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Attenuation of low frequency ground vibrations by means of resonant scattering of Rayleigh waves on heavy masses

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Summary
Although the main mechanisms of generating ground vibrations at source, e.g. by rail and road traffic, are now well understood, there are still very few investigations aimed to protect the affected buildings by influencing the propagation of ground vibrations, mainly Rayleigh surface waves, from a source to a receiver. A promising and cost effective method of screening the affected properties can be using heavy masses placed on the ground surface near the road (e.g. concrete or stone blocks, specially designed brick walls, etc). The principle of operation of such masses is based on the fact that their natural frequencies of vibration, which depend on the mass value and on the local ground stiffness, can be chosen within the frequency range of railway- or road-generated ground vibrations (normally from 5 to 50 Hz). When the mass is shaken under the impact of incident Rayleigh surface waves, it scatters the incident waves into the depth of the ground and at different directions on the surface, thus resulting in noticeable resonant attenuation of transmitted ground vibrations. Using suitable combinations of such mass scatterers, one can expect to achieve efficient vibro-isolation of affected buildings. While some initial efforts have been made in the past to investigate the above-mentioned mass scatterers, largely by means of numerical calculations, very little progress in understanding their behaviour has been made so far. The aim of the present paper is to give a brief introduction to the theory of resonant mass scatterers and to discuss some problems that still need to be considered to achieve a fuller understanding of their operation as means of damping of low frequency ground vibrations.

Introduction
One of the major sources of environmental noise and vibration pollution is ground vibration generated by rail and road traffic. Most of the energy of such vibrations is concentrated in the low frequency range (normally from 5 to 50 Hz). Ground vibrations cause significant disturbance for local residents and sensitive community buildings such as schools, hospitals, etc. They are also very disruptive for precise manufacturing, which is so important in our time of rapid development of high technology.
Theoretical investigations of ground vibrations from rail and road traffic undertaken during the last decade (see e.g. Krylov 1995, 1996, 1998, 2001a,b; Krylov et al. 2000; Sheng et al. 1999, 2003; Grundmann et al 1999; Watts et al. 2000; Takemiya 2001; Degrande et al. 2001) contributed to understanding the reasons why different levels of vibrations are generated at different conditions and parameters of vehicles and the infrastructure. In particular, it has been predicted theoretically (Krylov 1994, 1995, 1998) that especially large increase in railway-generated vibrations can occur if train speeds \( v \) exceed the velocity of Rayleigh surface waves in the ground \( c_R \). When this happens, a ground vibration boom takes place, similar to a sonic boom from supersonic aircraft.

In October 1997, a ground vibration boom has been observed for the first time on the newly opened high speed line from Gothenburg to Malmö in Sweden (see Madshus & Kaynia 1998). In particular, at the location near Ledsgärd, the Rayleigh wave velocity in the ground was only 45 m/s, so that an increase in train speed from 140 to 180 km/h lead to about 10 times increase in generated ground vibrations. This was in good agreement with the above-mentioned theoretical prediction.

Although the theory of ground vibrations from rail and road traffic is now well understood and some progress in their suppression at source has been achieved, there are still relatively few investigations aimed to protect the affected areas by influencing propagation of ground vibrations from a source to a receiver. We recall that most of the energy of generated ground vibrations is transmitted as Rayleigh surface waves. The ability to suppress ground vibrations on the propagation path would be especially important in the cases where it is difficult or impossible to reduce the intensity of generated ground vibrations at source.

Note that, in contrast to the case of ground vibrations, the progress in controlling air-borne noise generated by rail and road traffic by influencing wave propagation is much more noticeable. The most popular way of protecting built up areas in this case is acoustic screening, i.e. erecting anti-noise barriers close to the transport routes concerned. However, for ground vibrations induced by rail and road traffic, the complex nature of elastic fields propagating in the ground, which mostly contain surface Rayleigh waves, does not permit direct analogies of acoustic screening. Nevertheless, some measures resembling simple acoustic screening have been designed in the past for protection of built up areas against ground vibrations as well.

Among such measures one can mention specifically constructed protective trenches that can screen sensitive buildings from Rayleigh surface waves (see e.g. Degrande et al. 1996). Unfortunately, the above-mentioned protective trenches are very expensive to build and to maintain. Moreover, due to the specific features of Rayleigh wave scattering, they can not provide a full screening of Rayleigh waves even theoretically, regardless of how deep they are.

