Ground vibrations from tracked vehicles: theory and applications

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ABSTRACT: Over the last decade, much of attention was paid to the use of traffic-induced ground vibrations for the purposes of remote detection and monitoring of heavy military vehicles, such as tanks and armoured personnel carriers. A promising method of detection and identification is the one using the information extracted from ground vibration spectra generated by heavy military vehicles, often termed as their seismic signatures. The aim of the present work is to discuss the analytical approach to the prediction and interpretation of ground vibration spectra from heavy military vehicles. The analysis in this work is limited to tracked vehicles only. The modelling of track periodic irregularities (discontinuities) is considered from the point of view of their effect on the resulting dynamic forces applied from a vehicle to the ground and on the generated ground vibration spectra. Also, the effects of layered ground structure and of changes in wheel tyre compliance are considered for different values of ground and tyre parameters. It is shown that the obtained ground vibration spectra contain clearly identifiable peaks at frequencies associated with some characteristic vehicle parameters, such as track periodicity, distance between the nearest wheel axes, and vehicle speed. Although the relative amplitudes of these peaks vary with changes in layered structure, track irregularity profile, or tyre compliance, their positions on the frequency axis are robust and depend on track periodicity, distance between the nearest wheel axes, and vehicle speed only. A discussion follows on possible use of these spectral peaks for determination of these vehicle parameters and thus for vehicle identification.

KEYWORDS: Ground vibrations; Seismic signatures; Tracked military vehicles; Green’s function method.

1 INTRODUCTION

Research into generation and propagation of traffic-induced ground vibrations is usually associated with environmental issues concerning the development of quiet and environmentally friendly railways and roads (see e.g. [1-4]). More recently though, ground vibrations have been investigated also for the purpose of remote detection and monitoring of heavy military vehicles, such as tanks and armoured personnel carriers [5-10]. One of the first declassified projects, in which the roles of generated ground vibration spectra (also known as seismic signatures) have been studied experimentally, was the so-called Bochum Verification Project [5,6], that was open for participation of academic researchers from several European countries, USA and Russia. A typical application of this technology would be for UN peace keeping forces – to monitor agreed limits concerning weapons-free zones, etc. Autonomous acoustic and seismic sensors would provide covert monitoring and be independent of time-of-day or weather.

Earlier investigations into this topic employed either experimental methods [5,6] or purely numerical approaches (see e.g. [7,8]). In the recently published papers [9,10], a simplified analytical approach to the problem has been developed to predict fundamental characteristics of ground vibration spectra attributed to heavy military vehicles, such as tanks and armoured personnel carriers, traversing over flat homogeneous ground. The dynamic motion of typical heavy vehicles was analyzed in order to determine the forces applied from vehicles to the ground surface. These forces were then used for derivation of analytical expressions for generated ground vibrations, predominantly Rayleigh surface waves, using the Green’s function method. The advantage of such an analytical approach is that it assists in better understanding of basic properties of seismic signatures of heavy military vehicles and their dependence on different vehicle and ground parameters.

The aim of the present work is to present and further develop the analytical approach described in [10]. Like in [10], a simplified quarter car model representation for a heavy vehicle is considered. For this representation, the dynamic forces applied from a vehicle to the ground are derived and then used for calculation of generated ground vibration spectra. The analysis in the present work is limited to tracked vehicles only (tanks). An attention is paid to the modelling of the effect of track periodic irregularities (discontinuities) on the resulting dynamic forces applied from a vehicle to the ground, and eventually on generated ground vibrations. The analysis of generation and propagation of ground vibrations by the dynamic forces determined as above is extended to a more complex case of layered ground structure, and the effect of this factor on generated ground vibration spectra is considered for typical values of ground parameters. The effect of wheel tyre compliance is investigated as well.

It is shown that the predicted ground vibration spectra contain frequency peaks associated with such characteristic vehicle parameters as track periodicity and distance between the nearest wheel axles. A discussion follows on possible use...
of these spectral peaks for determination of these vehicle parameters and thus for vehicle type identification.

A comparison of the obtained theoretical results with published experimental data shows that the above-mentioned analytical approach is capable of producing ground vibration spectra of tracked military vehicles that are in a reasonably good agreement with the experimental ones. Some observed discrepancies, mostly at higher frequencies, could be attributed to excitation of ground vibrations by airborne engine noise via the mechanism of acousto-seismic coupling, which has not been taken into account in the above model. Some other unaccounted generation mechanisms are discussed as well in respect of their possible effects on generated ground vibrations.

