is much higher than for sub-Rayleigh train speed of 20 m/s. For train speed of 70 m/s exceeding the value of track critical velocity, $c_{min}$, a significant increase takes place at higher frequencies of generated ground vibration spectra. However, since the amplitudes of high-frequency components are generally low, the overall increase due to the track wave resonance is not large in comparison with the one associated with ground vibration boom. Typically the overall increase is only by 1.5-2 times.

3.4 Comparison with the experiments at Lädsgard
It was interesting to compare the above described theory with the recent observations made on the railway line from Gothenburg to Malmö (see the Introduction). For the rough estimate, the same parameters of TGV trains (instead of Swedish X2000) have been used, and calculations of the vertical ground vibration velocity averaged over the frequency range 0-50 Hz have been carried out (this roughly corresponds to a maximum peak velocity used in the observations). Being interested only in averaged ground vibration velocity, we did not take into account the actual four-layer ground structure present at the railway line from Gothenborg to Malmö (on the measurement site near Lädsgard). Instead, we used the reported low minimum value of Rayleigh wave velocity \( c_R = 45 \text{ m/s} \) as a parameter characterising an equivalent homogeneous ground. To facilitate the comparison of the predicted increase in ground vibration level with the observed one the amplitudes of generated ground vibrations were calculated in linear units (m/s).

The resulting amplitudes as functions of train speed have been calculated for two values of track critical velocity - \( c_{\text{min}} = 65 \text{ m/s} \) and \( c_{\text{min}} = 10000 \text{ m/s} \) (the latter very large value of \( c_{\text{min}} \) describes the hypothetical case when track dynamics effects can be completely ignored). Calculations show [15] that in both cases the predicted amplitudes of the vertical velocity component of generated ground vibrations change from \( 2 \times 10^{-5} \text{ m/s} \) at \( v = 140 \text{ km/h} \) (38.8 m/s) to \( 16 \times 10^{-5} \text{ m/s} \) at \( v = 180 \text{ km/h} \) (50 m/s). Thus, the estimated 8 times increase in ground vibration level following from the above theory for the train speeds and Rayleigh wave velocity considered is in reasonable agreement with the 10 times increase recently observed experimentally on the site near Lädsgard [3].

If train speed further increases and approaches or exceeds the track critical velocity \( c_{\text{min}} = 65 \text{ m/s} \) then the level of generated ground vibrations also becomes larger (by approximately 1.5-2 times, as compared to the case of absence of track dynamics effects). This increase is not as large as in the case of ground vibration boom. However, since it occurs in combination with the latter, this gives a noticeable amplification of the resulting ground vibration impact.

4. VIBRATIONS FROM HIGH SPEED TRAINS TRAVELLING UNDERGROUND

For high-speed underground trains the contribution of bulk shear and longitudinal elastic waves (S- and P-waves respectively) is often more essential than that of Rayleigh waves [19]. Obviously, the radiated S- and P-waves can also be significantly amplified if the train speeds are high enough and the conditions \( v > c_t \) or even \( v > c_l \) hold, in addition to the trans-Rayleigh condition \( v > c_R \) considered so far (we recall that \( c_R < c_t < c_l \)). In such cases these waves will be radiated into the ground as conical Mach waves propagating at the angles \( \Theta_t = \arccos(c_t/v) \) and \( \Theta_l = \arccos(c_l/v) \) relative to the track, in addition to the relatively low-amplitude Rayleigh waves radiated as quasi-plane waves along the surface at the angles \( \Theta = \arccos(c_R/v) \). The most likely contribution to the ground vibration boom from underground trains might be that of radiated S-waves since their velocity \( c_t \), being only about
10% higher than the velocity of Rayleigh waves, can be more easily exceeded by moving trains than the velocity of longitudinal (compressive) waves $c_l$.

In comparison with above-ground trains, the case of underground trains is more difficult for theoretical description, partly because of the influence of the tunnel geometry making the problem of constructing the corresponding Green's function extremely complex. We recall that physical meaning of the Green’s function for the problem under consideration is that it describes the ground vibrations generated by an individual sleeper which may be regarded as a point source in the low-frequency band. The approximate analytical approach described here considers the problem in the low-frequency approximation, i.e., the characteristic wavelengths of generated bulk acoustic waves in the ground are assumed to be essentially larger than the diameter of the tunnel. For simplicity, we only consider the case of homogeneous ground.