In the present paper, we consider a relatively little known, but promising and cost effective alternative to trenches as means of screening ground vibrations on the propagation path. This is using heavy masses placed on the ground surface, such as concrete or stone blocks. The principle of their operation is based on the fact that any surface disturbance, including stones or even simple topographic irregularities, affects Rayleigh wave propagation, causing scattering of the incident Rayleigh waves into bulk waves and into other surface Rayleigh waves propagating in different directions. For heavy masses placed on the ground, scattering can be especially strong at the natural frequencies of vibration of such masses resting on the elastic ground. These natural frequencies can be chosen within the low frequency range typical of railway- or road-generated ground vibrations. Then, when such resonant
masses are shaken in vertical and horizontal directions under the impact of incident Rayleigh waves, they scatter the incident Rayleigh waves into the depth of the ground and at different directions on the surface, thus resulting in significant resonant attenuation of transmitted Rayleigh waves.

While some initial efforts have been made to investigate the above-mentioned mass scatterers, mainly by means of numerical calculations, very little progress in understanding their scattering properties has been made so far. Therefore, this promising method of protection against environmental ground vibrations remains largely unexplored.

The aim of the present paper is to give a brief introduction to the theory of resonant mass scatterers and to discuss some problems that still need to be considered to achieve a fuller understanding of their operation as means of damping of low frequency ground vibrations.

1.0 Description of the method
The method of screening ground vibrations by heavy masses (e.g. by large concrete or stone blocks) placed on the ground surface has been first proposed about 20 years ago (see Jones et al. 1986; Ford 1990). Although the term ‘resonant scattering’ was not mentioned in the above-mentioned two papers, that employed numerical calculations in their studies, it was expected that natural vibration frequencies of such resting masses may play an important role in the behaviour of such masses as means of protection against propagating ground vibrations.

The natural frequencies of vibration of heavy masses resting on the ground \( f_0 = \frac{\omega_0}{2\pi} \) can be defined by the well-known general relationship \( f_0 = \frac{(K/M)^{1/2}}{2\pi} \), where \( K \) is the equivalent concentrated stiffness of the elastic ground associated with the resting mass of finite dimensions, and \( M \) is the mass value.

![Figure 1. Composition of decorative stones off Clifton Boulevard in Nottingham](image)
If \( f_0 \) is chosen, e.g. by selecting the appropriate value of \( M \), within the frequency range of rail- and road-generated ground vibrations (normally from 5 to 50 Hz), then, under the impact of incident Rayleigh surface waves, the mass will be shaken and the amplitudes of its vibrations will be strongly amplified at frequencies close to its natural frequency. Because of this amplification, the masses will scatter incident surface waves very efficiently both into the ground depth (as longitudinal and shear bulk elastic waves) and at different directions on the surface (into scattered Rayleigh surface waves). As a result of these processes, the energy of transmitted Rayleigh waves will be reduced, thus resulting in strong attenuation of transmitted ground vibrations.

The scattering properties of individual masses can be further enhanced to achieve the required protection of the built up areas by using their suitable combinations (arrays).

The important aspect of using heavy masses as means of protection against rail- and road-generated vibrations is that it may have an additional benefit for society. Namely, in addition to their scattering properties, the masses, such as natural stones or specifically designed concrete blocks, can be used also to enhance visual appearance of the area. For example, in some cities in the UK, architectural compositions of heavy natural stones are already used for this purpose without any thought of their possible complementary application for protection against traffic-induced vibrations (see, for example, Figure 1 showing the composition of decorative stones off Clifton Boulevard (A52) in Nottingham). Using specifically designed decorative stones, with the additional purpose of protecting built up areas against traffic-induced vibrations, would make life in the locations concerned not only aesthetically more pleasant, but also much quieter.

Another important practical aspect of using heavy masses as Rayleigh wave scatterers is that they can be built as specially designed brick or concrete walls that are constructed around boundaries of many properties anyway. In addition to their traditional security functions, such walls can be useful also for screening the enclosed properties against traffic-induced ground vibrations.

As was mentioned above, the initial research into using heavy masses for protection against ground vibrations has been carried out by applying numerical techniques (Jones et al. 1986; Ford 1990). It has been shown that large concrete blocks placed on the ground surface can act as vibration dampers. However, the physics of this process was not clarified, and a little progress in understanding the properties of such devices as resonant scatterers of Rayleigh surface waves has been made.