2 MODELLING OF DYNAMIC GROUND FORCES

2.1 General assumptions

Ground vibrations generated as a result of heavy vehicle motion over terrain can be attributed to the dynamic forces applied directly to the ground and to the conversion of vehicle-generated airborne sound into the ground motion via acousto-seismic coupling (see Ref. [6] for more detail). In the present paper, only the main type of directly applied forces is considered, namely the forces exerted to the ground as a result of vehicle wheel motion over track periodic irregularities.

2.2 Ground force spectra

To describe vehicle-induced ground force spectra, a well-known quarter car model (QCM) can be used to simulate the contact forces exerted to the ground as a result of a vehicle motion over surfaces characterised by the presence of geometrical irregularities (see Figure 1). For tracked vehicles moving over perfectly flat ground, these irregularities are due to the small gaps between track links. For wheeled vehicles, tyre treads can induce a similar effect.

Total vehicle mass is distributed evenly over all the wheel stations.

The road surface is considered as rigid for the purpose of finding ground forces, as are the track links of tracked vehicles.

For the QCM shown in Figure 1, the magnitude of the normal force \( F_t \) applied to the ground is equivalent to the force exerted by the compression of the tyre spring due to the vertical displacement of the wheel (tangential forces associated with vehicle acceleration or braking [11] are not considered in this work limited to movements at constant speeds only). Therefore, the solution for the dynamic response of the wheel \( z_a(t) \) to an input from the road irregularity \( z_r(t) \) has to be found to determine such ground forces.

In the time domain, the input into the quarter car model under consideration is the elevation changes \( z_r(t) \) as a result of the wheel’s passing over track linkages. To determine the function \( z_r(t) \) precisely represents quite a complex task. Strictly speaking, one has to consider the problem of wheel interaction (via a rubber tyre) with a tensioned chain of track links. Each of these links in turn interacts with the real ground, with the possibility of rotational motion in respect of the ground surface and pin contacts between the neighboring links. As a result of this complex interaction, the track links form a periodic irregular surface during the passage of a wheel. Undertaking such a rigorous approach to define the irregularity profile is beyond the scope of this work. Instead, we employ empirically based periodic irregularity profiles using suitable mathematical functions.

In our previous papers [9,10], the track-induced irregularity has been modelled as a simple half-sine periodic function. The variation in surface profile over which a wheel (modeled as a point contact) traverses has been estimated as a finite series of half sine pulses with a frequency \( \nu_0 \) corresponding to the ratio of the vehicle forward velocity \( V \) to the track pitch \( a \) : \( \nu_0 = V/a \). In the present work, we consider a more general model of surface profile with the period \( a \) formed by individual irregularities described by an arbitrary positive function of unit magnitude \( f(x) \). Expressing the horizontal distance \( x \) passed by the wheel over time \( t \) as \( x = Vt \), one can obtain the irregularity profile as a time function \( z_r(t) \) in which we consider only a finite number of periods \( M \) defined by a certain time interval \( T \) (period of spectral observation): \( M = V T / a \). By carrying out Fourier integration, this input signal can be represented also in the frequency domain: \( z_r(\omega) \). The corresponding expressions for \( z_r(t) \) and \( z_r(\omega) \) can be written in the form

\[
z_r(t) = z^0_r f(0) + f(t-a/V)+...+f(t-(N-1)a/V),
\]

\[
z_r(\omega) = \int_{-T/2}^{T/2} z_r(t) e^{-i\omega t} dt.
\]

For calculations of this work we use a rectangular periodic profile (Figure 2) that can be considered as a more realistic model for a track of periodicity \( a \) formed by constant thickness links of the effective length \( l \), with the dips between them having the effective length \( a-l \). Note that the magnitude...
of the irregularity profile $z_{r0}$ in Eqn (1) and in Figure 2 represents the magnitude of the dips, and it should not be confused with the track thickness which is substantially larger.