In the above mentioned low-frequency approximation the formal expression for the vertical component of the particle velocity of ground vibration spectra generated on the ground surface by trains travelling underground may be written as follows [19]:

$$v_z(0, y_0, \omega) = \left[ i \omega P(\omega)/4\pi \rho_0 \right] \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} \left[ 1 + \exp(iM\omega/v) \right] \exp\left[ i(\omega/v)(md + nL) \right] \times$$

$$\left( 1/r_m \right) \left[ \exp\left\{ (-\gamma_l \omega r_m/c_l) + i(\omega/c_l) r_m \right\} + \exp\left\{ (-\gamma_t \omega r_m/c_t) + i(\omega/c_t) r_m \right\} \right] \cos^2(\phi_m) \exp\left\{ (-\gamma_t \omega r_m/c_t) + i(\omega/c_t) r_m \right\} \sin^2(\phi_m) \right].$$

(9)

Here $H$ is the tunnel depth, $r_m = \left[ y_0^2 + (md)^2 + H^2 \right]^{1/2}$, $\cos(\phi_m) = H/r_m$ and $P(\omega)$ is a Fourier transform of $P(t)$ described by eqn (6). In writing eqn (9) we have taken into account attenuation in soil by replacing $1/c_l$ and $1/c_t$ in the exponentials by the complex values $1/c_l + i\gamma_l/c_l$ and $1/c_t + i\gamma_t/c_t$, where $\gamma_l, \gamma_t << 1$ are the constants describing the "strength" of dissipation of longitudinal and shear waves in the soil. All other notations are the same as in previous sections.

Calculations according to eqn (9) show that for underground trains travelling at conventional speeds the main contribution to the vertical component $v_z$ of the total ground vibration field at the ground surface is usually due to the radiated shear bulk waves rather than to the longitudinal bulk waves.

In the case of high-speed underground trains, the analysis of eqn (9) shows that it has two maxima for the values of observation angles satisfying the conditions: $\cos \Theta_s = c_l/v$ - for radiated shear waves, and $\cos \Theta_l = c_l/v$ - for radiated longitudinal waves. Since $\cos \Theta_s$ and $\cos \Theta_l$ must be less than 1 it follows from these conditions that the maxima can be achieved if the train speeds are high enough and the conditions $v > c_l$ or even $v > c_l$ hold. In such cases the corresponding waves waves will be radiated into the ground as conical Mach waves propagating at the angles $\Theta_s = \arccos(c_l/v)$ and $\Theta_l = \arccos(c_l/v)$ relative to the track. The most likely contribution to the ground vibration boom from underground trains might be that
of radiated S-waves since their velocity $c_s$, being only about 10% higher than the velocity of Rayleigh waves, can be more easily achieved by moving trains than the velocity of longitudinal (compressive) waves $c_l$.

The results of calculations of the amplitudes of generated ground vibrations at frequency 15 Hz (in dB re $10^{-9}$ m/s) as functions of the tunnel depth $H$ for $Y_0 = 30 \text{ m}$ show that ground vibrations generated by an underground train travelling at speed $v$ higher than shear wave velocity in the ground $c_s$ are essentially larger than those generated by the same train moving at conventional speed (the speed-related amplification of the total ground vibration field varies from about 50 dB to 20 dB for the tunnel depth $H$ changing from 2 m to 100 m).

For practical values of $H$ (less than 60-70 m) the contributions of shear waves is essentially higher than the contribution of longitudinal waves. For larger depths, the contributions of shear and longitudinal waves first become comparable with each other, causing an oscillatory behaviour of the resulting field versus $H$, and then the longitudinal waves prevail.

The results of calculations of total ground vibration spectra generated by the same TGV train comprising five carriages ($N = 5$) and travelling both above ground and underground at speeds $v = 13.8 \text{ m/s}$ and $80 \text{ m/s}$ demonstrate that in the case of ground vibration boom, which takes place at $v = 80 \text{ m/s}$ for both underground and above ground trains, a very large increase in generated ground vibrations is observed, albeit for an underground train this increase is less pronounced, especially at higher frequencies. The shapes of ground vibration spectra for underground and above ground trains travelling at the same speeds are very similar. This implies that the shapes of ground vibration spectra are determined mainly by track and train geometrical parameters rather than by the tunnel depth and consequently by types of predominantly generated elastic waves. We remind the reader that the low-frequency approximation used in this section to describe ground vibrations from underground trains is inaccurate for frequencies higher than 10-15 Hz. To improve the situation, one can take into account the next term in the series expansion of the Green’s function for the problem under consideration [20]. This term is proportional to the product of tunnel diameter and characteristic wavenumber.