In the same time, the theory of scattering of Rayleigh waves by different surface inhomogeneities is now well developed in respect of ultrasonic non-destructive testing applications and signal processing devices using surface acoustic waves (see e.g. the book of Biryukov et al. 1995 and references there). Moreover, during the last two decades some important advances have been made in understanding resonant scattering of surface Rayleigh waves, in particular for applications to solid state physics (see e.g. Maradudin et al. 1988; Plessky et al. 1991; Mayer (Garova) 1994). Therefore, it is quite natural to apply and develop the existing knowledge of Rayleigh wave resonant scattering to the specific case of heavy masses placed on the ground, with the purpose of protecting built environment against low frequency rail- and road-generated ground vibrations.
2.0 Theoretical approach
In this section, we briefly describe the outline of the rather general analytical
approach to the problem of resonant scattering of Rayleigh surface waves by heavy
masses, without any detailed derivations and discussions of the existing results.

Let us assume that a harmonic Rayleigh wave is incident upon a mass placed on
a flat ground surface \( z = 0 \) (see Figure 2), and let the incident Rayleigh wave be
characterised by the amplitudes of displacement \( u_0^i(r) \) (factor \( \exp(-i\omega t) \) is omitted).
Due to the interaction of the incident wave with the mass (also called the ‘scatterer’),
the total displacement field \( u_i \) in the elastic half space representing the ground
differs from the field of the incident wave, so that the scattered field is \( u_i^{sc} = u_i - u_0^i \).

![Rayleigh wave scattering by a heavy mass placed on the ground](image)

One of the possible ways to approach this self-consistent mathematical problem can
be based on the solution of the integral equation describing elastic wave scattering
by surface inhomogeneities. It can be shown (see Biryukov et al. 1995, p. 282-287)
that in the general case of Rayleigh wave scattering the total field \( u_m \) can be related
to \( u_m^0 \) by the following integral expression:

\[
 u_m(r) = u_m^0(r) + \int_C n_j \sigma_{ij}(r') G_{im}(r, r') d\mathbf{r}'. 
\]  

(1)

Here \( n_j = (0, 0, 1) \) is a unit vector normal to the ground surface, \( n_j \sigma_{ij}(r') = n_z \sigma_{iz}(r') \) are
non-zero stress components applied to the surface over the contact area \( C \) due to
the presence of a scattering mass, and \( G_{im}(r, r') \) is the dynamic Green's tensor for
an elastic half space, that satisfies the free boundary conditions everywhere at the
ground surface \( z = 0 \):
In the latter equation, \( c_{ijkl} \) are elastic constants of the medium (ground), which for simplicity is assumed isotropic.

If the scatterer-related stress components \( n_i \sigma_{ij}(r') \) can be expressed in terms of \( u_m \), which is generally a difficult problem that should be considered separately for each specific type of a scatterer, then by placing the point of observation \( r \) in Eqn (1) on the surface within the area of its contact with the scatterer \( C \), one can obtain the integral equation versus \( u \). Solving this equation analytically or numerically and substituting the obtained values of \( n_i \sigma_{ij}(r') \) again into the integral of Eqn (1), one can calculate \( u^{sc} = u_i - u_i^0 \) at any point on the surface or in the bulk of the ground.

As was mentioned above, the solution of the integral equation (1) is a difficult problem that depends on the dynamic properties of a scatterer, its size and geometry. However, in certain important cases the problem can be simplified substantially and the solution can be obtained analytically. In particular, in the case of heavy masses placed on the ground one can normally assume that the size of the mass is much smaller than the Rayleigh wave lengths of interest (low frequency approximation). Under such circumstances, one can ignore elastic deformations inside the mass and consider its movement as a whole. In the case of three dimensions such a simplified model is often called a 'point' model, and in the two-dimensional case - a 'line' model.

The non-zero stress components \( n_z \sigma_{iz}(r') \) applied to the surface over the contact area \( C \) in Eqn (1) then can be defined by the 2nd Newton's law:

\[
n_z \sigma_{iz}(r') = -(M/S) \omega^2 u_i , \tag{3}
\]

where \( S \) is the area of contact. Substituting Eqn (3) into Eqn (1), one can obtain the integral equation versus \( u_i \) in the area of contact. Solving Eqn (1) versus \( u_i \) and using the obtained value of \( u_i \) in Eqn (1) again, one can now calculate the scattering of Rayleigh waves into bulk longitudinal and shear waves and into other Rayleigh waves propagating in different directions on the ground surface. In particular, calculating Rayleigh waves scattered in backward and forward directions, \( u^{sc \text{back}} \) and \( u^{sc \text{forw}} \) respectively, one can obtain values of the complex amplitudes of reflected and transmitted Rayleigh waves.