As a 2-DOF system, QCM has a resonant response at both ‘wheel hop’ and ‘body bounce’ natural frequencies. To simplify the problem even more, one can consider the so-called ‘body still’ approximation [3,10] that reduces QCM to a 1-DOF system by freezing the low-frequency ‘body bounce’ mode of vibration (around 1-3 Hz). As a result, the problem is reduced to the analysis of only the wheel hop response to the displacement input from surface discontinuities that takes place at higher frequencies. This approximation is usually sufficient for calculation of generated ground vibrations, predominantly Rayleigh waves, since they are generated more efficiently at higher frequencies (see [10] for more detail).

Keeping this in mind, we use the simplified QCM, considering vehicle body as immobile in vertical direction and taking into account only axle vibrations. This 1-DOF simplified model comprises an unsprung (wheel) mass $M_w$ (this mass also includes masses of the suspension and of the shock absorber as well as a half of the axle mass) and two springs with constants $K_t$ and $K_s$ modelling respectively the rubber tyre compliance and the stiffness of vehicle suspension.

According to the above-mentioned simplified QCM, the expression for the Fourier transform $z_w(\omega)$ of an axle vertical displacement $z_w(t)$ can be written in the form [3]:

$$z_w(\omega) = z_r(\omega)Z(\omega).$$  \hspace{1cm} (3)

Here

$$Z(\omega) = \frac{\omega_h^2}{\sqrt{(\omega_h^2 - \omega^2)^2 + (2\omega \alpha)^2}} \exp\left[-i \tan^{-1}\left(\frac{2i\omega \alpha}{\omega_h^2 - \omega^2}\right)\right]$$  \hspace{1cm} (4)

is the frequency response function, where $\omega_h = (K/M_w)^{1/2}$ is the wheel hop resonant frequency, $\omega_t = (K_t/M_w)^{1/2}$ is the tyre ‘jumping’ resonant frequency, $\alpha = B/2M_w$ is a normalised damping coefficient, and $z_r(\omega)$ is the Fourier spectrum corresponding to the irregularity profile, $K = K_t + K_s$ is a combined elasticity of the irregularity profile, $K_t$ is the damping coefficient of suspension (tyre damping is neglected). Assume that the axle length is small as compared with wavelengths of generated ground vibrations and its centre is located at $x = 0$ and $y = 0$, the resulting normal concentrated force $F(t)$ applied to the axle to the ground can be written in the form

$$F(t) = 2K_t[z_w(t) - z_r(t)].$$  \hspace{1cm} (5)

where the factor 2 takes into account the presence of two wheels in an axle.

The Fourier transform of the vertical force applied from the vehicle to the ground, $F(\omega)$, is easily obtained from (5) via replacing $z_w(t)$ and $z_r(t)$ by their Fourier spectra and using Eqn (3):

$$F(\omega) = 2K_t[z_r(\omega)Z(\omega) - z_r(\omega)].$$  \hspace{1cm} (6)

Note that for tracked vehicles the values of tyre compliance $K_s$, characterising the elasticity of solid rubber coating of road wheels, are quite high in comparison with the stiffness of suspension $K_t$. In this case the wheel follows the track irregularities $z_r$ and the dynamic forces applied to the ground through the track are defined mainly by the stiffness of suspension $K_s$ (see [10] for more detail).

The QCM model described above depicts a single axle wheel displacement. To determine the ground force spectra observed due to the effects of multiple axles, a superposition of all wheel hop displacement responses should be taken. Obviously, the wheel hop response at each axle differs only by a phase shift that corresponds to the distance of the additional wheel axle (characterised by the integer number $n$) from the front axle, $E_{1n}$ divided by the vehicle forward speed $V$ [9,10]. The resulting expression for the vertical force $F_z(\omega)$ applied to the ground from the entire vehicle then takes the form

$$F_z(\omega) = F(\omega) \cdot \frac{1 + \exp(i\omega E_{12}/V) + \exp(i\omega E_{13}/V) + \cdots + \exp(i\omega E_{1N}/V)}{N},$$  \hspace{1cm} (7)

where $F(\omega)$ is the force spectrum for a single wheel axle defined by Eqn (6), and $N$ is the number of axles in a vehicle.