5. WAVEGUIDE EFFECTS OF THE EMBANKMENTS AND EFFECTS OF TRACK CURVATURE ON GENERATED GROUND VIBRATIONS

An interesting aspect that has to be taken into account when high-speed railway lines are built on the embankments is possible waveguide effects of the embankments on generated ground vibration boom [2]. The possibility of the embankments acting as waveguides for generated ground vibrations is closely related to the fact that Rayleigh surface waves associated with ground vibration boom are usually radiated at small angles relative to the track. This is why a dominant part of the radiated energy can be expected to be trapped and dissipated within the embankment itself, without significant leakage to the area outside. In what follows we briefly discuss the process of generating ground vibration boom by high-speed trains travelling along the tops of the embankments.
The effect of railway embankments on generated ground vibrations can be considered by means of construction of the specific Green's function for an elastic half space with an embankment. Such a Green's function must take into account the internal reflections of generated surface Rayleigh waves from the geometric boundaries between the embankment's top flat and side slop surfaces, and between side slop surfaces and the low ground. For simplicity, we consider only reflections from the boundaries between the embankment's top flat and side slop surfaces, assuming that side slop surface transfer smoothly to the ground.

The theory of Rayleigh wave reflection from the boundary between two surfaces intersecting at obtuse angle $\Theta$ has been earlier developed by the present author (see, e.g. [21]). For the problem under consideration, one can use this theory to represent the corresponding Green's function as an infinite sum of Rayleigh waves radiated by imaginary sources with the amplitudes defined by the reflection and transmission coefficients. This Green's function then can be used in the way described in previous sections to calculate ground vibrations generated by high-speed trains travelling along the embankments.

Calculations of the spatial distribution of ground vibration field (in arbitrary units) generated by a single axle load $2T = 200 \text{kN}$ travelling at speed $v = 138.8 \text{ m/s (500 km/h)}$ over 10 sleepers placed on the embankment of 8 m width and with the slop angle $\Theta_s = 150^\circ$ versus the flat top show that generated ground vibrations are propagating predominantly along the embankment where their amplitudes are much larger than in the outside area. This demonstrates that the embankments do act as waveguides for generated ground vibration boom, and the associated waveguide effects can significantly reduce the hazardous impact of ground vibration boom on the built environment.

If a track has curved parts to provide the possibility of changing direction of train movement, the wave fronts of ground vibrations generated under the conditions of ground vibration boom also become curved. This may result in focusing of generated ground vibrations at one side of the track and in the corresponding increase of their amplitudes.

Calculations of the spatial distribution of ground vibrations generated at the frequency component $f = 30 \text{ Hz}$ by a single axle load travelling along the curved track with the radius of curvature $R_0 = 100 \text{ m}$ show that for the load speed $v = 50 \text{ m/s}$ higher that Rayleigh wave velocity in the ground ($c_R = 45 \text{ m/s}$) a focusing of ground vibrations takes place. This results in the increase in ground vibration amplitudes by 1.5-2 times that must be taken into account.

6. CONCLUSIONS

The theory of generating ground vibrations by high-speed trains shows that if train speeds exceed the velocity of Rayleigh surface waves in the supporting soil a ground vibration boom occurs associated with a very large increase in amplitudes of generated vibrations. Crossing the track wave critical velocity results in further increase of generated ground vibrations, albeit not as dramatic as in the case of ground vibration boom.
Railway-generated ground vibration boom is a today’s reality for high-speed lines crossing soft soil, and so are “supersonic” or “trans-Rayleigh” trains. Builders and operators of high-speed railways must be aware of possible consequences of ground vibration boom and large rail deflections. The direct relevance of these phenomena is for railways built on soft soil, such as West Coast Line in Sweden and Channel Tunnel Rail Link in the UK. One can expect that similar problems will arise also on other sites, especially in the Netherlands with its very soft soils.

Problems associated with track-soil critical velocities may take place also for underground trains. In this case the first critical velocity is the velocity of shear bulk elastic waves, and the ground vibration boom for train speeds exceeding this velocity represent a Mach cone of shear waves radiated from the tunnel.

Waveguide effects of the embankments may cause concentration of radiated ground vibration energy around the track. This can reduce the ground vibration amplitudes outside the embankment. However, the associated vibrations of the track itself may increase tremendously in this case even at train speeds below the track critical velocity. Therefore, very large rail deflections can be observed at train speeds approaching the velocity of Rayleigh waves, rather than track critical velocity.

Under the condition of ground vibration boom, the effect of track curvature on the flat ground may result in focusing of generated ground vibrations at one side of the track and in the corresponding increase of their amplitudes.

It is too early on at this stage to foresee how the phenomenon of railway-generated ground vibration boom and its amplification due to track dynamics effects will be reflected in future standards on noise and vibration from high-speed trains. However, one can expect that such an important parameter as the Rayleigh wave velocity in the ground for the sites considered will be present in all these standards indicating maximal train speeds beyond which excessive ground vibrations can be expected.

ACKNOWLEDGEMENTS

This work was supported by the EPSRC Research Grant GR/M18249. The author is also grateful to Prof. G. Degrande for useful discussions of part of the material described in this paper and for drawing the author’s attention to some inaccuracies.

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