In the case of two-dimensional geometry (a line obstacle describing e.g. a stone or brick wall) one can introduce the reflection and transmission coefficients of Rayleigh waves as \( R = u^{sc \text{back}}/u_0 \) and \( T = 1 + u^{sc \text{forw}}/u_0 \).

From the point of view of mathematical solution, it is often convenient to rewrite Eqn (1) in the wave number domain, as the expressions for the Green's tensor are simpler in this case. The discussion of the corresponding mathematical aspects, however, is beyond the scope of this paper.

The important point to note is that all obtained scattered fields as functions of frequency, and hence the derived expressions for \( R \) and \( T \), show a resonant behaviour, i.e. they have resonant maxima around a certain value of frequency that can be associated with the above-mentioned mass resonant frequency \( f_0 = \).
These resonant maxima are normally broad enough, which allows to use such mass scatterers for protection against traffic-induced vibrations having relatively wide frequency spectra. Note that, in the framework of the above approach, the value of the equivalent stiffness $K$ associated with the ground elasticity is derived automatically as the result of the solution of the above-considered self-consistent problem of Rayleigh wave scattering in the elastic half space. However, for a quick estimation of the mass resonant frequency, the value of $K$ can be obtained also from a simpler model of a mass resting on Winkler foundation, using the known relationships between the stiffness of Winkler foundation and the real elastic moduli of the ground.

Using Winkler foundation model, one can estimate the expected change in resonant frequencies for scattering masses of different geometrical dimensions. In particular, one may assume for simplicity that a stone or a concrete block has a rectangular form, with one of the sides resting on the ground. Then, assuming that $S$ is the surface area of the contacting side of the rectangular block, $h$ is its height, $\rho$ is its mass density, and $k_w$ is the stiffness of Winkler foundation per unit area, one can obtain the following simple expression for the mass resonant frequency:

$$f_0 = \frac{(K/M)^{1/2}}{2\pi} = \frac{(k_w S/\rho S h)^{1/2}}{2\pi} = \frac{(k_w/\rho h)^{1/2}}{2\pi}.$$  \hspace{1cm} (4)

It is interesting that, according to this expression, $f_0$ does not depend on $S$. Therefore, for the same values of $k_w$ and $\rho$, the resonant frequency will be inversely proportional to the square root from $h$. For example, if all linear dimensions of a stone block of cubic form are reduced by 4 times, which means that its mass is reduced by $4^3 = 64$ times, its resonant frequency will increase by only 2 times. In practical terms this means that one would operate with resonant frequencies $f_0$ around 100 Hz for small stones with linear dimensions around 20 cm. As a typical wavelength of Rayleigh ground wave is around 1 m at 100 Hz it is expected that a point mass approximation will be sufficient for such small stones.

So far, the detailed calculations of the reflection and transmission coefficients of Rayleigh waves interacting with heavy masses have been carried out only for a two-dimensional case (see Plessky et al. 1991; Mayer (Garova) 1994). According to these calculations, for typical parameters of the two-dimensional mass scatterers and of the elastic half space the resulting absolute values of the reflection coefficient $|R|$ can be as high as 0.3-0.5 in a rather wide frequency range of about 40% around the resonant frequency of the mass scatterer $\omega_0$. In the same frequency range the absolute values of the transmission coefficient $|T|$ can be as low as 0.2-0.3, thus describing a significant (about 12 dB) resonant attenuation of transmitted Rayleigh waves. Most of the incident Rayleigh wave energy is thus scattered into bulk waves, according to the well-known energy conservation relationship $|R|^2 + |T|^2 + |B|^2 = 1$. Here $|B|^2$ is the fraction of the incident wave energy scattered into bulk longitudinal and shear waves (here for simplicity we do not take into account part of the energy transformed into heat).

### 3.0 Further development of the method

In spite of the above-mentioned basic understanding of the phenomenon of resonant scattering of Rayleigh waves on heavy masses resting on the ground, much work remains to be done to introduce this method into practice. First of all, there is a need
to develop a more comprehensive theoretical model of the phenomenon in question, especially in the light of the current knowledge of field distributions of traffic-induced ground vibrations, including those from high-speed trains and heavy lorries.