2.3 Calculation of ground force spectra

For calculation purposes, we will use the parameters of the Leopard-1 Main Battle Tank (MBT) (see Figure 3), for which a set of experimental results for generated ground vibration velocity spectra is available as part of the published works following from the Bochum Verification Project (BVP) [5,6].
The parameters that have been used in calculations of the present work are shown in Table 1.  
Note that there is a lack of published information about the values of suspension stiffness and tyre compliance for heavy military vehicles. The information that is available provides only some general characteristics, such as maximum speed, total vehicle mass, geometrical dimensions of tracks, etc. (see for example [12]). Therefore, the values of the above-mentioned two technical parameters in Table 1 have been obtained via theoretical estimates and by comparison with published parameters of similar vehicles (see e.g. [7, 8]). The uncertainty in these parameters though is not detrimental for calculations of ground vibration spectra as it affects only amplitudes of the frequency peaks but not their positions.

Table 1. Parameters of the Leopard-1 MBT used in calculations.

<table>
<thead>
<tr>
<th>Vehicle parameters</th>
<th>Symbols, Units</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total vehicle mass</td>
<td>$M_v$, kg</td>
<td>42500</td>
</tr>
<tr>
<td>Mass of wheel</td>
<td>$M_w$, kg</td>
<td>317</td>
</tr>
<tr>
<td>Number of wheels</td>
<td>$N_w$</td>
<td>14</td>
</tr>
<tr>
<td>Suspension stiffness</td>
<td>$K_s$, Nm$^{-1}$</td>
<td>$4.45 \times 10^5$</td>
</tr>
<tr>
<td>Tyre compliance</td>
<td>$K_t$, Nm$^{-1}$</td>
<td>$2.5 \times 10^6$</td>
</tr>
<tr>
<td>Suspension damping</td>
<td>$B_s$, Nsm$^{-1}$</td>
<td>$1.25 \times 10^5$</td>
</tr>
<tr>
<td>Vehicle velocity</td>
<td>$V$, ms$^{-1}$</td>
<td>3.9</td>
</tr>
<tr>
<td>Track/tread pitch</td>
<td>$a$, m</td>
<td>0.169</td>
</tr>
<tr>
<td>Magnitude of discontinuity</td>
<td>$z_{r0}$, m</td>
<td>0.01</td>
</tr>
<tr>
<td>Wheelbases</td>
<td>$E_i, i+1$, m</td>
<td>0.665</td>
</tr>
</tbody>
</table>

Figure 4 illustrates the resulting multi-axle ground force spectrum $F_{z, \text{mvw}}(\omega)$ for the Leopard-1 MBT (with 7 axles) calculated using the simplified quarter car model (QCM) and the rectangular model of track profile for $l = 0.9a$. Frequency resolution used in calculations was 0.4 Hz.

As expected, in Figure 4 the most significant force peaks are at the main frequency of excitation $f_v = V/a = 23.1$ Hz corresponding to the forward speed of the vehicle $V = 3.9$ m/s and the track pitch $a = 0.169$ m. The effect of multiple axles produces the additional peaks at different harmonics of the so-called wheel-base frequency $f_{wb} = V/E = 5.9$ Hz, where $E = E_i, i+1 = 0.665$ m is the distance between two neighbouring axles. Obviously, the amplitudes of all frequency peaks are proportional to the track hump height and depend on its shape.

Figure 4. Multiple-axle ground force spectrum calculated for the Leopard-1 MBT.

3 GENERATED GROUND VIBRATIONS

3.1 General comments

To calculate ground vibration spectra generated by the vehicle-induced ground forces one can use Green’s function method in which one can take into account only generated Rayleigh surface waves as they carry most of the radiated elastic energy (see Reference [3] for more detail). If the ground is modeled as a homogeneous elastic half space, then the vertical component of generated ground vibration velocity in the frequency domain, $v_z(\omega, \rho)$, can be described by the following expressions [3,10]:

$$v_z(\omega, \rho) = F_{z, \text{mvw}}(\omega) G_{zz}(\omega, \rho), \quad (8)$$

where

$$G_{zz}(\omega, \rho) = \left( \frac{2\pi}{k_R \rho} \right)^{1/2} \left( \frac{-i\omega}{k_R k_i} \right)^2 v_l \left( \frac{2\pi 4\pi^2}{k_R} \right) e^{-k_R \rho} e^{ik_R \rho - i3\pi/4} \quad (9)$$
is the Green’s function for particle velocity, or, if to use a more precise terminology, the component of the Green’s tensor describing vertical particle velocity in an elastic half space caused by a vertical unit force applied to the surface. Here $F_{zz}(\omega)$ is the multi-axle force spectrum defined by Eqn (6); $\rho = \rho(x,y)$ is the distance to the observation point; $k_R = \omega/c_R$ is the Rayleigh wave number, where $c_R$ is Rayleigh wave velocity in the ground; $F'(k_R)$ is the derivative $dF(k)/dk$ of the so-called Rayleigh determinant $F(k) = (2k^2 - k_{l,t}^2) - 4k^2\gamma V_1$ taken at $k = k_R$, where $V_1 = (k_{l,t}^2 - k_{l,t}^2)^{1/2}$ are unspecified expressions; $k_{l,t} = \omega/c_{l,t}$ are the wavenumbers of bulk longitudinal and shear seismic waves, where $c_{l,t}$ and $c_{l,t}$ are their phase velocities.

In writing Eqn (9) we have taken into account the attenuation of generated ground vibrations in the ground by replacing the wavenumber of a Rayleigh wave in an ideal elastic medium $k_R = \omega/c_R$ by the complex wavenumber $k_R' = k_R(1+i\gamma) = (\omega/c_R)(1+i\gamma)$. Here $\gamma << 1$ is a positive constant, called a loss factor, that describes the linear dependence of a Rayleigh wave attenuation coefficient on frequency $\omega$. For different types of ground $\gamma$ are in the range from 0.01 to 0.2. In what follows we will be interested only in amplitudes of generated ground vibrations $V_z(\omega) = |V_2(\omega)|$, ignoring the phase information.

As was mentioned above, the Green’s function described by Eqn (9) is valid for a homogeneous elastic half space. Strictly speaking, it is not applicable to a layered soil. In this situation one has to use a Green’s function for a layered elastic half space, as will be discussed below.

3.2 Effect of layered ground structure

To consider the influence of layered geological structure of the ground on generating ground vibrations, one should use the Green’s function for a layered elastic half space, instead of that for a homogeneous half space (see e.g. [13-15]). As a rule, such a function, that contains information about the total complex elastic field generated in a layered half space considered (including different modes of surface waves and modes radiating energy into the bulk (leaky waves)), cannot be obtained analytically. However, for description of ground vibration spectra generated by tracked vehicles, the problem can be simplified by considering the approximate Green’s function for a layered medium [16,17]. It takes into account the effects of layered structure on the amplitude and phase velocity of only the lowest order surface mode which goes over to a Rayleigh wave at higher frequencies.

We recall that in layered media surface waves become dispersive, i.e. their phase velocities $c_R$ depend on frequency: $c_R = c_R(\omega)$. For simplicity, it will be assumed in further consideration that the Poisson ratio $\sigma$ of the layered ground and the mass density $\rho_0$ are constant. Similarly to the Green’s function for a homogeneous half space $G_{zz}(\omega,\rho_0)$ (see Eqn (9)), one can construct its modification $G'_{zz}(\omega,\rho_0)$ describing approximately the effects of layered structure on generation and propagation of a lowest order surface Rayleigh mode.

According to [16,17], the approximate Green’s function for a layered medium $G'_{zz}(\omega,\rho)$ can be written in the form:

$$G'_{z}(\omega,\rho) = \left(\frac{2\pi}{k_R + \rho}\right) \frac{(1-i\omega)k_R'(k_R')^2V_1}{2\pi i F'(k_R')},$$

where $k_R' = \omega/c_R(\omega)$ is the wavenumber of the lowest order Rayleigh mode propagating with frequency-dependent velocity $c_R(\omega)$; terms $k_{l,t}' = \omega/c_{l,t}'(\omega)$ and $k_{l,t}' = \omega/c_{l,t}'(\omega)$ are the so-called ‘effective’ wavenumbers of longitudinal and shear bulk elastic waves at given frequency $\omega$ (these wavenumbers are inversely proportional to the longitudinal $c_{l,t}'(\omega)$ and shear $c_{l,t}'(\omega)$ wave velocities averaged over the ‘effective’ depth of Rayleigh wave penetration into the ground which is close to the Rayleigh wavelength). In the model under consideration, these velocities and the corresponding ‘effective’ shear modulus $\mu'$ are expressed in terms of frequency-dependent Rayleigh wave velocity $c_R(\omega)$ using the well known relations:

$$c_R(\omega)/c_{l,t}'(\omega) = (0.87 + 1.12\sigma)/(1+\sigma)$$

$$c_{l,t}'(\omega)/c_{l,t}'(\omega) = [(1 - 2\sigma)/2(1 - \sigma)]^{1/2}$$

$$\mu'(\omega) = \rho_0[c_{l,t}'(\omega)]^2.$$  

The term $q^l$ is defined as $q^l = [(k_R'^2 - k_{l,t}'^2)^{1/2}]$, and the factor $F_L(k_R'^2)$ is determined according to the following relationship [18]:

$$F_L(k_R'^2) = N(\sigma)(k_R'^2),$$
In our approach we will consider published values of the functions \( c_R(\omega) \), using where possible their simple analytical approximations.