Essential part of the work should be concentrated around developing the theory for real large stones and concrete blocks resting on the ground, in contrast to highly idealised models of point or line masses used in the previous theoretical works (see, e.g. Plessky et al. 1991; Mayer (Garova) 1994). Basically, one needs to know how to calculate the values of real resonant masses for given elastic parameters of the ground, expected levels and frequency contents of traffic-induced vibrations, building locations, etc. To answer these questions one can analyse the integral equation (1) in the general case of finite dimensions of a scatterer, taking into account its elastic deformations.

Basic solutions for single-mass scatterers discussed so far can be used for analysing the combined effects of combinations (arrays) of several scattering masses distributed over the ground surface to enhance their wave attenuation performance. It is well known that arrays of scatterers can multiply their individual action even if the scatterers are positioned in random. And this is especially so when the scatterers are distributed in such a way that the individual scattered fields are combined in phase. As the simplest type of such an array, one can consider a combination of stones of similar size placed in a straight line very close to each other. Obviously, such a combination is equivalent to a two-dimensional (a line) mass scatterer discussed above. If we combine several two-dimensional scatterers formed by individual stones placed in lines, then for simple incident waves, such as plane waves, the in-phase amplification of the resulting scattering can take place if the lines of scatterers are placed periodically (the so-called Bragg’s reflective array).

One should remember, however, that in the case of ground vibrations generated by rail and road traffic, incident waves may have much more complex distributions in space and time. Therefore, special investigation would be needed to find out optimal positions and mass distribution of individual scatterers in such complex cases.

The important exemption is the case of ground vibration boom generated by trans-Rayleigh trains mentioned in the Introduction to this paper. In this case the incident waves are quasi-plane waves propagating at the angles

$$\vartheta = \cos^{-1}(c_R/v)$$

relative to the track, so that Bragg’s reflective arrays can be directly applied for efficient damping of generated ground vibrations.

Finally, a series of experimental measurements of resonant scattering of Rayleigh waves on masses placed on the ground should be carried out. The aim of such experimental investigation is to validate the theoretical predictions for real situations of mass scatterers of arbitrary geometry.

It would be reasonable to carry out most of the experiments with relatively small masses in order to pass by difficulties associated with handling very heavy masses. This means that resonant frequencies of such smaller masses generally will be higher, and one will have to carry out the experiments at reduced scale.

The above-mentioned reduced-scale measurements of Rayleigh wave resonant scattering should be carried out for single stones and concrete blocks of different shapes and masses and for different combinations of such stones and blocks (arrays). These may include both combinations of spatially separated stones and straight and curved lines of closely placed stones (walls). As a particular case of lines of closed stones, specially built brick walls of different heights can be investigated, including the effects of wall parameters, such as height, width and mass density. Regarding measurements of scattering by arrays of masses in general, the main
attention should be paid to the experimental determination of the most efficient arrays and walls providing maximum values of Rayleigh wave attenuation.

Full-scale experimental investigations of resonant scattering of Rayleigh waves by large and heavy stones should be carried out as well. A major part of these investigations should be concentrated on validating the theory for large stones and concrete blocks resting on the ground, when Rayleigh wavelengths become comparable with stones' dimensions and simplified point mass models are no longer applicable.

4.0 Conclusions
According to the existing theoretical works, suitable combinations of heavy masses placed on the ground, such as large stones, concrete blocks, or specially designed brick and concrete walls can be used as efficient resonant dampers of Rayleigh waves generated by rail and road traffic. The main advantage of using such dampers is the expected low cost of their construction and maintenance.

The additional advantage of such scatterers is their ability to be used with a double purpose – as security walls at boundaries of properties or as architectural features aimed to enhance visual appearance of the area. In some cities, architectural compositions of heavy natural stones are already used for this purpose without any thought of their possible complementary application for protection against traffic-induced vibrations. Using such decorative stones with the additional purpose of protecting built up areas against traffic-induced vibrations would make life in the locations concerned not only aesthetically more pleasant, but also much quieter.

In spite of the above-mentioned basic understanding of the phenomenon of resonant scattering of Rayleigh waves by heavy masses, much work remains to be done to develop more comprehensive theoretical models of the phenomenon and to validate it experimentally.

References