The frequency-dependent Rayleigh wave velocity for the layered medium under consideration, comprising a top soft layer on a stiffer elastic half space, can be approximated as

\[
c_R(\omega) = (c_1 - c_2)\exp(-\alpha'\omega/2\pi) + c_2,
\]

where \( c_1 \) and \( c_2 \) are values of \( c(\omega) \) for \( \omega = 0 \) and \( \omega = \infty \) respectively, parameter \( \alpha' \) describes the ‘strength’ of dispersion (it depends on the characteristic layer thickness and on the difference between elastic moduli in the depth and on the surface of the ground).

Figure 5. Rayleigh wave dispersion curve for a layered ground used in calculations (solid line); dashed line indicates the value of Rayleigh wave velocity for a homogeneous ground with the parameters of the upper layer.

Figure 5 shows a typical Rayleigh wave dispersion curve \( c_R(\omega) \) calculated according to Eqn (15) for the value of the parameter \( \alpha' = \alpha'' = 0.02 \) Hz\(^{-1}\) in the case of a soft layer \( c_2 = 137 \) m/s placed on a stiffer ground \( c_1 = 198 \) m/s. The dashed curve indicates the value of Rayleigh wave velocity for a homogeneous half space with elastic properties of the upper layer. To plot this dashed curve (a horizontal line) the value of Rayleigh wave velocity for a homogeneous ground with the parameters of upper layer \( c_2 = 137 \) m/s was used in Eqn (15). The latter case describes the situation when the effect of a stiffer ground substrate can be neglected and the function \( c_R(\omega) = c_2 \) is determined entirely by the upper layer.

3.3 Results of the calculations and discussion

For the purpose of comparison of the results of the theoretical calculations with the experimental data obtained for the Leopard-1 MBT in the course of BVP [6], the selection of ground material constants had been chosen as consistent as possible with the ground parameters on the site of those experiments. The predominant soil type on the site of the experiments was sand, and thus the values of material parameters typical for this type of soil were used: mass density \( \rho_0 = 1800 \) kg/m\(^3\), Poisson’s ratio \( \sigma = 0.25 \), and the loss factor \( \gamma = 0.05 \). The values of Rayleigh wave velocity for a homogeneous half space with the parameters of upper and lower layers were chosen as \( c_2 = 137 \) m/s and \( c_1 = 198 \) m/s respectively, in accordance with the values used for plotting the curves shown in Figure 5.

Theoretical ground vibration velocity spectra for the Leopard-1 MBT at 11.8 m distance from the sensor and over time interval \( T = 0.5 \) s calculated for the cases of homogeneous and inhomogeneous ground are shown in Figure 6. One can see that in both cases the calculated ground vibration velocity spectra largely repeat the main features of the corresponding ground force spectrum (see Figure 4). These include major peaks at the track pitch frequency \( f_{tr} = V/a = 23.1 \) Hz and its harmonics as well as smaller peaks at harmonics of the wheel-base frequency \( f_{sub} = V/E = 5.9 \) Hz.

Including the effect of layered ground results in a slight decrease in ground vibration level at low frequencies (up to 6 dB). At medium and higher frequencies, spectra are almost indistinguishable. Note that the positions of frequency peaks are not influenced by the layered structure. These include peaks at the track pitch frequency \( f_{tr} \) and its harmonics as well as peaks at harmonics of the wheel-base frequency \( f_{sub} \).

Figure 7 shows the effect of tyre compliance \( K_t \) on generated ground vibration spectra. Calculations have been carried out for \( K_t = 2.5 \times 10^6 \) Nm\(^{-1}\) and \( K_t = 2.5 \times 10^7 \), and for the same parameters of layered ground as in Figure 6. It can be seen that in the case of \( K_t = 2.5 \times 10^7 \) the amplitudes of generated ground vibrations are generally higher. Especially large increase takes place around the second harmonic of the track pitch frequency, 46.2 Hz. This can be explained by the
amplification effect of the wheel hop resonance: the frequency $f_0 = \omega_0/2\pi$ is equal to 45.1 Hz in this case, in comparison with 15.3 Hz for $K_t = 2.5 \cdot 10^6$ Nm$^{-1}$. Note that the positions of all frequency peaks remain unchanged.

3.4 On the possibility of vehicle type identification

As it follows from Figures 6 and 7, the track pitch frequency $f_{tr}$ and the wheel base frequency $f_{wb}$ are robust and clearly identifiable from the positions of the peaks in the calculated frequency spectra, regardless of the effects of layered ground structure or other vehicle parameters, such as e.g. tyre compliance or shape of the track profile. This property of the calculated ground vibration spectra can be used for observation-based determination of such important vehicle parameters as the track pitch $a$ and the wheel base $E$, which could be used for the purpose of tracked vehicle type identification. Indeed, it follows from the above expressions that $a = V/f_{tr}$ and $E = V/f_{wb}$. By division of one of these expressions by another, one can obtain the following useful relationship:

$$\frac{E}{a} = \frac{f_{wb}}{f_{tr}}. \quad (16)$$

The ratio $E/a$ in general has different values for different types of tracked vehicles. Obviously, it can be easily determined from the observed ground vibration spectra according to Eqn (16) and thus used for vehicle type identification.

Even more possibilities to identify a tracked vehicle appear if not just the ratio but the absolute values of $a$ and $E$ can be determined separately. This can be done by measuring the vehicle velocity $V$ by independent means. For example, the value of $V$ can be determined using the same vehicle-generated Rayleigh waves by the well-known method of triangulation, employing three seismic sensors, perhaps from the same network, to determine the coordinates of the vehicle $x_r$, $y_r$ and $x_s$, $y_s$ in two different time instants $t_1$ and $t_2$, respectively. The velocity $V$ can then be determined as

$$V = \sqrt{(x_r - x_s)^2 + (y_r - y_s)^2} / (t_2 - t_1), \quad (17)$$

thus providing the absolute values of $a$ and $E$ to be used for identification.

3.5 Comparison with the experiment

A comparison of the above-mentioned theoretical results with the experimental spectrum of ground vibrations generated by the Leopard-1 MBT [6] and reproduced here with the kind permission in Figure 8 demonstrates that the theoretical spectrum predicts fairly well the experimental peaks associated with track and wheel axle periodicity. However, there are some discrepancies. In particular, the effect of fine structure in the spectrum associated with wheel-base frequencies $f_{wb}$ is not as well pronounced in the experiment (Figure 8) as in the theoretical spectra (Figures 6 and 7). Among possible reasons for that could be insufficient frequency resolution in the experimental spectrum and not perfect periodicity of the wheel axles.
seismic coupling [19,20] of engine-radiated sound that has not been taken into account in this investigation. Some of the observed differences between the obtained theoretical results and the experimental data could be attributed to inaccuracy in modelling the track irregularity profile, to uncertainty in the value of tyre compliance, and to ignoring rotational motion of a vehicle body. Also, various additional generation mechanisms that have not been taken into account in this work could play a part. In addition to the already mentioned acousto-seismic coupling, these are the effects of engine vibrations due to rotating unbalance, spatial variations in ground elastic parameters and mass density in horizontal direction, ground topography, etc. Further research would be required to explore the effects of these missing mechanisms on ground vibration spectra generated by tracked vehicles.

4 CONCLUSIONS

The results of this work show that analytical techniques based on the simplified quarter car model as well as on Green’s function method are capable to predict ground vibration spectra generated by tracked military vehicles that reproduce basic properties of experimental spectra.

The established relationships between some tracked vehicle parameters, such as track periodicity and distance between the nearest wheel stations, and characteristic frequency peaks in generated ground vibration spectra can be considered as a starting point for their possible use for vehicle type identification.

Some of the observed differences between the obtained theoretical results and the experimental data could be attributed to various additional generation mechanisms that have not been taken into account in this work. Further research is needed to explore the effects of these missing mechanisms on ground vibration spectra generated by tracked vehicles.

REFERENCES